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Modes of Interaction Between Divorced Parents[*]

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Abstract

It is not unusual for researchers as well as laypersons to refer to the behavior of noncustodial parents who comply with legally-determined child support obligations as "cooperative," even though this term would not typically be appropriate from the perspective of formal models of expenditures on public goods [the children in this case]. We develop a model in which compliance with child support orders *is* synonymous with cooperative behavior. The child support order imposed by institutional agents serves as focal point for the problem of dividing the gains from cooperation. If the gains from cooperation exceed the value of noncooperation for both parents, compliance results. The model is estimated using adminstrative data from the State of Wisconsin. Using model estimates we show that increasing enforcement activities may have weak effects on the welfare of children of divorced parents.

1   Introduction

In characterizing the behavior of divorced noncustodial parents under court orders to make child support transfers to custodial parents, it is not unusual to hear the claim that parents meeting their legal obligations are behaving "cooperatively." When viewing children as (quasi-) public goods, cooperative behavior among divorced parents would typically be taken to imply that efficient levels of expenditure on the children were made. In this paper, we develop and estimate a model of compliance with child support orders and expenditures on children in which compliance with child support orders is synonymous with efficient levels of expenditure on the child.

Of particular interest within our model is the role played by institutional agents, to be thought of as some amalgamation of judges, lawyers, case workers, and legislators [which for simplicity we will often times refer to as the judge]. As we will see in the data set utilized in this study, and as been found in numerous empirical studies on child support transfers, compliance with child support orders is far from perfect. This observation has led to numerous proposals to increase compliance rates through increased criminal penalties for noncompliance, automatic withholding of child support obligations from the noncustodial parent's paycheck, and others. We will show that the move from an environment in which institutional agents have little or no actual enforcement power to one in which their orders are always carried out does not necessarily lead to increases in expenditures on children.

In our model, the noncustodial parent is able to choose some level of money transfer to make to the custodial parent, while the custodial parent is assumed to make decisions regarding expenditures on the child; both parents have utility functions defined over their own private consumption and the consumption of the child. The solution to the parents' problem under noncooperation can be shown to be unique. Corresponding to any noncooperative equilibrium there exists a continuum of cooperative equilibria in which each parent's utility will be at least as high as it is in the noncooperative equilibrium; in any of these equilibria transfers from the noncustodial to the custodial parent and expenditures on the child will be higher than in the noncooperative equilibrium. If the child's welfare is an increasing function of the expenditures made upon it, the child's welfare will also be unambiguously higher under cooperation than under noncooperation. Divorced parents are assumed to recognize the potential welfare gain from cooperation and to trust their ex-spouse to faithfully implement their side of any agreed-

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upon cooperative solution.[1] The barrier to implementation of cooperative solutions is taken to be the choice of the particular cooperative solution to implement. This problem is potentially solved by the institutional agent in our model.

While the focus of the paper is most definitely on the behavior of divorced parents, some of the substantive issues we address carry over to the case of intact families as well. In particular, the problem of the selection of the particular cooperative solution to implement is one which can also be expected to impede the implementation of cooperative outcomes even when parents live together and exhibit some form of altruism toward one another. Some recent attempts to characterize heterogeneous modes of behavior within intact households [e.g., Lundberg and Pollak (1993)] have focused on the nature of the relevant threat point in a bargaining formulation as a key behavioral determinant. Their approach expands the scope of the original bargaining formulations of intrahousehold decision-making which were developed in Manser and Brown (1980) and McElroy and Horney (1981). However, all these approaches take as a given that the parents can agree to a bargaining mechanism to be used to define the cooperative solution. This is the problem we explicitly address here.

In a series of papers, Chiappori and his colleagues have developed an alternative to the bargaining framework within which intrahousehold allocations can be analyzed [see, e.g., Chiappori (1988a,1992), Bourguignon and Chiappori (1992)]. These authors have essentially made two criticisms of the bargaining approach: first, that it arbitrarily selects one equilibria from the Pareto frontier based on a particular axiomatic system, and, second, that it produces no empirically-testable implications [Chiappori (1988b)]. McElroy (1990) and Kooreman and Kapteyn (1990) have attempted to dispute this second claim by working out conditions for identification, estimation, and testing in bargaining-theoretic models of household behavior. In response to the first criticism, one may want to think of the assumption of a particular bargaining framework as an identifying piece of information. In the alternative empirical approach to household behavior taken in Bourguignon et al (1994), for example, the assumption of a particular bargaining framework is replaced with a "sharing rule," the purpose of which it is to select a unique point on the Pareto frontier. Whether one utilizes a given bargaining

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[1]This "trust" might arise from suitable adaption of folk theorem results when we view the static model presented here as a stage in a repeated game. See Del Boca and Flinn (1994a) for a discussion of some of these issues.

solution or a sharing rule, the assumption is that the parties have agreed to the use of some mechanism to select a unique outcome among the set of cooperative equilibria. It is this ability of divorced parents to commit to a mechanism to divide the rents which we question in this analysis. We think we have identified an avenue for resolving this implementation problem.

The starting point for our analysis is the theoretical and empirical literature on the behavior of divorced parents, and most especially Weiss and Willis (1985), which was the first attempt to formally model the behavior of divorced parents facing the problem of reaching efficient divorce settlements. The theoretical and empirical work of Del Boca and Flinn (1995) serves as an important reference point for comparing our model's implications and estimates. Perhaps the seminal paper in this area is that of Mnookin and Kornhauser (1979), who drew attention to the critical role played by legal and societal institutions in shaping the equilibria outcomes obtained by divorced parents. In one sense, this paper argues that their analysis did not go far enough - in disputes over the nature of the bargaining mechanism to be used, the institutional agent must step in to actually suggest a particular [efficient] outcome.

The remainder of the paper is organized as follows. Section 2 contains a formal description of the behavior of divorced parents in terms of transfers and expenditures on the children under noncooperation and under cooperation. Given the child support order, we then characterize the choice between the two modes of behavior. In Section 3 we discuss the institutional agent's role in leading divorced parents to cooperative equilibria. Section 4 contains the development of the econometric model used to recover the distribution of parental preferences in the population of divorced parents. A description of the data used to estimate the model and estimation results are presented in Section 5. Section 6 contains a welfare analysis, both analytical and empirical, of the effect of switching from a regime of no enforcement of orders to one of perfect enforcement. Section 7 concludes the paper with some parting thoughts.

2   Models of Decision-Making Among Divorced Parents

In this section we consider the manner in which divorced parents interact, particularly with respect to making expenditure decisions on child goods. We will allow parents the freedom to choose between behaving cooperatively

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or noncooperatively. As we shall use the term throughout the analysis, by cooperative behavior we mean that efficient levels of expenditure on the public good, the child, are made.

Because the ultimate goal of our analysis is to perform empirical work, we will work with simple specifications of parental preferences. For tractability and for reasons of comparability with Del Boca and Flinn (1995), we assume that each parent's preferences are defined over private consumption, c_p, $p \in \{m, f\}$, and the consumption of the child, k, and that these preferences can be represented by Cobb-Douglas utility functions

$$\alpha_p \ln(c_p) + (1 - \alpha_p) \ln(k), \ p \in \{m, f\}, \tag{1}$$

where m denotes the mother and f the father, and where each parent's preference parameter is contained in the open unit interval.

Our analysis [and data] refer to the case in which the mother has legal and physical custody of the child, by far the most common custody arrangement in the United States. Given this relationship, we view the mother as controlling expenditures on the public good, the child. The only way in which the father can affect the expenditures on the child is through transfers to the mother. Then the mother's choice variable is k and the father's is t, the transfer to the mother. We now turn to characterizing expenditures on the child and transfers to the mother from the father when the parents behave cooperatively and when they do not. In the ensuing discussion the decision rules of the parents in the two regimes will be denoted $\{k_j^*, t_j^*\}$, $j = C(\text{ooperative}), N(\text{oncooperative})$.

2.1 Noncooperative Behavior

We first determine expenditures on the child and transfers from the non-custodial to the custodial parent assuming noncooperative behavior. Even though the model is static, it is useful to think of decision-making as proceeding sequentially: given a transfer from the father, the mother decides how much to spend on the public good. [We will often refer to the noncustodial parent as the father and the custodial parent as the mother since this is the actual assignment in the vast majority of sole-custody cases.] The fathers use their knowledge of the mother's decision rule in deciding upon the size of the transfer to make. Conditional on the father's transfer, the mother solves the problem

$$\max_{k \in [0, y_m + t]} \alpha_m \ln(y_m + t - k) + (1 - \alpha_m) \ln(k), \tag{2}$$

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and the mother's decision rule for expenditures on the child given the transfer is simply

$$k_N^*(y_m + t) = (1 - \alpha_m)(y_m + t). \tag{3}$$

Given the mother's demand function for child goods, the father's problem is to choose the level of the transfer to make to her. His decision rule is given by:

$$t_N^*(y_m, y_f) = \arg \max_{t \in [0, y_f]} \alpha_f \ln(y_f - t) + (1 - \alpha_f) \ln(k_N^*(y_m + t)). \tag{4}$$

This rule can be explicitly stated as follows. Define the variable $\tau(y_m, y_f) = (1 - \alpha_f)y_f - \alpha_f y_m$. Then

$$t_N^*(y_m, y_f) = \begin{cases} 0 & \Leftrightarrow \quad \tau(y_m, y_f) \leq 0 \\ \\ \tau(y_m, y_f) & \Leftrightarrow \quad \tau(y_m, y_f) > 0 \end{cases}. \tag{5}$$

It is easy to show that there exists a unique Nash equilibrium to this Stackelberg game, namely $(\widetilde{k}, \widetilde{t})(y_m, y_f)$, where $\widetilde{k}(y_m, y_f) = (1 - \alpha_m)(y_m + t_N^*(y_m, y_f))$ and $\widetilde{t}(y_m, y_f) = t_N^*(y_m, y_f)$. The value of the mother's and father's utility maximization problems under noncooperation are then

$$V_m^N(y_m, y_f) = \alpha_m \ln(y_m - \widetilde{k}(y_m, y_f) + \widetilde{t}(y_m, y_f)) + (1 - \alpha_m) \ln(\widetilde{k}(y_m, y_f)) \tag{6}$$

and

$$V_f^N(y_m, y_f) = \alpha_f \ln(y_f - \widetilde{t}(y_m, y_f)) + (1 - \alpha_f) \ln(\widetilde{k}(y_m, y_f)), \tag{7}$$

respectively.

2.2 Cooperative Behavior

Due to the fact that expenditures on children are public goods, it is possible to increase the welfare of both parents (and the child, given that his or her welfare is an increasing function of his or her own consumption) if a cooperative equilibrium can be implemented. Any cooperative equilibrium will have the father transfer an amount greater than $\widetilde{t}(y_m, y_f)$ to the mother and will have the mother spend more than $\widetilde{k}(y_m, y_f)$ on the child. While the noncooperative equilibrium in this model is unique, there exist a continuum of cooperative equilibria. The problem of the selection of a particular

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cooperative equilibrium to implement is the topic of the following section. In this section we merely characterize the set of cooperative equilibria in a manner which will be useful for the econometric model which we develop below.

The Pareto frontier of cooperative welfare levels can be traced out in the following manner. Imagine that one of the parents, say the mother, has the ability to select both k and t, though her choices are constrained by the condition that the father obtains a welfare level of at least $\widetilde{V}_f$. Then this pseudo-choice problem of the mother can be represented by

$$\widetilde{V}_m(y_m, y_f, \widetilde{V}_f) = \max_{k,t} \alpha_m \ln(y_m + t - k) + (1 - \alpha_m) \ln(k) \qquad (8)$$

$$s.t. \ \alpha_f \ln(y_f - t) + (1 - \alpha_f) \ln(k) \geq \widetilde{V}_f,$$

where $\widetilde{V}_f$ is some predetermined constant which is to be thought of [for now] as the utility level guaranteed to the father using some unspecified mechanism which determines the division of the surplus from cooperation. The lower bound for this utility level is taken to be the utility he achieves by not cooperating, $V_f^N(y_m, y_f)$. Given the strict concavity of the parental utility functions, it is not difficult to demonstrate that the constraint in [8] will always be binding, and that the mother's welfare is a monotone decreasing function of the guaranteed welfare level of the father.[2] The maximum amount of welfare the father can achieve under cooperation is that level which would yield the mother exactly her noncooperative utility level. Then define the father's maximum utility, $\bar{V}_f^C$, implicitly by $\widetilde{V}_m(y_m, y_f, \overline{V}_f^C)$ $= V_m^N(y_m, y_f)$; then the father's *realizable* value of cooperation must satisfy $V_f^C \in [V_f^N, \overline{V}_f^C]$, where the dependencies on the parental income distribution have been dropped for notational simplicity.

We can solve for the cooperative levels of transfers and child good expenditures as a function of the parental income distribution and the division of the surplus from cooperation [represented by V_f^C] as follows. Since the welfare constraint on the pseudo-choice problem [8] solved by the mother is binding, we can write the constraint as

$$\alpha_f \ln(y_f - t) + (1 - \alpha_f) \ln(k) = V_f^C \qquad (9)$$

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[2]Assume the converse, so that the mother makes choices of k and t such that $U_f(y_f - t, k) > V_f^C$. By increasing t and holding fixed k the mother can increase her own welfare by reducing the father's, so the original allocation could not have been optimal.

$$\Rightarrow y_f - t \;\; = \;\; \exp(V_f^C/\alpha_f)\,k^{-\eta}$$
$$\Rightarrow t \;\; = \;\; y_f - R(V_f^C)\,k^{-\eta},$$

where $\eta \equiv (1-\alpha_f)/\alpha_f$ and $R(V_f^C) \equiv \exp(V_f^C/\alpha_f)$. Substituting this expression into the mother's utility function and solving for child goods expenditures yields

$$k^C(y_m, y_f, V_f^C) \;\; = \;\; \arg\max_k [\alpha_m \ln(y_m - k + y_f - R(V_f^C)k^{-\eta}) + (1-\alpha_m)\ln(k)]$$
$$= \;\; \arg\max_k [\alpha_m \ln(y_t - k - R(V_f^C)k^{-\eta}) + (1-\alpha_m)\ln(k)],$$

where $y_t \equiv y_f + y_m$ is total parental income. Thus the equilibrium expenditure level on the public good is given by the implicit function

$$k^C(V_f^C) = (1-\alpha_m)y_t - R(V_f^C)J[k^C(V_f^C)]^{-\eta}, \tag{10}$$

where $J \equiv (1-\alpha_m(1+\eta)) = 1-\frac{\alpha_m}{\alpha_f}$. We note that the equilibrium cooperative expenditure level on the child may be increasing or decreasing in the father's value of the problem depending on whether he is less or more "selfish" than the mother, i.e.

$$\frac{\partial k^C}{\partial V_f^C} \;\; < \;\; 0 \Leftrightarrow \alpha_m > \alpha_f \tag{11}$$
$$\frac{\partial k^C}{\partial V_f^C} \;\; > \;\; 0 \Leftrightarrow \alpha_m < \alpha_f,$$

with the partial derivative equal to 0 if parental preferences are identical.

Equilibrium transfers from the father to the mother can be written

$$t^C(V_f^C) = y_f - R(V_f^C)[k^C(V_f^C)]^{-\eta}.$$

Given this expression, we have that

$$\frac{\partial t^C}{\partial V_f^C} \;\; = \;\; \frac{1}{J}\frac{\partial k^C}{\partial V_f^C}$$
$$< \;\; 0,$$

since as we noted above in [11], the sign of the derivative of $\frac{\partial k^C}{\partial V_f^C}$ is the opposite of the sign of J

We know that in cooperative equilibrium

$$y_f - t^C(V_f^C) = R(V_f^C)[k^C(V_f^C)]^{-\eta}.$$

Substituting this expression into [10], we find

$$k^C(V_f^C) = \theta_1 y_f + \theta_2 y_m + \theta_3 t^C(V_f^C), \qquad (12)$$

where

$$\theta_1 = \frac{\alpha_m}{\alpha_f} - \alpha_m$$
$$\theta_2 = 1 - \alpha_m$$
$$\theta_3 = 1 - \frac{\alpha_m}{\alpha_f}$$

The expression given in [12] will turn out to be an extremely useful one - it gives the relationship between transfers, parental incomes, and expenditures by the mother on the child *along the Pareto frontier*. In Figure 1 we plot some examples of this linear function for a fixed parental income distribution (y_m, y_f); in this example we have set each parental income equal to its mean value in the sample, so that $y_m = 5.56$ and $y_f = 11.46$.[3] There are four cases represented in Figure 1, which correspond to four different pairs of parental preference parameters. In Figures 1.a and 1.d, parental weights on private consumption are the same; in Figure 1.a the common private consumption weight is .6 and in Figure 1.d it is .8. In these two cases, the cooperative level of child expenditures on the child is independent of the transfer from the father to the mother [since $\theta_3 = 0$ in this case]. In Figure 1.b, the mother values child expenditures more than the father [implying $\theta_3 > 0$]. The negative slope of the cooperative expenditure function in Figure 1.c results from the fact that $\alpha_m < \alpha_f$ in this case.

The set of transfers which could be observed in cooperative equilibrium is indicated on the horizontal axis of each graph. The size and location of this set varies greatly across the four examples. Comparing the cases in which $\alpha_m = \alpha_f$, the "cooperative" set of transfers is the interval [3.47,5.44] when $\alpha_p = .6$ [Figure 1.a], but shifts down to the interval [1.21,4.27] when both parents are relatively more selfish [$\alpha_p = .8$, Figure 1.d]. Comparing the cases in which one parent has an own-consumption utility weight of .6 and the other has an own-consumption utility weight of .8, we can see that the

[3]These represent monthly income figures expressed in 100s of 1980 dollars. Further descriptive statistics for the sample are presented in Section 5.

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cooperation sets depend critically on the identity of the more-selfish parent. When the mother is more selfish [Figure 1.c], the set of cooperative transfers is [4.09,6.90]; when the father is more selfish [Figure 1.b], the set shifts down to the interval [.79,3.21]. The variability in the cooperative sets of transfers, conditional on the observed parental income distribution, is one of the focal points of our analysis.

Before turning to the issue of the how divorced parents can be induced to cooperate, for future reference it will be useful to give a characterization of the cooperative solution to this public goods problem. Condition on a level of transfers from the father to the mother, $\hat{t}$. Then use the relation given in [12] to define a level of welfare for each parent associated with the transfer of $\hat{t}$ and expenditures on the child of $\hat{k}(\hat{t}) = \theta_1 y_f + \theta_2 y_m + \theta_3 \hat{t}$. Then define the welfare level of the father given $\hat{t}$ by

$$\hat{V}_f(\hat{t}; y_m, y_f) = \alpha_f \ln(y_f - \hat{t}) + (1 - \alpha_f) \ln(\hat{k}(\hat{t})),$$

and the welfare level of the mother by

$$\hat{V}_m(\hat{t}; y_m, y_f) = \alpha_m \ln(y_m + \hat{t} - \hat{k}(\hat{t})) + (1 - \alpha_m) \ln(\hat{k}(\hat{t})),$$

where $\hat{V}_p$ is equal to $-\infty$ when any argument in the utility function has a negative value. Then, *given* $\hat{t}$, two divorced parents with a given income distribution will cooperate if and only if *both* of the following weak inequalities are satisfied

$$
\begin{aligned}
\hat{V}_f(\hat{t}; y_m, y_f) &\geq V_f^N(y_m, y_f), &\qquad (13)\\
\hat{V}_m(\hat{t}; y_m, y_f) &\geq V_m^N(y_m, y_f).
\end{aligned}
$$

Given $\hat{t}$, y_m, and y_f, we define $C(\hat{t}, y_m, y_f)$ as the set of pairs of (α_m, α_f) such that [13] is satisfied. Thus, $C(\hat{t}, y_m, y_f)$ is the set of preference parameter pairs for which a cooperative equilibrium with transfers $\hat{t}$ and child good expenditures $\hat{k}(\hat{t})$ exists. The set C figures prominently in the remainder of the paper; further characteristics of C are discussed in the following section.

3 Implementing Cooperative Solutions

We argue that even when institutional agents have little enforcement power, indeed, even when they have none, they still can play a significant role in determining the form of the interaction between divorced parents. There

are many well-known problems connected with the implementation of co-operative solutions; here we focus on one often not emphasized and ignore other more commonly-discussed issues. In particular, we will view divorced parents as being willing to behave cooperatively and as being trustful that their ex-spouse will implement any mutually-agreed to cooperative equilibrium. The problem upon which we wish to focus concerns the *selection* of a particular cooperative equilibrium. While there currently exists a method-ological debate in the literature on intra-household bargaining [within intact households] on the proper way to estimate demand systems when the equilibrium outcome can in principle lie anywhere on the Pareto frontier, it is always assumed that the participants in the bargaining problem are able to choose an unique outcome or at least agree to the use of a mechanism which chooses it for them. We see this problem, particularly in the context of nonintact households, as being fundamental. We view the institutional agent as potentially being able to resolve this issue by "suggesting" a given allocation indirectly through the child support order. That is, let a point on the Pareto frontier be associated with a transfer of $\hat{t}$ and child-good expen-ditures of $\hat{k}$. Let the institutional agent's child support order s represent $\hat{t}$, so that when the father transfers s the mother's level of expenditure on the child will be $\hat{k}(s)$. The "judge" then serves as a sort of "focal arbitrator" in the language of Myerson (1991, p. 111), who goes on to state:

> [W]e say that an individual is a *focal arbitrator* if he can de-termine the focal equilibrium in a game by publicly suggesting to the players that they should all implement this equilibrium. Even though his suggestion may have no binding force, if each player believes that every other player will accept the arbitra-tor's suggestion, then each player will find it best to do as the arbitrator suggests, provided that the arbitrator's suggestion is an equilibrium. Thus, a game with a large set of equilibria is a game in which an arbitrator or social planner can substantially influence players' behavior.

An institutional agent having this role then "suggests" a particular co-operative equilibrium implicitly by announcing a child support order s. The parents determine whether or not the pair $(\hat{k}(s), s)$ implies utility values which lie on the Pareto frontier.[4] If they do, then this particular coopera-

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[4]Recall that we are only considering cooperative equilibria in which efficient expendi-tures on the child good are made, i.e., those (k, t) equilibria which produce utility realiza-

tive equilibrium is the one which is implemented. If $(\hat{k}(s), s)$ is not consistent with a cooperative equilibrium, then a noncooperative equilibrium is observed. In either case then, cooperative or not, since the focal arbitrator suggests only one value of s, the observed equilibrium outcome is unique. Under the model structure, we know that a cooperative equilibrium was implemented if the child support transfer is exactly equal to the order. If this is not the case, then the equilibrium is noncooperative.[5,6]

Under this interpretation of the interaction between divorced parents and institutional agents, it is easy to understand our concern with the set $C(\hat{t}, y_m, y_f)$ defined above. After replacing $\hat{t}$ with s, the set $C(s, y_m, y_f)$ defines all pairs of preference parameters (α_m, α_f) which are consistent with the parents "accepting" the suggestion of the judge and implementing the cooperative equilibrium. Some examples of the shape of this set are contained in Figure 2. Figure 2.a contains the plot of C for the case in which each of the three arguments have been set at their mean value in the sample. Figure 2.b plots C evaluated at the 10th percentile of the sample distribution for each argument, and Figure 2.c plots C with each argument corresponding to the 90th percentile of the sample distribution. In all cases the shapes of the cooperation set C are similar. There is some suggestion of nonconvexity, though this poses no problems for our estimation procedures or inference. The set C in the examples is always connected, and in fact it is possible to prove that C has this property for all values of the triple (s, y_m, y_f).[7] We note that the size of the sets C, as measured by area, varies somewhat across the three examples. [The size of C can also be viewed as the probability of compliance when the preference parameters are jointly distributed

tions which lie on the Pareto frontier.

[5]There are values of the preference parameters of the two parents for which the noncooperative equilibrium transfer would be exactly equal to the child support order. In such a case, one could not distinguish between the cooperative and noncooperative regimes. However, this set of preference parameters is of measure zero, and thus ignoring it poses no danger with respect to misspecification of the econometric model.

[6]Myerson's focal arbitrator facilitates selection of a particular noncooperative equilibrium from a multiplicity of possible noncooperative equilibria. Our focal arbitrator attempts to assist parents in selecting an efficient equilibrium from a continuum of such equilbria. If our focal arbitrator "fails," due to imperfect information regarding parental preferences, the result is a uniquely-determined noncooperative equilibrium. It is not clear what "failure" of a focal arbitrator would imply in the Myerson context.

[7]While connectedness may be a pleasing property for C to possess, all of our estimation procedures and policy experiments would be valid even if C was not connected. This is potentially important if one wished to estimate the model under alternative utility function specifications which may produce sets C which are not connected.

as independent uniform random variates on $[0,1]^2$.]

To illustrate the sizeable gains in child expenditure associated with cooperative outcomes, in Figure 3 we plot expenditures on the child as a function of the preference parameters of the parents for "average" sample parents. Expenditures on the child are greatest when neither parent values private consumption [i.e., $\alpha_m = 0$ and $\alpha_f = 0$], where $k = I_m + I_f$. The surface smoothly decreases as we move away from that point until we reach the set of points contained in C, at which point the surface displays the noticeable bulge produced by the relatively high levels of child good expenditure associated with cooperation. As the "selfishness" of the parents continues to increase, we move out of the set C and back into the low levels of expenditure associated both with selfishness and noncooperation.

There are a number of possible criticisms one can level against the conceptual framework utilized here. We discuss them and attempt to justify the equilibrium concept we employ in the context of the following example. Consider the case in which the parents have identical utility functions[8] given by

$$U_p = .767 \ln(c_p) + .233 \ln(k),$$

and let the parents have the mean incomes in the sample, $y_m = 5.56$ and $y_f = 11.46$. The noncooperative equilibrium would have the father transfer nothing to the mother, since $.233 y_f + .767 y_m < 0$. With a transfer of 0, the mother sets $k_N^* = (.233)(5.56) = 1.295$. Thus the noncooperative equilibrium is $(t_N^* = 0, k_N^* = 1.295)$. In the noncooperative equilibrium, the father's equilibrium utility level is $V_f^N = .767 \ln(11.46) + .233 \ln(1.295) = 1.931$, while the mother's equilibrium utility level is $V_m^N = .767 \ln(5.56 - 1.295) + .233 \ln(1.295) = 1.173$.

From [12], we know that when the preference parameters of the two parents are identical the cooperative level of expenditures on the child is fixed at $k^C = (1 - \alpha_p)(y_m + y_f) = 3.966$. The maximum level of transfers associated with a cooperative equilibrium is given by

$$
\begin{aligned}
V_f^N &= \alpha_p \ln(y_f - \bar{t}) + (1 - \alpha_p) \ln(k^C) \\
\Rightarrow \bar{t} &= y_f - \exp(V_f^N / \alpha_p)(k^C)^{(-\frac{1-\alpha_p}{\alpha_p})} \\
&= 3.301
\end{aligned}
$$

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[8]The preference parameter values correspond to the estimated average levels of α_m and α_f from Specification 3 below.

The minimal level of transfers associated with a cooperative equilibrium occurs at the point where the father gets the entire surplus, or

$$
\begin{aligned}
V_m^N &= \alpha_p \ln(y_m - k^C + \underline{t}) + (1 - \alpha_p) \ln(k^C) \\
\Rightarrow \underline{t} &= k^C - y_m - \exp(V_m^N / \alpha_p)(k^C)^{(-\frac{1-\alpha_p}{\alpha_p})} \\
&= 1.443.
\end{aligned}
$$

Then under our model structure, a cooperative outcome will occur whenever $s \in (1.443, 3.301)$. Beginning in the mid-1980s in Wisconsin, order guidelines specified that a noncustodial divorced parent of one child should pay 17% of their gross income to the custodial parent. In our case, this guideline would have resulted in an order of $(.17)(11.46) = 1.948$, which is an element of the "cooperation set" in this case. Therefore, given this order, our modeling assumptions result in a cooperative equilibrium of $(t = s = 1.948, \hat{k}(s) = 3.966)$.

It may be objected that other reasonable equilibria also exist. There exists an uncountable set Ψ consisting of pairs (t, k) which yield utility outcomes which Pareto dominate those associated with the noncooperative equilibrium, yet do not lie on the Pareto frontier. Parents facing the choice of implementation of some $(t, k) \in \Psi$ or the noncooperative pair (t_N^*, k_N^*) would always opt for the former. For example, one alternative equilibrium framework which also produces a unique equilibrium is the following. As above, assume that whenever $(\alpha_m, \alpha_f) \in C(s, y_m, y_f)$ the noncooperative equilibrium is not implemented. After receiving the amount s from the father, the mother chooses an expenditure level on the child sufficient to make the father no worse off then he would be in the noncooperative equilibrium.[9] Then the mother solves

$$
\max_k \alpha \ln(y_m + s - k) + (1 - \alpha) \ln(k) \tag{14}
$$

$$
\text{s.t.} \quad \alpha \ln(y_f - s) + (1 - \alpha) \ln(k) \geq V_f^N.
$$

Let the solution to [14] be denoted k^{**}. Since the constraint will always be binding, we have $k^{**} = \exp((V_f^N - \alpha \ln(y_f - s))/(1 - \alpha))$. In our example, $k^{**} = 2.393$.

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[9]One might think of this constraint on her choice as arising in the context of a repeated game, in which case failure to ensure that the father is at least as well off transferring s as behaving noncooperatively in any period would result in noncooperative behavior in all future periods.

There are some interesting differences and similarities between this alternative equilibrium framework and the one utilized in this paper. First, by construction, in both cases the event $t = s$ signals that a noncooperative equilibrium was not implemented. However, in the case of "cooperation," $k^{**} \leq \hat{k}(s)$. In our example, k^{**} is 84.7 percent greater than k_N^*, but is 40 percent less than $\hat{k}(s)$ − thus there are large differences in the distributions of gains from cooperation under the two equilibrium concepts. Given that $(\alpha_m, \alpha_f) \in C(s, y_m, y_f)$, under the alternative equilibrium concept, mothers do better, children worse, and fathers much worse. In fact, under the alternative equilibrium fathers would be indifferent between cooperative and noncooperative outcomes, which raises some questions as to the implementability of such an equilibrium.[10]

In the absence of data on expenditure levels k, it is not possible to empirically investigate which of these two equilibrium structures, if either, adequately captures the observed dependencies among the variables referenced by the model: y_m, y_f, t, k. In the estimation of household bargaining models [for intact families], the assumption of efficiency is key to identification of model parameters.[11] In the end, we have chosen to use an equilibrium framework which also imposes this condition, though the reader should bear in mind that the likely gains from cooperative behavior [in terms of expenditures on the public good] are larger than they would be if we also allowed for the implementation of cooperative equilibria which were not efficient.

We conclude this section with a brief discussion of an important issue, the determination of the ordered amount s. In the context of noncooperative equilibria, a best response of each agent is to behave according to the "suggested" equilibrium as long as they believe all the other agents will do likewise. In the case of the choice over cooperative equilibria, the best response argument needs to be modified somewhat. If we consider the model in the context of a repeated game to which Folk Theorem results apply, then implementing the particular cooperative outcome selected by $(\hat{k}(s), s)$ whenever $(\alpha_m, \alpha_f) \in C(s, y_m, y_f)$ is a best response if the only alternative

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[10] To ensure participation of the father in the alternative cooperative equilibrium, the mother could increase spending on the child to $k^{**} + \varepsilon$, where ε is some arbitrarily small positive amount. This solves the participation problem at the cost of introducing nonuniqueness through the indeterminancy of ε.

[11] In the two main approaches taken to the estimation of these models, the cooperative allocation of expenditures is assumed to be associated with Pareto efficient outcomes. A specific point on the frontier is chosen through the use of either a Nash Bargaining model or by adding a "sharing rule" which uniquely divides the cooperative surplus.

is the noncooperative outcome. This is true no matter how the amount s is determined. The only requirement we impose on the determination of s is that it be exogenous [i.e., not subject to the influence of the parents].[12] As is the case in most states currently, child support orders are typically issued under strict guidelines, which express order amounts as simple functions of the income of the noncustodial [and sometimes the custodial] parent and the number of children. Orders defined in this manner clearly satisfy our exogeneity condition.

4 Econometric Model

All divorced parents in our sample are assumed to have Cobb-Douglas utility functions as given in [1]. While we do not observe expenditures on the child, we do observe transfers. Given the parental incomes y_m and y_f, if an order is exactly complied with we learn that parental preference parameters belong to the cooperation set, $C(s, y_m, y_f)$. If the transfer is not equal to the order, we learn something about the father's preference parameter α_f, and conditional on this information we learn something about the mother's value of α_m. In particular, if we know the exact value of α_f in the noncooperative case, we know that the mother's value of α_m must be such that the parents did not choose to cooperate. We now derive the likelihood function for the model in detail.

Conditional on the child support order, the function of interest to us is the joint distribution of the preference parameters of the divorced mothers and fathers. Denote this distribution by $F(\alpha_m, \alpha_f; \theta)$, where θ is an unknown vector-valued parameter which completely characterizes the cumulative distribution function F; we further assume that F is continuously differentiable everywhere on $[0, 1]^2$ with the density of F given by f. Each pair of divorced parents have preference weights which are draws from F. It is reasonable to assume that these draws are independent across pairs of divorced parents. The assumption that each set of parents draw their preference weights from the same distribution essentially is made for purposes of identification. The likelihood function is formed on the basis of this random sampling assumption.

To describe the construction of the likelihood, it seems worthwhile to distinguish three cases. Case A is defined by transfers being strictly positive,

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[12]If s were endogenously determined, we would expect cooperative behavior always to be the end result.

but not equal to the ordered amount. This case includes both positive transfer levels less than the ordered amount and those greater than the ordered amount. Case B obtains when there is no transfer from the noncustodial parent to the custodial parent. Case C is defined as the cooperative case, in which child support transfers are exactly equal to orders.

Case A: $t > 0$, $t \neq s$.

When $t \neq s$, we know that the equilibrium is noncooperative, so that the transfer that the father makes to the mother is given by

$$t = (1 - \alpha_f)y_f - \alpha_f y_m. \tag{15}$$

Since we observe t, y_m, and y_f, this implies that the value of the father's preference parameter is

$$\alpha_f = \frac{y_f - t}{y_t}. \tag{16}$$

Now the joint density of the transfer made by the father and the preference parameter of the mother is given by

$$g(\alpha_m, t) = \left| \frac{\partial \alpha_f}{\partial t} \right| f(\alpha_m, \frac{y_f - t}{y_t}) \tag{17}$$

$$= \frac{1}{y_t} f(\alpha_m, \frac{y_f - t}{y_t}). \tag{18}$$

Given $t \neq s$, we know the value of α_f and we know that, conditional on that value, the mother's preference parameter is not contained in the set $C(s, y_m, y_f)$. Then the likelihood contributed by an observation of this type is

$$L_A(t, s, y_m, y_f) = \int_0^1 \chi[(\alpha_m, \alpha_f = \frac{y_f - t}{y_t}) \notin C(s, y_m, y_f)] \tag{19}$$
$$\times \frac{1}{y_t} f(\alpha_m, \frac{y_f - t}{y_t}) \, d\alpha_m,$$

where χ denotes the indicator function which assumes the value 1 when the logical statement is true and otherwise is equal to 0.

Case B: $t = 0$.

In this situation we only know that the noncooperative equilibrium is one in which the father would choose to make no transfer to the mother. For this to be true, we must have

$$\alpha_f \geq \frac{y_f}{y_t}. \tag{20}$$

Furthermore, for any value of α_f which satisfies this inequality, the value of the mother's preference parameter must not be contained in the cooperation set. Thus the probability of an observation of this type is

$$L_B(s, y_m, y_f) = \int_{\frac{y_f}{y_t}}^{1} \int_0^1 \chi[(\alpha_m, \alpha_f) \notin C(s, y_m, y_f)] \, f(\alpha_m, \alpha_f; \theta) \, d\alpha_m \, d\alpha_f.$$

(21)

Note that the lower limit of integration for the father's preference parameter is the right-hand side of [20]

Case C: $t = s$.

In this case, the fact that the parents are cooperating indicates that their preference parameters are contained in the set $C(s, y_m, y_f)$. The probability of this event can be written as

$$L_C(s, y_m, y_f) = \int_0^1 \int_0^1 \chi[(\alpha_m, \alpha_f) \in C(s, y_m, y_f)] \, f(\alpha_m, \alpha_f; \theta) \, d\alpha_m \, d\alpha_f.$$

(22)

The log likelihood for the sample is then given by

$$
\begin{aligned}
\ln L \;=\; & \sum_{S_A} \ln(L_A(t, s, y_m, y_f)) + \sum_{S_B} \ln(L_B(s, y_m, y_f)) \qquad (23) \\
& + \sum_{S_C} \ln(L_C(s, y_m, y_f)),
\end{aligned}
$$

where S_J denotes the set of sample members belonging to "class" J, $J = A, B, C$, and where individual subscripts have been omitted for simplicity. Estimation proceeds by first choosing a parametric family of distributions F with support $[0, 1]^2$. Once the family of distributions is chosen, the parameter space for θ, denoted Θ, is defined. The maximum likelihood estimator for θ is then defined by $\hat{\theta} = \arg\max_{\theta \in \Theta} \ln(L)$. Given the parametric distributions we have chosen, the usual regularity assumptions are satisfied so that the consistency and asymptotic normality of $\hat{\theta}$ is assured.

The model presented is undeniably a highly-structured one; it more or less has to be given our ambitious goal of identifying parental parameters based only on the relationship between child support orders and transfers. In such a situation, it is likely that the inferences we may draw concerning the distribution of parental preferences could be sensitive to the families of distributions we choose to estimate. To informally examine the sensitivity issue, and to learn something of a pragmatic nature concerning identification, we estimated the model under a variety of assumptions on F.

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In total we estimated six different specifications of the model, some of which were special cases of the others. The first and second specifications estimated were based on the power function distribution, and were predicated on the independence of α_m and α_f. The power function p.d.f. for parent p is given by

$$f_p(\alpha_p; \theta_p) = \theta_p \alpha_p^{\theta_p - 1}, \ \theta_p > 0, \ p = m, f. \tag{24}$$

In specification 1 we assume that the parameter $\theta_m = \theta_f$. In specification 2 we allow these parameters to be different.

In specifications 3 and 4 we continue to maintain the assumption that the parental preference parameters are independently distributed, and we assume that at least one parental preference parameter is distributed as a Beta. The Beta p.d.f. is given by

$$f_p(\alpha_p; \theta_p^1, \theta_p^2) = \frac{\Gamma(\theta_p^1 + \theta_p^2)}{\Gamma(\theta_p^1)\,\Gamma(\theta_p^2)}\, \alpha_p^{\theta_p^1 - 1} \, (1 - \alpha_p)^{\theta_p^2 - 1}, \ \theta_p^1 > 0, \ \theta_p^2 > 0. \tag{25}$$

Specification 3 is based on the assumption that the preference parameters for both parents are independent draws from the same Beta distribution (hence $\theta_m^1 = \theta_f^1$ and $\theta_m^2 = \theta_f^2$). In specification 4 we originally attempted to allow both parents to have preference parameters which followed different Beta distributions, but found that the log likelihood was ill-behaved in this case.[13] In response, we assumed that the father's preference parameter was distributed according to a Beta with the mother's preference parameter distributed according to a (one-parameter) power function distribution.

Our final family of distributional assumptions was based on the bivariate normal. Assume that $(x_m \ x_f)$ is drawn from the bivariate normal p.d.f. $f_{BVN}(x; \mu, \Sigma)$, where μ denotes the mean vector and Σ denotes the covariance matrix of the vector x. Then define

$$\alpha_p = \frac{\exp(x_p)}{1 + \exp(x_p)}, \ p = m, f.$$

The distribution of $(\alpha_m \ \alpha_f)$ is given by

$$
\begin{aligned}
f(\alpha_m, \alpha_f; \mu, \Sigma) \ = \ & \frac{1}{\alpha_m(1 - \alpha_m)} \frac{1}{\alpha_f(1 - \alpha_f)} \\
& \times f_{BVN}(\ln(\frac{\alpha_m}{1 - \alpha_m}), \ln(\frac{\alpha_f}{1 - \alpha_f}); \mu, \Sigma).
\end{aligned} \tag{26}
$$

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[13]We shall discuss our interpretation of this phenomenon when we present the results in the following section.

This specification allows us to relax the independence assumption on the parental preference parameters in a tractable way. The bivariate p.d.f. [26] has five parameters, two in the mean vector μ and three in the covariance matrix $\sum$. Not unsuprisingly we found that it was not possible to estimate all these parameters without further restrictions. We decided to restrict the marginal distributions of the preference draws to be identical, so that $\mu_m = \mu_f$ and $\sigma_m^2 = \sigma_f^2$. We impose this restriction in both specifications 5 and 6. In addition, specification 5 imposes the independence restriction $\sigma_{m,f} = 0$. The parameter $\sigma_{m,f}$ is unrestricted in specification 6.

5   Data and Estimation Results

The data used to estimate the model are identical to those which were used in Del Boca and Flinn (1995). Besides the fact that these are relatively unique, high quality administrative data, it is also of interest to see to what extent estimates of parental preferences differ across the two quite different modeling frameworks. Del Boca and Flinn (1995) viewed the parents as always behaving noncooperatively; in their model, the noncustodial agent only complied with child support orders when the noncompliance "cost" was sufficiently great. Del Boca and Flinn present estimates of the distribution of father's preference parameters and the cost of noncompliance distribution; in that model, the distribution of mother's preference parameters was not identified. In contrast, under the current specification the joint distribution of the preference parameters of mothers and fathers is identified. Thus in both models we can identify the marginal distribution of the father's preference parameter, and we will compare our estimates of this distribution.

The data consist of 222 divorce cases adjudicated in the years 1980-1982 in the State of Wisconsin. The original sample was drawn from court records involving divorce and paternity cases in 18 counties in the state. The final sample we utilize is a small portion of the original sample, which is due to extensive amounts of missing data in the original records and the stringent inclusion criteria we specify. In particular, we have included only divorce cases in which one child was involved, the mother was awarded sole legal and physical custody, and a child support order was in effect. The data consists of the reported monthly income of the mother and the father at the time of the final divorce stipulation,[14] and the child support order and recorded

[14]When the reported monthly income for either parent was less than \$280, we set it equal to \$280. Our reasoning was that potential welfare benefits or minimum wage employment

payment in the fifth month after the child support order took effect. We choose the fifth month so as to minimize start up problems which may have existed in the administrative recording system. The reported transfers in the fifth month of the order period were adjusted for a small number of cases when we deemed them likely to be mismeasured.[15] All variables are measured in terms of 1980 dollars.

Descriptive statistics for the sample are presented in Table 1; each entry for a variable reports the mean and standard deviation (in parentheses) for the sample or a particular subsample. We first note (from the bottom line of the table) that the majority of cases are either transferring nothing or are transferring exactly the ordered amount. Each of these groups represents about three-eighths of the sample. The remainder of the sample is almost evenly split between cases in which the father transfers a positive amount that is less than the order and cases in which he transfers more than the ordered amount.

The first column contains mean and standard deviations of the variables for the entire sample. We note that the mean income of the fathers is about twice as high as the mean income of the mothers. The average order is about 20 percent of the average income of fathers, while average transfers are about 13 percent of the average income of the father. There is substantial dispersion in the incomes of the fathers. This is in large part due to the presence of an outlier; the income value reported for this case, while large, is believable. We should also bear in mind that these are data which were reported to the court and that fathers had no reason to inflate their income reports.

There don't exist many striking differences in the means or standard deviations of the variables across payment groups. For the group comprised of fathers who transfer more than the ordered amount, which contains the outlier mentioned above, the mean income of fathers is high, certainly relative to the average child support award. For the group of individuals who pay exactly the ordered amount, the mean income of both parents is higher than

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could have yielded at least this level of income. This adjustment was made for 21.6 percent of the mothers and 1.3 percent of the fathers.

[15]A description of the adjustment procedure appears in Del Boca and Flinn (1995, p. 1253). Essentially, we recorded individuals as exactly complying with the order in the fifth month if (i) $t_5 = s_5$; (ii) $t_4 + t_5 = s_4 + s_5$; (iii) $t_5 + t_6 = s_5 + s_6$; or (iv) $t_4 + t_5 + t_6 = s_4 + s_5 + s_6$, where t_j and s_j denote the payments and orders in month j in the order period. Our reasoning was that if any of these conditions held, it was likely that the father was attempting to pay the ordered amount in the fifth month, and that the monthly transfer was misrecorded into an adjacent month or months.

the corresponding mean for the entire sample. Mean orders for this group are close to the mean for the entire sample. The group with the highest average orders are the "partial compliers," that is, those cases in which the father makes positive transfers but for less than the ordered amount. We also note that in this case both mean parental incomes are the lowest across the four groups. For the cases in which the father made no transfer, average values across orders and parental incomes are not very different than they are in the entire sample, though they are slightly lower for each of the three variables.

Table 2 contains estimates of the six model specifications described in the previous section. To enhance interpretation, we also present graphs of all the estimated marginal densities of the preference parameters of the parents; Figure 4 contains graphs of the preference parameter densities for the model specifications in which the distributions of preference parameters are restricted to be the same, and Figure 5 contains plots of the preference parameter densities when these densities are not restricted to be identical for mothers and fathers. Table 3 contains reports of the implied first and second moments of the preference parameter distributions and some measures of model fit.

In specification 1 the preference parameter draws of mothers and fathers are assumed to come from the same power function distribution, and the draws are assumed to be independent. The estimated density appears in Figure 4.a. We can see that the mass is concentrated on the right side of the support of the distribution; formal tests decidedly reject the null hypothesis that the density is uniform, for example. From Table 3 we learn that the mean private consumption weight (α_p) is .751, and the standard deviation of the distribution is .193.

Under specification 2 we continue to assume that the parents take independent draws from a power function distribution, but relax the assumption that the two distributions are identical. The graphs of the two marginal densities appear in Figures 5.a and 5.b. We see that the mother's density has substantially more mass in the right side of the support than does the father's. The mean private consumption weight for the mothers is now .836 as opposed to .750 for the fathers, which on the face of it would indicate that mothers tend to be less altruistic than fathers, a somewhat surprising result. However, we will argue that the identification of the mother's preference parameter distribution using only the data available to us is quite tenuous. In this case, their is strong evidence for this claim if we note the large standard error associated with the point estimate of the parameter δ_m [reported in

Table 2 under specification 2]. A likelihood ratio test of specification 2 versus 1 produces a test statistic of 2.282, which has a probability of .131 under the null hypothesis that $\delta_m = \delta_f$. Thus there is no statistical evidence that the marginal distributions of preference parameters differ across the parents when we restrict attention to the power function distribution.

Heuristically speaking, the practical identification issue concerning the recovery of the mothers' preference parameter distribution relates to the dependent variables over which the likelihood function is defined. There are really two dependent variables in the model, conditional on the order and the parental income distribution, which are the transfer and child expenditures. When the transfer made is not equal to the support order, little is learned concerned the mothers' preference parameter distribution, though especially for cases of partial compliance or when more is transferred than is ordered, we learn a substantial amount about the distribution of the father's parameter. It is never possible to recover the exact value of the preference parameter for the mother; in all cases, conditional on the size of the transfer, we can only assign the mother's parameter value to a relatively large set. This makes it difficult to learn the details of this distribution, especially given the modest sample size with which we are working. Clearly it would be possible to increase the precision with which we estimate the joint distribution of parental preferences if information both on transfers and expenditures on the child good were available. Unfortunately, there is no nationally representative data set available which has high quality information on both of these choice variables.[16]

In specification 3 we assumed that parental draws were independent, with both being generated from the same Beta distribution. The Beta nests the power function distribution (when $\delta_p^2 = 1$), so first we were interested in determining whether specification 3 provided a significantly better fit of the data than did specification 1. The likelihood ratio test statistic for this comparison is 3.968, which has a probability of .046 under the null hypothesis that $\delta_p^2 = 1$. Especially given the modest sample size, we took this as evidence in support of the Beta assumption. We can see from the

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[16]In 1987 the NLS72 questionaire contained a special module for divorced parents in the sample. Information was collected on custody type, visitation rates, incomes of the parents, child support orders and payments time-aggregated over a year, and some information on the frequency with which the noncustodial parent made various types of child-good expenditures. The expenditure information collected is relatively crude, and, more importantly, no information is available on the expenditures of the custodial parent. See Weiss and Willis (1993) for a detailed description and use of these data.

comparison of Figures 4.a and 4.c that the estimated density noticeably changes in shape, though from Table 3 we note that the first two moments of the distribution do not change markedly.

We then attempted to generalize the model by assuming that both parents drew from separate Beta distributions, though we were unsuccessful in obtaining convergence in this case. Roughly speaking, allowing the distribution of the mothers' preference parameters to depend on two unknown parameters asked too much of the data, especially given that the fathers' preference parameter distribution was allowed to depend on two unrelated parameters. We could only obtain convergence by restricting the mothers' distribution to be indexed by one parameter, and thus adopted the power function distribution assumption for their preference parameter draws. The preference parameter densities for the two parents are plotted in Figures 5.c and 5.d. Under this specification, there is considerable mass close to the value of 1 for the mothers. The mean α_m for the mothers is estimated to be .923 [from Table 3], though we hasten to point out the imprecision of our estimate of this distribution.. While the point estimate of the power function parameter is 11.941, the standard error is 9.553. This once again illustrates the difficulty of precisely estimating this distribution when only data on transfers are utilized. In any event, it is the case that the likelihood value associated with specification 4 is the lowest we obtained across the six specifications estimated.[17] For this reason, estimates from this specification continue to merit some attention.

The final two specifications were estimated under the assumption that the preference parameters were deterministic transformations of bivariate normal random variates; the transformation mapped both normal random variates into the unit interval. Given our experiences in estimating the other specifications, we decided to estimate both these specifications under the assumption that the marginal distributions of preference parameters for the parents were identical. In specification 5 we assume that the underlying normal random variables are independently distributed, while under

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[17]The likelihood ratio test of specification 4 against specification 1 [with an associated two restrictions] yields a test statistic of 10.472, which has a probability of .005 under the null. The likelihood ratio test statistic of 4 versus 2 [with one restriction] is 8.19, which has a probability value of .004 under the null. Specifications 3 and 4 are not nested, since in specification 4, $\delta_m^2 = 1$, though in 3 $\delta_m^1 = \delta_f^1$ and $\delta_m^2 = \delta_f^2$. Nevertheless, the log likelihood associated with specification 4 is considerably higher than the log likelihood associated with specification 3. In the policy experiment conducted below, we use estimates from both specifications 3 and 4.

specification 6 we allow them to be dependent.

Since the marginal densities estimated under specifications 5 and 6 were essentially identical, we only present the plot of the density associated with specification 5, which is exhibited in Figure 4.c. The shape of the density is quite similar to that associated with the restricted Beta estimated in specification 3 [which appears in Figure 4.b]. The only major difference in the two plots occurs around the value 1, where the density associated with 5 is constrained to come down to 0. Apparently this characteristic results in the transformed normals associated with specifications 5 and 6 fitting the data less well than the Beta and even the one parameter power function distribution [comparing the log likelihood values in Table 2]. When we estimate the model allowing for dependence of the draws, our estimate of the correlation coefficient of the underlying normal random variates is .143, though the standard error of the estimate is extremely large [.617]. This once again seems to reflect the difficulty of identifying the preference parameter distribution of the mothers, even though the marginal distributions have been restricted to be the same. While it is possible that the preferences of the divorced parents are independently distributed, one should probably have access to data on child expenditures as well as transfers before seriously trying to evaluate this hypothesis.

It is interesting to compare estimates of the fathers' preference parameter distribution from the noncooperative model presented in Del Boca and Flinn (1995) with those obtained here. First note the stability of the estimates of the first two moments of the distribution of α_f across the six specifications [which are reported in Table 3]. The estimated means range from .750 to .775. In Del Boca and Flinn (1995), estimates of mean α_f across four model specifications ranged from .756 to .796. Thus both models seem to be picking up similar features in the distribution of the data, even though the modelling framework itself is very different. If one wanted to more formally test between the two models, it seems that actual expenditure data would be indispensable. We shall return to this point in the conclusion.

We end this section with a brief consideration of measures of fit; since compliance with orders is central to our model, characteristics of predicted compliance probabilities on a case-by-case basis and over the sample are the focus of attention. For each case in the sample we first compute the predicted probability of compliance given the cases characteristics (s, y_m, y_f) using the estimated joint distributions of (α_m, α_f) from our six model specifications. The expression used to compute this probability for each case is the term which appears on the right-hand side of [22]. After computing this vector

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of probabilities, we did two things with it. The first was to compute the average in the sample, which, if the model is correctly specified, should equal the observed proportion of individuals in the sample for which the transfer is exactly equal to the order. The second exercise was an assessment of the degree to which the expected probability of compliance was related to the compliance outcome on a case-by-case basis. This was done by first rank-ordering the cases in terms of the predicted probability of compliance. We took the 38 percent of the sample with the highest predicted probabilities of compliance and denote these as predicted compliers.[18] For the set of cases for which compliance was predicted, we computed the proportion of cases for which compliance was observed. We also computed the number of noncompliance outcomes over the set of predicted noncompliers, and the total proportion of cases correctly classified.

The predicted proportion of cases in which compliance is observed in the sample, $\widehat{Pr}(C)$, is given in Table 3. Since the actual proportion of compliance cases is .38, we see that our specifications have underestimated the sample proportion, though for some specifications the level of agreement is relatively high [.337 for specification 2 and .325 for specification 4]. Since the population probability of complying is not explicitly used in the estimation and given the small sample size, we think the various specifications have done a reasonably good job of approximating the sample proportion.

That said, the ability to predict compliance on the individual level based on (s, y_m, y_f) is not good. $\hat{C}$ denotes the set of individuals who were predicted compliers, and $Pr(C|\hat{C})$ denotes the proportion of this set who actually complied. If membership in $\hat{C}$ and C were independent events, then $Pr(C|\hat{C}) = Pr(C) = .38$. Looking across the appropriate row in Table 3, we see that the conditional probability is actually greater than .38 for all the specifications, but never markedly so. The best specification in terms of this prediction criterion is 4, for which $Pr(C|\hat{C}) = .429$. We also present the proportion of the sample cases correctly classified. The highest proportion of correct classifications is .568 [associated with specification 4].

We don't believe the low predictive power of the model at the individual level is cause for undue concern for three reasons. First, the estimates used in assessing goodness of fit were not defined so as to maximize predictive accuracy in the manner we have examined it. If we were really concerned with predicting compliance as well as possible, a maximum-score type of es-

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[18]By taking the top 38 percent we forced the marginal predicted compliance distribution to equal the marginal observed compliance distribution.

timator should have been used. Second, based on past experience we know that it is difficult to predict discrete events using individual-level data. The predictive accuracy we obtain is comparable to what researchers often realize when studying the labor market participation decision of married women, for example. Third, the presence of an institutional agent may make it difficult to predict individual level outcomes when conditioning on (s, y_m, y_f). For example, if s were chosen by the institutional agent in a manner which reduces *ex ante* variability in the probability of cooperation, then the variance in the predicted probabilities of cooperation may mainly be attributable to sampling variability and minor forms of model misspecification. In such a case, which is not implausible, we should expect to find little relationship between compliance predictions and realizations.

In summary, we think that the model fits the data acceptably well given the parsimonious parameterization adopted and the rough agreement between our estimated preference parameter distributions and those obtained from somewhat similar exercises, such as Weiss and Willis (1993) and Del Boca and Flinn (1995). We now use these estimates to address an important policy question, the effect of increased enforcement on the distribution of welfare in nonintact households.

6 Welfare Effects of Perfect Enforcement

In the past few decades a number of proposals have been discussed and implemented which were designed to increase the amount of money transferred from noncustodial to custodial parents. For example, Chambers (1979) is an influential book which in large part is devoted to the problem of enforcement, the message being that tough criminal sanctions against noncomplying noncustodial parents could prove to be effective in inducing compliance. Given the costliness of implementing such sanctions, and a debate over how effective they actually were in facilitating transfers, recently attention has centered on more passive enforcement mechanisms such as mandatory withholding. Under mandatory withholding, a noncustodial parent with a child support obligation has his obligation automatically deducted from his paycheck on a regular basis. The deduction then makes its way to the custodial parent.[19]

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[19] Although it sounds as though mandatory withholding might lead to "perfect" compliance, this is far from the case [see, e.g., Garfinkel and Klawitter (1990) and Del Boca and Flinn (1994c)]. This seems largely due to the ability of parents under orders to hide

While enforcement on the face of it seems unambiguously desirable from a public policy perspective, in particular in terms of increasing the welfare of children, in this section we show that this may not be the case. The intuition for why a reallocation of income toward the mother may not increase expenditures on the child in the model we have exposited is relatively immediate. Mothers who want to receive a child support transfer greater than the noncooperative equilibrium transfer amount can only do so by spending more on the public good than they would in noncooperative equilibrium. Roughly speaking, when the order is guaranteed, mothers lose the incentive to spend at the cooperative level. Moreover, the role of the institutional agent shifts from focal arbitrator to income redistributor. The order changes from being a suggested efficient outcome when it lies in the "cooperation set" to the starting point of a noncooperative game in which the parental income distribution has shifted from (y_m, y_f) to $(y_m + s, y_f - s)$. In the no-enforcement environment, any transfer $t \geq 0$ can be observed, and some of these transfers are associated with cooperative equilibria [when $t = s$]; under perfect enforcement, $t \geq s$ and cooperative equilibria never exist.

To consider the effects of shifting from a nonenforcement regime to a perfect enforcement regime of the type we have described, it will help to think of a particular case characterized by the triple (s, y_m, y_f). In considering the effect of shifting from the no-enforcement environment to the perfect-enforcement environment Table 4 will prove useful; in this table we characterize the change in child expenditures [when shifting between environments] which is associated with pairs of transfer outcomes observed under both environments. This amounts to an implicit partitioning of the preference parameter space; within these partitions we can unambiguously sign the effect on child expenditures associated with switching environments. At the end of this discussion we will be able to directly define the partition of the (α_m, α_f) space on which we can derive unambiguous comparative statics results.

E1 When there are no transfers under either environment, total expenditures on the child are a fixed proportion $(1\text{-}\alpha_m)$ of the mother's income. Under perfect enforcement her income is $y_m + s$ as opposed to y_m under no enforcement, so $\Delta k > 0$.

E2 For a father to make no transfer under the no-enforcement environment, it must be the case that $\alpha_f \geq y_f/y_t$, and for him to make a transfer

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their employment and income from administrative agencies.

under perfect enforcement it must be the case that $\alpha_f < (y_f - s)/y_t$. The set of α_f for which these two conditions are satisfied is empty.

E3 Given that a positive transfer was made in a noncooperative equilibrium under no enforcement, total expenditures on the child were $k^N = (1 - \alpha_m)(1 - \alpha_f)y_t$. For the father to have made any transfer in the no-enforcement environment, it must have been the case that $\alpha_f < y_f/y_t$. Since he makes no [additional] transfer in the perfect-enforcement environment, it must be the case that $\alpha_f \geq (y_f - s)/y_t$. Then the maximum expenditure on the child in the no-enforcement environment is $(1 - \alpha_m)(y_m + s)$, which is the expenditure on the child in the perfect-enforcement environment when the father makes no additional transfer, implying that $\Delta k \geq 0$.

E4 When there exists a noncooperative equilibrium with positive transfers from the father to the mother both before and after the income redistribution, we are in the environment described by Warr (1983) and Bergstrom et al (1986). Income redistribution in noncooperative equilibria in which all agents make positive public good expenditures before and after the redistribution produce identical levels of expenditure on the public good. Therefore $\Delta k = 0$ in this case.

E5 There are two situations to consider when there is cooperation initially followed by a transfer equal to s under perfect enforcement. Say that the noncooperative equilibrium which would have existed under no enforcement was characterized by a positive transfer on the part of the father. Since the value of cooperation exceeded the value of noncooperation we have

$$V_f^C \geq V_f^N \qquad (27)$$

$$\Rightarrow \alpha_f \ln(y_f - s) + (1 - \alpha_f)\ln(k^C) \geq \alpha_f \ln(y_f) + (1 - \alpha_f)\ln(k^N)$$

$$\Rightarrow k^C \geq \left(\frac{y_f}{y_f - s}\right)^{\frac{\alpha_f}{1 - \alpha_f}}(1 - \alpha_m)(1 - \alpha_f)y_t,$$

while in the noncooperative perfect-enforcement equilibrium with $t = s$ expenditures on the child are equal to $(1 - \alpha_m)(y_m + s)$. Then the ratio of no-enforcement to perfect-enforcement expenditures on the child is

greater than or equal to

$$R_1(s) = \frac{\left(\frac{y_f}{y_f - s}\right)^{\frac{\alpha_f}{1 - \alpha_f}} (1 - \alpha_m)(1 - \alpha_f)y_t}{(1 - \alpha_m)(y_m + s)}. \tag{28}$$

Now for this case, $\alpha_f \in [(y_f - s)/y_t, y_f/y_t)$; since the numerator of [28] is an increasing function of α_f, the minimum value $R_1(s)$ can take occurs when $\alpha_f = (y_f - s)/y_t$. Then we have

$$R_1(s) \geq \left(\frac{y_f}{y_f - s}\right)^{\frac{y_f - s}{y_f + s}} > 0 \text{ for all } s, \tag{29}$$

so that $\Delta k \leq 0$.

The other situation occurs when the noncooperative transfer level in the no enforcement environment would have been 0. In this case, since there was cooperation, we know that

$$V_f^C \geq V_f^N \tag{30}$$

$$\Rightarrow k^C \geq \left(\frac{y_f}{y_f - s}\right)^{\frac{\alpha_f}{1 - \alpha_f}} (1 - \alpha_m)y_m, \tag{31}$$

and that $\alpha_f \geq y_f/y_t$. Since the expenditure under perfect enforcement on the child is given by $(1 - \alpha_m)(y_m + s)$, the ratio of child expenditures in the no-enforcement to the enforcement environment is greater than or equal to

$$R_2(s) = \frac{\left(\frac{y_f}{y_f - s}\right)^{\frac{\alpha_f}{1 - \alpha_f}} (1 - \alpha_m)y_m}{(1 - \alpha_m)(y_m + s)}. \tag{32}$$

Since the numerator of the right hand side of [32] is increasing in α_f, then

$$R_2(s) \geq \left(\frac{y_f}{y_f - s}\right)^{\frac{y_f}{y_m}} \frac{y_m}{y_m + s}. \tag{33}$$

The right-hand side of [33] is an increasing function of s. At $s = 0$ it assumes the value 1. Thus $\Delta k \leq 0$ for this case as well, and we have shown that when there are no additional transfers in the perfect-enforcement equilibrium, parents who cooperated in the no-enforcement environment will spend less on the child.

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E6 If the father would make additional transfers to the mother in the perfect-enforcement environment, then in the noncooperative equilibrium which would have held under no enforcement he also would have made positive transfers. Again using the results of Warr (1993) and Bergstrom et al (1986), expenditures on the public good must be the same before and after the income redistribution in the noncooperative equilibrium. Since expenditures on the public good under cooperation must have been greater in the no-enforcement environment, $\Delta k < 0$.

Now we can summarize our results in terms of partitions of the space of (α_m, α_f) pairs. Let $Q_1(s, y_m, y_f)$ be the set of preference parameters for which perfect enforcement will lead to a gain in expenditures on the child. These pairs are such that the father would make no additional transfer to the mother in the perfect-enforcement environment and also do not belong to the cooperation set, so $Q_1(s, y_m, y_f) = [(y_f - s)/y_t, 1) \cap {}^{\sim}C(s, y_m, y_f)$, where ${}^{\sim}C$ denotes the complement of C. On the set $Q_2(s, y_m, y_f)$ there is no change in expenditures on the child when we switch between the two environments, and we have $Q_2(s, y_m, y_f) = (0, (y_f - s)/y_t) \cap {}^{\sim}C(s, y_m, y_f)$. We have shown that anytime the parents were initially cooperating expenditures on the child would fall when switching to perfect enforcement. Then the set of preference pairs for which child expenditures unambiguously fall is $Q_3(s, y_m, y_f) = C(s, y_m, y_f)$.

The conclusions of this discussion are illustrated in Figure 6, once again where the arguments s, y_m, and y_f have been set at sample mean values. The figure plots the surface of the change in expenditures on the child when shifting from the no-enforcement equilibrium to the perfect enforcement one. The grid superimposed on the figure is at height zero, indicating no change.

The set of points for which there is no change belong to $Q_2(s, y_m, y_f)$, which appear on the right side of the plot. The part of the surface which rises above the grid are associated with preference parameter pairs in the set $Q_1(s, y_m, y_f)$. The largest gain in expenditures corresponds to the preference pair $(\alpha_m = 1, \alpha_f = 0)$. In this case, the entire [enforced] transfer from the father of 2.25 is spent on the child. The part of the surface which dips below the grid is associated with points in the set $Q_3(s, y_m, y_f)$. We have already show analytically that all points in Q_3 must be associated with decreasing expenditures on the child; in this example we see that these decreases can be quite sizable. While the largest increase in k is 2.25, the largest *decrease* in k is 3.15.

Since moving from no enforcement to perfect enforcement is associated

with gains in child welfare in some cases and decreases in others, a natural question to ask is what the *expected* effect is. To address this issue, we use estimates of the distributions of parental preferences associated with specifications 3 and 4. Using the estimated distributions, we computed the expected expenditures on the child in the case of no enforcement and the expected expenditure on the child in the noncooperative equilibrium associated with the income distribution $(y_m + s, y_f - s)$ for each case in our sample. The histogram of changes in expected expenditures on the child for the two specifications are presented in Figure 7. Using the estimates from specification 3, in moving from the no-enforcement to the perfect-enforcement equilibrium, the average gain in expected expenditures on the child is a relatively modest .135 [which is $13.50]. For 43 percent of the cases, the switch in environments was associated with a decrease in expected expenditures on the child, though for some sample cases the gain in expenditures due to the move to perfect enforcement was large [the maximum value in the sample is 2.41].

When the same exercise is repeated using the estimates from specification 4 the picture changes dramatically. The average change in expected expenditures becomes -.134, which is again modest, though now 88 percent of the cases experience a decline in expected expenditures. Most of these changes are small however, and for some cases a large gain in expected expenditures is observed once again [the maximum value of the change is 1.69 in this case].

The experiment we have conducted casts some doubt on the presumption that increases in enforcement will necessarily lead to better welfare outcomes for children. However, we should be quick to point out that the exercise we have conducted is a very limited one. Most importantly, we have looked at expenditure levels on the child when the size of the order is held fixed. It may or may not be the case that orders might be set very differently in the two types of environments we have examined. We are not able to speculate on how orders might change with the switch to a perfect-enforcement environment since we are not able to empirically characterize the utility functions of institutional agents.

It is of some interest to contrast the results of this exercise with a somewhat similar one conducted in Del Boca and Flinn (1995). In that model, where divorced parents never exhibited cooperative behavior, increases in enforcement activity lead to unambiguous increases in expenditures on the child *holding orders fixed*. Their empirical characterization of the preferences of institutional agents suggested that the net effect on child expendi-

tures would be small because institutional agents would reduce the order in response to increased enforcement. Our model suggests a theoretically ambiguous, but empirically small, effect of changes in enforcement regimes on child expenditures, but we are unable to suggest what the net effect would be because of the problem of identifying the preferences of institutional agents.

7   Conclusion

While individuals whose welfare is dependent upon the same public good can be expected to appreciate the value of cooperation, the problem of deciding upon a division of the surplus gained from cooperative behavior is likely to be a severe deterrent to its implementation. This problem may be especially severe in nonintact households, where divorced parents may feel some antipathy toward one another. An institutional agent who is able to suggest a specific cooperative solution to the public good problem can lead divorced parents to equilibria which Pareto-dominate those which they would attain in the absence of such an agent. Somewhat paradoxically, the institutional agent can only play this role if her ability to implement child support orders in the face of opposition by one or both parents is weak or nonexistent. When the institutional agent has the power to fully implement her orders, her ability to lead the parents to Pareto-improving equilibria is eliminated, though she gains increased power to shift welfare toward the parties to the divorce whom she favors.

This paper presents an attempt to estimate a model in which the form of the interaction between divorced parents, cooperative or noncooperative, is determined endogenously. The form the interaction takes in any particular case depends on the parental income distribution, the preferences of the parents regarding their own private consumption and the consumption of the child, and, most interestingly, the child support order itself. In fact, one can show that any pair of parents, no matter what their incomes and preferences, would choose to cooperate at some child support order level. It is interesting to note that parents may not cooperate not only because orders are too high, but also because they are too low.

We provide some evidence that the model is roughly able to reproduce the data at our disposal, and perform a comparative statics exercise that provides a cautionary tale regarding the presumed benefit of increases in enforcement activity on the welfare of children. In order to differentiate between the large number of behavioral models which may be developed

to explain interactions between divorced parents, information on the vector (y_m, y_f, k, t, s) is required. For example, in the model considered in this paper expenditures on the child are given by

$$
k = \begin{cases} (\frac{\alpha_m}{\alpha_f} - \alpha_m)y_f + (1 - \alpha_m)y_m + (1 - \frac{\alpha_m}{\alpha_f})t & (\alpha_m, \alpha_f) \in C(s, y_m, y_f) \\ (1 - \alpha_m)(y_m + t) & (\alpha_m, \alpha_f) \notin C(s, y_m, y_f) \end{cases}
$$

By contrast, the model estimated in Del Boca and Flinn (1995) implies that $k = (1 - \alpha_m)(y_m + t)$ for all values of t. This clear difference in the child good expenditure functions under the two models is easily testable given access to (y_m, y_f, k, t, s).[20] Aside from model testing, which is clearly critical given the sensitivity of policy implications to behavioral specifications, access to more complete data is required in order to adequately estimate any given model. This is clear from our experiences with the estimation of the joint distribution of parental preferences. In the one specification estimated which allowed for dependence between α_m and α_f, the estimates obtained suggested weak dependence. While it may be the case that there is little dependence between the preference parameters of *divorced* parents, it is at least equally possible that this result is a consequence of the tenuousness of the identification of the parameters characterizing the distribution of the mother's preference parameter. Heuristically speaking, this is due to the fact that we observe the endogenous variable chosen by the father (t) but not the one set by the mother (k). What we learn about the mother's preference parameter comes from its rather "indirect" effect on the father's transfer choice.

In spite of these limitations, the model does provide a coherent explanation of the empirical relationship between child support orders and transfers and the parental income distribution. The policy experiment conducted delivers an important message regarding the possible effects of increasing child support enforcement activities on the welfare of children.

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[20]Del Boca and Flinn (1994b) and Hernandez et al. (1992), using expenditure data from households headed by divorced mothers and data on the educational attainment of children of divorced parents, respectively, provide some evidence not inconsistent with this implication. In the data sets utilized in both papers, information on the child support order and the income of the noncustodial parent is not available, making the conclusions reached less than definitive.

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Table 1
Descriptive Statistics

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Variable	All	$t = 0$	$0 < t < s$	$t = s$	$t > s$
t	1.48 (1.70)	0	1.98 (1.45)	2.30 (1.48)	3.06 (1.92)
s	2.25 (1.49)	2.12 (1.54)	2.57 (1.53)	[same]	2.11 (1.24)
y_m	5.56 (2.56)	5.36 (2.24)	5.17 (2.51)	5.94 (2.86)	5.38 (2.50)
y_f	11.46 (8.76)	11.25 (7.68)	10.38 (3.89)	11.71 (6.59)	12.74 (18.63)
N	222	83	31	84	24
Proportion		.37	.14	.38	.11

Table 2
Maximum Likelihood Estimates of Preference Distributions
(Asymptotic Standard Errors in Parentheses)

Parameters	Specification					
	1	2	3	4	5	6
Power p.d.f.						
δ_m	3.018	5.092		11.941		
	(.214)	(2.029)		(9.553)		
δ_f	3.018	3.004				
	(.214)	(.217)				
Beta p.d.f.						
δ_m^1			2.572			
			(.280)			
δ_m^2			.783			
			(.094)			
δ_f^1			2.572	2.313		
			(.280)	(.278)		
δ_f^2			.783	.672		
			(.094)	(.093)		
Normal p.d.f.						
μ					1.666	1.679
					(.148)	(.150)
σ					1.420	1.445
					(.105)	(.117)
ρ					0	.143
						(.617)
$\ln L$	-358.463	-357.322	-356.479	-353.227	-358.841	-358.672

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Table 3
Evaluation of Behavioral Models

Statistic	Specification					
	1	**2**	**3**	**4**	**5**	**6**
$\overline{\alpha}_m$	.751	.836	.767	.923	.774	.774
$\overline{\alpha}_f$	.751	.750	.767	.775	.774	.774
$SD(\alpha_m)$	.193	.139	.203	.072	.204	.206
$SD(\alpha_f)$	.193	.194	.203	.209	.204	.206
$\widehat{Pr(C)}$	.314	.337	.293	.325	.316	.306
$Pr(C|\hat{C})$	.393	.393	.405	.429	.417	.417
$Pr(N|\hat{N})$	.630	.630	.638	.652	.645	.645
Prop. Correct	.541	.541	.545	.568	.559	.559

Table 4
Enforcement, Transfers, and Child Expenditures

	Perfect Enforcement	
No Enforcement	$t = s$	$t > s$
$t = 0$	$\Delta k > 0$ [E1]	not possible [E2]
$t > 0,\ t \neq s$	$\Delta k \geq 0$ [E3]	$\Delta k = 0$ [E4]
$t = s$ [Cooperation]	$\Delta k \leq 0$ [E5]	$\Delta k < 0$ [E6]