REVERSALS IN PEAK AND OFFPEAK PRICES

by

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I. INTRODUCTION

Can it be rational for the supplier of a monopoly service, such as transportation or public utilities, to charge a price during peak periods that is *lower* than the price in offpeak periods? Such a pricing possibility has been virtually ignored in economists' discussions of peak-load pricing. Yet, such pricing structures sometimes are found in practice. For instance, the daily peak in an electric utility is often generated mainly by business customers who pay less for service than residential customers whose evening demand is offpeak.

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Most theoretical treatments of peak-load pricing reach the conclusion that peak prices will be higher than offpeak prices. This result follows automatically in the standard model in which there is a welfare objective and the firm has constant returns to scale in production. We show that a higher peak price also occurs in a model in which the firm can levy a perfect two-part tariff.

What has not been recognized is that a peak price less than the offpeak price *can be optimal* in other models of firm behavior. Specifically, reversals can occur when:

(i) the firm has a welfare objective, but there are decreasing average costs in production, so that a profit constraint is explicitly or implicitly imposed;

(ii) the firm has a profit rather than a welfare objective, and faces a demand in the offpeak period that is sufficiently more *inelastic* than that during the peak so as to compensate at the margin for the attribution of capacity costs to the peak period;

(iii) the firm is subject to a rate-of-return constraint, which gives it an incentive to lower the rate to the peak users while keeping the offpeak rate at the monopoly level; and

(iv) the firm employs a multi-part pricing scheme, but one that is imperfect in extracting all the consumer's surplus.

Such models do not exhaust the situations that can lead to pricing reversals, but they do suggest that there are a number of important cases in which such reversals can be explained by rational economic processes.

The models are also important in a second respect, for in some of them, we are able to demonstrate that capacity may be expanded beyond the level that maximizes welfare. In particular, rate-of-return regulation can lead to such overexpansion of capacity, and may do so whether a single or a multi-part tariff is levied.

II. THE PROFIT VERSUS THE WELFARE OBJECTIVE

Welfare Maximization

Virtually all of the literature dealing with peak-load pricing principles seeks to determine the conditions necessary for maximization of social welfare in the sense of Pareto optimality or of maximization of consumer's plus producers' surplus.[1] We will begin by reviewing the results obtained with a simple version of this model.

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[1] See, especially, Boiteux [1960], Steiner [1957, 1971], Hirshleifer [1958], Wellisz [1963], Williamson [1966], Turvey [1968], Littlechild [1970], Mohring [1970], and Pressman [1970].

Our presentation assumes that there are two periods of equal duration and that demands in the two periods are not related. To make this seem sensible, think of the model as depicting two classes of customers, such as business users of power and residential users, whose demand for the facilities occurs during the day and evening, respectively.[1] The cost structure consists of a constant operating expense per unit of demand served, and a constant cost per unit of capacity. We shall suppose that the outcome in terms of capacity usage is that there is a distinct period of peak and a period of offpeak (rather than that there is a joint peak). These simplifying assumptions are made so that we can focus directly on the pricing reversal phenomenon, and so that straightforward graphical interpretations can be presented. The consequences of relaxing some of the assumptions are discussed later in the paper.

The mathematical formulation portrays the maximization of the sum of producer's and consumers' surplus, given by the integrals of demand curves less costs, as summarized by Model (1) in Table I, where $\overline{X}$ and $\underline{X}$ are demands in the peak ($\overline{X}$) and offpeak ($\underline{X}$) periods, respectively, with peak period

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[1] In essence, we have a model of price discrimination in which demands are sequential rather than simultaneous. For a suggestive daily load curve showing that the electricity example fits this description in at least an approximative manner, see Electric Power and Government Policy [1948], p. 38. Of course, in practice peaks in the electric utilities are always attributable, at least in part, to both business and to residential customers.

demand equaling capacity, $\overline{p}$ and $\underline{p}$ are prices in the peak ($\overline{p}$) and offpeak ($\underline{p}$) periods, respectively, c is the operating expense per unit of output (constant), k is the acquisition cost per unit of capacity (constant), r is the market cost of capital (constant), and the subscript W denotes values relating to the welfare-maximization objective.

Equations (2) and (3) indicate that peak price covers both the expense cost generated by the peak traffic and the marginal capacity cost, whereas offpeak price just covers its own expense cost. It is immediately obvious from (2) and (3) that

$$\overline{p}_W > \underline{p}_W$$

so that *if there are constant costs welfare maximization automatically requires the peak price to be higher than the offpeak price.* The familiar pricing rule has appeared.

The results are made clearer by Steiner's [1957] peak-load geometry as displayed in Figure 1. Equation (2) is just satisfied at point $\overline{W}$. Equation (3) is satisfied at $\underline{W}$. Since price is equated with the relevant costs in each case, and since costs are constant, the firm just exactly breaks even by supplying capacity $X = \overline{X}_W$ and total demand $D = \overline{X}_W + \underline{X}_W$ at prices $\overline{p}_W$ and $\underline{p}_W$ respectively.

Profit Maximization

Now let us contrast these results with those that follow when the firm's objective is to maximize profit rather than producer's plus consumers' surplus.

TABLE I

SUMMARY OF RESULTS

Model	Peak Price	Offpeak Price
Welfare Maximization (1) $\underset{\overline{X},\underline{X}}{\text{Maximize}}\ W = \int_0^{\overline{X}} \overline{p}\, d\overline{X} + \int_0^{\underline{X}} \underline{p}\, d\underline{X} - c(\overline{X}+\underline{X}) - rk\overline{X}$	(2) $\overline{p}_W = c + rk$	(3) $\underline{p}_W = c$
Profit Maximization (4) $\underset{\overline{X},\underline{X}}{\text{Maximize}}\ \pi = \overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) - rk\overline{X}$	(5) $\overline{p}_M = \frac{c+rk}{1-\frac{1}{\overline{e}}}$	(6) $\underline{p}_M = \frac{c}{1-\frac{1}{\underline{e}}}$
Increasing Returns in Capacity Provision (7) $\underset{\overline{X},\underline{X}}{\text{Maximize}}\ W = \int_0^{\overline{X}} \overline{p}\, d\overline{X} + \int_0^{\underline{X}} \underline{p}\, d\underline{X} - c(\overline{X}+\underline{X}) - K(\overline{X})$ subject to $\overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) - K(\overline{X}) \geq 0$	(8) $\overline{p}_I = \frac{(1+\lambda)(c+K')}{1+\lambda\left(1-\frac{1}{\overline{e}}\right)}$	(9) $\underline{p}_I = \frac{(1+\lambda)c}{1+\lambda\left(1-\frac{1}{\underline{e}}\right)}$
Rate-of-Return Regulation (10) $\underset{\overline{X},\underline{X},Z}{\text{Maximize}}\ \pi = \overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) - rk(\overline{X}+Z)$ subject to $\overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) - sk(\overline{X}+Z) \leq 0$	(11) $\overline{p}_G = \frac{c + rk - \frac{\lambda}{1-\lambda}(s-r)k}{1-\frac{1}{\overline{e}}}$	(12) $\underline{p}_G = \frac{c}{1-\frac{1}{\underline{e}}}$
Perfect Two-Part Tariff See Text	(23) $\overline{p}_T = c + rk$	(24) $\underline{p}_T = c$
Regulation of Rate of Return but not of Price Structure See Text	(25) $\overline{p}_R = c + rk - \frac{\lambda}{1-\lambda}(s-r)k$	(26) $\underline{p}_R = c$

Welfare Versus Monopoly Pricing

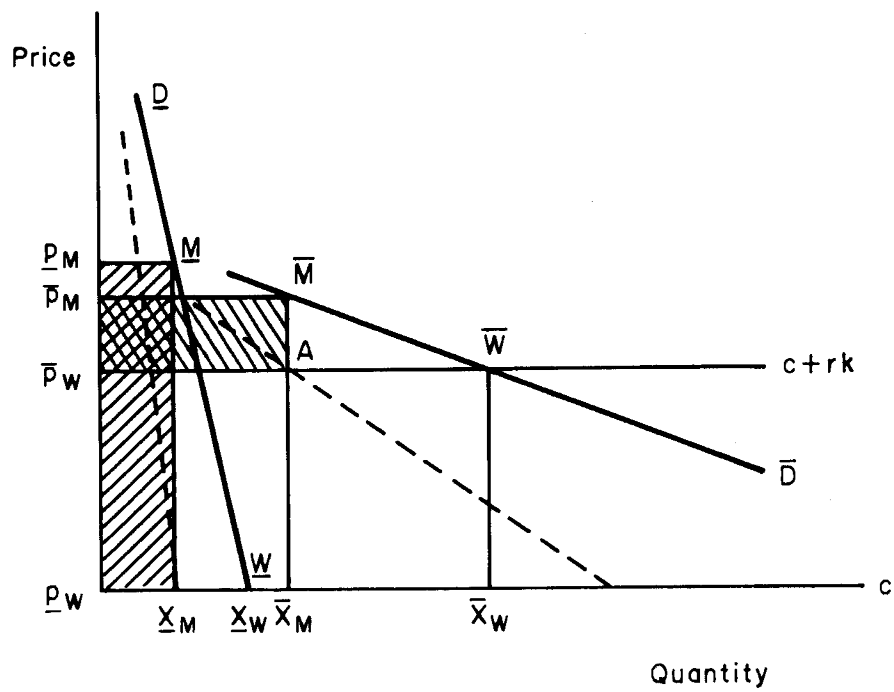

Figure 1

The relevant model is (4) in Table I, where the subscript M is used to denote values relating to the profit objective of the monopoly firm. The $\bar{e}$ and $\underline{e}$ are price elasticities of demand in the peak and offpeak periods, respectively.

When elasticities are equal, (5) and (6) show that peak and offpeak prices are in the same ratio as the marginal costs of peak and offpeak service, and thus offpeak price in this case will always be lower than peak price. However, offpeak rates are *higher* than peak rates whenever

$$c\left[\frac{1}{1-\frac{1}{\underline{e}}}\right] > (c+rk)\left[\frac{1}{1-\frac{1}{\bar{e}}}\right] \quad .$$

Thus, *offpeak price can be higher than peak price if offpeak service has demand that is sufficiently more inelastic than the peak service so as to compensate for the larger marginal costs attributable to the peak service. It is clear that the size of rk relative to c is also important in the determination of pricing reversals. If marginal capacity costs are large relative to marginal operating expenses, then the pricing reversal is less likely to occur.*

In terms of the electricity example, the pricing reversal may occur if rate schedules are "a result of pricing policies based on value of service."[1] Residential users place a higher value on such service (i.e., have a more inelastic demand) than do the business users since large business customers are more likely to be able to generate their own electricity or to convert to alternative sources of energy.

The monopoly solution is described graphically in Figure 1. Equations (5) and (6) are satisfied at points $\overline{M}$ and $\underline{M}$ respectively, with $\underline{p}_M > \overline{p}_M$ so that the offpeak price is higher and the reversal occurs. Profits are given by the sum of the two striped rectangles $\underline{p}_M\underline{M}X_M\underline{p}_W$ and $\overline{p}_M\overline{M}A\overline{p}_W$, with the height of the former rectangle being given by the difference between offpeak price and operating expense, and the height of the latter rectangle being given by the difference between peak price and the sum of operating and capacity expense. Notice, however, that it is not correct to refer to rectangle $\underline{p}_M\underline{M}X_M\underline{p}_W$ as "offpeak profit" and $\overline{p}_M\overline{M}A\overline{p}_W$ as "peak profit". Since peak and offpeak users share some common capacity, any attribution of costs and profits runs into the familiar joint-cost problem.

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[1] Davidson [1955], p. 215. Notice also that the pricing reversal might be explained if the cost of distributing electricity to the residential customers (the c term) were higher than the distribution cost to the business customer. See, for example MacAvoy and Noll [1973]. Another complication that can arise in the real situation is that the peak and offpeak periods of the generating and distribution systems might occur at different times.

Welfare Maximization with Increasing Returns in Capacity Provision

Most firms for which the peak-load pricing problem is relevant have increasing returns to scale in the provision of capacity,[1] that is, capacity costs are given by $K(\overline{X})$, $K' > 0$, $K'' < 0$. Figure 2 illustrates the situation. At the output pair $(\underline{W},\overline{W})$ where welfare is maximized, price equals marginal cost for each user, $\underline{p}_W = c$ and $\overline{p}_W = c + K'$. Under such a scheme the firm operates at a loss whose magnitude is given by the dotted area $B\overline{W}\overline{p}_W$. If a single price is all that can be levied for each type of customer, and if the firm is to breakeven in its operations, the appropriate model is given by (7) in Table I.

The pricing rules (8) and (9) can be interpreted as stating that the deviation of price from marginal cost for each period should be proportional to the marginal deficit incurred by the last unit of output in that period.[2] It is clear that

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[1] See Mohring [1970] for another discussion of this problem, which neglects, however, the pricing reversal phenomenon. See also Mowery [1970].

[2] These rules are essentially the same as those derived in the more general model of optimal departures from marginal cost pricing, as described in Baumol and Bradford [1970] and Boiteux [1971]. It is no accident that pricing rules of the Baumol and Bradford variety have appeared in our model. By implicitly assuming that we know which of the periods is peak and which is offpeak (as we have in our models), it has been possible to avoid explicit introduction of the capacity constraints, $\underline{X} \leq X$ and $\overline{X} \leq X$. This is precisely the trick that is needed to reduce the peak-load pricing problem to a special case of an optimal departures from marginal cost pricing problem.

Increasing Returns to Capacity

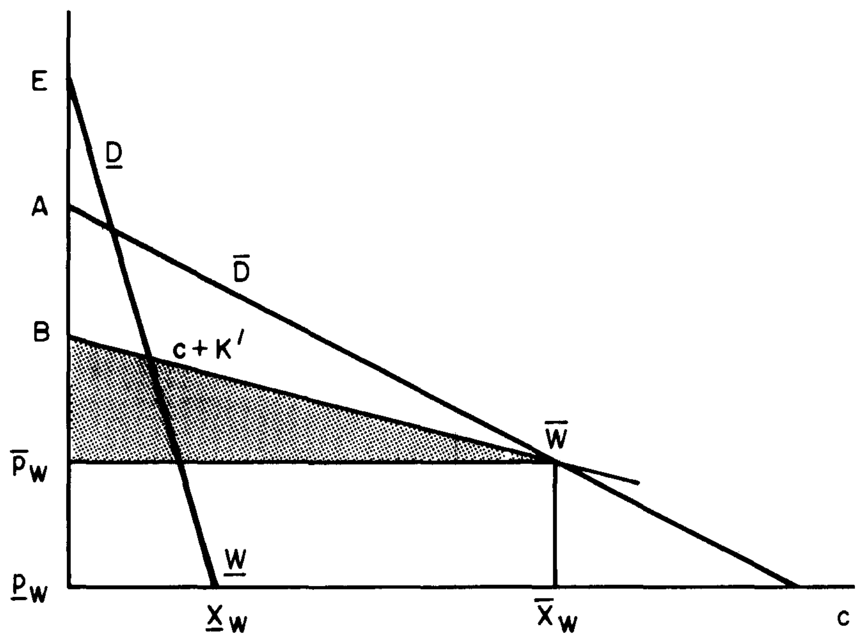

Figure 2

we can obtain from (8) and (9) an equation that indicates the conditions under which the pricing reversal $\underline{p}_I > \overline{p}_I$ occurs just as we did previously. Furthermore, precisely the same qualitative statements about elasticities hold in the breakeven welfare-maximization model as held in the profit-maximization model.

III. THE REGULATED FIRM

Breakeven Regulation

We now consider a peak-load pricing model with profit maximization as the objective but with a regulatory constraint on the firm's behavior. If the constraint binds the firm in such a way that its total revenue just covers its total cost, including the cost of capital, then no unique solution to the peak-load pricing problem exists. With the profit goal, any pair of peak and offpeak prices consistent with the zero-profit constraint is equally desirable to management.

Rate-of-Return Regulation

If the Averch-Johnson [1962] assumption of a fair return, s, larger than the market cost of capital r, is made, then a unique pair of peak and offpeak prices can be attained. Model (10) in Table I follows Bailey [1972] but uses the idea from Bailey [1973] that operation off the production frontier is a possible alternative for the firm.[1] We use λ to denote the multiplier associated with the rate-of-return constraint, Z for the number of units of capital wasted, and the subscript G to denote the solution prices for the model.

Equations (11) and (12) give the rules for peak and offpeak prices. Equation (12) is precisely the same as the rule followed by the unregulated profit-maximizing firm (c.f., Equation (12) and Equation (6)). Thus, in the absence of cross-elasticity effects, (12) says that the offpeak quantity and price are identical to that which an unregulated monopoly would attain; the constraint does not affect the offpeak policy.[2]

The entire effect of regulation is reflected instead in peak price changes and/or in the possibility of operating off the production frontier. Equation (11)

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[1] The idea that the firm might choose to operate off its production possibility frontier in a peak-load pricing model also appears in Riley [1972]. See also Zajac [1972]. In model (10) the regulator is assumed not to be able to distinguish between $\overline{X}$ and Z and thus permits a fair return s on either type of capital expense. If the regulator is instead assumed to detect and disallow all Z, the conclusions change slightly, as we will shortly show.

[2] Note, however, that if there are cross-elasticities, the solution to (12) depends on the value of s, since there is a term coupling peak and offpeak demands

$$\overline{X} \frac{\partial \overline{p}}{\partial \underline{X}} .$$

Hence, the act of regulation may be expected to affect offpeak quantity and price in any case where a customer utilizes the service in both peak and offpeak periods.

gives the peak-period effect. Marginal revenue is set equal to something less than the marginal cost in the peak period. Intuitively, because constrained profits increase with the level of capacity, the firm passes on the benefits of regulation to those users whose increased demand will cause an increase in capacity. *Thus, the peak period price under rate-of-return regulation will be lower than the peak period price in a model of unconstrained profit maximization (c.f., (11) and (5)). Accordingly, the likelihood is increased of finding the pricing reversal described in Section II.*

This model also has some interesting implications as to the capacity that is provided.[1] Let us denote the surplus of offpeak revenues over offpeak operating costs as

$$\underline{S} = \underline{pX} - c\underline{X} ,$$

and let us assume for the moment that the firm operates on the production frontier ($Z_G = 0$). Then the constraint in model (11) can be rewritten

$$\underline{S} + \bar{p}_G\bar{X}_G - c\bar{X}_G - rk\bar{X}_G = (s-r)k\bar{X}_G ,$$

which can be used to determine the conditions under which the peak-period capacity of the regulated monopoly would

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[1] See Wellisz [1963] for the earliest treatment of this matter.

exceed or fall short of the socially optimal level of capacity.

This equation tells us that once the surplus from the offpeak period has been determined, the firm portrayed by this model would choose the peak-period quantity (= capacity) and price to bring its *overall* profits in line with the constraint. In choosing its peak-period capacity, the firm would thereby determine its rate base and the level of allowable profits under the constraint. The surplus from the offpeak period (already determined) and the surplus or deficits from the peak period (equal to peak revenues minus the sum of peak operating costs and total capacity costs) must together sum to the total allowable profits under the constraint.[1]

More specifically, if the offpeak surplus is very large, the firm would lower its peak price below c + rk, and thus run a deficit on its peak period operations. It would do this in order to absorb its offpeak surplus and at the same time to expand its capacity and thus expand its allowable profits. The firm would continue to lower its price and expand its capacity until the allowable profits on the expanded rate base equal the offpeak surplus less the peak period deficit. The rate-of-return constraint has given the firm an incentive to absorb its extra profits through expanded peak capacity rather than changing its

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[1] In the more general case where each class of customers uses the facilities during both peak and offpeak periods, the regulatory constraint will almost certainly affect offpeak prices to at least some extent (see, for example, Bailey [1972] and Currie [1973]).

offpeak price to reduce profits. If, on the other hand, the offpeak surplus is comparatively small, the firm will raise its peak price and make a surplus on its peak operations, until the sum of the offpeak and peak surplus comes into equality with the allowable profits (which are being reduced because the higher price causes a reduction in capacity).

These last results expand considerably on one of Bailey's principal conclusions. Bailey shows [1972, p. 676] that the peak period capacity may exceed the welfare maximization level if cross-elasticity is not zero. Here, we see a more general result: *Peak period capacity exceeds the welfare maximization level whenever the surplus from the offpeak period is large enough to require deficits in the peak period.*[1] *In the other case, in which there is some surplus in the peak period, the regulated firm will provide a larger capacity than would an unregulated monopolist,* $\overline{X}_M < \overline{X}_G$, *but below that which welfare maximization produces,* $\overline{X}_G < \overline{X}_W$. The crucial variables determining whether or not the peak capacity exceeds the welfare maximizing level are the size of the offpeak surplus and the *margin* between s and r. As the margin between s and r is reduced, the constrained level of profits declines, and it becomes more likely that the

[1] Kafoglis [1971] also recognizes the possibility that expansions in capacity beyond the welfare maximizing level can occur in regulatory models when there is price discrimination between classes of customers.

offpeak surplus will have to be absorbed by an expansion of peak period capacity and consequent peak period deficits. Note, though, that if the margin between s and r narrows because s is lowered, peak-period capacity will expand; whereas, if the margin narrows because r rises, the peak-period capacity will not change but the welfare maximizing capacity level will decline. These last results follow from the constraint in model (10), in which s, but not r, is a determinant of the peak-period capacity.[1]

We now examine the possibility of $Z > 0$, i.e., the possibility that the firm is operating off its production frontier by employing excess capital. If $Z > 0$, then the Kuhn Tucker condition on the variable Z requires that $\lambda = \frac{r}{s}$.[2] Substituting into (11), we can see that this happens only when

$$\bar{p} = \frac{c}{1 - \frac{1}{e}} .$$

The common sense of this equation is straightforward. The firm wishes to expand its capacity to absorb its offpeak

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[1] A similar sort of result is derived in Bailey [1973, Chapters 6 and 8] in which it is shown that in an Averch-Johnson [1962] model of rate-of-return regulation the optimal capital stock increases as the fair return s is lowered toward r, but does not change if the regulator holds s fixed and the cost of capital r fluctuates with changes in market conditions.

[2] The Kuhn-Tucker conditions on Z are that $k(-r+\lambda s) \leq 0$, and that $Zk(-r+\lambda s) = 0$.

surplus. If it is worthwhile to expand capacity at the point where the marginal revenue equals marginal operating cost, then the best way of expanding capital further is to add capital that is not used productively. Such an addition of unproductive capital does not necessitate any price drop, whereas adding productive capacity means that the price and marginal revenue on peak operation has to drop and the firm's marginal expenditure on operating costs exceeds the additional revenue it brings in.[1] For the rest of this section, we shall suppose that $MR_G > c$ at the optimal solution, so that the operation off the production frontier is not a profitable alternative.

The results under rate-of-return regulation may be seen graphically in Figure 3. As before, the profit-maximizer's solution is denoted $\overline{M}$ and $\underline{M}$, and the welfare solution is $\overline{W}$ and $\underline{W}$. The horizontal line $c + sk$ is constructed so that the difference between it and the $c + rk$ line is precisely the permitted return $(s-r)k$ per unit of capacity.

For the regulated firm, constrained profits at any given peak period output $\overline{X}_G$ (= capacity) would be equal to $(s-r)k\overline{X}_G$, or the solid rectangle $\overline{p}_W BFE$. If, as in the diagram, the surplus from the offpeak period, the striped rectangle $\underline{p}_G G\underline{X}_G\underline{p}_W$, exceeds $\overline{p}_W BFE$, the firm would set a price $\overline{p}_G$ for the peak period that was below $c + rk$ until the deficits, dotted rectangle

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[1] Of course, if the regulator detects and disallows unproductive capital, such expenditures only subtract from profits and do not add to the rate base. In this case, adding productive capital would always be preferable.

Pricing Under Rate-of-Return Regulation

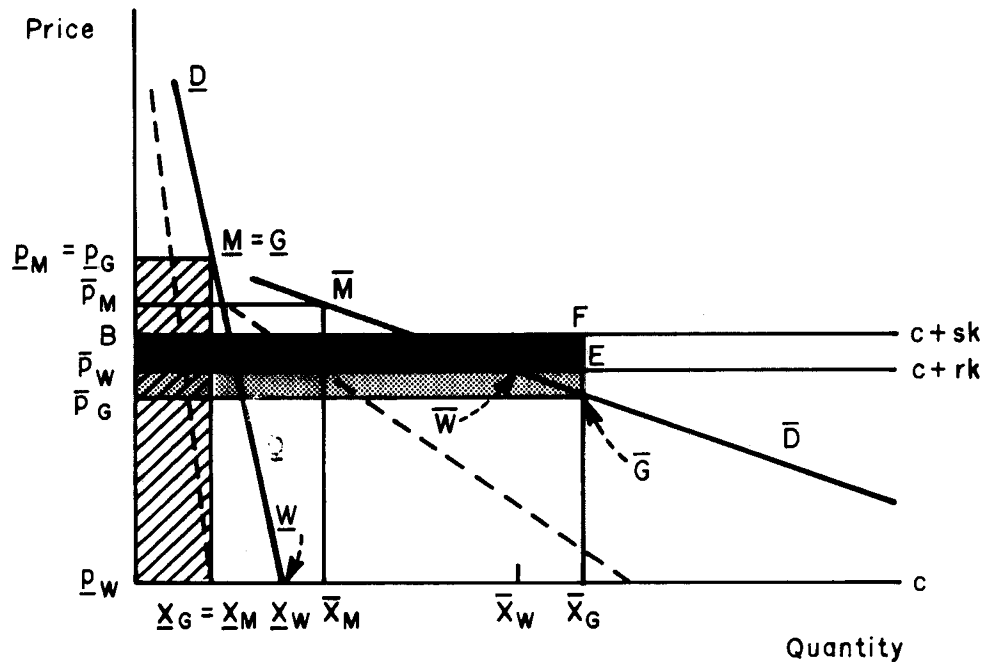

Figure 3

$\bar{p}_W E\bar{G}\bar{p}_G$, were such that $\underline{p}_G \underline{GX}_G \underline{p}_W - \bar{p}_W E\bar{G}\bar{p}_G = \bar{p}_W BFE$. This last condition can also be restated as $\underline{p}_G \underline{GX}_G \underline{p}_W = \bar{p}_W BFE + \bar{p}_W E\bar{G}\bar{p}_G$ or $\underline{p}_G \underline{GX}_G \underline{p}_W = BF\bar{G}\bar{p}_G$, i.e., the striped rectangle is equal to the sum of the solid and dotted rectangles in Figure 3. In effect, then, regulated firm finds that point on its peak demand curve $\bar{G}$ at which the product of the *margin* between c + sk and $\bar{p}_G$, times the $\bar{X}_G$ that is forthcoming at that $\bar{p}_G$, is just equal to the offpeak surplus. This result has a simple interpretation in common sense terms: c + sk represents the maximum that the firm could charge for peak sales if it had no offpeak surplus. But if the firm has an offpeak surplus, it cannot charge the full c + sk for its peak operations. Hence, it drops its peak price until the difference between c + sk and its peak price, times the peak quantity, can just absorb the offpeak surplus.

As is clear from the diagram, if the margin (s-r)k (the distance FE) is smaller than the distance $F\bar{G}$, the firm will be running a deficit in the peak period and the regulated firm will be providing peak capacity that is larger than that which maximizes welfare. The opposite will be true if the distance FE is larger than the distance $F\bar{G}$. Note, though, as was argued before, if r moves relative to s, the peak capacity offered does not change (though the welfare maximizing level changes). Thus, in the diagram, if r increases so that the margin (s-r)k is diminished, the peak output $\bar{X}_G$ does not change. This is true because $\bar{X}_G$ is still the only point at which $BF\overline{Gp}_G = \underline{p}_G\underline{GX}_G\underline{p}_W$. All that is implied is that the firm is running larger losses on its unchanged peak output and its overall profits are reduced. If, on the other hand, s declines $BF\overline{Gp}_G$ is no longer equal to $\underline{p}_G\underline{GX}_G\underline{p}_W$, and $\overline{X}_G$ has to increase.

IV. TWO-PART TARIFFS

Perfect Two-Part Tariff

The preceding sections have described models in which only one charge is levied on peak users and one on offpeak users. In practice, however, firms such as public utilities usually have a set of multiple prices or block tariffs with prices falling in steps as greater volumes are used. To reflect such a system of prices, we now introduce a model of peak-load pricing in which a customer charge as well as a usage charge is levied.

To avoid complicating the model, we suppose that the firm can charge an entrance fee or customer charge to each class of customers, as well as a peak and an offpeak usage price. Furthermore, we assume that all customers in a particular customer class have identical demand curves.[1]

If the firm has complete freedom in setting the customer fees, $\overline{E}$ and $\underline{E}$, and the usage fees, $\overline{p}$ and $\underline{p}$, the model becomes

[1] This is the same sort of assumption as is made in MacAvoy and Noll [1973]. The cases where customers have different demand curves and/or where entrance fees can be tailored to individual customers has been described in Oi [1971]; however, Oi does not include peak and offpeak pricing considerations.

$$\underset{\overline{E},\underline{E},\overline{X},\underline{X},Z}{\text{Maximize}} \quad \pi = \overline{E} + \underline{E} + \overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) - rk(\overline{X}+Z)$$

$$\text{subject to } \overline{E} \leqq \int_0^{\overline{X}} \overline{p}(\overline{X})d\overline{X} - \overline{p}(\overline{X})\overline{X} \tag{13}$$

$$\underline{E} = \int_0^{\underline{X}} \underline{p}(\underline{X})d\underline{X} - \underline{p}(\underline{X})\underline{X}$$

$$\pi \leqq (s-r)k(\overline{X}+Z)$$

The constraints on the entrance fees assert that these cannot exceed the accumulated consumers surplus above the marginal usage charge to the particular customers. For, otherwise, the customer would purchase none of this particular product or service. If $\overline{\alpha}$ and $\underline{\alpha}$ denote the Lagrange multipliers for the entrance fee constraints, the Lagrangian for model (13) becomes

$$\begin{aligned}\phi(\overline{E},\underline{E},\overline{X},\underline{X},Z,\lambda,\overline{\alpha},\underline{\alpha}) = {} & (1-\lambda)[\overline{E} + \underline{E} + \overline{p}(\overline{X})\overline{X} + \underline{p}(\underline{X})\underline{X} - c(\overline{X}+\underline{X}) \\ & - rk(\overline{X}+Z)] + \lambda(s-r)k(\overline{X}+Z) \\ & + \overline{\alpha}\left[\int_0^{\overline{X}} \overline{p}(\overline{X})d\overline{X} - \overline{p}(\overline{X})\overline{X} - \overline{E}\right] \\ & + \underline{\alpha}\left[\int_0^{\underline{X}} \underline{p}(\underline{X})d\underline{X} - \underline{p}(\underline{X})\underline{X} - \underline{E}\right].\end{aligned} \tag{14}$$

The Kuhn-Tucker conditions are

$$\phi_{\overline{E}}:\quad 1-\lambda \leqq \overline{\alpha},\ \overline{E}(1-\lambda-\overline{\alpha}) = 0 \tag{15}$$

$$\phi_{\underline{E}}:\quad 1-\lambda \leqq \underline{\alpha},\ \underline{E}(1-\lambda-\underline{\alpha}) = 0 \tag{16}$$

$$\phi_{\overline{X}}:\quad (1-\lambda)\left[\overline{p}(\overline{X}) + \overline{X}\frac{d\overline{p}}{d\overline{X}} - c - rk\right] + \lambda(s-r)k - \overline{\alpha}\left(\overline{X}\frac{d\overline{p}}{d\overline{X}}\right) = 0 \tag{17}$$

$$\phi_{\underline{X}}:\quad (1-\lambda)\left[\underline{p}(\underline{X}) + \underline{X}\frac{d\underline{p}}{d\underline{X}} - c\right] - \underline{\alpha}\left[\underline{X}\frac{d\underline{p}}{d\underline{X}}\right] = 0 \tag{18}$$

$$\phi_{Z}:\ -rk + \lambda sk \leqq 0,\ Zk(-r+\lambda s) = 0 \tag{19}$$

$$\phi_{\overline{\alpha}}:\ \overline{E} \leqq \int_0^{\overline{X}} p(\overline{X})d\overline{X} - \overline{p}(\overline{X})\overline{X},\ \overline{\alpha}\left(\overline{E} - \int_0^{\overline{X}} \overline{p}(\overline{X})d\overline{X} - \overline{p}(\overline{X})\overline{X}\right) = 0 \tag{20}$$

$$\phi_{\underline{\alpha}}:\ \underline{E} \leqq \int_0^{\underline{X}} \underline{p}(\underline{X})d\underline{X} - \underline{p}(\underline{X})\underline{X},\ \underline{\alpha}\left(\underline{E} - \int_0^{\underline{X}} \underline{p}(\underline{X})d\underline{X} - \underline{p}(\underline{X})\underline{X}\right) = 0 \tag{21}$$

$$\phi_{\lambda}:\ \pi \leqq (s-r)k(\overline{X}+Z),\ \lambda[\pi - (s-r)k(\overline{X}+Z)] = 0 \tag{22}$$

A number of interesting results emerge from these conditions. First, in the absence of regulation, $\lambda = 0$ and, from (15) and (16), $\overline{\alpha} = \underline{\alpha} = 1$. Then, by (20) and (21), the *firm sets the customer for each class of customers so as to precisely equal the entire area above the usage price and below the demand curve.* Substituting $\overline{\alpha} = \underline{\alpha} = 1$ into (17) and (18), and simplifying, we find

$$\overline{p}_T = c + rk \qquad (23)$$

$$\underline{p}_T = c \qquad (24)$$

so that *under a two-part tariff welfare pricing is preserved: the usage prices to the peak and offpeak customers just exactly compensate at the margin for their respective additional costs.* Stated differently, the elasticity terms that appeared in Equations (5) and (6) have disappeared, and the welfare maximization results of Section II have re-emerged.

This result holds also if K' replaces rk as the marginal cost of capacity. Thus, in decreasing average cost industries, a two-part tariff enables the firm to be financially viable without having to depart (optimally or otherwise) from marginal cost pricing. Welfare pricing ($\overline{W}$ and $\underline{W}$ in Figure 2) is the outcome, and yet the firm is not supported by taxation out of the public treasury.[1]

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[1] This point was made by Coase [1970], p. 118 and by Buchanan [1966], p. 470; also, see Oi [1971], and Buchanan [1953]. Wallace Oates pointed out to us that for those in the field of public finance, welfare maximization implies a "regulatory" policy which actually leads to the welfare-maximizing outcome, which of course is precisely what happens in the two-part tariff situation.

The two-part tariff is indeed "perfect" both in extracting all of the consumers' surplus, and because the investment decision, consumption decision, and resource allocation decision are all made at the correct margin. However, the income distribution is now very different, since the utility has captured all of the social surplus[1] (the peak surplus $A\overline{Wp}_W$ in Figure 2 plus the offpeak surplus $E\underline{Wp}_W$ less the capacity deficit $B\overline{Wp}_W$.)

Regulation of Rate of Return but not of Price Structure

One way to redistribute the income is to impose a rate-of-return constraint. Such a constraint has been included in Model (13). The model is thus one in which the regulator permits the firm to set its own price structure and insists only that the rate of return end up at the level deemed fair. When regulation is effective, $0 < \lambda < 1$ from (19). From (15) and (16), this means that $\overline{\alpha}$ and $\underline{\alpha}$ are both nonzero, so that the equalities must hold in the entrance fee constraints (20) and (21): as before, the customer charges are set so as to absorb the entire surplus.

If we solve for the offpeak usage price by substituting $\underline{\alpha} = 1 - \lambda$ into (18), we achieve the same result as (24): the offpeak user is charged his marginal

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[1] For an interesting treatment of income distribution in this context, see Feldstein [1972].

running cost. To solve for the peak price, substitute $\bar{\alpha} = 1 - \lambda$ into (17) to obtain

$$\bar{p}_R = c + rk - \frac{\lambda}{1-\lambda}(s-r)k \qquad (25)$$

Hence, the price to the peak user is set below the sum of marginal operating and capacity costs and *capacity will always be in excess of the welfare-maximizing level. In essence, the regulated utility absorbs the surplus from the customer charges entirely by expanding capacity.*

This expansion of capacity will be productive as long as $\bar{p}_R > c$. To see this, note that if waste is positive ($Z > 0$), then $\lambda = r/s$, so that from (17), price must have been driven down to marginal operating cost. As long as price is above this marginal cost, the firm prefers to add productive capacity, for by doing so it is simultaneously earning a larger surplus from the peak customer charge and is adding to the rate base. If price is below marginal running costs, then a deficit is incurred on marginal operation and it is cheaper, as long as the regulator permits Z to enter the rate base, for the firm to add unused capital.

These results indicate that, *if the regulator does not distinguish productive from nonproductive capital, the pricing reversal discussed in previous sections cannot occur.*

The price to offpeak users is set equal to marginal operating cost, and the price to the peak user is set above or equal to this level.

Figure 4 displays the equilibrium graphically. The regulated firm sets the same offpeak customer charge (equal to the area of the triangle $B\underline{X}_R\underline{p}_R$) and the same price, $\underline{p}_T$ as the unregulated firm. The offpeak surplus, along with that of the peak users (given by the triangle $A\overline{W}\overline{p}_W$) yield profits in excess of those allowed at the welfare-maximizing level of capacity, i.e., $A\overline{W}\overline{p}_W + B\underline{X}_R\underline{p}_R > HF\overline{W}\overline{p}_W$. Hence, the utility lowers peak price and expands capacity until at capacity $\overline{X}_R$ the total profit has been reduced (by the dotted triangle $\overline{W}E\overline{R}$) sufficiently to bring about equality in the regulatory constraint. The equilibrium price $\overline{p}_R$ is clearly below the welfare-maximizing level.

<u>Regulation of Price Structure</u>

In practice, the fixed charges levied by utilities appear to fall far short of the full consumer's surpluses. It is likely that there are formal or informal limits imposed by regulators on the fixed charges that can be levied.[1] Let us suppose that the customer fees are set so as to take into

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[1] These limits may arise in part because customers have different demand curves. In order to make the service available to everyone, the entrance fee cannot exceed the level which the customer with the smallest consumers' surplus is willing to pay. See Oi [1971].

Two - Part Tariff and Rate - of - Return Regulation

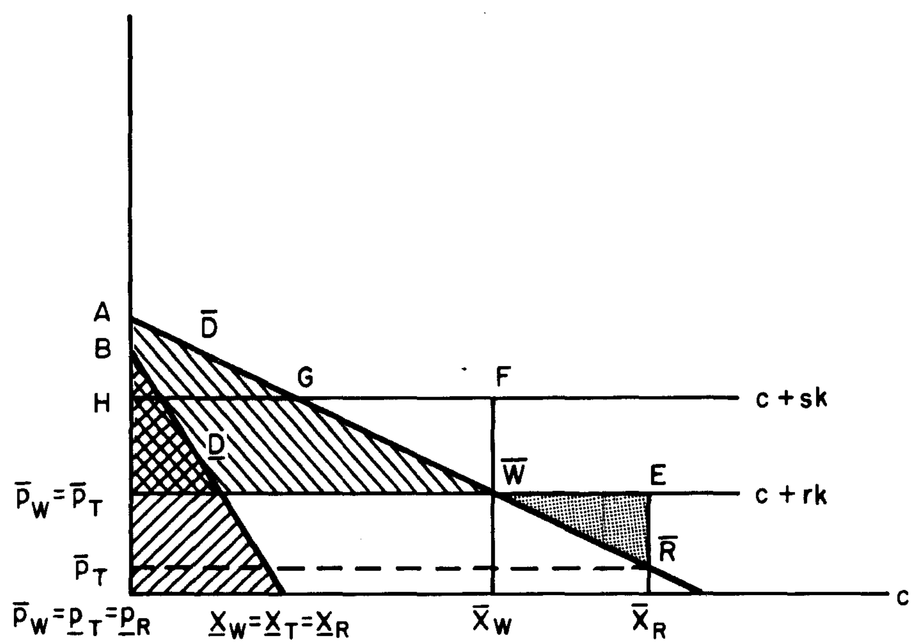

Figure 4

account the large fixed capital costs in the industries, but that they are limited to levels below those at which all the surplus is absorbed. The interesting feature of the analysis when we construct the model in this way is that we revert to conclusions that closely resemble those in Section III. The elasticities of the different classes of customers are again evident, and the pricing reversal possibility reappears.

Figure 5 illustrates the situation. The customer charge to the offpeak user is assumed to be limited by the regulator to area AGB. A profit-maximizing firm does not set usage price equal to marginal operating cost for this offpeak period, but instead sets marginal revenue equal to cost, for by doing so, the offpeak surplus $\underline{p}_S M \underline{X}_S Q$ can be gained in addition to the customer charge, AGB. It is for this reason that the elasticity effects show up in the first-order conditions.[1]

Similarly, the firm will treat the customer fee of the peak users, triangle LOH in Figure 5, as an endowment of profit, and will proceed to adjust the usage price to the peak user until the regulatory constraint is satisfied.

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[1] More specifically, inspection of Equations (15) - (22) indicates that if $\bar{E}$ and $\underline{E}$ are limited to amounts unrelated to the elasticities of demands, these elasticities will reappear in the first order conditions determining the prices for peak and offpeak service. Elasticity effects would only be modified if the regulator permitted the customer fee to be in excess of a monopoly surplus triangle such as $AM\underline{p}_R$ in Figure 5.

Pricing With Limited Customer Charge

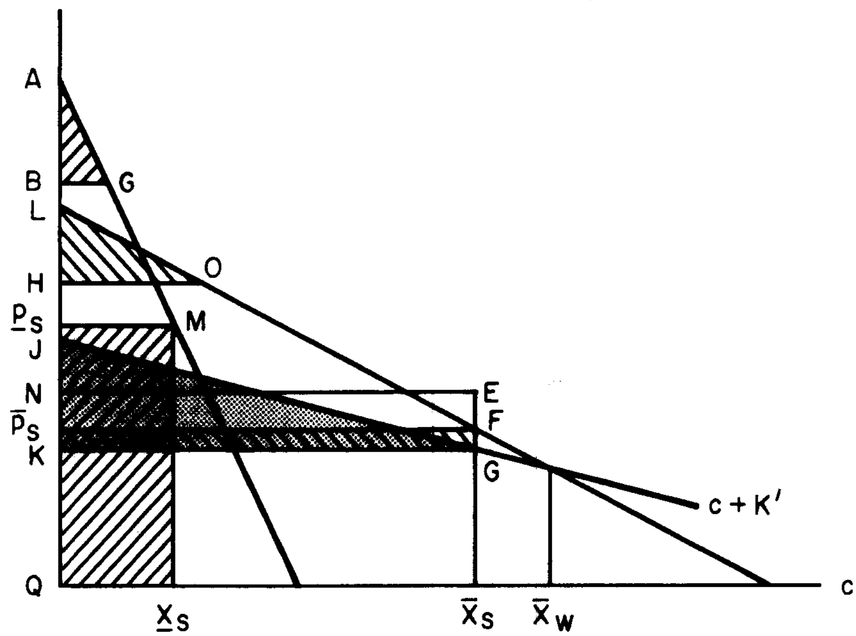

Figure 5

If the fair return is such that the profit permitted by the constraint is given by rectangle NEGK, then this rectangle must equal the sum of the surplus from the customer charges AGB + LOH, the surplus from the offpeak usage charge, $\underline{p}_S M\underline{X}_S Q$, the deficit or surplus from the peak period (the surplus $\bar{p}_S FGK$), less the deficit JGK incurred by the increasing returns to capacity, that is,

$$NEGK = AGB + LOH + \underline{p}_S M\underline{X}_S Q + \bar{p}_S FGK - JGK.$$

V. SUMMARY

We have demonstrated in this paper that it is theoretically possible for offpeak prices to be higher than peak prices for an unconstrained profit-maximizing firm, for a welfare-maximizing firm with increasing returns to scale, for a profit-maximizing firm subject to a regulatory constraint on its rate of return and for a firm that can levy a two-part tariff but where there is a constraint on the size of its customer charge. In the first two instances, the pricing reversal comes about because of the inverse elasticity rule, familiar from the optimal departures from marginal cost pricing literature. In the latter cases, the reversal comes about because the rate-of-return constraint encourages price reductions to peak-period users rather than to offpeak users. As a result, the pricing reversal becomes ever more likely to occur as the constraint is tightened. Furthermore, as the constraint is tightened, it becomes more likely that there can be an expansion of capacity beyond the welfare-maximizing level.

In the electricity example, these last results imply that for customer charges of approximately the same size, state regulatory agencies that have tighter rate-of-return requirements may encourage lower usage prices to peak business users while the prices charged to offpeak residential users remain substantially unchanged. An additional implication is that effectively regulated privately owned electric utilities might grant larger price reductions for business users than those instituted by government-owned electrics.

Either of these views suggest a possible method of empirically testing whether actual behavior in the electric utility industry agrees with that predicted by the theoretical models of peak-load pricing reversals. Recent evidence confirms that such agreement may be found. Peltzman, in contrasting the behavior of regulated and government owned electrics finds that "residential output is about 1/3 greater ... [while] industrial output is twice as large in privately owned as in government owned utilities...

[so that] the magnitude of the output effect of ownership is, in fact, smallest for residential customers."[1] The relatively larger price reductions to the business users in the regulated electrics is just what our theory would predict.

[1] Peltzman [1971], p. 138. Strictly speaking, as Peltzman pointed out to us in correspondence, his "result was based upon more finely developed price discrimination by private as opposed to public utilities, rather than upon any difference in average residential and industrial rates." However, he goes on to state "the thrust of your argument seems to me to imply that regulated utilities will be motivated to seek special rates for the more elastic demands within as well as between customer groups."

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