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# *ON THE DESIGN OF OPTIMAL ORGANIZATIONS USING TOURNAMENTS: AN EXPERIMENTAL EXAMINATION*

by
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# On The Design of Optimal Organizations Using Tournaments: An Experimental Examination

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## Abstract

This paper investigates multi-person tournaments both theoretically and experimentally. It asks, and attempts to answer three questions: 1) As the size of a tournament grows through replication (i.e. at any level as the number of large prizes grow proportionally with the number of people at that level), what happens to the effort of agents? 2) If the size of the tournament is held fixed, what happens to effort levels as the fraction of large prizes in it changes? 3)If discrimination exists within a tournament in the sense that a fraction $\zeta$, of the workers are discriminated against by having to significantly outperform non-discriminated workers in order to get a large prize, what happens to output as tournaament size increases with the fraction $\zeta$ held constant? We find that while the results of our experiment support the theory in most instances, the deviations from the theory we find raise a series of interesting questions about the proper design of corporate compensation mechanisms.

KEY WORDS: Tournaments, Economic Experiments, Incentives
JEL CLASSIFICATION : C91, J33, C72

## 1. Introduction

Any modern hierarchical organization implicitly motivates its agents by having them compete for promotions which get progressively more scarce as one moves up through the ranks. Lambert, Larcker , and Weigelt (1993) study a set of 303 large publicly traded corporations and find that the structure of organizational incentives closely resembles that of an intra-firm tournament in which promotions are used to motivate agents to work and compensation is set at each stage to maintain incentives. Despite their common (implicit or explicit) use, tournaments have met with relatively little empirical or experimental study. The few investigations that exist, (see Nalbantian and Schotter (1997, forthcoming), Bull Schotter and Weigelt (1987), Schotter and Weigelt (1992), Cappelli and Cascio (1991), Ehrenberg and Bognanno (1990) tend to lend support for the theory, yet many questions remain unanswered. [1]

For example, if workers compete for promotions at any level within the organization what is the optimal fraction of workers to promote? Having too few promotions, one might think, may lead to discouragement while having too many, so that the probability of promotion is great, may lead to shirking. Is the effort level of workers homogenous of degree zero for proportionate increases in the size of the organization? Are large work groups better than small ones and if so what factors are important in determining work-group size (i.e. the size of the tournament to run)? Finally, a whole host of questions arise for organizational design when discrimination exists at the workplace, i.e., when certain workers have to be virtual superstars in order to get a promotion. For instance, if discrimination exists are its effects most damaging to organizational output in larger versus smaller organizations?

These questions are formalized as follows:

1) **Organizational Replication:**

As the size of the organization grows through replication (i.e. at any level as the number of promotions grows proportionally with the number of people at that level), what happens to the effort of agents? For instance, do agents work more or less hard in organizations with 10 employees and 5 promotion possibilities than ones with 100 agents and 50 promotion possibilities?

2) **Prize Structure:**

If the tournament size is held fixed, what happens to effort levels as the fraction

[1] For a discussion of the theory see Lazear and Rosen (1981), O'Keefe, Viscusi, and Zeckhauser, (1984), Green and Stokey (1983), Nalebuff and Stiglitz ( 1983) and others.

of promotions in the organization changes? For instance, do agents work more or less hard in a 10 worker organization with 3 promotion possibilities than they do in the same organization with 6 promotion possibilities?

3) **Organizational Size and Discrimination:**

If discrimination exists within an organization in the sense that a fraction $\zeta$, of the agents are discriminated against by having to significantly outperform non-discriminated agents in order to get the same promotion, what happens to organization output as organization size increases with the fraction $\zeta$ held constant?

Theory gives us some insight in answering these questions as follows. According to the theory of tournaments, the equilibrium effort level for identical agents engaged in symmetric tournaments does not change as the size of these tournaments increases. Tournament equilibrium effort is homogeneous of degree zero in tournament size. Second, for organizations where the random shocks are uniformly distributed, the theory predicts that the prize structure in the tournament should not effect equilibrium effort levels. Hence, at interior equilibria, a ten person tournament with three large and seven small prizes (i.e., three promotion possibilities) should elicit the same effort level as one with seven large and three small prizes.

Finally, and perhaps most surprisingly, as the size of unfair tournaments increases (i.e., tournaments where some fraction of the population is discriminated against), the effort levels of agents approaches that of symmetric fair tournaments. Put differently, the efficiency loss of discrimination decreases as the tournament size increases even with fairly small tournaments.

Obviously these results are of significance for the optimal design of organizations yet little attention has been paid to them. We hope to take a step to rectify this situation here. To do this we investigate a set of 2, 4, and 6 person experimental tournaments and compare the behavior of our subjects across them. While financial constraints prevented us from increasing our laboratory organizations beyond a size of 6, we still find behavior fairly consistent with that predicted by the theory. More precisely, in answering the three questions posed above we find that as the size of the laboratory tournament grows through replication from size 2 to 4 to 6 the mean effort level of subjects remains constant. This result, therefore, seems to contradict the commonly held belief that people tend to work less hard in large impersonal organizations as opposed to small ones.

The same is true of the mean effort levels of subjects in 6-person tournaments as the prize distribution changes from having 3 small and 3 large prizes to one having 2 small and 4 large ones. There is, however, a significant decrease in effort when we move to the 4-large and 2-small prize structure.. The implication of this result is striking since, if effort levels decrease in tournaments with relatively more large prizes, the per-unit cost of output significantly increases. A profit maximizing manager should therefore choose that prize structure with the smallest number of large prizes consistent with the desired output goal. Finally, in unfair tournaments, holding the fraction of people discriminated against constant but allowing the tournament to grow from 4 to 6 subjects, leads to an increase in the mean effort levels of subjects as predicted by the theory. This result indicates that the efficiency cost of discrimination is greatest in smaller rather than larger organizations. However, counter to the theory, no significant difference in behavior was found when comparing 2-person and 6-person tournaments. We discuss this anomaly in the paper.

The policy implications of this discrimination result are mixed. On the one hand, if efficiency is the sole reason why society is concerned about discrimination, an assumption that is probably far from the truth, then this result implies that enforcement efforts of the government might be best aimed at small rather than large firms since it is there that these discrimination costs are greatest. On the other hand, however, as Becker (1957) has suggested, if employers have a taste for discrimination which is not exercised when the efficiency cost of doing so is too high, then we might expect more discrimination in large firms than in small ones since it is in large firms that the opportunity cost associated with discrimination are smallest. This would imply that enforcement efforts on the part of the government should be aimed at large firms since (as Becker (1957) points out) the market would police small firms exercising an irrational and costly taste for discrimination.

In this paper we will proceed as follows: In Section 2 we will quickly review the theory relevant to our experiments and prove four theorems which answer, at least theoretically, the three questions posed above. These theorems will provide us with three hypotheses which we will use to organize the results of our experiments. In Section 3 we present our experimental design, while in Section 4 we present and discuss our results. Finally, in Section 5 we offer some concluding remarks.

## 2. Section 2: Tournaments and Their Equilibria

### 2.1. A Simple Tournament Model

Consider the following n-person tournament with n identical agents i = 1,2,.....,n each having the same utility functions separable in the payment received and the effort exerted.

$$u(p, e) = w(p) - c(e_i), i = 1, 2, ...., n, \tag{2.1}$$

where p denotes the non-negative payment to the agent, $e_i$ is the agent's non-negative effort level. The positive and increasing functions $w(\cdot)$ and $c(\cdot)$ are, respectively, concave and convex. Agent i chooses a level of effort from a closed and bounded set on the real line. This effort is not observable to anyone except agent i but generates an output $y_i$ according to

$$y_i = f(e_i) + \xi_i, \tag{2.2}$$

where the production function $f(\cdot)$ is concave and $\xi_i$ is a random shock drawn independently for each agent i from an identical and continuous density function defined on a common closed and bounded support. All other agents have a similar technology and face an identical decision problem. In this tournament there will be n prizes (i.e. the number of prizes is equal to the number of participants in the tournament) each of which can take one of two values, M or m, with M > m. A prize structure $\lambda^T = (\lambda , 1- \lambda)$ is defined by a fraction, $\lambda$, indicating the fraction of large prizes existing in the tournament with $1 - \lambda$ being the fraction of small prizes.

The rules of the tournament are as follows: After outputs are determined for each of the n agents, let $S_i$ be the set of agents j in the tournament such that $y_i > y_j$. In other words, $S_i$ is the set of agents who agent i "beats" in the sense of having a larger output. Let $C(S_i)$ be the cardinality of $S_i$. In a tournament with prize structure $\lambda^T$ the payment to agent i is M > 0 if $C(S_i) \geq (1 - \lambda)$n and is m < M if $C(S_i) < (1-\lambda)$n. Put differently, the agents with the top $\lambda n$ outputs get the large prize M (the promotion), the remaining $(1-\lambda)$n agents get m.

In some tournaments, a subset of agents are favored in the sense that they do not have to perform as well as disfavored or discriminated against agents in order to win a large prize. This can be modelled by adding a constant k to their output so that whatever their effort is, their "effective" output is guaranteed to be larger by k. Such a constant is not added to the output of disfavored agents. Viscusi, Zeckhauser, and O'Keeffe (1984) call such tournaments unfair with k being the discrimination factor. Given any vector e = ($e_1$,....,$e_n$) of effort choices by the agents, agent i's probability of winning M can be denoted by $\pi$($e_i$,$e_{-i}$) where $e_{-i}$ is the vector e with the ith agent's effort level deleted. Thus i's expected payoff from such a choice is

$$Ez_i(e_i, e_{-i}) = \pi(e_i, e_{-i})w(M) + (1 - \pi(e_i, e_{-i}))(w(m)) - c(e_i), i = 1, 2, ...., n \quad (2.3)$$

where $\pi(e_i, e_{-i})$ is the probability of winning a large prize for agent i given the effort choices, $e_{-i}$, of his n-1 competitors.

The above tournament defines a game with payoffs given by (2.3) and a strategy set E given by the feasible set of effort choices, which we assume is a closed interval on the real line. The theory of tournaments restricts itself to the game's pure strategy Nash equilibria and in much of what we do here we will restrict ourselves even further to the symmetric pure strategy equilibria where $e^* = e^*_1 = ... = e^*_n$ . With suitable restrictions on the distribution of random shocks and the utility functions posited above, a unique pure strategy symmetric Nash equilibrium will exist for all of the tournaments we will be dealing with in our experiments (see Theorems 2 and 4 below). The theory requires the specification of the utility function, the production function, the distribution of shocks and the prize structure. One specification we use in our experiment below is the following:

$$U_i(p_i, e_i) = p - e^2/c, i = 1, 2, ..., n. \quad (2.4)$$

$$y_i = e_i + \xi_i, i = 1, 2, ...., n, \quad (2.5)$$

where c > 0 and $\xi_i$ is distributed uniformly over the interval [-q, q], q > 0 and independently across agents. $e_i$ is restricted to lie in [0, 100]. In particular, the agents' expected payoff in the tournament is:

$$Ez_i(e_i, e_{-i}) = m + \pi(e_i, e_{-i})[M - m] - e_i^2/c \quad (2.6)$$

We call this our Experimental Specification and note it is defined by the parameters $\Gamma = \{M, m, q, c, k, \phi(\xi_i), i = 1, 2, ...., n\}$, where M, m, q, k, and c are defined above and $\phi(\xi_i)$ is a uniform density function determining each independent realization $\xi_i$. At the unique interior pure strategy Nash equilibrium, each agent's first order condition must be fulfilled,

$$\frac{\partial Ez_i}{\partial e_i} = \frac{\partial \pi(e_i, e_{-i})}{\partial e_i} \cdot [M - m] - 2e_i/c = 0 \quad (2.7)$$

$$or \ \frac{\partial \pi(e_i, e_{-i})}{\partial e_i} \cdot [M - m] = 2e_i/c \quad (2.8)$$

This first order condition has a simple explanation. On the left hand side we have the marginal benefit to a tournament participant from increasing his or her effort level. Obviously, this is equal to the increase in the probability of winning caused by the effort increase $\frac{\partial\pi(e_i,e_{-i})}{\partial e_i}$ –the marginal probability of winning– multiplied by the net benefit of winning, [M-m]. The right hand side is simply the marginal cost of effort. The second order conditions guarantee that this is indeed a maximum. Using a uniform distribution with support [-q,+q], at a symmetric equilibrium where $e_i = e_j = e^*$, we know that $\frac{\partial\pi(e_i,e_{-i})}{\partial e_i} = \frac{1}{2q}$. Hence the first order condition becomes,

$$\frac{1}{2q} \cdot [M - m] = 2e_i/c \tag{2.9}$$

which, when solved for the optimal e*, yields,

$$e* = \frac{(M - m)c}{4q} \tag{2.10}$$

Note that whenever the equilibrium marginal probability of winning is $\frac{1}{2q}$ for all agents, the equilibrium effort level is defined by (2.10). We will use this fact later. In our experiments we parameterize this model by setting q = 60, M = 2.04, m=0.86, q = 60, and c = 15,000, which determines e*=73.75.

### 2.2. Some Theoretical Results

The model presented above yields some interesting results when the number of participants in the tournament and its prize structure are allowed to vary. In addition, the introduction of discriminatory behavior on the part of the tournament organizer has interesting implications as well both for organizational efficiency and social policy. What we will do in this subsection is to prove a number of simple theorems pertaining to symmetric and asymmetric tournaments. To state our results efficiently, however, we need to make a few distinctions about types of symmetries and asymmetries we are concerned with here.

Tournaments are characterized by three factors: the characteristics of the agents functioning in them, i.e. their utility and cost of effort functions, the tournament parameters, i.e. the prize distribution $\lambda^T$, M, c, and m, and finally the fairness of the rules, i.e. whether k is equal to, greater than, or less than zero for any subset of players. We will call a tournament **Fully Symmetric** if all factors mentioned above are symmetric. For example, a tournament is fully symmetric if all players have identical cost functions, $\lambda = 1/2$ so that there is an equal number of large and small prizes, and k = 0 for all i = 1, 2, ....., n so that there is no discrimination against any group. The asymmetries we are interested in occur when an asymmetry is introduced into one of our three factors leaving the other two untouched. We will call a tournament **Unfair** if $\lambda = 1/2$, all agent cost functions are identical, but $k > 0$ for some subset of agents. Likewise, a tournament is **Prize-Asymmetric** if k = 0 for all i = 1, 2, ...., n, all cost functions for agents are identical, but $\lambda \neq \frac{1}{2}$, so the number of large prizes is not equal to one half the number of agents in the tournament. Finally, we call an equilibrium symmetric if $e_i = e^*$ for all i = 1, 2, ...., n.

With these concepts defined we now state and prove three results which furnish the hypotheses to be tested in our experiments.

**Theorem 1.** *Let $e^* = (e_1{}^*, e_2{}^*, \ldots, e_n{}^*)$ be a symmetric equilibrium for a fully symmetric tournament with n players using our experimental specification. Then, ceteris paribus, $e^*$ remains a symmetric equilibrium for that tournament as n increases. In addition, holding n constant, $e^*$ remains a symmetric equilibrium for any prize-asymmetric tournament that can be derived from that tournament by allowing $\lambda$ to vary away from $\lambda = 1/2$ which satisfies the participation constraint of the agents.*
***Proof.*** *: See Appendix 1*

Note that Theorem 1 answers both of our questions about organizational replication and prize structure (questions 1 and 2) posed in the Introduction since it shows that, at a symmetric equilibrium, agent effort in tournaments should remain constant as the size of the tournament increases through replication or, alternatively, as the size is held constant and the distribution of prizes is varied. (In fact, both n and $\lambda n$ can be varied simultaneously with $e^*$ remaining constant). The second result seems counter intuitive, at least to the untrained, who might tend to think in terms of total and not marginal probabilities. For example, a common mistake might be to think that, ceteris paribus, as the number of large prizes in a tournament is increased, equilibrium effort levels should fall since the probability of winning one of those prizes has increased and hence high effort levels would be a costly waste. The trick to seeing that effort levels should remain constant, however, is seeing that while the total probability of winning increases, the marginal probability remains constant. This is the main intuition of the proof.

Theorem 1 is constructed under the assumption that a symmetric interior equilibrium exists. To prove such a result we offer the following theorem.

**Theorem 2.** *Any Fully Symmetric or Prize Asymmetric Tournament with our experimental specification has a unique symmetric interior pure strategy equilibrium if [M-m]c < 4q·100.*[2]
***Proof.*** *: See Appendix 1.*

Our next result deals with unfair tournaments and the impact that size has on effort levels in such tournaments.

**Theorem 3.** *Consider a fully symmetric tournament of size n with $\frac{n}{2}$ large prizes and with a symmetric equilibrium e*. From this fully symmetric tournament create an unfair tournament again with $\frac{n}{2}$ large prizes and with $\frac{n}{2}$ agents receiving $k > 0$, $0 \leq k \leq 2q$. Let ê be the symmetric equilibrium associated with this unfair tournament. Then as n increases from 2, to 4, and eventually to 6, ê increases monotonically toward e*. (Remember, from Theorem 1, e* is invariant to increases in the size of the tournament).*
***Proof.*** *: See Appendix 1.*

This theorem answers the third question posed in the Introduction since it proves that as n becomes large, the efficiency effects of discrimination get small since equilibrium effort levels rise to their non-discriminatory levels.

**Remark:**

Without going through a full induction, it should be obvious from the proof of Theorem 3 that this result can be generalized to demonstrate that when random shocks are uniform, as $n \to \infty$, $\hat{e} \to e^*$. The strategy to prove this result is simple. As we know from the first order condition (2.8), whenever the marginal probability of winning is equal to $\frac{1}{2q}$ for all agents, the equilibrium effort level for agents in a symmetric equilibrium is $e^* = \frac{(M-m)c}{4q}$. Hence, if we can show that as n gets large the marginal probability of winning in an unfair tournament converges to $\frac{1}{2q}$ for all agents, we would have proven our result. However, as we see in the proof of Theorem 3, the marginal probability of winning in an unfair tournament with $\frac{n}{2}$ large prizes and $\frac{n}{2}$ favored agents (receiving a $k > 0$) can be written as $\frac{\partial(\Pr(i \text{ wins}))}{\partial e_i} = \frac{1}{2q} - \frac{k^{\frac{n}{2}}}{(2q)^{\frac{n}{2}+1}}$ for all agents. With $k < 2q$, this term converges to $\frac{1}{2q}$, as $n \to \infty$.

[2]While this condition does guarantee the existence of an interior equilbrium, one still has to check a global condition to guarantee that , for each agent, the payoff at e* is sufficent to induce him or her to participate in the tournament as opposed to setting $e_i = 0$ and receiving a payment of m for sure. This condition is satisfied if $\sqrt{c\lambda[M-m]} > e*$.

**Theorem 4.** *:Assuming the appropriate second order conditions are satisfied, all unfair tournaments satisfying our experimental specification have a unique symmetric pure strategy equilibrium.*

### 2.3. Hypotheses To Be Tested

Theorems 1 and 3 define a set of three null hypotheses which serve as the basis for the discussion of our experimental results. Stated in terms of the experiments actually performed, these hypotheses are stated as follows:

**Hypothesis 1**: The effort levels observed in fully symmetric 2, 4, and 6 person tournaments are identical.

**Hypothesis 2**: The effort levels observed in 6-person tournaments with 2 large, 3 large, and 4 large prizes, respectively, are identical.

**Hypothesis 3**: The effort levels observed in unfair tournaments increase monotonically as we move from 2 to 4 and eventually to 6 person tournaments and approach the fully symmetric equilibrium effort levels.

Note the consequences of these hypotheses for organizational design and efficiency issues. If Hypothesis 1 is correct, the output observed in organizations whose incentive programs define fully symmetric tournaments should not vary as the size of the organization increases. Hence, if you have a firm with 100 workers, you can get as much output from your workers by organizing them into ten 10-person tournaments each with five large and five small prizes as you can by creating one large 100-person tournament with 50 large and 50 small prizes. Obviously, which you do will simply depend on which type of organization is cheaper to administer. Hypothesis 2 goes even further. It claims that not only does the size of the organization make no difference, but the composition of the prizes doesn't matter either. This means that, subject to retaining the interior equilibrium of the tournament, equilibrium effort by workers should be invariant to the composition of large and small prizes. Clearly, a profit maximizing manager would, given this result, always opt for as few large prizes as possible since the labor cost to the firm is defined as the sum of the prizes. Finally, Hypothesis 3 indicates a strong preference for large organizations when it is suspected that discrimination is being practiced within the organization. The reason is that if Hypothesis 3 is correct, in the face of discrimination output actually increases as the size of the organization increases. Coupled with Hypothesis 2, these two Hypotheses indicate that large organizations with few large prizes should define the profit maximizing structure for organizations suspected of having discrimination within their ranks.

# 3. The Experiments Performed and the Experimental Design

Eight experiments examined the effects of organizational size and incentive structure on players effort in tournaments. Experimental parameters are shown in Table 1. The decision and random number range, cost function, and fixed payment structure (M and m) were identical across all eight experiments. Experiments 1 - 3 utilized fully symmetric tournaments to test Hypothesis 1. All experimental tournaments were identical except for the number of participants with Experiment 1 testing 2-person, Experiment 2, 4-person, and Experiment 3, 6-person tournaments. Experiments 3 - 5 tested Hypothesis 2. All three experiments were identical 6-person tournaments except for a change in the tournaments prize structure. In Experiment 3, there were 3 large- and 3 small-fixed payments; Experiment 4 used 4 large- and 2 small-fixed payments; and Experiment 5 used 2 large- and 4-small fixed payments. We tested Hypothesis 3 using Experiments 6 - 8. All these experiments were unfair (asymmetric) tournaments, with $k = 25$. They were identical in all experimental parameters except for tournament size. Experiment 6 used 2-person, Experiment 7, 4-person tournaments, and Experiment 8, 6-person tournaments. All had $\lambda = \frac{1}{2}$.

## 3.1. Experimental procedures

286 subjects were recruited both from New York University and University of Pennsylvania undergraduate classes. They were randomly assigned to a time slot, and instructed to meet at the behavioral laboratory. The behavioral laboratory consisted of cubicles that made it difficult to see others during the experiment. Subjects were randomly assigned to a cubicle, given instructions (See Appendix), and the instructions were publicly read. The instructions basically said the following: For this experiment, subjects would be randomly assigned with a specified number of other subjects (either 1, 3, or 5 depending on the tournament size). These subjects would be their "group member(s)". Group members remained the same during the entire experiment, and their physical identities were not revealed. Subjects were told the amount of money they earned was a function of their decisions, their group members' decisions, and the realization of a random variable as described by the rules of the tournament. They were then given cost-of-effort tables and told that all subjects had identical tables and instructions. In each round, their task was to chose an effort level (to which a random component was

added by the computer or by pulling a chip from a bag of chips, if the experiment was a 2-person experiment and hence performed by hand). After each round, they were shown their effort, their random realization, and whether they earned a large or a small fixed payment. They learned nothing about the effort levels or the random realizations of their cohorts. All parameters were common knowledge except the identity of pair member(s).

All 2-person tournaments were conducted manually while 4 and 6 person tournaments were run using computer terminals. A more detailed set of experimental procedures for our 2-person experiments can be found in Bull et al. (1987). Appendix 2 presents the instructions for one of our 6 person experiment–experiment 5.

## 4. Results

To organize our data we present experimental results to test of our three hypotheses. Experimental results are shown in FIGURES 1-4, and TABLES 2-5. FIGURES 1, 2 and 4 show the round-by-round subject mean effort choices across experiments. FIGURE 3 shows the organizational cost per unit of effort in 6-person symmetric tournaments. TABLE 2 presents summary statistics of observed behavior in experiments 1-3. TABLE 3 and 4 do the same for experiments 3-5. TABLE 5 shows summary statistics of observed behavior in experiments 6-8.

### 4.1. Hypothesis 1 - Organizational replication of symmetric tournaments

Hypothesis 1 states that observed behavior in fully symmetric tournaments should be invariant to tournament size. Hence, we expect observed effort levels in Experiments 1, 2, and 3 to be identical and equal to 73.75. FIGURE 1 and TABLE 2 compare subject behavior across 3 organizational replications – 2-person, 1 large prize; 4-person, 2 large prizes; and 6-person, 3 large prizes. To test Hypothesis 1 we investigate whether observed behavior deviates from predicted.

First we test whether mean subject choices in each organization significantly differ from that predicted (73.75). A round-by-round Wilcoxon signed rank test does not reject the hypothesis that observed effort levels in each organizational setting came from a population with a mean of 73.75.[3] As shown in TABLE 2,

[3]The Wilcoxon signed-rank test requires a symmetric distribution of data. Using a Kolmogorov-Smirnov test, we could not reject the hypothesis that the data were drawn from a

the highest observed deviation from predicted effort level in rounds 1-10 is 9.44 (in all three experiments, the highest deviation occurs in round 1), and in rounds 11-20, 9.40. Across all symmetric tournaments (i.e., across all fully symmetric tournaments of size 2, 4 and 6) , the mean deviation from the predicted effort level in rounds 11-20 is 4.6.

We then test whether mean subject choices in each experiment were significantly different from each other. Using pair-wise round-by-round Mann-Whitney tests we cannot reject the hypothesis that observed effort levels in each setting came from identical populations.

Symmetric tournament theory (and Hypothesis 1) states that agents should not care about organizational size in choosing effort levels, as long as the proportion of large payments remains constant. Our results support this prediction. Subjects exhibited similar behavior in our 2, 4, and 6 person tournaments; tournament size did not significantly impact behavior.

For tournament organizers (i.e., managers), the results imply the cost per unit of output does not significantly differ across fully symmetric tournaments of different sizes. If managers need to efficiently use resources, then one important design issue is tournament size – should one use several small tournaments, or a single large one? If the fixed costs associated with setting up tournaments do not significantly vary as tournament size increases, then managers should prefer larger tournaments to smaller ones since they would economize on these fixed costs. This is especially true if the presence of fixed costs implied economies of scale in using large tournaments. Our results indicate that since effort levels are invariant to tournament size, larger tournaments are more cost effective than a series of small ones. In addition, (as we will see in the test of Hypothesis 3, when discrimination exists in the organization, there is an additional reason why larger tournaments migth dominate – they raise output per worker at the equilibrium

### 4.2. Hypothesis 2: Effort and compensation design

Hypothesis 2 states that effort levels in experiments 3, 4, and 5 (three 6-person tournaments with different prize distributions, 3M-3m, 4M-2m, 2M-4m) should be identical and equal to 73.75. Compensation designs of 4M-2m, 3M-3m, and 2M-4m should elicit identical effort levels. FIGURE 2 and TABLE 3 show observed results. We test for similarity using a pair-wise round-by-round Mann-Whitney test. While the test confirms similar behavior using designs of 3M-3m and 2M-4m

normal, hence symmetric, distribution. All statistical tests used a significance level of percent.

– we cannot reject the hypothesis that observed effort levels in any round came from identical populations, this is not true in comparing the design of 4M-2m with 3M-3m and 2M-4m. Observed effort levels in tournaments with 4M-2m are significantly below those with either 3M-3m or 2M-4m as Figure 2 implies. A Mann-Whitney test confirms this for every round.

We next test whether observed behavior differs from that predicted. As previously discussed, a round-by-round Wilcoxon test shows observed behavior in a 3M-3m tournament is as predicted. A similar test confirms that observed effort in a 2M-4m tournament is as predicted[4], while effort choices in a 4M-2m design are significantly lower than predicted.

Although theory predicts agents should exert equal effort under any of our 3 designs, results indicate subjects exert significantly less effort with a relatively large number of high payments (4M).

The explanation for this failure, we suspect, involves an inability on the part of subjects, and perhaps people in general, to sufficiently understand the difference between total and marginal quantities.[5] In this case, we feel subjects confuse the total probability of winning in these tournaments with their associated marginal quantities. In many instances such a mistake may not lead one too far astray, but here it seems to. For example, with a 3M-3m design, the expected equilibrium probability of winning is .5; this increases to .666 with 4M-2m, and decreases to .33 with 2M-4m. However, at the equilibrium the marginal probability remains constant at $\frac{1}{2q} = \frac{1}{120}$ (given our parameters – see Theorem 1).

If subjects look at total probabilities rather than marginal, what are the incentives in the 4M-2m and 2M-4m designs? In the former, there is an incentive to reduce effort levels since, at the equilibrium, you are expecting to receive a large fixed payment with what might appear to be an excessively high probability, 66.6%. By lowering your effort level you reduce the probability of winning to a still acceptable lower levels but, in the process, increase your payoff conditional on winning since lowering your effort level reduces your cost. This marginal cost saving in effort is substantial when the equilibrium calls for an effort level of 73.75.

To illustrate this point, imagine you are an assistant professor competing with 5 others of identical ability for tenure, and are told 4 of you will receive it, i.e., the tenure rate in your department is 80%. With such odds you might logically reduce

[4]Using a round-by-round Wilcoxon test this finding is not supported for round 12 only, where results are marginally significant at the .10 level.

[5]Actually, even trained economists can, at first glance, miss the intuition here and think that effort levels should decline aas the proportion of large prizes is increased.

your effort level. Intuitively, one thinks, while I have to work very hard to be the first or second best, I don't have to work that hard to be fourth best. Subjects in 4M-2m tournaments appear to have used this line of reasoning. Clearly, they were willing to trade off lower probabilities of winning (reducing them toward 50%) for reductions in effort costs.

Incentives in the 2M-4m tournament aren't so clear – if one wants to increase the probability of winning here one must increase effort, but this leads to higher, not lower, costs. In addition, since the cost function is relatively steep at 73.75, any increase in effort is associated with significantly higher marginal costs. Subjects may get discouraged because of their low probability of winning and reduce effort levels in order to lower costs. To a minor extent this is not what appears to have happened. Though effort levels are not significantly different between the 3M-3m and 2M-4m designs, they are still higher in the 3M-3m design in 18 of the 20 rounds. And, mean effort in the 2M-4m design is lower than predicted in 14 rounds.

So in both prize asymmetric tournaments (4M-2m, 2M-4m) we see similar effects for opposite reasons. In the 4M-2m subjects may believe it is easier to win, so they reduce effort levels; in the 2M-4m design they think it is more difficult to win so they slightly reduce effort levels.

### 4.3. Organizational Costs

Managers who design compensation schemes that elicit similar effort for lower costs are obviously using their managerial resources more efficiently. One measure of organizational efficiency is organizational cost per unit of effort. Holding effort constant, organizations want to economize on the payments they need to make to achieve that effort level. Payment design (changing the prize distribution) is one of many variables that managers use to modify behavior. In theory, while the three tested prize distributions all elicit effort of 73.75, the equilibrium organizational cost per unit of effort differs – for 4M-2m, \$.0223, for 3M-3m, \$.0197, and for 2M-4m, \$.0170.

If observed behavior (and hence organizational costs) differ from that predicted at the equilibrium, an interesting empirical question is how do they differ and do they do so in a systematic manner. Realized organizational costs are presented in FIGURE 3 and TABLE 4. Obviously, from looking at Figure 3 we see that the 4M-2m design leads to the highest cost per unit of effort, both because of the large number of large fixed payments and (as can be seen in Figure 2), because of the lower effort levels exerted. Further, as shown in TABLE 4, in all of the last ten rounds, the actual cost per effort unit was higher than predicted in the 4M-2m design.

The comparison of actual costs between the 3M-3m and 2M-4m design is interesting. We previously discussed how subjects exerted slightly more effort in the 3M-3m relative to the 2M-4m design. This would seem to imply that managers are better off using a symmetric (equal number of high and low payments) compensation design. However, there is obviously an inherent advantage in offering a smaller number of high payments, since this design reduces the tournament's cost. We see the predicted difference between the two designs is .0027 per unit of effort. TABLE 4 shows that although the actual cost difference is smaller than that predicted (since the 3M-3m design elicits slightly higher effort), the 2M-4m design still results in lower costs. That is, while the cost differential between the two designs is reduced, the 2M-4m design still produces a lower cost per unit of effort – an average of .0179 over the last ten rounds compared to .0187 for the 3M-3m design.

### 4.4. Hypothesis 3: Organizational replications of unfair tournaments

We test the effect of discrimination on subject behavior by fixing the discrimination level (k = 25) across three different size organizations – 2, 4, and 6 person. Hypothesis 3 states that the effort levels in the 6-person tournament are higher than in the 4-person tournament, which, in turn, is higher than in the 2-person tournament, i.e. the loss in efficiency is due to discrimination monotonically decreasing as organization size increases. The predicted effort level in organizations of size 2 is 58.39, of size 4, 70.55, and of size 6, 73.08. The theory also predicts identical effort choices for advantaged and disadvantaged subjects. FIGURE 4 and TABLE 5 compare subject behavior across these unfair tournaments.

A round-by-round Mann Whitney test for rounds 11 - 20 confirms no significant differences in choice levels of advantaged and disadvantaged subjects in the 6-person tournament. Further, a Wilcoxon test cannot reject the hypothesis that choices in these rounds came from a population with a mean of 73.08. Tests for the 4-person tournament show similar results (using a mean of 70.55). However, behavior in the 2-person tournaments is different from that predicted. Effort levels of disadvantaged and advantaged subjects significantly differ in 7 of the last 10 rounds, and we can reject the hypothesis effort levels were drawn from a population with a mean of 58.39 in every one of the last 10 rounds. In fact, as we can see in Figure 4, in 13 of their 20 rounds effort levels were higher in the 2-person experiment than in the 4 or 6 person experiment which is counter to the theoretical predictions.

We also tested whether observed behavior is significantly different across these unfair tournaments. Although the spread in predicted effort is small (58.39, 70.55, 73.08), we still find some differences in observed behavior. In comparing the 2-person tournament with the 4, we observe significant differences in 8 of the last ten rounds. In comparing the 4-person tournament with the 6, we find significant behavioral differences in 5 of the last ten rounds (12, 14, 15, 18, 19). Finally, we find no significant differences in behavior in the 2- and 6-person tournaments.

In summary, our results are partially counter to the predictions of theory. While theory predicts an increase in effort levels as the size of the organization increases, our results support this prediction only when going from a 4- to a 6-person tournament; they do not support it when moving from a 2- to 4-person tournament or even from a 2-to-6 person tournament; effort levels in a 2-person tournament were significantly higher than predicted.

We do not find this result discouraging, however, since the theory fails to predict well only in the two-person case, a case we feel may be special. This is because a two-person tournament, especially one where one agent is discriminated against, has many of the characteristics of a game of status where part of a subject's pay-off is derived from winning ("beating" one's opponent).[6] While this phenomenon may exist in 4 and 6 person tournaments as well, we feel they are sufficiently impersonal to allow subjects to treat their competition more parametrically and avoid this effort matching behavior. Since the results for the 4 and 6 person tournaments were consistent with the theory, we consider this interpretation a viable explanation.

Results also indicate a wider choice of effort levels in unfair 6-person tournaments than in their symmetric counter-part. Comparing TABLES 2 and 5, we see the mean standard deviations are much higher in the unfair tournament.

## 5. Section 5: Discussion and Conclusion

This paper develops basic properties and tests the descriptive validity of multi-person tournaments using laboratory experimental techniques. Overall, observed

[6] Similar results, of disadvantaged subjects trying too hard in two-person tournaments, were seen by Bull Schotter and Weigelt (1987) . Hence, our results here are consistent with those previously recorded.

behavior is similar to that predicted by the theory. This supports the conclusions of earlier two-person tournament experimental studies (Bull, Schotter, and Weigelt, 1987; Schotter and Weigelt, 1992).

We focus on individual tournament behavior and its associated impact on organizational cost. Our results show that tournament design does affect both concerns. What we find is that with respect to Hypothesis 1, behavior is indeed invariant to organizational replication, i.e. effort levels remain constant as the size of the organization grows. With regard to Hypothesis 2, behavior was not always found to be invariant to the prize structure. More specifically, as predicted, behavior was invariant in the 3M-3m and 2M-4m designs, but not in the 4M-2m design, where effort levels were significantly lower than predicted. Tournament size, as stipulated in Hypothesis 3, does seem to reduce inefficiency effects of discrimination, thereby reducing its opportunity costs. This was particularly true in the move from 4 to 6 person tournaments, but not substantiated in comparisons to 2-person tournaments..

While it is hard to extrapolate from small laboratory experiments to large corporations, our results have implications for public as well as corporate policy. With respect to public policy there is empirical evidence showing discrimination rates are high in large organizations. One study finds men constitute 95% of senior managers in the top 500 service and 1000 industrial companies; and 97% of them are white (Redwood, 1996). Another, finds only 1 Fortune 1000 company without a white male CEO (Leinster, 1988). Others find significant discrimination across a variety of large organizations ranging from Fortune 500 firms, (see Greenhaus et al. (1990) and Tokunga and Tracy (1996) to the U.S. Army (see Baldwin, 1996).

Public policies against discrimination already focus on large organizations as our results suggest they should. For example, small firms are exempt from filing EEOC reports if they have less than 25 employees. This policy is consistent with the theory of tournaments and Becker's (1957) theory of discrimination. These theories imply that if the efficiency loss of output due to discrimination is great when firms are small, a small firm with a taste for discrimination will be forced out of existence by less discriminatory rivals. On the other hand, if the efficiency loss of discrimination is small for large firms, as tournament theory suggests, a firm with a taste for discrimination may easily survive. Medoff (1985) provides evidence that this emphasis on large firm enforcement has been properly directed. He reports that the rate of discrimination has significantly decreased since equal opportunity policies were enacted since the 1960.

With respect to corporate policy, our results have implications for the proper design of organizational incentives. One crucial determinant of a good manager is his/her ability to design effective incentive structures. Such structures should modify agent behavior in the desired way, and minimize the associated labor costs. Our theoretical results indicate that since equilibrium effort levels are invariant to the prize structure (for interior equilibria) the optimal prize structure is one which minimizes the number of prizes consistent with maintaining an individual participation constraint. Our empirical results go even further since they indicate that when too many large prizes are dangled above the noses of agents, they tend to ease off in their effort. Since it is costly to provide such a large number of large prizes, on a behavioral level it again appears as if the steeper organizational pyramids are better for worker motivation (at least up until the point where the probability of promotion becomes so low that agents "drop out" and provide no effort at all). We hope our results can aid managers in better understanding the properties of tournament design and help them construct more efficient tournaments.

Finally, larger tournaments have several advantages. They reduce the decrease in output due to discriminaton while economize on the fixed costs invlved in setting up a number of small tournaments.

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# 7. Appendix

## 7.1. Appendix 1: Proofs

**Theorem 1:Proof.**

We will prove that $\frac{\partial\pi(e_i,e_{-i})}{\partial e_i} = \frac{1}{2q}$ at the symmetric interior equilibrium of all prize-asymmetric tournaments. Hence, from the first order condition (2.8) we know that $e^* = \frac{(M-m)c}{4q}$ and is independent of n and $\lambda$ as asserted in the statement of the Theorem. Consider a prize-asymmetric tournament with $\lambda$n large prizes (of course when $\lambda = 1/2$ we are in a fully symmetric tournament). At the symmetric equilibrium of this tournament (which we prove exists in Theorem 2) all agents choose the same effort level e*. Hence who wins the $\lambda$n large prizes, M, is determined strictly by the sample of realization of uniform random draws from [-q, +q]. Hence, let $\xi$= ( $\xi_1, \xi_2, ...., \xi_n$) be the sample of random realizations and $w$ = ($w_1, w_2, ...., w_n$ ) be the transformation of these random variables into order statistics with $w_n > w_{n-1} > ...., > w_1$. Consider now an agent contemplating a "small" increase in his effort level above e*.[7] We want to calculate the marginal probability of winning a large prize associated with such an increase. Obviously if the agent contemplating the effort increase received one of the top $\lambda$n order statistics, he will win a large prize with probability 1 and hence an increase in effort will have no marginal impact. The same is true for agents contemplating such an increase and receiving order statistic $w_{[\lambda n-2]}$ or worse, since they are sure losers even after such a contemplated increase. It is then only that agent who has received $w_{[\lambda n-1]}$ that can actually change his probability of winning by increasing his effort above e*. Consequently, the marginal probability of winning for any agent contemplating an increase in his effort is equal to the probability that that agent draws $w_{\lambda n-1}$ times the marginal increase in the probability of winning for that particular agent. Let us call the agent receiving $w_{\lambda n-1}$ agent i and the agent who has received $w_{\lambda n}$ agent j. To derive i's marginal probability of winning define z = ($w_{\lambda n}$) - ($w_{\lambda n-1}$) e.g. z is the difference between the n$^{th}$ and the n-1$^{st}$ order statistic. When each $\xi_i$ is drawn independently from a uniform distribution, it can be shown (See Reiss (1989) that z is distributed as

$$f(z) = \frac{n}{2q^n} \times (2q - z)^{n-1}, \ z \in [0, 2q].) \tag{7.1}$$

When ($e_i$ - $e_j$) < 2q, the probability that agent i wins a large prize is [8]

$$Pr([w_{\lambda n} - w_{\lambda n-1}] < [e_i - e_j]) \times \Pr(\varepsilon_i = w_{\lambda n-1}). \tag{7.2}$$

Recognizing that $\Pr(\varepsilon_i = w_{n-1}) = \frac{1}{n}$ we get,

$$\frac{1}{n}\int_0^{e_i-e_j} \frac{n}{(2q)^n} \times [2q-z]^{(n-1)} dz = (\frac{-1}{n(2q)^n} \cdot (2q-z)^n \mid_0^{(e_i-e_j)} = \frac{1}{n} - \frac{[2q-(e_i-e_j)]^n}{n \cdot (2q)^n}. \tag{7.3}$$

[7] With a continuous underlying density function the possibility of a tie in the value of any two consequtive order statistics is zero

[8] When $(e_i - e_j) \geq 2q$, the probability of winning is, of course, zero.

Taking the derivative with respect to $e_i$, we find

$$\frac{\partial(\text{Pr i wins})}{\partial e_i} = \frac{(2q - e_i + e_j)^{n-1}}{(2q)^n}, \tag{7.4}$$

which when evaluated at $e_i = e_j = e^*$ equals 1/2q. Since this expression is independent of both n and $\lambda$(n), we have proven Theorem 1. ■

**Theorem 2:Proof.**

From the proof of Theorem 1 we know that given a uniform distribution for shocks, whenever agents all choose the same effort level, the common marginal probability of winning is $\frac{1}{2q}$. If a unique equilibrium is to exist the first order condition must be satisfied for each agent at the same $e_i$. Substituting $\frac{1}{2q}$ *for* $\frac{\partial \pi(e_i, e_{-i})}{\partial e_i}$ in (2.8) we see that for each agent i

$$\frac{\partial E z_i}{\partial e_i} = \frac{1}{2q}[M - m] - \frac{2e_i}{c} = 0 \tag{7.5}$$

or

$$\frac{2e_i}{c} = \frac{[M - m]}{2q}. \tag{7.6}$$

The left hand side of this equation is a linear increasing function of $e_i$ which is identical for all agents since they have identical cost of effort functions. The right hand side is a constant. Hence the existence of an $e_i = e^*$ for i = 1,2,....,n, that satisfies this condition is guaranteed . In addition, from the description above it is clear that this $e^*$ must be unique and is an interior solution if [M-m]c < 4q·100. ■

**Theorem 3:Proof.**

**Case 1: n = 2**

Consider a two-person unfair tournament with two agents i = 1,2, where k = $k_1$,$k_2$ >0. Given the linear nature of production, output for i is $y_i = e_i + k_i + \varepsilon_i$, i = 1, 2. Agent 1 beats agent 2 if $y_1 > y_2$,or $e_1 + k_1 + \varepsilon_1 \geqslant e_2 + k_2 + \varepsilon_2$. Without loss of generality assume that $e_1 + k_1 > e_2 + k_2$ so that 1 is the agent with the highest potential output. Given 1's random realization $\varepsilon_1$,whether 1 wins depends upon whether 2's random realization $\varepsilon_2$ is such that $\varepsilon_2 < (e_1 + k_1 - e_2 - k_2) + \varepsilon_1$.

The probability that $\varepsilon_2$ is this small determines the conditional probability of winning for 1 given a realization of $\varepsilon_1$ : $\Pr(1 \text{ wins}|\ \varepsilon_1) = \int_{-q}^{e_1+k_1-(e_2+k_2)+\varepsilon_1} \frac{1}{2q} d\varepsilon_2$,or

$\Pr(1 \text{ wins}|\ \varepsilon_1) = \frac{q+(e_1+k_1)-(e_2+k_2)+\varepsilon_1}{2q} = \frac{1_2+\varepsilon_1}{2q}$,where $1_2 = q+(e_1+k_1)-(e_2+k_2)$.

Note that with uniform distributions over the $\varepsilon's$, $\frac{q+(e_1+k_1)-(e_2+k_2)+\varepsilon_1}{2q} = \frac{1}{2} + \frac{(e_1+k_1)-(e_2+k_2)}{2q} + \frac{\varepsilon_1}{2q}$,which says that the conditional probability of winning given

$\Pr(1 \text{ wins}|\ \varepsilon_1) = \frac{q+(e_1+k_1)-(e_2+k_2)+\varepsilon_1}{2q} = \frac{1_2+\varepsilon_1}{2q}$, where $1_2 = q+(e_1+k_1)-(e_2+k_2)$.

Note that with uniform distributions over the $\varepsilon's$, $\frac{q+(e_1+k_1)-(e_2+k_2)+\varepsilon_1}{2q} = \frac{1}{2} + \frac{(e_1+k_1)-(e_2+k_2)}{2q} + \frac{\varepsilon_1}{2q}$, which says that the conditional probability of winning given any $\varepsilon_1$ is equal to $\frac{1}{2}$ plus a term proportional to the difference in the effort levels modified by the discrimination terms, plus another term proportional to $\varepsilon_1$, where the factor of proportionality is $\frac{1}{2q}$.

To find the unconditional probability of winning we must integrate over all possible realizations of $\varepsilon_1$. However, since we have assumed that $(e_1 + k_1) > (e_2 + k_2)$, we know that for all realizations of $\varepsilon_1 > (e_2 + k_2) + q - (e_1 + k_1) = 2_1$ agent 1 can not lose, i.e. the probability of winning is equal to 1. Call the region of random realizations $\varepsilon_i \in [(e_2 + k_2) + q - (e_1 + k_1),\ q]$ the Sure Win Interval (SWI). To find the full or unconditional probability of winning we now integrate over two regions,

$\Pr(1\text{wins}) = \int_{2_1}^{q} \frac{1}{2q} d\varepsilon_1 + \int_{-q}^{2_1} \frac{(1_2+\varepsilon_1)d\varepsilon_1}{(2q)^2} = \frac{1}{2} + [\frac{(e_1+k_1)-(e_2+k_2)}{2q}] - \frac{[(e_1+k_1)-(e_2+k_2)]^2}{8q^2}$.

In a similar fashion,

$\Pr(2\text{wins}) = \int_{-2_1}^{q} \frac{(2_1+\varepsilon_2)}{(2q)^2} d\varepsilon_2 = \frac{1}{2} - [\frac{(e_1+k_1)-(e_2+k_2)}{2q}] + \frac{[(e_1+k_1)-(e_2+k_2)]^2}{8q^2}$

These formula yield expressions for the marginal probability of winning which are,

$\frac{\partial(\Pr(1\text{wins})}{\partial e_1} = \frac{1}{2q} - \frac{(e_1+k_1)-(e_2+k_2)}{(2q)^2}$,

$\frac{\partial(\Pr(2 \text{ wins})}{\partial e_2} = \frac{1}{2q} - \frac{(e_1+k_1)-(e_2+k_2)}{(2q)^2}$.

Note that these marginal probability of winning functions are identical so that if we evaluate them at $e_1 = e_2$, $k_1 > 0$, $k_2 = 0$, we find $\frac{\partial(\Pr(i \text{ wins}))}{\partial e_i} = \frac{1}{2q} - \frac{k_i}{(2q)^2}$, i = 1,2.

Using this as the expression for the marginal probability of winning in (2.8) yields e* = 58.39 when k = 25.

**Case 2: n = 4**

The analysis for n = 4 follows identically from the n = 2 case except it is more tedious. Assume that there are four agents, 1, 2, 3, 4 in a tournament with two large and two small prizes and with k = ($k_1$,$k_2$,$k_3$,$k_4$ ). Assume, again without loss of generality, that $e_1 + k_1 \geqslant e_2 + k_2 \geqslant e_3 + k_3 \geqslant e_4 + k_4$, and let us calculate the unconditional probability that any agent i will win.

Looking at agent 1 first we find,

$\Pr(1 \text{ wins}) = \int_{3_1}^{q} \frac{1}{2q} d\varepsilon_1 + \int_{4_1}^{3_1} \frac{1}{2q}[\frac{(1_2+1_3+2\varepsilon_1)}{(2q)} - \frac{(1_2+\varepsilon_1)(1_3+\varepsilon_1)}{(2q)^2}]d\varepsilon_1 + \int_{-q}^{4_1} \frac{1}{2q}[\frac{(1_2+\varepsilon_1)(1_3+\varepsilon_1)}{(2q)^2} + \frac{(1_2+\varepsilon_1)(1_4+\varepsilon_1)}{(2q)^2} + \frac{(1_3+\varepsilon_1)(1_4+\varepsilon_1)}{(2q)^2} - 2(\frac{(1_2+\varepsilon_1)(1_3+\varepsilon_1)(1_4+\varepsilon_1)}{(2q)^3}]d\varepsilon_1$.

Here the first term on the right is the sure win area while the other intervals of integration represent the probability of winning if the random realization $\varepsilon_1$ falls into other intervals which requires beating competitors in various combinations.

For agents 2, 3, and 4, a similar analysis yields the following probability of winning formulas:

$$\Pr(2\text{wins}) = \int_{3_2}^{q} \frac{1}{2q} d\varepsilon_2 + \int_{4_2}^{3_2} \frac{1}{2q}\left[\frac{(2_1+2_3+2\varepsilon_2)}{(2q)} - \frac{(2_1+\varepsilon_2)(2_3+\varepsilon_2)}{(2q)^2}\right] d\varepsilon_2$$
$$+ \int_{-2_1}^{4_2} \frac{1}{2q}\left[\frac{(2_1+\varepsilon_2)(2_3+\varepsilon_2)}{(2q)^2} + \frac{(2_1+\varepsilon_2)(2_4+\varepsilon_2)}{(2q)^2} + \frac{(2_3+\varepsilon_2)(2_4+\varepsilon_2)}{(2q)^2}\right.$$
$$\left. -2\left(\frac{(2_1+\varepsilon_2)(2_3+\varepsilon_2)(2_4+\varepsilon_2}{(2q)^3}\right)\right] d\varepsilon_2 + \int_{-q}^{-2_1} \frac{1}{2q}\left[\frac{(2_3+\varepsilon_2)(2_4+\varepsilon_2)}{(2q)^2}\right] d\varepsilon_2,$$

$$\Pr(3\text{ wins}) = \int_{4_3}^{q} \frac{1}{2q}\left[\frac{(3_1+\varepsilon_3)}{(2q)} + \frac{(3_2+\varepsilon_3)}{(2q)} - \frac{(3_1+\varepsilon_3)(3_2+\varepsilon_3)}{(2q)^2}\right] d\varepsilon_3$$
$$+ \int_{-3_1}^{4_3} \frac{1}{2q}\left[\frac{(3_1+\varepsilon_3)(3_2+\varepsilon_3)}{(2q)^2} + \frac{(3_1+\varepsilon_3)(3_4+\varepsilon_3)}{(2q)^2} + \frac{(3_2+\varepsilon_3)(3_4+\varepsilon_3)}{(2q)^2}\right.$$
$$\left. -2\left(\frac{(3_1+\varepsilon_3)(3_2+\varepsilon_3)(3_4+\varepsilon_3)}{(2q)^3}\right)\right] d\varepsilon_3 + \int_{-3_2}^{-3_1} \frac{1}{2q}\left[\frac{(3_2+\varepsilon_3)(3_4+\varepsilon_3)}{(2q)^2}\right] d\varepsilon_3,$$

$$\Pr(4\text{ wins}) = \int_{-4_1}^{q} \frac{1}{2q}\left[\frac{(4_1+\varepsilon_4)(4_2+\varepsilon_4)}{(2q)^2} + \frac{(4_1+\varepsilon_4)(4_3+\varepsilon_4)}{(2q)^2}\right.$$
$$\left. + \frac{(4_2+\varepsilon_4)(4_3+\varepsilon_4)}{(2q)^2} - 2\left(\frac{4_1+\varepsilon_4)(4_2+\varepsilon_4)(4_3+\varepsilon_4)}{(2q)^3}\right)\right] d\varepsilon_4 + \int_{-4_2}^{-4_1} \left(\frac{(4_2+\varepsilon_4)(4_3+\varepsilon_4)}{(2q)^2} d\varepsilon_4\right.$$

Since we are interested in the marginal probabilities of winning we must differentiate these terms with respect to the relevant effort levels. Letting $e_{k1} = e_1 + k_1$ ,$e_{k2} = e_2 + k_2$ $e_{k3} = e_3 + k_3$, $e_{k4} = e_4 + k$ ,this yields:

$$\frac{\partial(\Pr(1\text{ wins}))}{\partial e_1} = \frac{1}{2q} - \frac{1}{(2q)^3}[(e_{k1} - e_{k2})(e_{k1} - e_{k3}) + (e_{k1} - e_{k2})(e_{k1} - e_{k4})$$
$$+(e_{k1} - e_{k3})(e_{k1} - e_{k4})] + \frac{(e_{k1}-e_{k2})(e_{k1}-e_{k3})(e_{k1}-e_{k4})}{(2q)^4},$$
$$\frac{\partial(\Pr(2\text{ wins}))}{\partial e_2} = \frac{1}{2q} - \frac{1}{(2q)^3}[(e_{k2} - e_{k3})(e_{k2} - e_{k4}),$$
$$\frac{\partial(\Pr(3\text{ wins})}{\partial e_3} = \frac{1}{2q} - \frac{1}{(2q)^3}[(e_{k3} - e_{k1})(e_{k3} - e_{k2}),$$
$$\frac{\partial(\Pr(4\text{ wins}))}{\partial e_4} = \frac{1}{2q} - \frac{1}{(2q)^3}[(e_{k4} - e_{k1})(e_{k4} - e_{k2}) + (e_{k4} - e_{k1})(e_{k4n} - e_{k3})$$
$$+(e_{k4} - e_{k2})(e_{k4} - e_{k3})] - \frac{(e_{k4}-e_{k1})(e_{k4}-e_{k2})(e_{k4}-e_{k3})}{(2q)^4}.$$

Evaluating these expressions at $e_1 = e_2 = e_3 = e_4 = e^*$, and, without loss of generality, letting $k_1 = k_2 > 0$ (favored agents) and $k_3 = k_4 = 0$,(non-favored agents), we find that these expressions collapse to $\frac{\partial(\Pr(i\text{ wins}))}{\partial e_i} = \frac{1}{2q} - \frac{k^2}{(2q)^3}$.

The trick to understanding why our marginal probability of winning expressions take this form is realizing that $(e_{ki} - e_{kj})$ is positive and equal to k only when i is a favored agent and j is non-favored since in that case ($e_{ki} = e_i + k_i$ , $e_{kj} = e_j$) and therefore $(e_{ki} - e_{kj})$ = k, whereas if i and j were both favored or non-favored then $(e_{ki} - e_{kj}) = 0$. Hence, the marginal probability of winning equals the marginal increase in the sure win interval that results from an increase in effort minus the marginal probability of winning that results from an agent's ability to beat all agents of the other type (a marginal increase in effort has no impact on your ability to beat members of your own type). All terms involving mixtures of types are zero and drop out of the expression. Again, using the marginal probability of winning expression in (2.8) we find e* = 70.55 when k = 25.

**Case 3: n=6**

For the case where n = 6, we will simply write down the expressions for the marginal probability of winning deriving them in an analogous fashion as we have before.

Take the case where we have six players indexed i = 1 ,2 ,3 4, 5, 6, assume $k_1 = k_2 = k_3 = k$, $k_4 = k_5 = k_6 = 0$, and, without loss of generality , $e_1 + k_1 \geqslant e_2 + k_2 \geqslant e_3 + k_3 \geqslant e_4 + k_4 \geqslant e_5 + k_5 \geqslant e_6 + k_6$. Using the same method as in cases 1 and 2, and setting $e_{ij} = (e_i + k_i) - (e_j + k_j)$, we can derive the marginal probability of winning functions for these agents as:

$\frac{\partial(\Pr(1\text{ wins}))}{\partial e_1} = \frac{1}{(2q)^6}[(\mathbf{2q})^5 - (2q)^2(e_{12}e_{13}e_{14} + e_{12}e_{13}e_{15}$
$+e_{12}e_{13}e_{16} + e_{12}e_{14}e_{15} + e_{12}e_{14}e_{16} + e_{12}e_{15}e_{16} + e_{13}e_{14}e_{15} + e_{13}e_{14}e_{16} + e_{13}e_{15}e_{16}$
$+\mathbf{e_{14}e_{15}e_{16}}) + 3(2q)(e_{12}e_{13}e_{14}e_{15} + e_{12}e_{13}e_{14}e_{16} + e_{12}e_{13}e_{15}e_{16} + e_{12}e_{14}e_{15}e_{16}$
$+e_{13}e_{14}e_{15}e_{16}) - 3(e_{12}e_{13}e_{14}e_{15}e_{16})]$.

Note that the only $e_{ij}$ term in this expression that is nonzero is the one in bold so,

$\frac{\partial(\Pr(1\text{ wins}))}{\partial e_1} = \frac{1}{(2q)^6}[(\mathbf{2q})^5 - (2q)^2k^3] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$.

$\frac{\partial(\Pr(2\text{ wins}))}{\partial e_2} = \frac{1}{(2q)^5}[(\mathbf{2q})^4 - (2q)(e_{23}e_{24}e_{25} + e_{23}e_{24}e_{26} + e_{23}e_{25}e_{26} + \mathbf{e_{24}e_{25}e_{26}})$
$+3e_{23}e_{24}e_{25}e_{26}]$.

Again, noting that only bold expressions are non-zero and equal to k, we find,

$\frac{\partial(\Pr(2\text{ wins}))}{\partial e_2} = \frac{1}{(2q)^5}[(\mathbf{2q})^4 - (2q)k^3] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$.

$\frac{\partial(\Pr(3\text{ wins}))}{\partial e_3} = \frac{1}{(2q)^4}[(\mathbf{2q})^3 - \mathbf{e_{34}e_{35}e_{36}}] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$.

$\frac{\partial(\Pr(4\text{ wins}))}{\partial e_4} = \frac{1}{(2q)^4}[(\mathbf{2q})^3 - \mathbf{e_{41}e_{42}e_{43}}] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$.

$\frac{\partial(\Pr(5\text{ wins}))}{\partial e_5} = \frac{1}{(2q)^5}[(\mathbf{2q})^4 - (2q)(\mathbf{e_{51}e_{52}e_{53}} + e_{51}e_{52}e_{54} + e_{51}e_{53}e_{54} + e_{52}e_{53}e_{54})$
$+3e_{51}e_{52}e_{53}e_{54}] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$.

$\frac{\partial(\Pr(6\text{ wins}))}{\partial e_{6}6} = \frac{1}{(2q)^6}[(\mathbf{2q})^5 - (2q)^2(\mathbf{e_{61}e_{62}e_{63}} + e_{61}e_{62}e_{64} + e_{61}e_{62}e_{65} + e_{61}e_{63}e_{64}$
$+e_{61}e_{63}e_{65} + e_{61}e_{64}e_{65} + e_{62}e_{63}e_{64} + e_{62}e_{63}e_{65} + e_{62}e_{64}e_{65} + e_{63}e_{64}e_{65})$
$+3(2q)(e_{61}e_{62}e_{63}e_{64} + e_{61}e_{62}e_{63}e_{65} + e_{61}e_{62}e_{64}e_{65} + e_{61}e_{63}e_{64}e_{65} + e_{62}e_{63}e_{64}e_{65})$
$-3(e_{61}e_{62}e_{63}e_{64}e_{65})] = \frac{1}{2q} - \frac{k^3}{(2q)^4}$. ■

**Theorem 4:Proof.**

Note that the marginal probability of winning function is identical for all agents in these unfair tournaments and equal to $\frac{\partial(\Pr(i\text{ wins}))}{\partial e_i} = \frac{1}{2q} - \frac{k^{\frac{n}{2}}}{(2q)^{\frac{n}{2}+1}}$. This fact allows us to proceed as we did in the proof of Theorem 2. Substituting

$\frac{1}{2q} - \frac{k^{\frac{n}{2}}}{(2q)^{\frac{n}{2}+1}}$ for $\frac{\partial \pi(e_i, e_{-i})}{\partial e_i}$ in (2.8) we see that for each agent i

$$\frac{\partial E z_i}{\partial e_i} = \left(\frac{1}{2q} - \frac{k^{\frac{n}{2}}}{(2q)^{\frac{n}{2}+1}}\right)[M - m] - \frac{2e_i}{c} = 0. \tag{7.7}$$

For any fixed n this condition can be written as

$$\frac{2e_i}{c} = \mathcal{B}[M - m], \text{ where } \mathcal{B} = \left(\frac{1}{2q} - \frac{k^{\frac{n}{2}}}{(2q)^{\frac{n+1}{2}}}\right). \tag{7.8}$$

The left hand side of this equation is a linear increasing function of $e_i$ that is identical for all agents since they have identical cost of effort functions. The right hand side is a constant. Hence the existence of an $e_i$ = e* for i = 1,2,....,n, that satisfies this condition is guaranteed . Second order conditions and conditions guaranteeing an interior equilibrium must, of course, also be checked.

## 7.2. Appendix 2: Instructions

### Instructions for =2, k=0, n = 4 Experiment

#### Introduction

This is an experiment in decision making. The instructions are simple; if you follow them carefully and make good decisions, you could earn a considerable amount of money, which will be paid to you in cash.

As you read these instructions you will be in a room with a number of other subjects. Each subject has been randomly assigned an ID number.

The experiment consists of 20 decision rounds. In each decision round you will be grouped with three other subjects by a random drawing of ID numbers. These three subjects will be called your "group members." Your group members will remain the same throughout the entire experiment. The identity of your group members will not be revealed to you.

Your group members are of two different types: "Blue" and "Green." Out of the 4 people in your group, 2 are Blue and 2 are Green. When the experiment begins, the computer will randomly assign you to be either a "Blue" or "Green" type, and one of the people running the experiments will tell you what type you have been assigned. Each individual will remain the same type throughout all rounds of the experiment.

Experimental Procedure

In the experiment you will perform a simple task. Attached to these instructions is a sheet called your "Decision Costs Table." This sheet shows 101 numbers from 0 to 100 in the first column. These are your Decision Numbers. All subjects have the same"Decision Costs Table."

Associated with each Decision Number on the Decision Costs Table are Decision Costs for the Blue and Green. Note that for each type, Blue and Green, the higher the Decision Number chosen, the greater is the associated cost.

Note also, however, that these costs differ by type. If your type is Blue, take your Decision Costs from the second (middle) column. If your type is Green, your Decision Costs come from the third column. Green Decision Costs are higher than Decision Costs from type Blue individuals.

Your computer screen should look like the following:

GAME: TOURNAMENTS

PLAYER ID# :

ROUND DECISION # RANDOM # TOTAL # COST EARNINGS

In each decision round the computer will ask each subject to choose a Decision Number. Therefore, you and your group members will each separately choose one Decision Number. Using the number keys at the top of the keyboard, you will enter your selected number and then hit the Return (Enter) key. To verify your selection, the computer will then ask you the following question:

Is Your number? [Y/N]

If you want to select the number shown as your Decision Number, hit the Y key. If not, hit the N key and the computer will ask you to select a number again. You do not need to hit the Return Key after entering Y or N. After you have selected and verified the Decision Number, this number will be recorded on the screen in Column 2, and its associated cost will be recorded in Column 5.

After you have selected your Decision Number, the computer will ask you to generate a random number. You do this by hitting the space bar (the long key at the bottom of the keyboard). Hitting the bar causes the computer to select one of the 121 numbers that fall between -60 and +60 (including 0). Each of these 121 numbers has an equally likely chance of being chosen when you hit the space bar. Hence, the probability that the computer selects, say, +60, is the same as the probability that it selects -60, 0, -15 or +23.

Each subject will follow the same procedure, so that each subject generates his or her random separately. After you hit the space bar, the computer will record your random number on the screen in Column 3.

**Calculation of Payoffs**

Your payment in each decision round will be computed as follows. After you select a Decision Number and generate a random number, the computer will add these two numbers and record the sum on the screen in Column 4. We will call the number in column 4 your "Total Number." After every members of your group has had his or her Total Number recorded, the computer will compare all of the Total Numbers. On the basis of this comparison, the computer will tell you whether you receive the "Fixed Payment" 2.04 or the Fixed Payment 0.86.

Recall that there are 4 members in your group. If your Total Number is one of the 2 highest in your group, then you will receive the Fixed Payment 2.04. If it is one of the two lowest, you will receive the Fixed Payment 0.86. If any group members have the same Total Number and it makes a difference in the Fixed Payment allocated, then the computer will randomly determine which of these "tied" members receives the high Fixed Payment. For example, if both you and another group member have the same Total Number, and that Total Number places you on the "borderline" of receiving the high Fixed Payment or the low one, the computer will randomly decide which group member gets the Fixed Payment and which group member receives the lower one. Think of this procedure as though the computer is assigning "heads" to one group member, "tails" to the other, and then flipping a coin. If "heads" turn up, the group member assigned "heads" receives the high Fixed Payment.

Whether you receive the high Fixed Payment or the low Fixed Payment depends only on whether your Total Number if greater than the Total Numbers of at least 2 other group members. IT DOES NOT DEPEND ON HOW MUCH GREATER IT IS.

The computer will record (on screen in Column 6) which Fixed Payment you receive. If you receive the high Fixed Payment (2.04), then "M" will appear on Column 6. If you receive the low Fixed Payment (0.86), "m" will appear.

After indicating which Fixed Payment you receive, the computer will subtract your associated Decision Cost (Column 5) from this Fixed Payment. This difference represents your earnings for the round. The amount of your earnings will be recorded on the screen in Column 6, right next to the letter ("M" or "m") showing your Fixed Payment. The earnings of your group members will be calculated in exactly the same way.

**Continuing Rounds**

After Round 1 is over, you will perform the same procedures for Round 2, and so on for 20 rounds. In each round you will choose a Decision Number (though, of course, you may choose the same one), you will again generate a random number by pressing the space bar, your Total Number will be compared to the Total Numbers of the other members of your group, and the computer will calculate your earnings for the round.

Example of Payoff Calculations

In a 4-person group with 2 large and 2 small fixed payments, suppose that group member A2 is Green and Group members A1 , A3 , and A4 are Blue. Then the following might occur:

Group member A1 (Blue) chooses Decision Number 60 and generates random number 10.

A 1s Total Number is therefore equal to 70.

Group member A2 (Green) chooses Decision Number 53 and generates random number 2.

A2's Total Number is therefore equal to 55.

Group member A3 (Blue) chooses Decision Number 79 and generates random number -28.

A3s Total Number is therefore equal to 51.

Group member A4 (Blue) chooses Decision Number 31 and generates random number 33.

A4's Total Number is therefore equal to 64.

Since there are two high Fixed Payments, group members A1 and A4 would each receive 2.04. From these Fixed Payments, A1 would subtract 0.24 (the cost of Decision Number 60 for Blue) and A4 would subtract 0.06407 (for Decision Number 31). Group members A2 and A3 would each receive the low Fixed Payment, 0.86; A2 would subtract 0.37454 (the cost of Decision Number 53 for Green); A3 would subtract 0.41607.

Note that the Decision Cost subtracted in Column 5 is a function only of the Decision Number selected; i.e., your random number does not affect the amount subtracted. Also, note that your earnings depend on the following: the Decision Number you select (both because it contributes to your Total Number and because it determines the amount to be subtracted from your Fixed Payment), the Decision Numbers your group members select, your generated random number, and your group members' generated random numbers.

When Round 20 is completed, the computer will ask you to press any key on its keyboard. After you do this, the computer will add your earnings from each of the 20 rounds, multiply the total by 0.5 and subtract a fixed cost of $2.00 from this sum. We will then pay you this amount. You are free to make as much money as possible.

DECISION COST TABLE[2]

| Decision Number | Blue Cost | Green Cost | Decision Number | Blue Cost | Green Cost |
|---|---|---|---|---|---|
| 0 | 0.00000 | 0.00000 | 50 | 0.16667 | 0.33333 |
| 1 | 0.00007 | 0.00013 | 51 | 0.17340 | 0.34680 |
| 2 | 0.00027 | 0.00053 | 52 | 0.18027 | 0.36053 |
| 3 | 0.00060 | 0.00120 | 53 | 0.18727 | 0.37453 |
| 4 | 0.00107 | 0.00213 | 54 | 0.19440 | 0.38880 |
| 5 | 0.00167 | 0.00333 | 55 | 0.20167 | 0.40333 |
| 6 | 0.00240 | 0.00480 | 56 | 0.20907 | 0.41813 |
| 7 | 0.00327 | 0.00653 | 57 | 0.21660 | 0.43320 |
| 8 | 0.00427 | 0.00853 | 58 | 0.22427 | 0.44853 |
| 9 | 0.00540 | 0.01080 | 59 | 0.23207 | 0.46413 |
| 10 | 0.00667 | 0.01333 | 60 | 0.24000 | 0.48000 |
| 11 | 0.00807 | 0.01613 | 61 | 0.24807 | 0.49613 |
| 12 | 0.00960 | 0.01920 | 62 | 0.25627 | 0.51253 |
| 13 | 0.01127 | 0.02253 | 63 | 0.26460 | 0.52920 |
| 14 | 0.01307 | 0.02613 | 64 | 0.27307 | 0.54613 |
| 15 | 0.01500 | 0.03000 | 65 | 0.28167 | 0.56333 |
| 16 | 0.01707 | 0.03413 | 66 | 0.29040 | 0.58080 |
| 17 | 0.01927 | 0.03853 | 67 | 0.29927 | 0.59853 |

## DECISION COST TABLE (CONTINUED)

| Decision Number | Blue Cost | Green Cost | Decision Number | Blue Cost | Green Cost |
|---|---|---|---|---|---|
| 18 | 0.02160 | 0.04320 | 68 | 0.30827 | 0.61653 |
| 19 | 0.02407 | 0.04813 | 69 | 0.31740 | 0.63480 |
| 20 | 0.02667 | 0.05333 | 70 | 0.32667 | 0.65333 |
| 21 | 0.02940 | 0.05880 | 71 | 0.33607 | 0.67213 |
| 22 | 0.03227 | 0.06453 | 72 | 0.34560 | 0.69120 |
| 23 | 0.03527 | 0.07053 | 73 | 0.35527 | 0.71053 |
| 24 | 0.03840 | 0.07680 | 74 | 0.36507 | 0.73013 |
| 25 | 0.04167 | 0.08333 | 75 | 0.37500 | 0.75000 |
| 26 | 0.04507 | 0.09013 | 76 | 0.38507 | 0.77013 |
| 27 | 0.04860 | 0.09720 | 77 | 0.39527 | 0.79053 |
| 28 | 0.05227 | 0.10453 | 78 | 0.40560 | 0.81120 |
| 29 | 0.05607 | 0.11213 | 79 | 0.41607 | 0.83213 |
| 30 | 0.06000 | 0.12000 | 80 | 0.42667 | 0.85333 |
| 31 | 0.06407 | 0.12813 | 81 | 0.43740 | 0.87480 |
| 32 | 0.06817 | 0.13653 | 82 | 0.44827 | 0.89653 |
| 33 | 0.07260 | 0.14520 | 83 | 0.45927 | 0.91853 |
| 34 | 0.07707 | 0.15413 | 84 | 0.48040 | 0.94080 |
| 35 | 0.08167 | 0.16333 | 85 | 0.48167 | 0.96333 |
| 36 | 0.08640 | 0.17280 | 86 | 0.49307 | 0.98613 |
| 37 | 0.09127 | 0.18253 | 87 | 0.50460 | 1.00920 |
| 38 | 0.09627 | 0.19253 | 88 | 0.51627 | 1.03253 |
| 39 | 0.10140 | 0.20280 | 89 | 0.52807 | 1.05613 |

DECISION COST TABLE (CONTINUED)

| Decision Number | Blue Cost | Green Cost | Decision Number | Blue Cost | Green Cost |
|---|---|---|---|---|---|
| 40 | 0.10667 | 0.21333 | 90 | 0.54000 | 1.08000 |
| 41 | 0.11207 | 0.22413 | 91 | 0.55207 | 1.10413 |
| 42 | 0.11760 | 0.23520 | 92 | 0.56127 | 1.12853 |
| 43 | 0.12327 | 0.24653 | 93 | 0.57660 | 1.15320 |
| 44 | 0.12907 | 0.25813 | 94 | 0.58907 | 1.17813 |
| 45 | 0.13500 | 0.27000 | 95 | 0.60467 | 1.20333 |
| 46 | 0.14107 | 0.28213 | 96 | 0.61440 | 1.22880 |
| 47 | 0.14727 | 0.29453 | 97 | 0.62727 | 1.25453 |
| 48 | 0.15360 | 0.30720 | 98 | 0.64027 | 1.28053 |
| 49 | 0.16007 | 0.32013 | 99 | 0.65340 | 1.30680 |
| | | | 100 | 0.66667 | 1.33333 |

FIGURE 1

Mean Effort Choices in 2, 4, 6 Person Fully Symmetric Tournaments

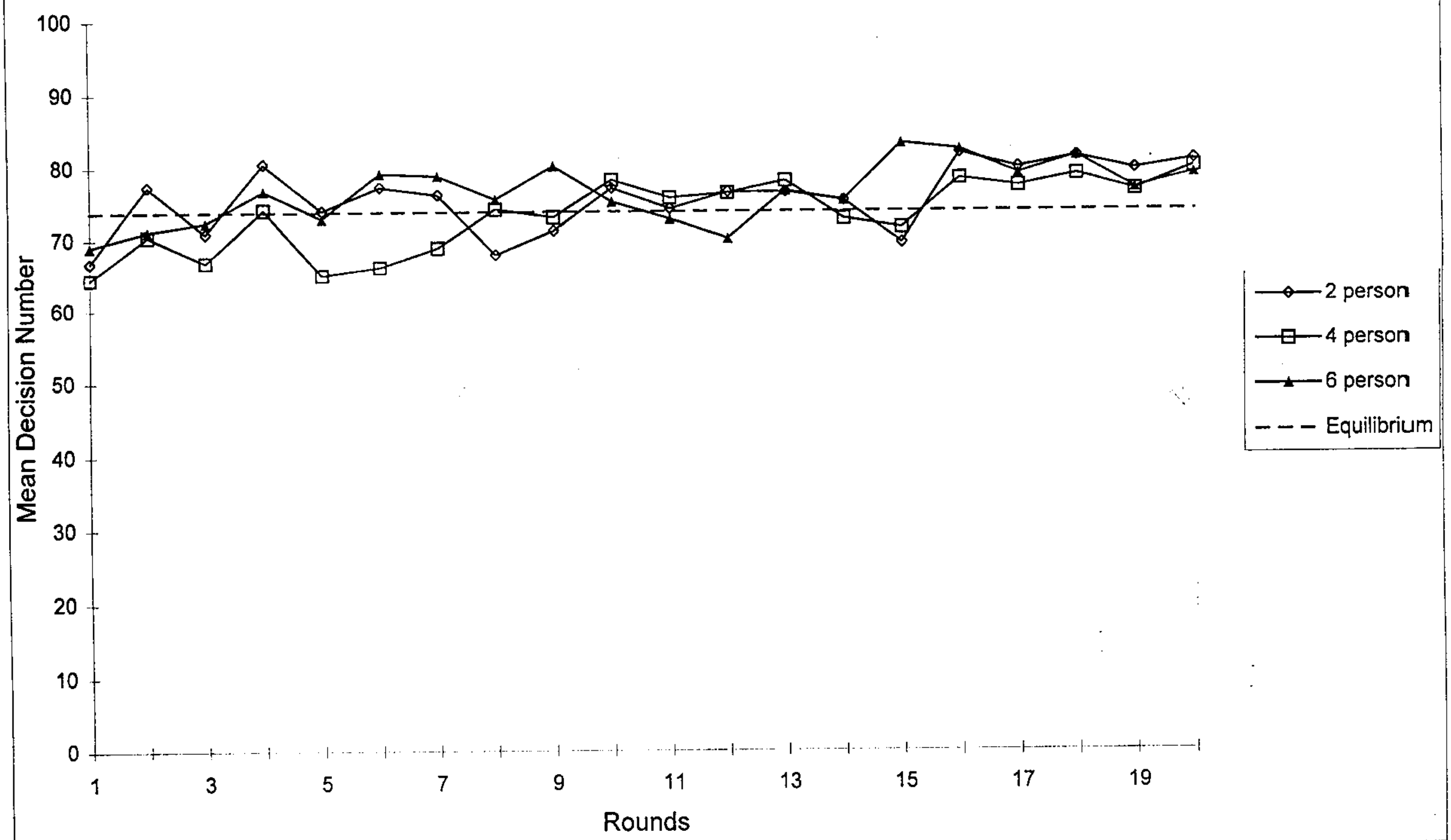

FIGURE 2

Mean Effort Choices in 6-Person Prize-Asymmetric Tournaments ($\lambda$ = 1/2, 1/3, 2/3)

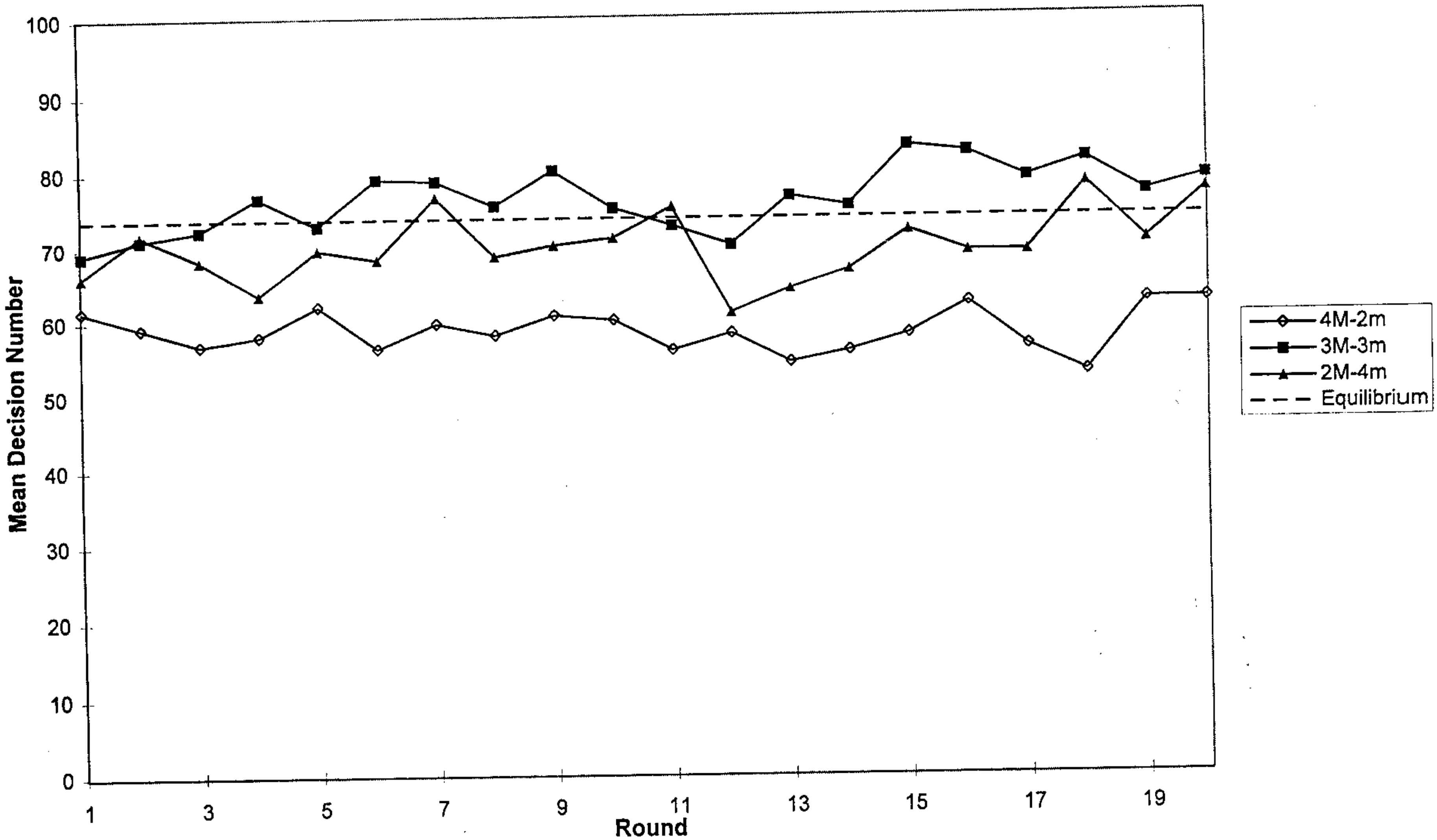

FIGURE 3

Organizational Cost per Unit of Effort in 6-Person Prize-Asymmetric Tournaments

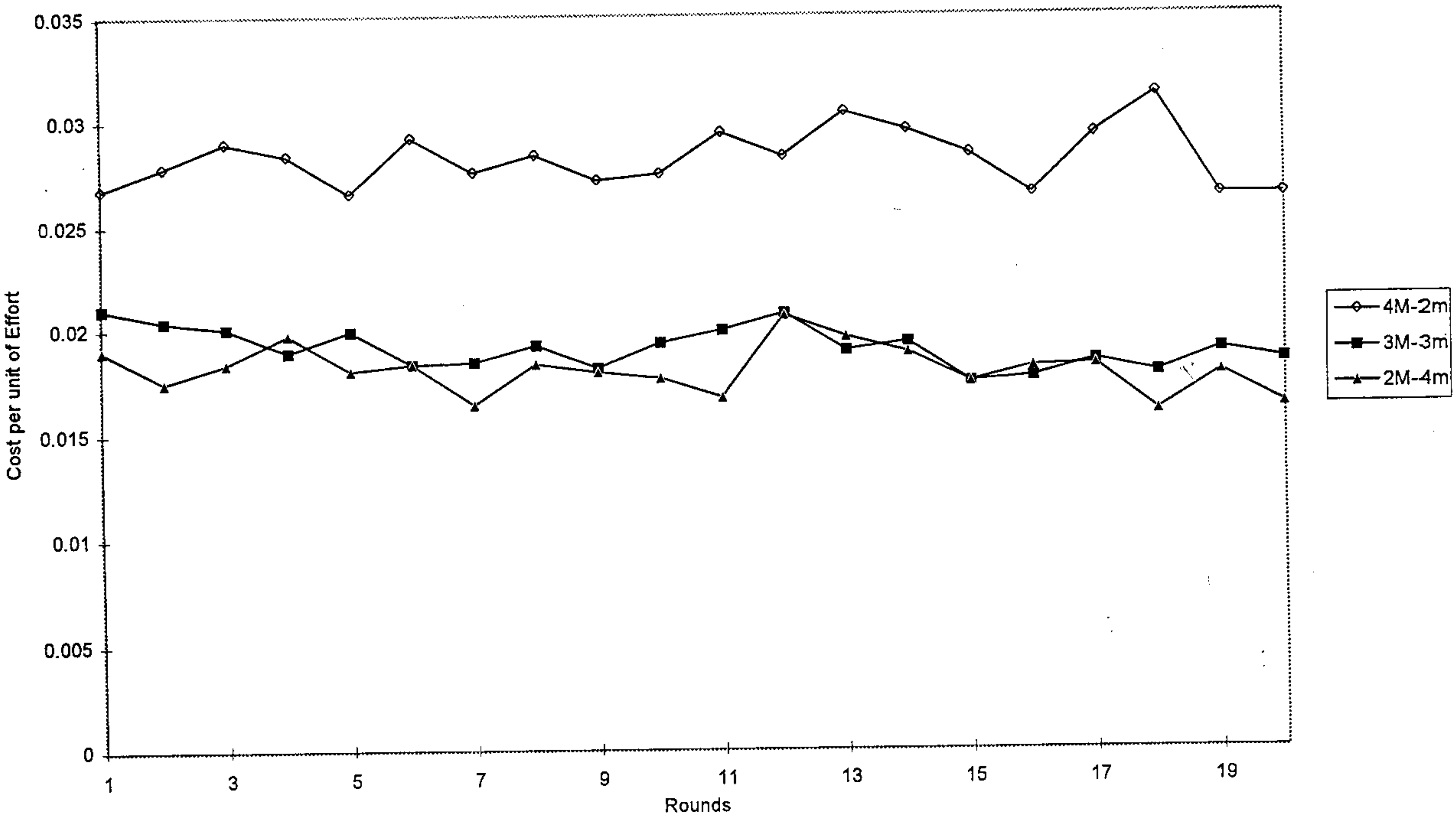

TABLE 1

Experimental Parameters

| Experi-ment | Tourna-ment size | Decision number range | Cost function | Random # range | M | m | Number of M | Predicted Equilibrium Advan. - | Disadvan. | Number of subjects |
|---|---|---|---|---|---|---|---|---|---|---|
| Symmetric tournaments | | | | | | | | | | |
| 1 | 2 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 1 | 73.75 | | 24 |
| 2 | 4 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 2 | 73.75 | | 52 |
| 3 | 6 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 3 | 73.75 | | 24 |
| 4 | 6 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 4 | 73.75 | | 24 |
| 5 | 6 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 2 | 73.75 | | 24 |
| Asymmetric tournaments - ½ of subjects advantaged, (K = 25) | | | | | | | | | | |
| 6 | 2 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 1 | 58.39 | 58.39 | 18 |
| 7 | 4 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 2 | 70.55 | 70.55 | 36 |
| 8 | 6 | (0 - 100) | $e^2/15,000$ | (-60, 60) | $2.04 | $0.86 | 3 | 73.08 | 73.08 | 42 |

TABLE 2

Organizational replications of fully symmetric tournaments

| | | Mean decision numbers | | | Maximum deviation | | Mean deviation | Mean Standard Deviations | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Experiment | predicted | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 | Rounds 1 - 10 | Rounds 11 - 20 | Rounds 11 - 20 | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 |
| 2-person N = 24 | 73.75 | 73.87 | 77.91 | 80.75 | 6.95 | 9.0 | 5.3 | 25.04 | 24.75 | 23.46 |
| 4-person N = 52 | 73.75 | 70.12 | 76.47 | 79.77 | 9.25 | 6.0 | 3.7 | 9.83 | 9.49 | 5.51 |
| 6-person N = 24 | 73.75 | 75.07 | 77.59 | 78.85 | 9.44 | 9.4 | 4.8 | 7.04 | 7.88 | 2.8 |

TABLE 3

6-person prize-asymmetric tournaments - different compensation designs

| | | Mean decision numbers | | | Maximum deviation | | Mean deviation | Mean Standard Deviations | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Experiment | predicted | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 | Rounds 1 - 10 | Rounds 11 - 20 | Rounds 11 - 20 | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 |
| 2M - 4m N = 18 | 73.75 | 69.42 | 70.25 | 77.10 | 10.22 | 12.88 | 5.3 | 6.18 | 7.84 | 15.42 |
| 3M - 3m N = 24 | 73.75 | 75.07 | 77.59 | 78.85 | 6.95 | 9.44 | 4.8 | 7.04 | 7.88 | 2.83 |
| 4M - 2m N = 24 | 73.75 | 59.18 | 57.77 | 62.33 | 17.42 | 20.97 | 16.1 | 15.7 | 19.63 | 16.18 |

TABLE 4

Observed cost per unit of effort in prize asymmetric tournaments

| Compensation design | Predicted cost/unit | Average Rounds 1 - 10 | Average Rounds 11 - 20 | Round 20 Cost | Number of rounds below predicted in Rounds 11 - 20 |
|---|---|---|---|---|---|
| 2M - 4m | .0170 | .0181 | .0179 | .0163 | 3 |
| 3M - 3m | .0197 | .0194 | .0187 | .0184 | 8 |
| 4M - 2m | .0223 | .0279 | .0286 | .0264 | 0 |

TABLE 5

Organizational replications of asymmetric tournaments
½ of organizational subjects - advantaged

| | | Mean decision numbers | | | Maximum deviation | | Mean deviation | Mean Standard Deviations | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Experiment | predi cted | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 | Rounds 1 - 10 | Rounds 11 - 20 | Rounds 11 - 20 | Rounds 1 - 10 | Rounds 11 - 20 | Round 20 |
| 2-person N = 18 | 58.39 | 65.27 | 66.58 | 65.33 | 14.66 | 12.61 | 7.95 | 25.16 | 28.91 | 32.22 |
| 4-person N = 36 | 70.55 | 53.89 | 59.19 | 62.58 | 21.61 | 15.8 | 11.35 | 22.14 | 23.62 | 24.50 |
| 6-person N = 42 | 73.08 | 62.65 | 64.60 | 67.76 | 13.69 | 12.56 | 8.48 | 13.21 | 14.91 | 12.49 |