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BLACK-MARKETS FOR CURRENCY, HOARDING ACTIVITY AND POLICY REFORMS

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Black-Markets for Currency, Hoarding Activity and Policy Reforms

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Abstract

In the former Soviet Union and throughout Eastern Europe, black-market exchange rates and second-economy prices often are interpreted by policy-makers as indicative of post-reform levels. However, these exchange rates and prices can provide highly-biased signals for policy setting. These biases are especially important when exchange rates fixed on the basis of these signals are expected to play a nominal anchor role during stabilizations. This paper traces the paths and biases in black-market exchange rates, second-economy prices, hoarding stocks, and privately-held dollars balances after policy-initiatives or other changes in the economic environment are implemented. The stimuli studied are official exchange-rate adjustments, price reforms, foreign-aid packages, altered risks of monetary confiscation or currency reforms, and goods-supply related initiatives. We provide the conditions under which announcements of reform lead short-run prices or exchange rates to overshoot or to undershoot their long-run equilibrium levels.

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I. Introduction

Governments sometimes interpret current black-market exchange rates as indicative of post-reform equilibrium rates. For purposes of nominal exchange-rate targeting, policy-makers may set the fixed official exchange rates to align with current levels of black-market exchange rates. This interpretation of exchange rate signals is misleading.[1] When current black-market rates reflect anticipated changes in policies and when there is gradual adjustment of stock variables to policy initiatives, the black-market exchange rates are biased indicators of long-run equilibrium conditions.

This paper presents a theoretical investigation of the relationship between economic-reform initiatives and the black markets for dollars and goods in the former Soviet Union and Eastern Europe. In the East, restrictions still hamper free-market transactions in goods and financial assets. Financial markets are under-developed and few investment assets and outlets for savings are available to individuals. The existence of pervasive shortages and rigidities, the vestiges of central planning and controlled prices, have led the second-economy or black-market production structures to operate in tandem with black markets for hard currencies. Within this setting, the analysis first shows the direction of response of black-market exchange rates, second-economy prices, hoarding stocks, and privately-held dollars balances to reform initiatives. Second, the analysis shows the extent to which black-market exchange rates are poor predictors of future post-reform exchange rates.

Due to the paucity of financial instruments, the proliferation of consumption-goods shortages and the uncertainty about their future supplies, one principal form of investment and intertemporal transfer of wealth is through purchases of storable consumption goods.[2] Other important vehicles for savings are domestic-currency balances (or savings accounts) and foreign exchange holdings. In the recent period, the

[1]In addition, such policy rules can lead to instability as in Kharas and Pinto (1989).

[2]On Soviet savings activity, also see Klachko (1988), Ofer and Pickersgill (1980), and an updated discussion by Alexeev (1991).

extreme uncertainty of valuation of domestic currencies and of goods availability has dramatically increased the desire to accumulate foreign currency, mainly dollars, and any available consumer goods. For example, in the former Soviet Union, this saving activity traditionally had taken the form of currency, savings deposits, 3 percent lottery bonds, consumer durables and jewelry [Ofer and Pickersgill,1980].

As a consequence of these financial structures, the theory presented herein incorporates this feature of incomplete financial markets. The investment opportunities available to the consumer/ investors include hoarding of goods, hard-currency accumulation, and domestic-currency savings. Since our model introduces a demand for second-economy goods that arises from both investment and consumption motives, rather than from consumption motives only, a range of reform initiatives have significantly different effects than those predicted by existing studies. This broader approach is better-suited for the reforming economies of Eastern Europe and the former Soviet Union. In contrast to the conventional approaches, actual and anticipated goods-market initiatives, in addition to initiatives aimed directly at financial markets, simultaneously affect both exchange rates and prices, and the long-run state of both goods markets and financial markets. This facilitates a rich discussion of the dynamic effects of a range of anticipated and unanticipated reforms on the amount of hoarding of consumption goods, on inflation in the second-economy, on the black-market exchange rate, and on black-market trade volumes in transition economies.

The theory builds on two themes developed in other contexts. The first body of literature models the determinants of black-market exchange rates in developing countries. Motivated in particular by the experiences of Latin American economies, these models interpret black-market currency demands as arising from "portfolio" motives, wherein changes in the risk, return and stocks of various financial assets drive the currency price.[3] [de Macedo, 1982 and 1987; Dornbusch et al., 1983] Black markets for currencies arise, in part, from their functions as conduits for flight capital into foreign investments. Recent theoretical extensions to this literature have explored the fiscal ramifications of policies intended to unify black and official market exchange rates, and the feedback into black-market premia [Pinto, 1991].

The second related body of literature is the well-known theory of goods and financial-market interactions, as developed in the dynamic-adjustment models based on the seminal Dornbusch exchange-rate overshooting framework. In these models, there are two distinct markets: the market for financial assets and the market for goods. In the context of the reforming economies of Eastern Europe, this literature provides a useful vehicle for analyzing the dynamic effects of changes in the financial and goods market structures. Nonetheless, even in this literature, in which important contributions are made on financial market reforms by Calvo and Frenkel (1991) and on exchange rate unification by Agenor and Flood (1992), the assumption of distinct markets is maintained in efforts to study the role of policy initiatives in centrally-planned economies when there are imperfections in domestic financial markets.

The separability of the goods markets and financial markets, even in the very short-run, is an inappropriate assumption for the economies of Eastern Europe and the former Soviet Union. Reform of the production structure, anticipated or actual, will alter the attractiveness of the hoarding of goods. Since hoarding demand is determined alongside the demand for the other assets treated as stores of value, "real-side" reforms will be reflected immediately in asset prices, including the price of foreign currency in the black markets. Likewise, currency-market reforms impinge on the activities of goods markets. Therefore, in the theoretical analyses of this paper, we explore the

[3] Dollars are held by individuals as a means of diversifying a "portfolio" of assets to maximize their expected return on "invested" wealth and minimize the variance of these returns.

effects of policy initiatives without imposing the assumption that goods markets and financial markets are separable.

Depending on the reforms announced (and their credibility), the black-market exchange rate either can undershoot or overshoot the post-reform equilibrium exchange rate. For example, in the former Soviet Union, a dramatic example of exchange-rate overshooting occurred in February 1992. This panic episode reportedly was triggered by a shortage of cash rubles in circulation and by the announcement by the central government that (via foreign-exchange market intervention) the ruble would be appreciating against the dollar. In mid-February the ruble appreciated from 135 rubles per dollar to 60 to 65 rubles per dollar. At the same time, there was a dramatic increase in efforts to purchase consumer durables as an alternative to foreign exchange as a store-of-value. The reversal of the decline in the value of dollars in Moscow occurred as people who had been queuing to sell dollars realized that goods purchased for dollars in hard-currency shops actually were cheaper than in some ruble shops. The demand for dollars recovered. Within a month, by mid-March, the ruble returned to close to its mid-February level, much less sharply appreciated than in the very short run. The path of the value of the ruble against the dollar was predictable. Despite the enthusiastic proclamations by policy-makers in Russia, the theory of exchange-rate determination shows that the observed short-run gains in the value of the ruble against the dollar did not provide any lasting signals about the long-run health of the ruble.

Motivated by such events, and by noting that policy-makers tend to place considerable weight on observed short-run exchange rates, we show the dynamic effects of specific reform initiatives on the amount of hoarding of consumption goods, on inflation in the second-economy, on the black-market exchange rate, and on black-market trade volumes in transition economies. This provides important intuition about the biases inherent in exchange rates and in prices between the time of announcement of a pending or actual event and the overall adjustment of the economy to that event.

It is easy to provide additional examples of this misuse of black-market exchange rates for purposes of official exchange-rate targeting. In Poland the official zloty-dollar exchange rate initially was priced at an undervalued level, at precisely the rate prevailing in the free market [Nuti, 1991]. Moreover, "the free market rate became an important and popular institution, an indicator of the state of the economy, while the dollar became the only stable reference point in an otherwise rapidly changing economic environment" [Olechowski and Oles, p.158, 1991]. In the former Soviet Union, the black-market exchange rate was used as a reference point for the crawling-peg exchange-rate regime introduced between July 1990 and October 1991 [Goldberg, 1992]. In Bulgaria, the black-market exchange rates played an important role in the reform discussions that took place in early 1991. Initially, reformers had expected that "the introduction of convertibility would mean acceptance of an exchange rate close to the present black market rate" [Dimitrov, p.255 1991]. After liberalization of the exchange rate and to the surprise of the reformers, "in spite of the sharp initial price rise, however, the exchange rate then appreciated from 25 leva to the dollar to the range of 15 to 18 leva to the dollar by June 1991" [Dimitrov, p.263 1991].[4]

Another contribution of the theory is the insight it provides into the meaning of "monetary overhang". Under underdeveloped financial markets, the risk-adjusted returns on money may not be dominated by those on alternative investment assets. The portfolio model is used to generate equations for the theoretically "optimal" demand for money balances in the transition economy. These demands are distinct from those generated by current transactions motives and are not related exclusively to stochastic future consumption opportunities.[5]

The overall plan of the paper is as follows. First, in Section II we develop the model of the black-market for foreign exchange and its relationship with goods-market

[4]See also Kharas and Pinto (1989) on Bolivia.

[5]For further discussion of the monetary-overhang issues see Dornbusch and Wolf (1990) and Alexeev (1991).

activities. This model is applied to determine, theoretically, the effects of various announced and enacted real and monetary reforms. Section III presents the adjustment paths of black-market exchange rates, second-economy goods prices, dollar balances in private portfolios, and hoarding stocks. Section IV concludes with discussions of policy implications and potentially useful extensions of this work. As noted, the results are important for stabilization efforts, since policy-makers may improperly interpretate current black-market exchange-rate levels as indicative of post-reform equilibrium levels.

II. The Model: Black Markets for Foreign Currencies and Goods Hoarding

Two aspects of the demand and supply for (black) foreign exchange and for second-economy consumer goods are considered. The first aspect is derived from "financial" or investment motives for holding foreign currency and hoarding goods, and is subsumed under the heading of "portfolio" or speculative currency-demands. This is developed in Section II.A. The second aspect is related to current consumption needs and is subsumed under the heading of transaction demands with smuggling. Section II.B formulates transactions and consumption demands, along with adjustment equations in goods markets and in the second-economy current-account. Section III considers the dynamic effects of the reform initiatives.

II.A. Portfolio Demands for Foreign Currency in the Black Market

The consumer/investor chooses an optimal division of his wealth among domestic savings, foreign savings, and hoarding of goods. The investment assets have some trend rates of return, and each of these returns have a stochastic element (represented by a Brownian-motion diffusion term). Below, we present the choices available to consumer/investors and the solution to their optimal investment decisions.

The technical details of the derivation of optimal demands for the respective assets are provided in Appendix I.

Domestic savings, denoted by $\boldsymbol{B}$, have an expected nominal return of i_b equal to the interest paid on domestic-currency savings deposits. There is some risk σ_b associated with this form of savings: it is possible that the account will be confiscated in a monetary reform.[6] One example of this risk is the reduction in East German household savings that occurred in the process of German unification. Alternatively, if domestic currency is held in the form of cash balances ($i_b=0$), there is a risk that cash holdings outside of the savings accounts will be invalidated, as in the January 1991 "monetary reform" in the former Soviet Union.

Another asset available to the consumer/investor is foreign-currency accumulated through the black market. Throughout the paper we refer to this foreign exchange as dollars. In practice, dollars, along with Finnish marks, are popular in the former Soviet Union. Austrian shillings and German marks are popular transaction currencies in Czechoslovakia and Hungary. The domestic-currency value of these foreign-exchange holdings, denoted by *SF*, is the product of the quantity of dollars held, *F*, multiplied by the black-market exchange rate *S*, defined as units of domestic currency per unit of foreign exchange (rubles per dollar). Domestic residents are prohibited from opening interest-bearing foreign-currency accounts.[7] Consequently, foreign-exchange holdings do not yield a nominal return.[8] The valuation and rate-of-return on these dollar stocks depends on the process defining the short-run black-market

[6]In terms of the theory, this risk is represented as the standard deviation of the diffusion process on returns on domestic currency assets. Analogous interpretations will apply to other risk measures introduced.

[7]This assumption is not important for our results and easily could be relaxed.

[8]One could easily introduce expected costs of obtaining and later liquidating black-market foreign exchange, as consumer/investors pay a range of fees to money dealers. Also, it is possible to introduce a parameter reflecting the risks associated with holding foreign exchange, due to the likelihood of being caught by authorities and penalized.

exchange rate. The expected rate of depreciation of the ruble against the dollar is given by ϕ and the standard deviation of this forecast is σ_s.

The third investment opportunity is through hoarding storable goods, G, valued at second-economy prices P_g. The expected return on hoarding is the expected rate of increase in durable-goods prices, π, (i.e. the inflation rate on domestic goods), less the cost of obtaining or storingthese goods, c_g, over the same interval of time for which the inflation series is measured. This cost can be interpreted as the opportunity cost of waiting in queues or searching for the goods [see Stahl and Alexeev (1985) and Weitzman (1991)]. The scale of uncertainty in the forecast of future goods availability and future inflation is subsumed within the index σ_g.

These investments are made out of available nominal wealth stock W, defined by:

$$W = B + P_g G + SF \tag{1}$$

and the portfolio shares allocated to the respective assets are denoted by:

$$\lambda_1 = P_g G/W\,,\ \lambda_2 = SF/W\,,\ \text{and}\ \ \lambda_3 = (1-\lambda_1-\lambda_2) = B/W\,. \tag{2}$$

The objective of the consumer/ investor is to choose portfolio shares optimally in order to maximize the expected real return on his portfolio while minimizing its variability.

$$\underset{\lambda_1\,,\,\lambda_2}{\text{maximize}} \qquad U = E\left(\frac{dw}{w}\right) - \frac{1}{2}R\cdot Var\left(\frac{dw}{w}\right) \tag{3}$$

where R is the coefficient of relative risk aversion, $w = W/Q$ is real wealth, and Q is the price index of the consumption basket. Consumption occurs over two types of goods, domestic goods and imported goods.

Domestic goods enter the price index with weight α.[9] Imported goods enter the price index with weight (1-α). Importers are price-takers in the market for foreign

[9]The domestic goods can be hoarded, consumed internally, or exported. If exported, these goods are traded at prices determined in domestic markets, so that the exporters are not price-takers in world-markets.

products. The aggregate price index is a weighted average of the price of domestic goods, P_g, and the imported-goods price, SP^* with $P^*=1$ hereafter. The domestic import price follows the black-market exchange rate.

Table 1	Variable Definitions and Description
B	Domestic financial assets (savings accounts or money)
i_b	Expected nominal return on B
σ_b	Risk associated with nominal returns i_b(monetary confiscation possible)
S	Black-market exchange rate (domestic currency per unit of foreign)
F	Quantity of foreign currency in circulation in black markets
ϕ	Expected nominal depreciation rate in black market
σ_f	Risk or uncertainty associated with black-market depreciation forecast
P_g	Second-economy goods prices
G	Net supply of hoardable goods in the second economy
π	Expected inflation rate on second-economy goods
σ_g	Uncertainty associated with inflation or goods-availability forecast
W	Nominal wealth
Q	Domestic aggregate price index
α	Share of domestic goods in the domestic price index
P^o	Official-sector/ controlled prices on goods
S^o	Official exchange rate
γ	Food aid or supply supplements in goods markets
c_g	Cost of obtaining or holding second-economy goods for hoarding (ex. depreciation or lost wages)

The derived optimal-portfolio shares are provided in the technical appendix. There are two components of the optimal demands for the respective assets: the optimal minimum-variance and optimal speculative portfolios. The assets demanded through the minimum-variance portfolio are precisely equal to their shares in the future consumption baskets of individuals: αW is minimum-variance investment in real goods, while $(1-\alpha)W$ is the minimum-variance holding of foreign exchange. A short position is taken in domestic currency assets to offset the long minimum-variance position in the other two investment alternatives. However, this does not imply that negative stocks and demands for domestic currency (i.e. net borrowing) prevail. The second component of the optimal portfolio demands, the speculative portfolio demands for domestic-currency assets, would be required to be positive and at least as great as the negative minimum-variance demands. As shown in the technical appendix, this implies that domestic interest rates must be at least as great as the returns on alternative assets, net of a linear combination of their expected variances and covariances.

For conciseness and brevity, at this point it is sufficient to present only the signs of the asset-demand elasticities with respect to various parameters of financial markets. Appendix Table A1 presents the full set of equations and results on optimal portfolio shares. Asset demands are normal, so that $\lambda_1^{\pi} > 0$ and $\lambda_2^{\phi} > 0$. The higher the expected inflation (π) on the hoarded goods (the lower the expected future goods availability) the higher the current speculative-portfolio demand for durable goods (λ_1). This responsiveness is decreasing in the relative-risk-aversion of the investor, since the source of demand for durables is the investor's attempts at minimizing the variance of his investment portfolio. The responsiveness of portfolio demands to changes in π is influenced by the variances and covariances of the returns on alternative assets, but not by the expected returns on those assets.

Changes in the risk associated with returns to hoarding or holding domestic currency have first-order and second-order effects on investment demands. For example, the first-order effect of increased supply uncertainty, through σ_g^2, operates to directly reduce the portfolio demand for goods (i.e. hoarding) and to increase the demand for foreign and domestic currency.[10] The second-order effect of increased supply uncertainty works through altering the responsiveness of asset demands to other policy and process changes: for example, as σ_g^2 rises, the expansionary effect on goods hoarding of increasing π is reduced.

Table 2 **Asset Demand Functions: Signs of Demand Elasticities**

effect on demand for \		σ_b^2	σ_g^2	i_b	c_g	ϕ	π
goods-hoarding	λ_1	+ [1]	- [2]	-	-	-	+
dollars	λ_2	+ [1]	+ [3]	-	+	+	-
home-assets	λ_3	- [1]	+ [3]	+	+	-	-

[1]: Unambiguously satisfied if $\lambda_3 > 0$.

[2]: Unambigously satisfied if $\pi - c_g - i_b > \dfrac{\rho_{gs}\left(\sigma_b^2 + \sigma_s^2 - \rho_{gs}\right)}{\left(\sigma_b^2 + \sigma_s^2\right)}$

[3]: $\dfrac{\partial \lambda_2}{\partial \sigma_g^2} < 0$ requires $\alpha < \dfrac{1}{A}\begin{pmatrix} -\left(\sigma_b^2 + \sigma_s^2\right)\left(\pi - c_g - i_b - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}\right) \\ +\left(\sigma_b^2 + \rho_{gs}\right)\left(\phi - i_b - (1-\alpha)\sigma_s^2 - \alpha\rho_{gs}\right) \end{pmatrix}$

where $A = \left(\sigma_b^2 + \sigma_s^2\right)\left(\sigma_b^2 + \sigma_g^2\right) - \left(\sigma_b^2 + \rho_{gs}\right)^2$

Likewise, consider the impact of increasing the risk of monetary reform, σ_b^2. Under reasonable conditions, this policy initiative either will increase the demand for hoarding and for holding foreign currency as investment assets.

[10]It is theoretically possible to reverse the sign of these elasticities, but this is only likely to occur with $\lambda_3 < 0$.

These derivations are important for two immediate reasons. First, they link hoarding-demands for goods and demands for foreign exchange to actual and expected changes in prices, policies, and perceived risks of monetary and goods-supply shocks to the economy. Second, the demand elasticities formally are shown to be sensitive to a variety of policy parameters. Another level of importance for these parameters arises below, when they are combined with a specification of flow relationships to determine equilibria in the second-economy for goods and in the market for dollars. In the short-run, exchange rates and prices adjust to ensure asset-market equilibrium, although excess supply or demand may occur in goods markets and second-economy trade imbalance may develop. The medium run supply response enables stock adjustments to occur in amounts of goods hoarded and in the amount of dollars accumulated or decumulated through illicit cross-border trades. In turn, these stock adjustments trigger the exchange rate and price adjustments required for maintaining asset-market equilibrium. In the long-run, asset markets, second-economy goods markets, and the illicit trade balance are in equilibrium.

The asset-market component of market equilibrium, achieved through second-economy exchange rate and price adjustments, can be presented in reduced form. Collecting all demand influences other than ϕ and π into the vector Z, and multiplying the optimal-portfolio shares by stocks of investable wealth, the conditions for stock equilibrium in the asset markets are given by:

$$\lambda_1(\phi, \pi, Z)(B + P_g G + SF) = P_g G \tag{4}$$

$$\lambda_2(\phi, \pi, Z)(B + P_g G + SF) = SF \tag{5}$$

where $Z = (\sigma_b^2, \sigma_g^2, i_b, c_g)$.

Equations (4) and (5) provide two equations in two endogenous unknowns, S and P_g. Both the black-market exchange rate and the second-economy price of goods are assumed to be perfectly flexible (where the goods-price flexibility also contrasts with the more conventional approaches). The values of these variables that imply equilibrium in these asset markets depend on the type of expectations that regulate ϕ and π. We will examine two cases: i) assuming that expectations are static, $\phi = \pi = 0$; and ii) assuming perfect foresight, wherein consumer/investors anticipate the future values of S and P_g that will result after the market responds to initial disturbances.

B. Transactions/ Consumption Demands on the Black Market

To complete the specification of markets for foreign exchange and storable goods, it is necessary to introduce non-portfolio-related flows. In this respect, the Soviet Union and Eastern European countries do not differ fundamentally from other countries with active parallel-markets. For the foreign-exchange market, as discussed in depth by Agenor (1992), there are five possible sources of illegal foreign-currency flows. These are the under-invoicing or smuggling of exports, over-invoicing of imports, flows from foreign tourists, and diversion of remittances through non-official channels. Surpluses or deficits in the second-economy trade balance, generated by changes in the net supply of private dollars, are represented by equation (6).

$$\dot{F} = t\left(\frac{S}{P_g}, \frac{S}{S^o}\right) \quad \text{where } t_1 = t_{\frac{S}{P_g}} > 0 \text{ and } t_2 = t_{\frac{S}{S^o}} > 0 \tag{6}$$

Transactions/ consumption demands for dollars arise from the desire to purchase foreign goods for immediate consumption. As the relative price of smuggled imports S/P_g rises, the relative demand for domestic goods rises and demand for foreign goods falls (domestic availability of dollars increases). This leads to an improvement in the second-economy trade balance and accumulation of dollars, so that $t_1 = t_{S/P_g} > 0$.

The availability of foreign currency in the domestic economy also is influenced by the black-market exchange-rate premium, S/S^o, defined as the ratio of the black-market exchange rate to the official exchange rate offered to tourist or would-be emigrants. Increases in the black-market currency premium improve the second-economy trade balance by increasing the transactions-supply of dollars, so that in equation (6) $t_2 = t_{S/S^o} > 0$. These flows arise from tourist activities and from exporters who are paid in foreign exchange.[11]

The determinants of changes in goods availability in the second economy are shown in equation (7).

$$\dot{G} = g\left(\frac{S}{P_g}, \frac{P_g}{P^o}, \gamma\right) \text{ where } g_1 = g_{\frac{S}{P_g}} < 0, g_2 = g_{\frac{P_g}{P^o}} > 0, \text{ and } g_3 = g_\gamma > 0 \tag{7}$$

Two relative prices influence the net supply of goods. First, the consumption demand for second-economy goods is increasing (net supply is decreasing) in the relative price of imported to second-economy prices of domestic goods, S/P_g, so that $g_1 = g_{S/P_g} < 0$. Second, the net supply of goods is increasing in the ratio of the free-market price P_g to the government-controlled price of goods, P^o. Demand for secondary market goods decreases in P_g and supply increases in P_g. The supply response to P_g is attributed to increased theft of goods from the official sector. Thus, the overall effects of increased P_g are unambiguous.

The impact of changes in government-controlled prices is less clear-cut. First, an increase in official prices can lead to a reduction in the supply diverted to secondary markets, since the premium to theft is reduced. In the official sector, more goods are available but at higher prices. Hence, demand in the second-economy declines as P^o increases. The supply effects dominate, so that $g_2 = g_{P_g/P^o} > 0$. Also relevant for goods-availability are supply shocks. These can be domestically induced or generated by foreign-aid flows in the form of goods. Represented by γ, these supplement domestic supplies so that $g_3 = g_\gamma > 0$.[12]

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[11]See Dornbusch et al. (1983). Note also that the function representing the net supply of dollars easily can be modified to account for the considerable obstacles that limit East-West trade. Historically, these obstacles have included prohibitive border-taxes, tariffs, and more subtle costs of illicit activity.

[12]The black-market premium on currency is not introduced explicitly. Instead, the price of goods in unofficial or free internal markets relative to the controlled price on state goods is relevant here since goods can be diverted from internal official goods-markets to internal unofficial goods-markets.

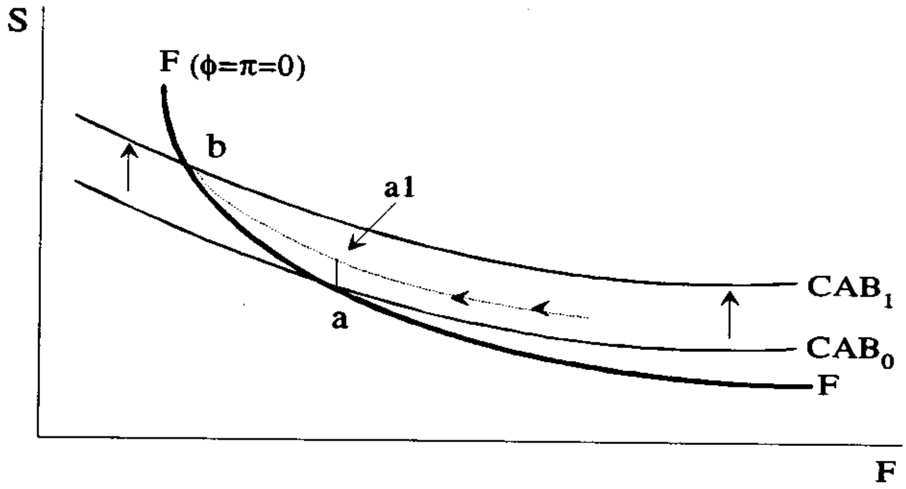
The long-run is characterized by equilibrium in the current account (i.e. balanced smuggling) $\dot{F} = 0$, and in the second-economy goods market (i.e. no additions or reductions of hoarding stocks), $\dot{G} = 0$. Also, in the long-run, $\phi = \pi = 0$.[13]

III. Dynamic Effects of Reform Initiatives

In this section we apply the model to trace the dynamic effects of a range of reform initiatives. These initiatives are: i) Official/ Tourist exchange-rate devaluation (increase in S^o); ii) Relaxation of the state-controlled prices of goods, either through one-shot price adjustment or liberalization (increase in P^o); iii) Altered supply of domestic goods via production reform or international aid (increase in γ); iv) Altered restrictiveness of monetary policy (change in B or i_b); v) Increased risk of monetary reform through domestic-currency confiscation (increase in σ_b); and vi) Increased variability of goods supplies (increase in σ_g).

Before tracing these results, intuition about the adjustment process driving the price and stock variables can be drawn from examining only one dimension of the model in our paper. Suppose we fix the price and quantity of second-economy goods at constant levels, and examine only the activity in black markets for foreign currency. Ignoring the goods market, the demand for dollars is the only asset-market equilibrium condition (FF), and the current-account balance $\left(\text{CAB} = \dot{F} = 0\right)$ is the only flow equilibrium condition. Figure 1 shows these curves in the space of S, the black-market exchange rate, and F, the dollars accumulated through second-economy trade imbalances.

[13]While we use $\phi=\pi=0$ as the steady-state condition of our model, in practice these inflation and depreciation rates will return to some "normal" rates determined by forces outside of the scope of the model, which need not equal zero. For example, these forces would include the dynamic path of money creation. Any discussion herein of short-run and long-run activity clearly is limited to the specific forces triggered by the events analyzed within this paper.



- Figure 1-

An unanticipated devaluation of the official exchange rate reduces the availability of currency supplied to the black market, shifting upward the CAB curve from CAB_0 to CAB_1. Instantaneously, the exchange rate adjusts from a to restore asset-market equilibrium. Under static expectations, since the asset-market equilibrium curve does not shift, the exchange rate is unchanged in the short-run. Under rational expectations, agents expect future depreciation of the domestic currency. This immediately raises the demand for foreign currency and leads to a black-market depreciation from a to a1. In both static and rational expectations cases, the economy runs a second-economy current-account deficit and decumulates dollars. This process of dollar reductions in private (or black-market) tillers and gradual black-market exchange rate depreciation continues until the new equilibrium is attained at b. At this long-run steady state, the economy has decumulated enough foreign exchange and depreciated the black-market rate sufficiently to restore equilibrium in both asset markets and in external (second-economy) accounts.

This type of dynamic adjustment also occurs in the full model presented herein, although it's dimensionality prevents graphical representation. In our case, there are two variables (both black-market prices and exchange rates) that adjust instantaneously to yield asset-market equilibrium. The dynamics of the real as well as the nominal black-market exchange rates are specified, since the paths of both nominal exchange rates and prices are determined. In addition, there are two flow adjustment-equations that dictate the accumulation of **G** and **F** along the path to the long-run equilibrium.[14]

For example, consider the effect of an unanticipated official exchange-rate devaluation. Under the static-expectations assumption, neither of the asset-market-equilibrium conditions shift and there is no instantaneous equilibrating adjustment of either S or P^g. However, the second-economy current account moves into deficit. The depreciation of the black rate lowers the net available supply of goods in the internal goods-markets. Ultimately, the resulting stock and price adjustments lead to a reduced internal supply of goods, reduced dollar holdings, and increased prices of both goods and foreign exchange. Both the real and the nominal exchange rates are depreciated in the long run. Under perfect foresight, the unanticipated official devaluations lead to short-run jumps in both exchange rates and prices. Although the long-run is the same in the perfect foresight and static expectations scenarios, the transition path to the long-run will depend on whether or not short-run overshooting occurs.

Below, we develop the dynamics of price and stock adjustments to the forementioned range of policy initiatives. While it is assumed throughout that policy changes are unanticipated, this is not crucial for the results. Announcements of policy changes in advance of their actual enactment serve to initiate the adjustment process at an earlier point in time, without altering the long-run equilibrium of the system.[15] However, we do distinguish between the response to reform initiatives when alternative expectations-formation mechanisms are imposed. Solutions are provided under both static-expectations and perfect-foresight assumptions.

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[14]It is assumed that the market chooses the stable and convergent trajectory. As in all models of this type, this assumption may not be valid.
[15]See Wilson (1979).

III A. Effects of Policy Initiatives Under Static Expectations

In the very short-run, under static expectations, the only policies that cause instantaneous adjustments of prices and exchange rates are those that directly shift asset demands or supplies. Such policies include changes in the supplies of domestic credit (*B*) or changes in the risk-return characteristics of any of the investment assets. The short-run and long-run effects of these policy initiatives under the static-expectations assumption are summarized in Table 3. Also shown are the long-run equilibrium effects of policy initiatives on foreign currency held by the private second-economy and on the overall supply of storable goods (i.e. hoarding).

An increase in the supply of *B* in agent's portfolios implies that weights on assets held in investor portfolios are in disequilibrium at current prices. Demands for *F* and *G*, the other investment assets, increase, instantaneously driving up the prices of both foreign exchange and goods. In the very short run, the jumps in black-market exchange rates and in prices overshoot their long-run levels. Therefore, after the initial jump, the medium-run adjustment process is described by deflation of prices and recovery of the value of the domestic currency in foreign-exchange markets. The initial price adjustment triggered imbalances in net-consumption in goods markets and illicit external accounts. This ultimately leads to an accumulation by households of both dollars and stocks of storable goods.

Under static-expectations, an official-exchange-rate devaluation (increase in S^o) does not immediately influence asset-market supplies or demands, and hence does not instantaneously affect exchange rates or prices in the second economy. However, the official devaluation reduces the attractiveness to tourists of supplying of foreign currency in the second economy, and *F* stocks decline. The consequent imbalances in investor portfolios (due to excess demand for available *F*) are eliminated by adjustments in asset prices. Although there are no short-run jumps in prices or black-market exchange rates, the medium-run is characterized by inflation in goods prices and persistent black-market exchange-rate depreciations. The long-run effects on exchange rates and prices are presented in the Appendix. The gradual effect of the initial official exchange-rate devaluation is a reduction in foreign-currency holdings by households, a decline in household hoarding stocks, and depreciated black-market exchange rates and the increased prices of hoarded goods.

Table 3 **Static Expectations Effects of Policy Initiatives**

	Short-Run Impact on		Long-Run or Steady-State Impact on			
Policy Initiative	S	P_g	S	P_g	F	G
Price Reform (P^o)	0	0	+	+	-	-
"Food" Aid (γ)	0	0	-	-	+	+
Official Devalu. (S^o)	0	0	+	+	-	-
Bond sale (B)	+	+	0	0	+	+
Confiscat. threat (σ_b^2)	+	+	0	0	+	+
Supply uncertainty(σ_g^2)	+ [1]	- [1]	0	0	+ [1]	- [1]

+ ⇒ increase. - ⇒ decrease. 0 ⇒ no effect.

[1]: See Table 2 for necessary and sufficient conditions on this partial derivative.

Analogous explanations can be provided for the adjustment processes following the other shocks presented in Table 3. One parameter of particular interest is that of σ_b^2, the effects of increased risk of monetary reform. As long as positive real money balances are held for portfolio purposes, an increase in the risk of monetary reform will cause substitution away from money balances and domestic savings, driving up the price of hoarded goods and foreign currency. These short-run exchange rate and price responses unambiguously overshoot the long-run effects, wherein S and P_g return to their initial levels. Following an increase in σ_b^2, there is a jump depreciation of the nominal black market exchange rate and a price increase. The short-run real exchange rate response depends on the asset shares in the economy. The medium run is characterized by domestic-currency appreciation and price deflation, while agents accumulate both hard-currency balances and increase their goods hoarding.

IIIB. Black-Markets and Hoarding Adjustments Under Perfect Foresight

The more interesting dynamic effects are derived under the assumption that agents have perfect foresight.[16] Herein, policy initiatives that affect goods markets, cross-border illicit activity, and asset markets all have instantaneous effects. Agents understand that the economy ultimately will adjust to the policy changes, and they temper their immediate reactions accordingly.

As formalized below, the short-run perfect foresight movements of exchange rates and prices are functions of the extent of the substitutability and complementarity among demands for foreign assets, demand for hoarding, and demand for domestic assets. The results also are functions of the share of each asset in the domestic portfolio and the variances and covariances of the asset returns. The short-run effects of policy initiatives under perfect foresight, and the conditions that must be satisfied for short-run overshooting of long-run equilibria by exchange rates and prices, are summarized in Table 4. The contents of this table reflects our interest in two questions: (i) What is the short-run impact (in terms of direction) on black-market exchange rates of policy initiatives? and (ii) Do we observe that short-run exchange rates overshoot or undershoot long-run equilibria? This latter question has bearing on whether depreciation (or appreciation) or inflation (or deflation) will characterize the medium-run adjustment. Table 4 does not reproduce the long-run or steady-state effects of

[16]The complete mathematical derivations of these results are given in Technical Appendix T2.

policy initiatives under rational expectations: these are the same as those already presented in Table 3 for the static-expectations case.

Table 4 Perfect Foresight Effects of Policies on S and P_g

	Short-Run Impact	Relation to Long-Run Equilibrium Value
Price Reform (P^o)		
on Exchange Rates	+	Short-Run Overshooting if *Condition 1* is satisfied.
on Prices	+	Short-Run Overshooting if *Condition 3* is satisfied.
Comments		In the long-run, $0 < \frac{\partial S}{S} < \frac{\partial P_g}{P_g}$
"Food" Aid (γ)		
on Exchange Rates	-	Short-Run Overshooting if *Condition 2* is satisfied.
on Prices	-	Short-Run Overshooting if *Condition 4* is satisfied.
Comments		In the long-run, $\left\lvert\frac{\partial S}{S}\right\rvert < \left\lvert\frac{\partial P_g}{P_g}\right\rvert$
Official Deval. (S^o)		
on Exchange Rates	+	Short-Run Overshooting if *Condition 1* is satisfied.
on Prices	+	Short-Run Overshooting if *Condition 3* is satisfied.
Comments		In the long-run, $0 < \frac{\partial P_g}{P_g} < \frac{\partial S}{S}$
Bond sale (B)		
on Exchange Rates	+	Short-run exchange-rate overshooting.
on Prices	+	Short-run price overshooting.
Comments		In the long-run, changes in B are neutral with respect to S and P_g.
Confiscation risk (σ_b)		
on Exchange Rates	+	Short-run exchange-rate overshooting.
on Prices	+	Short-run price overshooting.
Comments		In the long-run, changes in σ_b are neutral with respect to S and P_g.
"Supply" risk (σ_g)		
on Exchange Rates	+	Short-run exchange-rate overshooting.
on Prices	-	Short-run price overshooting.
Comments		In the long-run, changes in σ_g are neutral with respect to S and P_g.

Those real shocks to the economy that are neutral with respect to exchange rates and prices, changes in B, σ_b and σ_g, lead to identical adjustment dynamics under the static-expectations and perfect-foresight cases. By contrast, when shifts occur in P^o, S^o and γ, short-run movements in exchange rates are triggered by the anticipation of future movements in exchange rates and prices. These short-run movements are given by:

$$\left.\begin{bmatrix} \partial S \\ \partial P_g \end{bmatrix}\right|_{SR} = \frac{W}{\lambda_3 FG}\left.\begin{bmatrix} G\left(\left(\lambda_1^{\phi}\lambda_2 + \lambda_2^{\phi}(1-\lambda_1)\right)\partial\phi + \left(\lambda_1^{\pi}\lambda_2 + \lambda_2^{\pi}(1-\lambda_1)\right)\partial\pi\right) \\ F\left(\left(\lambda_2^{\phi}\lambda_1 + \lambda_1^{\phi}(1-\lambda_2)\right)\partial\phi + \left(\lambda_2^{\pi}\lambda_1 + \lambda_1^{\pi}(1-\lambda_2)\right)\partial\pi\right) \end{bmatrix}\right|_{\text{for } \partial P^o, \partial S^o, \partial\gamma} \tag{8}$$

In the system provided in (8), short-run expected proportional movements in S and P^g (i.e. ϕ and π), by assumption, are proportional to the long-run expected changes in these variables.[17] This is expressed in (9).

$$\left.\frac{\partial S}{S}\right|_{LR} = \partial\phi \text{ and } \left.\frac{\partial P_g}{P_g}\right|_{LR} = \partial\pi \tag{9}$$

By eq.(9) and some algebraic manipulation, we can rewrite the short-run movements in exchange rates and prices in terms of elasticities conditions:

$$\left.\begin{bmatrix} \dfrac{\partial S}{S} \\ \dfrac{\partial P_g}{P_g} \end{bmatrix}\right|_{SR} = \left.\begin{bmatrix} \dfrac{\lambda_2^{\phi}}{\lambda_2} - \dfrac{\lambda_3^{\phi}}{\lambda_3} & \dfrac{\lambda_2^{\pi}}{\lambda_2} - \dfrac{\lambda_3^{\pi}}{\lambda_3} \\ \dfrac{\lambda_1^{\phi}}{\lambda_1} - \dfrac{\lambda_3^{\phi}}{\lambda_3} & \dfrac{\lambda_1^{\pi}}{\lambda_1} - \dfrac{\lambda_3^{\pi}}{\lambda_3} \end{bmatrix}\begin{bmatrix} \dfrac{\partial S}{S} \\ \dfrac{\partial P_g}{P_g} \end{bmatrix}\right|_{\text{LR for } \partial P^o, \partial S^o, \partial\gamma} \tag{10}$$

Under perfect foresight, official exchange-rate devaluations instantaneously lead to jump depreciations of the black-market exchange rate and increases in prices of hoarded goods. These jumps were not observed in the static-expectations case. However, the absence of jumps also would occur in the perfect foresight case if the official devaluation had been announced in advance. At the announcement date, the

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[17] It is possible to complicate this expectations rule by permitting current one-period-ahead expectations of price and exchange rate changes to be non-unity fractions of long-run expected changes. While this modification is technically not difficult, it unnecessarily complicates the discussion without adding much additional intuition.

jump would have occurred.[18] Moreover, the effect on short-run exchange rates is proportionately greater than the effect on hoarded goods prices. Likewise, increases in official sector goods prices and goods-supply shocks have greater (absolute) effects on prices than on exchange rates under perfect foresight, since agents react to expected long-run movements in these variables. The market response to changes in B, σ_b and σ_g is analogous under the static-expectations and perfect-foresight cases, since agents understand the long-run neutrality of these variables with respect to S and P_g.

To determine whether the perfect-foresight short-run equilibria imply over- or under-shooting of long-run (second-economy) exchange rates and prices, for the changes in P^o, S^o, γ , B, σ_b and σ_g, we compare the long-run changes in S and P^g with the expectations-based changes that initiate the dynamic adjustments of these prices. The explicit long-run effects on S and P^g are derived from the matrices presented in Appendix II.

We focus on the conditions for exchange-rate overshooting without providing an extensive discussion of the issue of second-economy price overshooting. This emphasis is motivated by the specific purpose of understanding the conditions under which black market exchange-rates overstate or understate long-run equilibrium values and therefore provide biased signals to policy-makers who interpret the observed black-market exchange rates as representing long-run equilibrium levels. Nonetheless, conditions for price overshooting are provided for reference at the end of the section.

The likelihood of overshooting depends both on which shocks barrage the economy and on the elasticities of demand in both asset and goods markets. Since both official exchange-rate devaluation and official goods-price increases lead to long-run black-market exchange-rate depreciations, *Condition 1*, below, for short-run exchange-rate overshooting, determines whether $\left.\frac{\partial S}{S}\right|_{SR} > \left.\frac{\partial S}{S}\right|_{LR}$. By contrast,

[18]For a related discussion see Agenor and Flood (1992).

since increases in food-aid (or goods-supply increments) lead black-market exchange rates to appreciate in the long-run, for *Condition 2* the inequality in the overshooting condition will be reversed .[19]

Condition 1: *For official-price increases and official exchange-rate devaluations to lead to short-run exchange-rate overshooting, the necessary and sufficient condition is*

$$\left(\frac{\lambda_2^{\phi}}{\lambda_2}-\frac{\lambda_3^{\phi}}{\lambda_3}\right)\frac{\partial S}{S}\bigg|_{LR}+\left(\frac{\lambda_2^{\pi}}{\lambda_2}-\frac{\lambda_3^{\pi}}{\lambda_3}\right)\frac{\partial P_g}{P_g}\bigg|_{LR}>\frac{\partial S}{S}\bigg|_{LR} \tag{11}$$

Condition 2: For positive goods-supply shocks and increased food availability to lead to short-run exchange-rate overshooting, the necessary and sufficient condition is

$$\left(\frac{\lambda_2^{\phi}}{\lambda_2}-\frac{\lambda_3^{\phi}}{\lambda_3}\right)\frac{\partial S}{S}\bigg|_{LR}+\left(\frac{\lambda_2^{\pi}}{\lambda_2}-\frac{\lambda_3^{\pi}}{\lambda_3}\right)\frac{\partial P_g}{P_g}\bigg|_{LR}<\frac{\partial S}{S}\bigg|_{LR} \tag{12}$$

Exchange-rate overshooting in response to official price increases or official exchange-rate devaluations depends on the expected long-run movements of exchange rates and prices, and on the price elasticities of demand for foreign currency and domestic currency within domestic "financial" markets. Both official price increases and official exchange-rate devaluations lead to long-run increases in black-market exchange rates (depreciations) and second-economy prices. In the long-run, the official exchange-rate devaluation has a greater proportional expansionary impact on exchange rates, while the official price increase has a larger proportional expansionary effect on prices.

Effects of Increased Domestic Credit or Increased Risk of Monetary-Reform: Both increased risk of monetary reforms (increases in σ_b^2) and increased availability of domestic investment assets (*B*) lead to short-run exchange-rate overshooting. These policy shocks lead to non-expectations-based short-run shifts in asset demands, but are

[19]In the technical appendix we discuss the stability conditions of this model. As a four variable nonlinear differential equation system, it requires specific parameters for decisive statements on stability.

neutral in the long-run with respect to black-market exchange rates and prices. Consequently, the static and rational expectations behavior of these prices are analogous: there are immediate jumps in exchange rates and prices which always overshoot long-run equilibrium levels. Over time these movements are reversed and, through compensating stock adjustments, exchange rates and prices return to initial levels. The conditions for each direction of overshooting, and the subsequent reversals over the medium run, were interpreted and presented in Table 2.

Effects of Increased Uncertainty of Food or Goods Supplies: Increased uncertainty about the future availability of food or goods supplies is analogous to increased uncertainty about future goods prices in the second economy. In the steady state, this increase in σ_g is neutral with respect to S or P_g. Thus, the overshooting that occurs in the short run is identical for both the static expectations and perfect foresight cases. In the very short-run, there is a jump depreciation of both the nominal and the real black-market exchange rate. This is followed by a period of gradual appreciation as the economy accumulates dollars through surpluses in illicit trade.

Exchange-Rate Overshooting with Official Devaluation: Unambiguous statements on necessary and sufficient conditions for overshooting following official devaluation requires parameterization of the model. Such parameterization presumably would differ across economies and is not imposed herein. Short of knowing these elasticities and asset proportions within the economy, we are able to provide some limited insights by returning to the microfoundations of asset demands derived within the portfolio problem of Section II.

The inequality of Condition 1 can be rewritten as:

$$\frac{1}{\lambda_2}\left(\lambda_2^{\phi}\frac{\partial S}{S}+\lambda_2^{\pi}\frac{\partial P_g}{P_g}\right)-\frac{1}{\lambda_3}\left(\lambda_3^{\phi}\frac{\partial S}{S}+\lambda_3^{\pi}\frac{\partial P_g}{P_g}\right)>\frac{\partial S}{S} \tag{11'}$$

which, when expanded using the derivations from the optimal portfolio problem, provides the following insights:

i) Role of i_b: Given any set of risk-return and asset-supply combinations in an economy, a high enough level of i_b exists which guarantees exchange-rate overshooting. If exchange-rate overshooting does occur, the short run depreciation of the domestic currency in black markets will be followed by a period of exchange-rate appreciation. At lower levels of i_b, exchange-rate overshooting following official devaluation may occur, but is not guaranteed.

ii) Role of c_g: All else equal, the higher is c_g (the cost of obtaining storable goods) the less likely that exchange-rate overshooting will occur in response to official exchange-rate devaluations. This result arises because high c_g implies relatively high shares of domestic and foreign-currency assets in the investor's portfolio. Proportionately, portfolio adjustments in response to an S^o increase are smaller so overshooting is less likely. As c_g falls to lower levels,[20] overshooting may at some point occur in response to S^o but this is not guaranteed.

iii) Role of R, σ_b and σ_g: Without more detailed information on the elasticities and asset shares in the economy, we cannot make unambiguous statements about the linkages between these variables and the likelihood of exchange-rate overshooting following official exchange-rate devaluation.

Exchange-Rate Overshooting with Official Price Reform: In the long run, official-sector price increases lead to proportionately higher increases in second-economy prices relative to nominal exchange-rate depreciations. Thus, while nominal rates depreciate in the long run, real exchange rates appreciate.

In contrast to the case of official exchange-rate devaluation, exchange-rate overshooting after state-sector price reform requires comparison of the size of

[20]The cost of acquiring and/ or storing hoarded goods is assumed exogenous in our model.

proportional exchange rate and price increases, in addition to comparisons of the coefficients on these terms. There arise two cases for which overshooting may be analyzed: 1) the first case has the same conditions and interpretations as those provided in the section on exchange-rate overshooting following official rate devaluation. This occurs when inequality (13) is satisfied:

$$\left(\sigma_b^2+\sigma_g^2\right)\frac{t_1}{P_g}-\left(\sigma_b^2+\rho_{gs}\right)\left(\frac{t_1}{P_g}+\frac{t_2}{S^o}\right)>0 \tag{13}$$

which is calculated by using the long-run proportionate changes in prices and exchange rates provided in the Appendix, then substituting in derived elasticities formulas, and finally solving. This inequality is more likely to be satisfied when σ_g^2 is high relative to ρ_{gs}, and when the second-economy trade balance is not highly sensitive to the black-market premium on currency (t_2 is small).

Otherwise, 2) the second case arises when the inequality in eq.(13) is reversed. Therefore, when σ_g^2 is low relative to ρ_{gs}, and when the second-economy trade balance is highly sensitive to the black-market premium on currency, the following observations hold regarding the probability of exchange-rate overshooting in response to official-sector price reform:

i) Role of i_b: The higher is i_b, the less likely that exchange-rate overshooting will occur in response to an anticipated P^o increase. For lower i_b, exchange-rate overshooting may occur but this is not guaranteed.

ii) Role of c_g: The effect of c_g (the cost of obtaining storable goods) on the likelihood of overshooting purely depends on the parameterization of the economy.

Exchange-Rate Overshooting with Supply Shocks in Goods Markets: These goods-supply shocks lead to long-run declines in second-economy goods prices. In the long-run, the domestic currency appreciates in nominal terms in the black-markets, but depreciates in real terms. The sufficient conditions for short-run overshooting following supply shocks in goods markets, such as those that would be attributed to an increase in food aid, depend on the sign of the coefficients of the left-hand side and right-hand side terms in *Condition 2*.

As in the scenario of an official price reform, two analytical cases are possible for interpreting the likelihood of exchange-rate overshooting.[21] The first case occurs when

$$\left(\sigma_b^2+\sigma_g^2\right)\frac{t_1}{P_g}-\left(\sigma_b^2+\rho_{gs}\right)\left(\frac{t_1}{P_g}+\frac{t_2}{S^o}\right)>0 \tag{14}$$

(recall eq. 13) and has exactly the same interpretation as in the case of the effects of official exchange-rate devaluation. The second case, where the sign of this inequality is reversed, has exactly the same interpretation as that in the analysis of the effects of official-sector price reforms.

Finally, for completeness, provided below are the conditions for short-run second-economy prices to overshoot their long-run levels in response to the range of policy initiatives.

Condition 3: *For official-price increases and official exchange-rate devaluations to lead to short-run second-economy price overshooting, the necessary and sufficient condition is*

$$\left(\frac{\lambda_1^{\phi}}{\lambda_1}-\frac{\lambda_3^{\phi}}{\lambda_3}\right)\frac{\partial S}{S}\bigg|_{LR}+\left(\frac{\lambda_1^{\pi}}{\lambda_1}-\frac{\lambda_3^{\pi}}{\lambda_3}\right)\frac{\partial P_g}{P_g}\bigg|_{LR}>\frac{\partial P_g}{P_g}\bigg|_{LR} \tag{15}$$

Condition 4: *For positive goods-supply shocks and increased food availability to lead to short-run exchange-rate overshooting, the necessary and sufficient condition is*

[21]Notice that an official price increase and a positive supply shock in goods markets have proportional but oppositely signed short-run influences on exchange rates and free-market prices.

$$\left(\frac{\lambda_1^{\phi}}{\lambda_1}-\frac{\lambda_3^{\phi}}{\lambda_3}\right)\frac{\partial S}{S}\bigg|_{LR}+\left(\frac{\lambda_1^{\pi}}{\lambda_1}-\frac{\lambda_3^{\pi}}{\lambda_3}\right)\frac{\partial P_g}{P_g}\bigg|_{LR}<\frac{\partial P_g}{P_g}\bigg|_{LR} \tag{16}$$

The analysis of price overshooting proceeds analogously to that presented for exchange-rate overshooting.

IV. Concluding Remarks

This paper has documented the signals that black-market exchange rates and second-economy prices give to policy-makers and markets during different stages of economic reform. In contrast to previous studies, we use a framework that introduces hoarding activity and therefore integrates the real and financial sectors of a developing economy. This facilitates analysis of a variety of policy initiatives, including some particularly pertinent to the former Soviet Union and Eastern Europe.

A clear undertanding of the signals from black markets is especially important for the reforming economies of Eastern Europe and the former Soviet Union. In these countries, exchange rates *vis-a-vis* Western currencies often are interpreted as indicative of some measure of success in economic reforms. While this interpretation may be valid under some circumstances, we have shown that short-run appreciations of the domestic currency may simply be a by-product of relative speeds of adjustment of prices and asset stocks within the economy. Short-run black-market exchange-rate appreciations (and therefore reduced black-market premia) may by caused by a number of forces, including: the inflow of food aid, appreciations of official exchange rates, reduced risk of monetary confiscations, and increased confidence in the availability of future supplies of goods. At any point in time, exchange rates and prices reflect speculative and transitional demands for currency and for goods and need not reflect long-run market-equilibria. The biases (relative to long-run equilibria) implicit in each of these prices depend on the structure of announced and implemented policy initiatives operating in the economy in transition. Moreover, the adjustment paths for black-market exchange rates and prices have clear, qualitative dependencies on the structure of investment-demand elasticities and the size of various assets relative to the total set of investment assets available to households.

While we have provided the specific conditions for overshooting of exchange rates and prices in response to policy announcements and initiatives, the real application of the theory is to the experiences of countries of Eastern Europe and the former Soviet Union, where both goods hoarding and black-markets for foreign currency are pervasive. Given specific features of an economy in transition, it may be possible to conduct useful empirical studies of the extent of these biases, as well as of expected exchange-rate and price overshooting.

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Technical Appendix: T1

The problem of the consumer/ investor is captured by the objective function:

$$\underset{\lambda_1,\ \lambda_2}{\text{maximize}} \qquad U = E(dw/w) - (R/2)Var(dw/w) \tag{t1}$$

where w= W/Q is real wealth, Q is the price index of the consumption basket, and R is his coefficient of relative risk aversion. Nominal wealth, W, is the sum of stock holdings of domestic currency bonds, B, foreign currency holdings, SF, and domestic goods hoarded, P^gG. The risks and returns of the various processes are given by

$$dB/B = i_b\, dt + \sigma_b\, dZ_b \tag{t2}$$

$$dS/S = \phi\, dt + \sigma_s\, dZ_s \tag{t3}$$

$$d(P^gG)/P^gG = (\pi - c_g)dt + \sigma_g\, dZ_g \tag{t4}$$

so that the portfolio shares allocated to the respective assets are given by:

$$\lambda_1 = P^gG/W\,,\ \lambda_2 = SF/W\,, \text{ and } \lambda_3 = (1-\lambda_1-\lambda_2) = B/W \tag{t5}$$

The process driving the price index is:

$$dQ/Q = \pi_q\, dt + \sigma_q\, dZ_q \tag{t6}$$

Using Ito's Lemma, the stochastic differential and its expected valuation and variance are derived:

$$\begin{aligned} dw/w = {} & (1-\lambda_1-\lambda_2)(dB/B) + \lambda_2(dF/F + dS/S) + \lambda_1(d(P^gG)/P^gG) - (dQ/Q) \\ & + [-(1-\lambda_1-\lambda_2)(dB/B)(dQ/Q) + \lambda_2[(dS/S)(dF/F) - (dF/F)(dQ/Q) - (dS/S)(dQ/Q)] \\ & -\lambda_1(d(P^gG)/P^gG)(dQ/Q) + (dQ)^2/Q^2\ ] \end{aligned} \tag{t7}$$

If the risks associated with domestic currency savings accounts are uncorrelated with other stochastic processes, the expected value and variance of portfolio holdings are given by:

$$E(dw/w) = (1-\lambda_1-\lambda_2)(i_b) + \lambda_2\phi + \lambda_1(\pi - c_g) - (\pi_q) + [\lambda_2(\rho_{sf} - \rho_{fq} - \rho_{sq}) - \lambda_1(\rho_{gq}) + \sigma_q^2\,]$$

$$var(dw/w) = (1-\lambda_1-\lambda_2)^2(\sigma_b^2) + \lambda_2^2(\sigma_s^2) + \lambda_1^2(\sigma_g^2) + (\sigma_q^2) + 2\lambda_1\lambda_2(\rho_{gs}) - 2\lambda_1(\rho_{gq}) - 2\lambda_2(\rho_{qs}). \tag{t8}$$

where ρ_{ij} represents the covariances of processes i and j.

The optimal share of household wealth invested in durables for hoarding purposes (λ_1) and the optimal share of wealth dedicated to portfolio demands for foreign currency (λ_2) are determined by differentiating the resulting objective function with respect to λ_1 and λ_2.

$$\begin{aligned} \lambda_1 = {} & A^{-1}[-(\sigma_b^2+\rho_{gs})(\sigma_b^2+\rho_{sq}) + (\sigma_b^2+\rho_{gq})(\sigma_b^2+\sigma_s^2) \\ & + R^{-1}\{(-i_b - c_g + \pi - \rho_{gq})(\sigma_b^2+\sigma_s^2) - (-i_b + \phi - \rho_{sq})(\sigma_b^2 + \rho_{sg})\}\,] \end{aligned} \tag{t9}$$

$$\lambda_2 = A^{-1}[-(\sigma_b^2+\rho_{gq})(\sigma_b^2+\rho_{gs}) + (\sigma_b^2+\sigma_g^2)(\sigma_b^2+\rho_{sq})$$

$$+ R^{-1}\{(-i_b + \phi - \rho_{sq})(\sigma_b^2+\sigma_g^2) - (-i_b - c_g + \pi - \rho_{gq})(\sigma_b^2+\rho_{gs})\}] \qquad (t10)$$

where $A = (\sigma_b^2 + \sigma_s^2)(\sigma_b^2 + \sigma_g^2) - (\sigma_b^2 + \rho_{gs})^2$

To formalize the link between the goods prices and the black-market exchange rate, we introduce the domestic price index:

$$Q = P_g^{\alpha} P_T^{(1-\alpha)} \qquad (t11) \qquad (t12)$$

where α is the weight of the price of domestic goods P_g and $(1-\alpha)$ is the weight of the imported-goods price P_T in the consumer price index. Using the variances, covariances and expected values of the inflation series, the optimal portfolio shares solve to:

$$\lambda_1 = \alpha + \frac{1}{AR}\begin{pmatrix} (\sigma_b^2 + \sigma_s^2)(\pi - c_g - i_b - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}) \\ -(\sigma_b^2 + \rho_{gs})(\phi - i_b - (1-\alpha)\sigma_s^2 - \alpha\rho_{gs}) \end{pmatrix}$$

$$\lambda_2 = (1-\alpha) + \frac{1}{AR}\begin{pmatrix} (\sigma_b^2 + \sigma_g^2)(\phi - i_b - (1-\alpha)\sigma_s^2 - \alpha\rho_{gs}) \\ -(\sigma_b^2 + \rho_{gs})(\pi - c_g - i_b - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}) \end{pmatrix} \qquad (t12)$$

$$\lambda_3 = -\frac{1}{AR}\begin{pmatrix} (\sigma_g^2 - \rho_{gs})(\phi - i_b - \sigma_s^2) \\ +(\sigma_s^2 - \rho_{gs})(\pi - c_g - i_b - \rho_{gs}) \end{pmatrix}$$

where $A = (\sigma_b^2 + \sigma_s^2)(\sigma_b^2 + \sigma_g^2) - (\sigma_b^2 + \rho_{gs})^2$

In order for individuals in this economy with hoarding to hold positive money balances, there is a necessary condition on the size of domestic interest rates:

$\lambda_3 > 0$ requires $i_b > \dfrac{(\sigma_g^2 - \rho_{gs})(\phi - \sigma_s^2) + (\sigma_s^2 - \rho_{gs})(\pi - c_g - \rho_{gs})}{\sigma_s^2 + \sigma_g^2 - \rho_{gs}}$

Table TI: Short-Run Comparative Statics Results for Asset-Demand Equations

Sensitivity of Hoarding Demands

$$\frac{\partial \lambda_1}{\partial \pi} = \frac{\left(\sigma_b^2 + \sigma_s^2\right)}{AR} > 0 \qquad \frac{\partial \lambda_1}{\partial \phi} = -\frac{\left(\sigma_b^2 + \rho_{gs}\right)}{AR} < 0$$

$$\frac{\partial \lambda_1}{\partial \sigma_b^2} = -\frac{\left(\sigma_s^2 - \rho_{gs}\right)}{A^2 R}\begin{pmatrix}\left(\sigma_s^2 - \rho_{gs}\right)\left(\pi - c_g - i_b - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}\right) \\ +\left(\sigma_g^2 - \rho_{gs}\right)\left(\phi - i_b - (1-\alpha)\sigma_s^2 - \alpha\rho_{gs}\right)\end{pmatrix} > 0$$

$$\frac{\partial \lambda_1}{\partial \sigma_g^2} = -\frac{\left(\sigma_b^2 + \sigma_s^2\right)}{A^2 R}\begin{pmatrix}\left(\sigma_b^2 + \sigma_s^2\right)\left(\pi - c_g - i_b - \rho_{gs}\right) \\ -\left(\sigma_b^2 + \rho_{gs}\right)\left(\phi - i_b - \sigma_s^2\right)\end{pmatrix}$$

$$\frac{\partial \lambda_1}{\partial c_g} = -\frac{\left(\sigma_b^2 + \sigma_s^2\right)}{AR} < 0 \qquad \frac{\partial \lambda_1}{\partial i_b} = -\frac{\left(\sigma_s^2 - \rho_{gs}\right)}{AR} < 0$$

Sensitivity of Portfolio Demands for Black-Market Dollars

$$\frac{\partial \lambda_2}{\partial \pi} = -\frac{\left(\sigma_b^2 + \rho_{gs}\right)}{AR} < 0 \qquad \frac{\partial \lambda_2}{\partial \phi} = \frac{\left(\sigma_b^2 + \sigma_g^2\right)}{AR} > 0$$

$$\frac{\partial \lambda_2}{\partial \sigma_b^2} = -\frac{\left(\sigma_g^2 - \rho_{gs}\right)}{A^2 R}\begin{pmatrix}\left(\sigma_s^2 - \rho_{gs}\right)\left(\pi - c_g - i_b - \alpha\sigma_g^2 - (1-\alpha)\rho_{gs}\right) \\ +\left(\sigma_g^2 - \rho_{gs}\right)\left(\phi - i_b - (1-\alpha)\sigma_s^2 - \alpha\rho_{gs}\right)\end{pmatrix} > 0$$

$$\frac{\partial \lambda_2}{\partial \sigma_g^2} = \frac{\left(\sigma_b^2 + \sigma_s^2\right)}{A^2 R}\begin{pmatrix}\left(\sigma_b^2 + \rho_{gs}\right)\left(\pi - c_g - i_b - \rho_{gs}\right) \\ -\left(\sigma_b^2 + \sigma_g^2\right)\left(\phi - i_b - \sigma_s^2\right)\end{pmatrix}$$

$$\frac{\partial \lambda_2}{\partial c_g} = \frac{\left(\sigma_b^2 + \rho_{gs}\right)}{AR} > 0 \qquad \frac{\partial \lambda_2}{\partial i_b} = -\frac{\left(\sigma_g^2 - \rho_{gs}\right)}{AR} < 0$$

$A = (\sigma_b^2+\sigma_g^2)(\sigma_b^2+\sigma_s^2) - (\sigma_b^2+\rho_{gs})^2 > 0$ and demands are normal.

Appendix T2

T2.1. Long-Run Equilibrium: Comparative-Statics Matrix

$$\begin{bmatrix} \left(t_1(P_g)^{-1}+t_2(S^o)^{-1}\right) & -t_1 S(P_g)^{-2} & 0 & 0 \\ g_1(P_g)^{-1} & g_2(P^o)^{-1}-g_1 S(P_g)^{-2} & 0 & 0 \\ -\lambda_1 F & (1-\lambda_1)G & -\lambda_1 S & (1-\lambda_1)P_g \\ (1-\lambda_2)F & -\lambda_2 G & (1-\lambda_2)S & -\lambda_2 P_g \end{bmatrix} \begin{bmatrix} \partial S \\ \partial P_g \\ \partial F \\ \partial G \end{bmatrix}$$

$$= \begin{bmatrix} t_2 S(S^o)^{-2} & 0 & 0 & 0 & 0 \\ 0 & g_2 P_g(P^o)^{-2} & 0 & 0 & -g_3 \\ 0 & 0 & W\lambda_1^Z & \lambda_1 & 0 \\ 0 & 0 & W\lambda_2^Z & \lambda_2 & 0 \end{bmatrix} \begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial Z \\ \partial B \\ \partial \gamma \end{bmatrix}$$

Long-run Comparative Statics Results

$$\begin{bmatrix} \partial S \\ \partial P_g \\ \partial F \\ \partial G \end{bmatrix} = \begin{bmatrix} + & + & 0 & 0 & - \\ + & + & 0 & 0 & - \\ - & - & c1 & + & + \\ - & - & c2 & + & + \end{bmatrix} \begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial Z \\ \partial B \\ \partial \gamma \end{bmatrix}$$

where

$$c1 = -(\det A)^{-1}\left(\lambda_2\lambda_1^z + \lambda_2^z(1-\lambda_1)\right)WP_g\left(a_{11}a_{22}-a_{21}a_{12}\right)$$

$$c2 = -(\det A)^{-1}\left(\lambda_1\lambda_2^z + \lambda_1^z(1-\lambda_2)\right)WS\left(a_{11}a_{22}-a_{21}a_{12}\right)$$

and a_{ij} are the elements of the A matrix on the left-hand side of the long-run comparative statics system. The determinant of this submatrix is positive unambiguously. Note that the policy changes denoted by **Z** are defined by the vector: **Z** $=(\sigma_b^2, \sigma_g^2, i_b, c_g)$. For example, the long-run effects of an official exchange-rate devaluation are:

$$\left.\frac{\partial S}{S}\right|_{LR} = \frac{t_2\left(S/S^o\right)}{\kappa}\left(g_2\frac{S}{S^o}\frac{P_g}{P^o} - g_1\frac{S}{P_g}\right) \text{ and } \left.\frac{\partial P_g}{P_g}\right|_{LR} = \frac{t_2\left(S/S^o\right)}{\kappa}\left(-g_1\frac{S}{P_g}\right)$$

where $\kappa = t_1 g_2 P_g S^o + t_2 g_2 P_g^2 - t_2 g_1 P^o S > 0$ and $t_1 = t_{S/P_g} > 0$ and $t_2 = t_{S/S^o} > 0$.

The long-run proportionate changes in prices and exchange rates attributed to an increase in official sector prices are given by:

$$\left.\frac{\partial S}{S}\right|_{LR} = \frac{g_2 S^o P_g^{\,2} P^{o^{-1}}}{\kappa}\left(\frac{t_1}{P_g}\right) \text{ and } \left.\frac{\partial P_g}{P_g}\right|_{LR} = \frac{g_2 S^o P_g^{\,2} P^{o^{-1}}}{\kappa}\left(\frac{t_1}{P_g}+\frac{t_2}{S^o}\right)$$

Short-run Comparative-Statics Matrix (Static-Expectations Case)
(assuming that dF=dG=0 in the very short run.)

$$\begin{bmatrix} -\lambda_1 F & (1-\lambda_1)G \\ (1-\lambda_2)F & -\lambda_2 G \end{bmatrix}\begin{bmatrix} \partial S \\ \partial P_g \end{bmatrix} = \begin{bmatrix} 0 & 0 & W\lambda_1^z & \lambda_1 & 0 & \lambda_1 S & -(1-\lambda_1)P_g \\ 0 & 0 & W\lambda_2^z & \lambda_2 & 0 & -(1-\lambda_2)S & \lambda_2 P_g \end{bmatrix}\begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial Z \\ \partial B \\ \partial \gamma \\ \partial F \\ \partial G \end{bmatrix}$$

Short-run Results (Summary)

$$\begin{bmatrix} \partial S \\ \partial P_g \end{bmatrix} = \begin{bmatrix} 0 & 0 & SZ & + & 0 & - & 0 \\ 0 & 0 & PZ & + & 0 & 0 & - \end{bmatrix}\begin{bmatrix} \partial S^o \\ \partial P^o \\ \partial Z \\ \partial B \\ \partial \gamma \\ \partial F \\ \partial G \end{bmatrix}$$

where det K = $-\lambda_3 FG < 0$ and

$$SZ = -(\det K)^{-1} WG\left(\lambda_2\lambda_1^z + \lambda_2^z(1-\lambda_1)\right)$$
$$PZ = -(\det K)^{-1} WF\left(\lambda_1\lambda_2^z + \lambda_1^z(1-\lambda_2)\right)$$

Short-run Comparative-Statics Matrix (Perfect Foresight Case)

$$\begin{bmatrix} -\lambda_1 F & (1-\lambda_1)G \\ (1-\lambda_2)F & -\lambda_2 G \end{bmatrix} \begin{bmatrix} \partial S \\ \partial P_g \end{bmatrix} = \begin{bmatrix} 0 & 0 & W\lambda_1^{\pi} & \lambda_1 & 0 & \lambda_1 S & -(1-\lambda_1)P_g & \lambda_1^{\phi} & \lambda_1^{\pi} \\ 0 & 0 & W\lambda_1^{\pi} & \lambda_2 & 0 & -(1-\lambda_2)S & \lambda_2 P_g & \lambda_2^{\phi} & \lambda_1^{\pi} \end{bmatrix} \begin{bmatrix} \partial S^{o} \\ \partial P^{o} \\ \partial Z \\ \partial B \\ \partial \gamma \\ \partial F \\ \partial G \\ \partial \varphi \\ \partial \pi \end{bmatrix}$$

where, for example, the general form for the perfect-foresight effects of policy changes (which supplement the static-expectations effects) is given by:

$$\begin{bmatrix} \partial S \\ \partial P_g \end{bmatrix} = \frac{W}{\lambda_3 FG} \begin{bmatrix} G\left(\left(\lambda_2\lambda_1^{\phi} + (1-\lambda_1)\lambda_2^{\phi}\right)\partial\phi + \left(\lambda_2\lambda_1^{\pi} + (1-\lambda_1)\lambda_2^{\pi}\right)\partial\pi\right) \\ F\left(\left(\lambda_1\lambda_2^{\phi} + (1-\lambda_2)\lambda_1^{\phi}\right)\partial\phi + \left(\lambda_1\lambda_2^{\pi} + (1-\lambda_2)\lambda_1^{\pi}\right)\partial\pi\right) \end{bmatrix}$$

this system is relevant and self-inclusive for all unanticipated policy changes other than δZ and δB. δZ and δB are neutral with respect to exchange rates and prices in the steady-state, and have very short-run effects beyond those generated from expected movements in exchange rates and prices.

Each policy change yields a distinct $\delta\phi$ and $\delta\pi$ pair that is computed from the results of the steady-state comparative-statics matrix. Before inputting those pairs into these formulas, it is useful to consider the signs of their coefficients in the formulas provided above. For this analysis, we appeal to the comparative-statics of the asset-market/ portfolio problem.

Stability of the Dynamic System

Is it likely that the reforming economies will respond to market stimuli and over time converge on these long-run equilibrium outcomes? A conclusive answer to this question is nearly impossible to provide within the highly nonlinear dynamic system of our model. Accordingly, computational constraints lead us to analyze the local rather than global properties of the dynamic system. Even within the local analysis, we are not able to present much more than this system itself, which must be analyzed with parameter values drawn from a specific economy.

For local stability analysis, we begin with the following four equations:

$$\lambda_1^{\phi}\left(\frac{S_{t+1}^{e}-S_t}{S_t}\right)+\lambda_1^{\pi}\left(\frac{P_{t+1}^{e}-P_t}{P_t}\right)=\frac{P_tG_t}{W}-\lambda_1^{Z}Z \tag{s1}$$

$$\lambda_2^{\phi}\left(\frac{S_{t+1}^{e}-S_t}{S_t}\right)+\lambda_2^{\pi}\left(\frac{P_{t+1}^{e}-P_t}{P_t}\right)=\frac{S_tF_t}{W}-\lambda_2^{Z}Z \tag{s2}$$

with $W=B+S_tF_t+P_tG_t$

$$\dot{F}=t\left(\frac{S}{P_g},\frac{S}{S^o}\right)\quad \text{where } t_1=t_{\frac{S}{P_g}}>0 \text{ and } t_2=t_{\frac{S}{S^o}}>0 \tag{s3}$$

$$\dot{G}=g\left(\frac{S}{P_g},\frac{P_g}{P^o},\gamma\right)\quad \text{where } g_1=g_{\frac{S}{P_g}}<0,\ g_2=g_{\frac{P_g}{P^o}}>0, \text{ and } g_3=g_{\gamma}<0 \tag{s4}$$

Assume perfect foresight, so that the expected future value is equal to the actual future value. Then, using equations (s1) and (s2) to solve for the percentage changes in S and in P. After we solve for the change in S and the change in P (after multiplying each of the lines through by S and P respectively), we combine these in vector notation with the equations (s3) and (s4) representing changes in F and G. Using this system of equation, we can determine the Jacobian matrix of the system. In theory, this would be used to solve for eigenvalues and the roots of the dynamic system. In practice, without assigning values to the functional forms in our system and conducting country specific simulations, there is little that can be said about the stability of the system as a whole.

(s1) and (s2) can be rewritten as:

$$\begin{bmatrix}\dot{S}\\ \dot{P}\end{bmatrix}=(ad-bc)^{-1}\begin{bmatrix} d\left(\frac{P_tG_t}{W}-Z1\right)-b\left(\frac{S_tF_t}{W}-Z2\right)\\ -c\left(\frac{P_tG_t}{W}-Z1\right)+a\left(\frac{S_tF_t}{W}-Z2\right)\end{bmatrix}$$

which we now can combine with (s3) and (s4) to yield the nonlinear dynamic system:

$$
\begin{bmatrix} \dot{S} \\ \dot{P} \\ \dot{F} \\ \dot{G} \end{bmatrix} =
\begin{bmatrix}
(ad-bc)^{-1} d\left(\frac{P_t G_t}{W} - Z1\right) - b\left(\frac{S_t F_t}{W} - Z2\right) \\
(ad-bc)^{-1} - c\left(\frac{P_t G_t}{W} - Z1\right) + a\left(\frac{S_t F_t}{W} - Z2\right) \\
t\left(\frac{S}{P_g}, \frac{S}{S^o}\right) \\
g\left(\frac{S}{P_g}, \frac{P_g}{P^o}, \gamma\right)
\end{bmatrix}
$$

Linearization of this dynamic system yields the following Jacobian matrix, where $\alpha = (ad\text{-}bc)^{-1}$. The four eigenvalues of this system then would need to be determined.

$$
\begin{bmatrix}
\alpha\begin{pmatrix} d\left(\frac{PG}{W} - Z1\right) - b\left(\frac{SF}{W} - Z2\right) \\ + \frac{SF}{W}\left(-d\frac{PG}{W} - b\left(1 - \frac{SF}{w}\right)\right) \end{pmatrix} & \alpha\frac{SG}{W}\left(d\left(1 - \frac{PG}{W}\right) + b\frac{SF}{W}\right) & \alpha\frac{SP}{W}\left(d\left(1 - \frac{PG}{W}\right) + b\frac{SF}{W}\right) & \alpha\frac{SS}{W}\left(d\left(-\frac{PG}{W}\right) - b\left(1 - \frac{SF}{W}\right)\right) \\
\alpha\begin{pmatrix} -c\left(\frac{PG}{W} - Z1\right) + a\left(\frac{SF}{W} - Z2\right) \\ + \frac{SF}{W}\left(c\frac{PG}{W} + a\left(1 - \frac{SF}{W}\right)\right) \end{pmatrix} & \alpha\frac{SG}{W}\left(-c\left(1 - \frac{PG}{W}\right) - a\frac{SF}{W}\right) & \alpha\frac{SP}{W}\left(-c\left(1 - \frac{PG}{W}\right) - a\frac{SF}{W}\right) & \alpha\frac{SS}{W}\left(-c\left(-\frac{PG}{W}\right) + a\left(1 - \frac{SF}{W}\right)\right) \\
g_1\frac{1}{P} + g_2\frac{1}{S^*} & -g_1\frac{S}{P^2} & 0 & 0 \\
t_1\frac{1}{P} & -t_1\frac{S}{P^2} + t_2\frac{1}{P^*} & 0 & 0
\end{bmatrix}
$$

To conduct the stability analysis, it is necessary to parameterize the system using values relevant for the specific economy under study.