THE ROLE OF INFORMATION IN DESIGNING SOCIAL POLICY TOWARDS EXTERNALITIES

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This paper analyzes the externality problem afresh, taking explicit account of the information structure. The analysis is conducted from the viewpoint of agents who are affected by externalities but who are imperfectly informed about the parties who generate them. Information collection is costly, and the amount gathered is determined endogeneously to avoid prior specification of either perfect or no personalized information.[1]

The result of this approach is a new class of abstract allocation mechanisms that outperform those previously studied, when information is costly, and that model many extant social programs. The presented mechanisms combine an impersonal market price with programs to allocate the good to selected groups of consumers. Each program offers a special price to all applying agents who are validated to possess certain characteristics or to exhibit the requisite signals. A suitable application fee ensures that only qualified agents self-select into the costly screening phases of the programs. This process of self-selection economizes on information costs to such an extent that price discrimination can be regarded as a viable instrument of social policy. In fact, existing social programs (e.g., Medicaid, R&D grants, and scholarship funds) do contain the key elements of price discrimination and self-selection.

Most of the paper is devoted to analysis of the socially optimal allocation mechanism under the assumption that the component programs are designed and implemented by an idealized government. Such centralization may, in fact, be the only feasible approach when the effects of the externalities are diffuse and when interpersonal transactions costs are high. Otherwise, however, given the delineation of property rights, an Agent for those affected by the externalities would have the incentive and ability to implement the programs that combine self-selection and price discrimination. We use the socially optimal set of government run allocative mechanisms as a benchmark for the evaluation of the programs that would be optimal for the private Agent.

We work with the simplest possible model. There is a particular discrete activity which each agent either does or does not perform and which requires a single input with no other uses. This input has a constant marginal cost of production, c. Every agent is characterized by his private benefit, or reservation price, B, for the activity (or its input) and by the total external impact, E, (in money units) from his performing the activity. The values of B and E corresponding to each agent are assumed to be exogeneous and independent of the allocative events endogeneous to the model.

Consumption of the input good, or alternatively, performance of the activity, results in externalities. We restrict attention to the case of positive externalities and assume that for each agent both B and E are nonnegative.[2] The normative framework chosen does not distinguish among costs and benefits that accrue to different individuals and to the government.

We assume that the continuous joint distribution of B and E in the population of agents, h(B,E), is freely available information. Agents themselves know their own B's and will sometimes be assumed to know their own E's and other identifying characteristics as well. However, the characteristics of individuals cannot be identified without costly screening, testing, and sorting processes.

In Section II, we consider the benchmark case in which the government develops no personalized information. Here, the government can only set a single, nondiscriminatory, market price for the input good, and we analyze its welfare optimal level. (We call this the pure price subsidy case.) We show that this scheme generally leads to a poor allocation. The basic reason is that a market price, p, can only sort agents by means of their reservation prices into a half-space set of buyers with $B \geq p$, whereas the agents whose consumptions are socially desirable form the set with $B + E \geq c$.

In Section III we develop the theme that society need not confine itself to the instrument of a single market price. Instead, it can break the bounds of the pure price subsidy scheme by utilizing a carefully controlled amount of personalized information to achieve a limited degree of price discrimination. Optimally chosen special prices offered to self-selecting subgroups of agents balance allocative gains against information costs. These programs allocate the good to some agents whose consumptions bring them only small private benefits but which yield large beneficial externalities.

In Section IV, we study an allocation mechanism which implements perfect price discrimination, over an optimally chosen subset of the population, by validating self-selected agents' B's and E's. The program is called conditional perfect price discrimination (abbreviated to cppd). We show that no matter how costly it is to discover an agent's B and E, it is always socially advantageous to append a program of cppd to any other allocative scheme. Moreover, we prove that when information is costly, a program of conditional perfect price discrimination, together with an optimized market price below marginal cost, strictly outperforms classical perfect price discrimination.

Maintaining the presence of cppd, in Section V we study the desirability and market price effects of utilizing other arbitrarily specified signals to sort the population into groups facing distinct prices. We show that as the costs of personalized information drop from infinity to zero, the optimal market price rises from the pure subsidy case level up to marginal cost.

In Section VI we see that specializing the analysis to signals which reveal agents' E's generates a program of conditional perfect internalization. This mechanism is allocatively superior to conditional perfect price discrimination, but it may incur significantly greater information costs. We prove that when information is costly, a properly chosen mixture of conditional perfect internalization and conditional perfect price discrimination, together with a subsidized market price, strictly outperforms a complete program of classical internalization.

Section VII analyzes the behavior of an individual agent who acts as if he were the sole recipient of the externalities, given that all are entitled by the legal system not to perform the activity. Assuming that the agent faces the same information structure and costs as did the government in the earlier sections, we show that the decentralized allocation is socially optimal if the private benefits of all consumers can be costlessly revealed. However, if it is costly to ascertain the private benefits of consumers with B's near c, then the decentralized allocation systematically deviates from the socially optimal one.

By way of conclusion, Section VIII reviews the information structures of the allocation mechanisms introduced in the previous sections, recapitulates the crucial informational assumptions, and begins the discussion of the consequences of relaxing them.

II. Pure Price Subsidy

In a pure price subsidy scheme the only instrument used by the government (or center) to effect the allocation of the input is its price. An agent purchases the input and performs the activity if and only if his reservation price is not less than the impersonal market price. Each agent who performs the activity adds his B + E to social welfare; the social cost is the constant marginal cost, c, of the input. Thus, at price p, the money measure of the net social welfare resulting from the allocation of the good is

(1) $$W = \int_p^\infty \int_0^\infty (B+E-c)h(B,E)dEdB \ .$$

This implies

(2) $$\frac{\partial W}{\partial p} = - \int_0^\infty (p+E-c)h(p,E)dE = [c - p - m(E|p)] \int_0^\infty h(p,E)dE \ ,$$

where $m(E|B)$ is the average externality from the consumptions of the agents with reservation price B.

We assume throughout that $h(B,E) > 0$ over the relevant region. Then, the first order condition for an interior $(p>0)$ optimal choice of price is

(3) $$p + m(E|p) = c \ .$$

Equation (3) says that the interior optimal price is below marginal cost by an amount equal to the average externality from those agents for whom $B = p$. These agents are the marginal buyers of the input and it is their E's that would be lost to society if the price were marginally raised.

Another way to view (3) is to consider as a function of B, $B + m(E|B)$. This is the average gross social value of the consumption of the input by all agents with reservation price B. The first order optimality condition is that <u>at $B = p^*$ (the optimal price), $B + m(E|B) = c$, or in words, marginal cost equals the average social benefit from the marginal</u>

buyers. Figure 1 pictures a $B + m(E|B)$ function and shows that p^* may be found at its intersection with the horizontal at c.

The second order conditions ($\partial^2 W/\partial p^2 < 0$) for a p satisfying (3) to be optimal is that

$$(4) \qquad d[B + m(E|B)]/dB > 0 \quad \text{at} \quad B = p \ .$$

Thus, p* can only be an interior optimum if the average gross social value function is increasing at that point. Since the buyers are those with $B > p$, (3) and (4) together say that the average social benefit exceeds marginal cost for the first infra-marginal buyers, and inversely for the first infra-marginal non-buyers.

Conditions (3) and (4) are necessary only for an interior solution to the maximization of (1). The extremes of a zero price, or an ineffective price above all reservation prices cannot be ruled out a priori. For example, if the average gross social benefit function were everywhere above the marginal cost line ($B + m(E|B) > c$, for all B), then $p^* = 0$ would be indicated.[3]

The market price should exclude all consumption if $B + m(E|B) < c$ for all B with positive weight in h. On the other hand, p* should be effective if $B + m(E|B) > c$ for the largest B's in the population. In particular, since we are restricting ourselves to activities with non-negative E's, p* should be effective and not above c if there are any agents with $B \geq c$. If there were no such agents, it would suffice to set $p = c$ for p* to be ineffective and to exclude all consumption. Thus, we need never consider market prices above c.

The optimal market price will generally fail to perfectly allocate the input, for two fundamental reasons. First, the agents with a particular B will usually have diverse E's, some below and some above the average, $m(E|B)$. This variance will lead to misallocation, even with an interior optimal price p* (satisfying (3) and (4)) which equates marginal cost to marginal average social benefit. In Figure 2, the horizontally shaded region contains the (E,B) points of agents who will purchase the good because their B's exceed p*, despite the fact that their purchases are socially injurious since $B + E < c$. The vertically shaded set represents agents who will not buy, since $B < p^*$, despite the social desirability ($B+E > c$) of their purchase. These misallocations are an unavoidable consequence of the fact that while the social desirability of consumption depends on B + E, a price system can only sort agents on the basis of their B's.

Second, a market price can only sort agents into the groups with $B \geq p$ (buyers) and $B < p$ (non-buyers). While this would be the appropriate dichotomization if the $B + m(E|B)$ function were monotone increasing in B, it becomes a serious limitation otherwise. If the gross average social value curve crosses the horizontal at c more than once, then any market price must misallocate the good (in either direction) with respect to an entire group of agents with the same B.[4] If the $B + m(E|B)$ function is decreasing over the population's B's, and crosses the horizontal at c, (4) shows that the only candidates for optimality are a zero price and an ineffectively large price. Either one would misallocate the good to all those agents with B's in an entire interval. These welfare losses would occur even if there were no variance in the distribution of the E's of the agents with each B. The realistic presence of such variance compounds the misallocation with the effect discussed above.

To circumvent the misallocations caused by the rigidity of the pure price subsidy allocation scheme, society offers inputs to externality producing activities at special low rates (perhaps zero), through a variety of programs, to a variety of groups, while a regular market price is maintained for everyone else.[5] Such allocative programs screen agents for signals indicative of their membership in a subset of the population with a distinctive distribution of B and E. Then, a special price which is optimal for this subset can be presented to the agents exhibiting the proper signals.

III. Screening Mechanisms and Self-Selection

Let us suppose that the government knows not only the distribution h(B,E) in the whole population, but that it also knows the distribution $h^{\alpha}(B,E)$ in the subpopulation of agents who exhibit the unalterable[6] characteristic α. If the government could costlessly identify those agents with α, then the overall allocation would surely be (weakly) improved by using the signal α to dichotomize the population into groups facing different market prices for the good. Each group-specific price could be set optimally with (1)-(4) separately applied to the group's distribution $h - h^{\alpha} \equiv h^{\bar{\alpha}}$ and to h^{α}.

Generally, though, it is costly to verify that an agent has the characteristic α, and it is not clear that these costs won't outweigh the welfare gains from the price discrimination. It is not necessary, however to test the characteristics of all agents.

The center can induce agents to self-select into the screening program on the basis of their own known characteristics. The government can require a small fee of those applying for the α-test (or force people to incur a transactions cost), and announce that only those who are validated to exhibit α will receive a special price. Then, assuming that people believe both the announcement and the accuracy of the testing procedure, only agents who do actually have the characteristic α will apply for the test. Further, among the agents with α, only those who find the special price attractive enough to induce purchase will have the incentive to enter the program. These agents must, indeed, be tested so as to assure the credibility of the program.[7]

Given that agents are to be offered the special price p_α if they select themselves into the α testing program, the social welfare function is

$$(5) \quad W = \int_{p}^{\infty} \int_{0}^{\infty} (B+E-c)h^{\bar{\alpha}}(B,E)dEdB + \int_{p_\alpha}^{\infty} \int_{0}^{\infty} (B+E-c-s)h^{\alpha}(B,E)dEdB \ .$$

Here, s is the cost of validating α and, due to the self-selection, it is only incurred for the agents with characteristic α who have $B \geq p_\alpha$. Of course, the testing program would attract no one if $p_\alpha > \dot{p}$, and (5) must be optimized subject to that constraint. Note, however, that if the constraint were binding, p_α^* would equal p^*, society would gain nothing from the sorting procedure, information costs would be borne needlessly, and the α testing program would be wasteful.

Then, assuming that the α-test is desirable, the $p_\alpha \leq p$ constraint is not binding, and the first order conditions for interior optimal prices are [analogous to (3)]:

(6) $$p + m^{\bar{\alpha}}(E|p) = c$$

and

(7) $$p_\alpha + m^{\alpha}(E|p_\alpha) = c + s$$

Here, $m^{\bar{\alpha}}(E|p)$ is the mean E among the agents in the group without characteristic α who have reservation price p, and $m^{\alpha}(E|p_\alpha)$ is the mean E in the α group of those with $B = p_\alpha$. (7) shows that in the α-group, the average gross social benefit of the marginal buyers must equal marginal production cost plus the marginal information cost. Eliminating buyers by raising p_α saves both production costs, and the costs of validating the α signals of those agents.

Intuitively, an α-testing program would be worthwhile if most agents with α were socially desirable consumers ($B+E > c+s$) and if most wouldn't have been consuming without the costly α-test ($B<p^*$). Figure 3 pictures a subset of the population which meets these criteria.[8] Further, worthwhile screening programs must be based on truly informative signals; i.e., those characteristic of population subsets with genuinely distinctive distributions of B and E. In particular, if α is an uninformative signal, then $h^{\alpha}(B,E) = kh(B,E)$ and $m^{\alpha}(E|B) = m^{\bar{\alpha}}(E|B)$. Examination of (5) reveals that in this case at the unconstrained welfare maximum, $p_\alpha > p$. This result follows:

<u>Proposition</u>: If $h^{\alpha}(B,E)$ is proportional to $h(B,E)$, then screening on α is undesirable.

In general, there will be a large set of signals α_i that can be used to sort out population subgroups, each with a different testing cost s_i. The center is faced with the problem of choosing the best set of sorting signals, as well as the best corresponding group-specific prices. When many signals are utilized to sort the population, agents who can qualify for more than one special price will select themselves into the program offering the lowest one. From the social point of view, this will be adverse selection if the screening cost of the chosen program is not the smallest of the screening costs of the programs available to the agent. In view of this effect, in addition to the combinatorics of the problem, little more can be usefully said, in general, about the optimal choice of signals with which to sort the population. In the next section, however, we treat an important special case in which there is no adverse selection.

IV. Conditional Perfect Price Discrimination

In this section we show that irrespective of the signals that are used to sort agents into groups offered special prices, it will always (weakly) increase social welfare to offer, in addition, a program of conditional perfect price discrimination. This program screens self-selected agents for the social desirability of their consumptions with costly tests which reveal both their B's and their E's. These agents are then offered personalized prices just below their reservation prices.[9]

We model conditional perfect price discrimination (henceforth cppd) by assuming that agents know both their own E's and their own B's,[10] and that these can both be validated at a cost of t per agent. The government announces that for a small fee, an agent may apply for a special price which will be just below his measured B. The government will test the agent's E and B, and if and only if they meet the announced standards will the application fee be returned and the special price offered. We embody the announced standards in the index function $\phi(B,E)$, which will be set optimally. $\phi(B,E) = 1$ if the agents with characteristics B,E are to be offered a perfectly discriminatory price, and $\phi(B,E) = 0$ otherwise.

Suppose that signals $\alpha_1,\ldots,\alpha_m$ are being used to sort the population into groups facing prices $p_1,\ldots,p_m$, and that the remaining agents face the market price p. We consider adding to this scenario the cppd program. It will attract almost no one (in the measure theoretic sense) who was formerly buying at one of the prices p, $p_1,\ldots,p_m$, $(B \geq p_i)$, because cppd offers a personalized price just below B. Thus, the increment to W from the cppd scheme is

$$(8) \qquad \Delta W = \int_0^p \int_0^\infty \phi(B,E)[B+E-c-t]g(B,E)dEdB \ ,$$

where $g(B,E) \equiv h(B,E) - \sum_{\substack{1\leq i\leq m \\ B\geq p_i}} h^{\alpha_i}(B,E) \geq 0$, the distribution function of agents who are not buying at any of the special prices $p_1,\ldots,p_m$.

To find the optimal standards for the program, consider the first variation in ΔW due to changing $\phi(B,E)$ from 0 to 1. This is:

$$\delta(\Delta W) = [B+E-c-t]g(B,E) \ , \quad \text{for} \quad B < p \ .$$

Hence, clearly, the center should set $\phi(B,E) = 1$ iff $B + E \geq c + t$. Only agents whose gross social value of consumption exceeds both production and information costs should be encouraged to participate in cppd. With the announced standards so established, (8) shows that $\Delta W \geq 0$, and that $\Delta W > 0$ if the set of agents with $B + E > c + t$ who are not buying at p, $p_1,\ldots,p_m$ has positive measure. We have proven:

<u>Theorem 1</u>: Under our assumptions, it is (always weakly and sometimes strictly) socially desirable to append a program of conditional perfect price discrimination to any other allocative mechanism.

It is remarkable that perfect price discrimination, derived by Pigou in a theoretical context with costless information, should remain socially desirable (when modified in a natural way) no matter how costly information is. However, when there are information costs, cppd alone, without any α signaling, strictly outperforms the classical version, even though only the latter results in a perfect allocation of the good.

Theorem 2: If $t > 0$, conditional perfect price discrimination, with an optimal market price below marginal cost, strictly outperforms complete perfect price discrimination.

Proof: Under the classical version of perfect price discrimination, all agents are tested, and those with $B + E > c$ are given the good at or below their reservation prices. With information costs, it is clearly superior to set a market price of c. This eliminates the need to test those with $B > c$, without changing the allocation of the good. Further, it is more advantageous yet to introduce self-selection and to make the criterion $B + E > c + t$ to eliminate the agents whose social benefits fall short of the full social marginal costs. We have arrived now at cppd with $p = c$. Consider social welfare with cppd and no α screening as a function of market price:

$$W = \int_p^\infty \int_0^\infty (B+E-c)h(B,E)dEdB + \int_0^p \int_{\max(c+t-B,0)}^\infty (B+E-c-t)h(B,E)dEdB ,$$

$$\partial W/\partial p = - \int_0^{\max(c+t-p,0)} (p+E-c)h(p,E)dE - t \int_{\max(c+t-p,0)}^\infty h(p,E)dE .$$

For $p \geq c$, $\partial W/\partial p < 0$, as long as $t > 0$. Thus, to save on information costs, the optimal price is below c, despite the consequent misallocation of the good. Q.E.D.

It follows, of course, that if the α_i are desirable signals with which to sort the population, then classical price discrimination can be further improved upon when information is costly. Due to Theorem 1, we shall maintain the assumption of the existence of a cppd program and turn to the questions of the desirability of sorting programs and their effect on the optimal market price.

V. General Screening: Its Desirability and Its Impact on Market Price

With cppd, a market price p, and special prices $p_i < p$ for agents with signals α_i, the welfare function is

$$(9)\quad W = \int_p^\infty \int_0^\infty (B+E-c)g(B,E)dEdB + \sum_{i=1}^m \int_{p_i}^\infty \int_0^\infty (B+E-c-s_i)h^{\alpha_i}(B,E)dEdB$$

$$+ \int_0^p \int_{\max(c+t-B,0)}^\infty (B+E-c-t)g(B,E)dEdB \ .$$

Here, as in (8), g(B,E) is the distribution function of agents not buying at any of the special prices p_i, and $h^{\alpha_i}(B,E)$ is the distribution of agents buying at p_i.[11] We consider the welfare impact of establishing another special price, p_{m+1}, for agents with another characteristic α_{m+1}. Since agents with α_{m+1} may also have some of the other characteristics α_i, the total impact is generally difficult to gauge. Thus, we deal with a series of special cases which clarify the allocative issues.

Suppose that the set of agents with α_{m+1} (henceforth denoted A_{m+1}) has empty intersections with the sets of agents buying at the special prices p_i, at the market price p, and through the cppd program. Then, the desirability of screening on α_{m+1} can be judged irrespective of the other programs.

In this case, p_{m+1} should be set optimally in the same manner as was the pure subsidy price in Section II. However, here, the relevant population group is A_{m+1}, the pertinent distribution is $h^{\alpha_{m+1}}(B,E)$, and the associated social marginal cost is $c + s_{m+1}$. The net contribution to welfare at p^*_{m+1} can be written

$$(10)\qquad \int_{p^*_{m+1}}^\infty \left[B + m^{\alpha_{m+1}}(E|B) - c - s_{m+1}\right] \int_0^\infty h^{\alpha_{m+1}}(B,E)dEdB \ .$$

This expression exposes the fact that the net contribution is critically dependent upon the level of the gross average social benefit function of the group with characteristic α_{m+1}. If the consumptions of these agents do not yield large externalities, then their exclusion by the market price is completely appropriate. However, the definitive test of the desirability of the α_{m+1} program is whether the optimal p_{m+1}, ascertained with the methods of Section II, is effective; meaning it is below the market price, p. That is, no special price should be offered to the agents with α_{m+1} if the p^*_{m+1} which maximizes (10) is above p.

Application of the envelope theorem shows that the derivative of the optimized net welfare contribution with respect to s_{m+1} is $-\int_{p^*_{m+1}}^{\infty}\int_0^{\infty} h^{\alpha_{m+1}} dE dB$, or minus the number of agents buying at the special price. Thus, the desirability of the α_{m+1} program diminishes rapidly with increasing information cost.

To begin the process of removing the simplifying assumptions, let some agents with the α_{m+1} signal be potential participants in cppd. The allocations of the good to these agents will not be affected by the new program, but those with $B > p_{m+1}$ will have the incentive to purchase via the α_{m+1} scheme instead of through cppd. If $s_{m+1} > t$, then this will be adverse selection, and, in this case, all agents who are desirable consumers through the α_{m+1} track ($B+E \geq c+s_{m+1}$) would otherwise have been purchasing through the less socially costly cppd. This is the essence of the argument[12] which establishes:

<u>Theorem 3</u>: It is never desirable to sort agents by means of a signal more costly to validate than B and E.

If, instead, $s_{m+1} < t$, as we assume henceforth, then information costs are saved by the α_{m+1} program. Taking this to the extreme, if $A_{m+1} \subset \{a|B(a) + E(a) \geq c + s_{m+1}\}$,[13] then the new mechanism improves the allocation, economizes on information costs and is unambiguously desirable. More generally, if the group $A_{m+1} - \{a|B(a) + E(a) \geq c + t\}$, viewed as having the marginal cost $c + s_{m+1}$, has an effective optimal pure subsidy price,[14] then it is desirable to screen the group A_{m+1} for the signal α_{m+1}.

Complicating matters further, let some agents with α_{m+1} have B's greater than the market price. With an operational α_{m+1} program, $p_{m+1} < p$, and so these agents will choose the α_{m+1} track, incurring new information costs while leaving their consumption decisions unchanged. Such adverse selection makes the new program less desirable than it would otherwise be. Now the allocative gains and the information cost savings relative to cppd must be balanced against these extra information costs. Similarly, when A_{m+1} intersects other A_i's, although no new effects on the allocation of the good arise, information costs are affected by agent's choices among the tracks open to them. The α_{m+1} program is more or less desirable as s_{m+1} is below or above the information costs of other programs with participating groups which intersect A_{m+1}.

In general, the desirability of a screening program depends on the level of p, and obversely, the presence of a program affects the optimal p. Having already considered the former effect, we now study the effect on p* of adding the α_{m+1} track, holding constant the existence of the other α_i programs.

Working from (9), we see

$$(11)\quad \partial W/\partial p = -\int_0^{\max(c+t-p,0)} (p+E-c)g(p,E)dE - (t)\int_{\max(c+t-p,0)}^{\infty} g(p,E)dE .$$

Assuming an interior optimum ($p^* > 0$ and effective), at p^*, $\frac{\partial W}{\partial p} = 0$ and $\frac{\partial^2 W}{\partial p^2} < 0$. In the spirit of standard comparative statics, if the addition of the α_{m+1} program increases the $\partial W/\partial p$ function, then it follows (if $\partial^2 W/\partial p^2 < 0$ in the relevant range) that p^* increases.

If $A_{m+1} \cap \{a|B(a) = p\} = \Phi$, where p is optimal without the α_{m+1} program, then nothing in (11) is altered by the addition of the new program. Otherwise, (11) changes due to the decrease in the $g(p,E)$ function. If most of the agents in A_{m+1} who have $B = p$ (the marginal buyers at the market price) are socially desirable consumers ($B+E > c$), then the decrease in $g(p,E)$ from the addition of the α_{m+1} track increases $\partial W/\partial p$, and hence increases p^*.

As pictured in Figure 4, the benefit of raising p is the extension of the vertically shaded region of agents whose consumptions are socially undesirable and who are, indeed, not consuming.

One cost of raising p is the extension of the diagonally shaded region of agents who are desirable consumers but who are excluded by both the market price and by cppd (this cost and the aforementioned benefits are captured in the first term of (11)). The other cost is the extension of the expensive cppd program (horizontally shaded region) into the area of agents who would otherwise have been buying at the market price (this is the second term of (11)). The addition of the α_{m+1} program insulates the agents in A_{m+1} from the effects of changing p. As drawn, the new track removes none of the benefits of a price increase, while it diminishes the costs. Thus, here, the addition of the α_{m+1} mechanism raises the optimal market price. For it to lower p*, A_{m+1} would have to be concentrated (at B = p*) inside the B + E = c line, among agents whose consumptions are undesirable. This would also tend to make the α_{m+1} program an undesirable addition.

In the same vein, consider the impact on p* from the implementation of cppd, given the tracks α_i. Without cppd, $\partial W/\partial p$ is $-\int_0^\infty (p+E-c)g(p,E)dE$, and

$$\left(\frac{\partial W}{\partial p}\right)_{\text{cppd}} - \left(\frac{\partial W}{\partial p}\right)_{\substack{\text{no}\\ \text{cppd}}} = \int_{\max(c+t-p,0)}^{\infty} (p+E-c-t)g(p,E)dE \geq 0 .$$

Thus, assuming $\frac{\partial^2 W}{\partial p^2} < 0$ in the relevant, range,[15] the addition of the socially desirable cppd program raises the optimal market price.

Performing ordinary comparative statics, but holding constant the existence of the α_i programs, we see that

$$\frac{\partial^2 W}{\partial p \partial t} = - \int_{\max(c+t-p,0)}^{\infty} g(p,E)dE < 0 ,$$

and that $\frac{\partial p^*}{\partial t} < 0$. Thus, as the costs of cppd diminish and more agents (those not purchasing through other programs for whom $B+E \geq c+t$) are screened for their B's and E's, the optimal market price increases. Of course, (11) shows that as long as $t > 0$, $\partial W/\partial p \leq 0$ for $p \geq c$, so $p^* \leq c$. If $t = 0$, since any market price below c can mar the perfect allocation from complete costless price discrimination, p should be set at or above c. Thus we have

Theorem 4: With positive information costs, as the costs of cppd fall to zero, the optimal market price rises to marginal cost.

Putting this result together with the observations made above concerning the effect on p* of the addition of desirable α_{m+1} or cppd programs, we have this observation:

Proposition: The addition of socially desirable screening programs tends to increase the optimal market price towards marginal cost.[16]

Moving in the opposite direction, we know that as the information costs s_i increase, the desirability of screening on α_i diminishes. When $c + s_i > B + m^{\alpha_i}(E|B)$, for all B represented with positive measure in A_i, we know that the α_i program is socially undesirable. If the $B + m^{\alpha_i}(E|B)$ functions are all bounded above on their relevant domains, then there are information costs s_i big enough to make all the α_i programs undesirable.

With no α_i programs, g(p,E) is replaced by h(p,E) in (11) and we have

$$(12)\quad \partial W/\partial p = -\int_0^{\max(c+t-p,0)} (p+E-c)h(p,E)dE - (t)\int_{\max(c+t-p,0)}^{\infty} h(p,E)dE \ .$$

We show that (12) approaches (2), $\partial W/\partial p$ without cppd, in the limit as $t \to \infty$, for each relevant p ($p \leq c$), given that $m(E|p) < \infty$. Clearly, the first term of (12) approaches (2) for any fixed p. By Cauchy's Condition, $m(E|p) < \infty$ implies $\lim_{t\to\infty} \int_t^{\infty} Eh(p,E)dE = 0$ for any fixed p. Then, with $p \leq c$,

$$\int_t^{\infty} Eh(p,E)dE \geq \int_t^{\infty} (t)h(p,E)dE \geq \int_{t+c-p}^{\infty} (t)h(p,E)dE \geq 0 \ ,$$

and it follows that $\lim_{t\to\infty} (t)\int_{c+t-p}^{\infty} h(p,E)dE = 0$.

Thus, as $t \to \infty$, the behavior of W with respect to p (in the relevant region) with cppd becomes identical to that in the pure price subsidy program. Putting this together wtih the comparative statics and with the observation above about sufficiently large costs of screening the α_i's, we have

Theorem 5: As all information costs increase without bound, the optimal market price falls to the optimal pure subsidy price.

Briefly, the rationale behind these results is that when screening is impossible, or very costly, the full task of correcting the allocation for the externalities falls to the market price. While the market mechanism incurs no information costs, it can only sort the population coarsely into sets of the form $\{a|B(a) \geq p\}$ and $\{a|B(a) < p\}$. Unless the distributions of E conditional on B have no variance and have an increasing $B + m(E|B)$, dichotomizations of this form result in relatively poor allocations. Since the market price has to be held down to permit the consumption of high E agents, many low E agents purchase the good as well. Screening on selected signals allows the population to be more finely sorted. As personalized information costs drop, groups of high E agents can be identified and offered special prices. This removes part of the allocative burden from the market price and permits it to rise to exclude undesirable consumption. In the limit, as information costs fall to zero, all externalities can be corrected with full knowledge. Then, and only then, the market price loses all trace of subsidy and becomes equal to production marginal cost.

VI. Conditional Perfect Internalization

In this section, we specialize the analysis to signals which perfectly reveal agents' E's. Such signals are, of course, necessary for the implementation of the classical scheme designed to correct externalities through internalization of spillover effects.

Suppose then that on a self-selection basis, an E verification test is offered to those with E's of some prespecified value. The cost of this test, e, is less than t, the cost of identifying both B and E. Denote by p(E) the special price offered to the agents whose E's are validated by this test. Then, the social benefit, net of testing costs, which accrues from these agents is

$$W(E) = \int_{p(E)}^{\infty} (B+E-c-e)h(B,E)dB \ . \tag{13}$$

For this E, the optimal price p(E) becomes

$$p^*(E) = \begin{cases} (c+e) - E & \text{if} \quad E < c + e \ , \\ 0 & \text{if}^{17} \quad E \geq c + e \ . \end{cases} \tag{14}$$

Equation (14) is a special case of the optimal price rule developed earlier and is analogous to Equation (7). Indeed, for a group of agents all having a given E, the gross average social value function is simply $B + m(E|B) = B + E$.

We shall show that selling the input at p*(E) is equivalent to achieving the perfect internalization which Coase, among others, recommends as the solution to the externality problem. Consider an agent with the requisite particular value of E. Since the government requires deposit of a testing fee which is refundable only upon purchase at the special price, the agent refrains from testing unless his $B \geq p^*(E) = \max[0,c+e-E]$. If his B exceeds the input price, the agent will test, purchase the input, and will be left with a net surplus of

$$0 \leq B - p^*(E) = \begin{cases} B + E - (c+e) & \text{if} \quad E < c + e \\ B & \text{if} \quad E \geq c + e \ . \end{cases} \tag{15}$$

This process of self-selection yields a perfect allocation, relative to the information cost e. If $E \geq c + e$, then, of course, all agents in the group are socially desirable consumers, and the consequent zero special price achieves the correct outcome. Otherwise, the buyers are those with $B \geq c + e - E$, precisely the agents whose gross social benefits of purchase exceed the true social marginal costs.

The classical internalization program also yields a perfect allocation, relative to its information structure. The government, assumed to know each agent's E, offers a subsidy payment of his E to any agent who will buy the input at a price equal to social marginal cost. This is equivalent to each agent being offered a personalized price of (marginal cost) - E, exactly the content of (14).

However, when information is costly to obtain, we shall soon see that it is not socially advantageous to offer all agents E-tests and the special prices in (14). Instead, this option will be optimally made available to only a proper subset of agents, conditional on the size of their E's. Hence, we label this allocative program conditional perfect internalization (abbreviated to cpi).

With some agents excluded from cpi, there will be a market price, below c, set optimally for its effect on the allocation of the good to them. Thus, with imperfect information, the classic version of internalization is suboptimal both because it relies on a marginal cost market price and because of its informational requirements. Far better, with costly information,is conditional perfect internalization, due to its personalized prices, self-selection, and optimally selected set of potential participants.[18]

Let us consider the allocative and informational consequences, as illustrated in Figure 5, of introducing cpi for a particular level of E. Because $t > e$, every agent with that E who is eligible to participate in cppd can achieve a larger surplus from the cpi program, will switch, and thereby save society $t - e$ in screening costs. Moreover, those agents with $t + c > B + E \geq e + c$ will now purchase the good and thereby increase social welfare. The undesirable countervailing effect of cpi is that agents who are given the option to purchase the input at $p(E) < p$ will leave the impersonal market -- if their $B \geq p$ -- and will offer themselves for testing, at a social cost of e per agent, without a concommitant improvement in the input's allocation.

The tradeoff between allocative efficiency and informational efficiency is explicit when we consider the expression which measures the gain (or loss) in social welfare that results from the extension of cpi to a particular value of E. In light of Theorem 1, this first variation, δW, is calculated relative to a pre-existing program of cppd. It reduces to:

(16)

$$\delta W = \int_{c+t-E}^{p} (t-e)h(E,B)dB - \int_{p}^{\infty} eh(E,B)dB + \int_{p(E)}^{c+t-E} (B+E-c-e)h(E,B)dB \ .$$

If the first variation is positive (negative), cpi is socially advantageous (disadvantageous) as compared with cppd for this particular E.

Note that for $E \leq c + e - p$, the special price $p(E) \geq p$, and consequently cpi is ineffective for $0 \leq E \leq c + e - p$. Furthermore, it can be seen from (16) that for E close to and above $c + e - p$, δW is negative. Consequently, for these values of E it is not desirable to offer cpi and so, in our framework, the classical mechanism which offers internalization to all is strictly suboptimal.

Theorem 6: With costly information, conditional perfect internalization, offered to an appropriate proper subset of agents, combined with a program of cppd and a subsidized market price, strictly dominates complete perfect internalization.

VII. Decentralization

In the preceding sections, we approached the problem of allocation of resources with externalities from the vantage point of a benevolent government or center. Whereas the allocative mechanisms relied upon consumer sovereignity, the center itself was posited to play an essential role in them. The center was assumed to design the screening programs and the subsidized market price optimally for the standard social objective function. Indeed the center appears to be the appropriate locus for policy towards externalities whose effects are diffuse.

However, our analysis would have remained unchanged were the external effects concentrated on one agent. Yet, in such a case, absent transactions and information costs, Coasian analysis indicates that government intervention is unnecessary. With clear assignment of property rights, self-interested individuals' own actions would effect the socially optimal resource allocation.

In this section, we analyze the behavior of an individual Agent who acts as if he were the sole recipient of the externalities generated by others' actions. The allocation of property rights is such that all are entitled not to perform the activity. We maintain the analytic structure utilized in the earlier sections; assume that the Agent and the center face the same information costs, and find that with arbitrarily small information costs, the decentralized allocation systematically deviates from the socially optimal one.

Within our framework, the Agent can implement all the allocative mechanisms previously analyzed. He can successfully offer the input in the impersonal market at any price below c. (Of course, no one would buy from him at a price above the competitive level of c.) Further, like the center, he can offer a special price to any self-selecting agent who is validated to possess the requisite signal. Moreover, the Agent always finds it desirable to append a cppd program to any other allocative mechanisms he implements.

From a participant in cppd the Agent gains E at a screening cost of t, and a transfer cost of c-B. The net gain is E+B-c-t, which is guaranteed to be non-negative by the construction of the program.

Now, maintaining the existence of cppd, let us consider the Agent's incentive to screen on the signals $\alpha_1,\ldots,\alpha_m$ and his determination of the corresponding special prices $p_1,\ldots,p_m$ together with the market price, p. Assuming that $p_i < p$ (so that screening on α_i is effective), the Agent's private benefit for feasible $p \leq c$ is,

$$
\begin{aligned}
\Pi = & \int_0^p \int_{c+t-B}^{\infty} (E+B-c-t)g(B,E)\,dE\,dB \\
(17) \qquad & + \sum_i \int_{p_i}^{\infty} \int_0^{\infty} (E+p_i-c-s_i)h^{\alpha_i}(B,E)\,dE\,dB \\
& + \int_p^{\infty} \int_0^{\infty} (E+p-c)g(B,E)\,dE\,dB \ ,
\end{aligned}
$$

where as in (8) and (9), $g(B,E)$ is the distribution of agents not buying at any of the special prices, p_i.

Equation (17) reflects the assumption that the Agent receives as a private benefit the E corresponding to each consumer of the good. The Agent pays a subsidy of c-p for each unit purchased in the impersonal market, and incurs a net cost of $c+s_i-p_i$ on sales to those with signal α_i. The difference in the contribution made by each consumer to social welfare (in equation (5)) and to Π is B-p, or $B-p_i$, depending on where he buys. This difference is the consumer's net benefit (surplus) which is a legitimate part of social welfare, but which is of no concern to the Agent.

The structural deviation between social welfare and the Agent's private benefit tends to cause the prices that are optimal for the former to be lower than those optimal for the latter. This can be seen by comparing the partial derivatives of (9) and (17) with respect to p and p_j. The latter is calculated for a program that does not draw consumers from other special programs or cppd.

(18)
$$\frac{\partial \Pi}{\partial p} = \int_{c+t-p}^{\infty} (E+p-c-t)g(p,E)dE - \int_{0}^{\infty} (E+p-c)g(p,E)dE + \int_{p}^{\infty}\int_{0}^{\infty} g(B,E)dEdB.$$

(19)
$$\frac{\partial \Pi}{\partial p_j} = -\int_{0}^{\infty} (E+p_j-c-s_j)h^{\alpha_j}(p_j,E)dEdB + \int_{p_j}^{\infty}\int_{0}^{\infty} h^{\alpha_j}(B,E)dEdB$$

Comparison of (18) and (19) with the derivatives of (5) reveals that when calculated at the same points and given that the same g(B,E) functions are relevant,

(20) $$\partial\Pi/\partial p > \partial W/\partial p \text{ and } \partial\Pi/\partial p_j > \partial W/\partial p_j \ .$$

Together with the fact that $\partial^2 W/\partial p_j \partial p = \partial^2 \Pi/\partial p_j \partial p = 0$, (20) suffices to establish the following result.

Theorem 7[19]: Suppose that p^*, the welfare optimal market price, is less than c, and that the welfare and privately optimal sets of screening programs are the same. Then $\hat{p}$, the privately optimal market price, is greater than p^*. Similarly, $\hat{p}_j > p_j^*$, if no consumer with characteristic α_j is eligible for another special price.

The intuitive explanation of this theorem is that the Agent, unlike the center, views as a loss the price dependent subsidies of c-p and $c-p_j$ given to the consumers. Consequently, the Agent is more reluctant than is the center to lower any price.

The hypothesis of Theorem 7 that $p^* < c$ is needed because, otherwise, p^* and $\hat{p}$ would both equal c due to the fact that $\hat{p}$ cannot exceed c. In fact, it can be seen that $\hat{p} = c$ if information costs are small.

Theorem 8[20]: There exists $\hat{t} > 0$ such that for $t \leq \hat{t}$, the Π maximal market price is c.

The result says that while the social optimum requires a subsidized market price whenever there are any screening costs, under decentralization, for some range of information costs, all subsidizations will be effected through special screening programs.

In fact, it is easy to see that if t = 0, the Agent's objective, Π, is maximized by sole reliance on cppd. Of course, in this case, the resulting allocation is socially optimal and decentralization obviates government intervention. Unexpectedly, the result does not hinge on the costless availability of all information.

The following theorems show that in the present framework, it is the availability of information about consumers' private benefits that is critical for the social efficacy of decentralization. This is surprising because, in the model considered here, any deviations between the social and private optima arise from the externalities, and thus one would expect that only costless information about them would allow the two optima to coincide. However, this surprise stems from a confounding of two distinct comparisons. In general, information costs cause the social optimum to deviate from the one that would be realized if information were free. Moreover, information costs may cause a divergence between the social optimum and the decentralizated allocation. Our results in Section VI indicate that it is costless information about consumers' externalities that is required to close the former gap while the results below show that it is costless information about consumers' private benefits that is required to close the latter one.

Theorem 9 (Coase): In the present framework, if it is costless to ascertain the private benefit, B, of each consumer, then the allocations that are optimal for the Agent and for social welfare coincide.

To prove the theorem, note that the Agent has incentive to use the costless information that reveals each consumer's B in order to extract the maximal feasible price, B, from each subsidized buyer of the input. Consequently, both the center and the Agent receive the same benefit, net of information costs, from the provision of the input to each consumer, namely B+E-c. Thus, whatever complex of screening mechanisms is optimal for W is also optimal for Π.

However, we show in the Appendix that the social efficiency of decentralization generally fails to hold when information is costly. Nevertheless, it should be noted that this and our other results may depend critically on the particular assignment of property rights inherent in our model.

Theorem 10[21]: Decentralization is socially suboptimal if it is costly to discover the private benefits of all consumers whose B's are close to and below c.

VIII. The Information Structure

In this section we discuss some of the effects of relaxing our assumptions on the information structures of the allocation mechanisms discussed throughout the paper.

In our model, agents must believe in the precise accuracy of the tests, because otherwise, the self-selection processes become imperfect. Moreover, people have incentives to attempt to manipulate the tests if they believe that it is possible. For example, a participant in cppd will gain from a downward misrepresentation of his private benefit, as long as his test scores satisfy $B + E \geq c + t$. Of course, this is also true in the classical price discrimination schemes, and one can appeal to cheat-proof tests to resolve the problem.

For our mechanisms to be effective, there must be no "black market" for the resale of the good by those who buy it at special prices. An agent with a private benefit below the market price would prefer resale to consumption. But resale would deny society the external benefits from that agent's consumption, and would vitiate the allocative program.[22] Such a black market could not exist if the good were, by nature, non-transferrable (such as personalized educational tuition vouchers). It might also be prevented by requiring evidence of the good's use from agents who purchased at a special price.[23]

Nonetheless, with no black market and with credibly accurate cheat-proof tests, our mechanisms do not deny, and in fact, they rely upon the strength of individuals' selfish incentives. Agents are only asked to behave rationally and consistently with their own welfare.

There is an interesting obverse class of allocative programs which we reject because they are not compatible with individual rationality. These test agents for undesirability of consumption, reward them for refraining from the activity, and distribute the good freely to the remaining desirable consumers. Although in some cases such a program might economize on screening costs, it leaves strong incentives for circumvention. We view as prohibitively high the costs of preventing the undesirable consumers from using inputs which are freely available to others. In contrast, we view as reasonably small the costs of barring the black market transactions which would threaten to ruin our self-selection schemes.

In fact, these costs would be zero if the programs were effected ex-post, after the activity had been undertaken. Ex-post, the government would be observing the realized values of B and E and using them to compute the optimal personal subsidies. Here, since the subsidies are irrevocably tied to the actual performance of the activity, rather than to the promise of its performance, any black market possibilities disappear.

Ex-post versions of the mechanisms, heretofore assumed to be ex-ante, can be easily constructed. The government announces, before all purchasing decisions, that sometime later, after the completion of the activity, testing and transfer programs will be implemented. These programs are to have the same structure as the ex-ante ones, except that wherever personalized prices were offered in the ex-ante mechanisms, transfers will be made in the ex-post schemes. The transfers will be set equal to the difference between the market price and the special personalized price of the ex-ante mechanism, and will be only given to those who have performed the activity. In this scenario, agents are assumed to correctly anticipate their later subsidies and to make their purchasing decisions accordingly. Again, to assure the credibility of ongoing programs, all promises of government actions must be fulfilled. Then, other things equal, the allocations and information costs resulting from the ex-post mechanisms are identical to those of the ex-ante versions.

However, it seems likely that the parameters and the effectivenesses of the two classes of programs would differ. It is plausible that more informative signals are available for ex-post than for ex-ante screening.[24] Both this effect and the more easily solved resale problem argue for the adoption of ex-post programs. On the other hand, the allocation resulting from such a program may be distorted by capital market imperfections. Agents with low B's and high E's who anticipate large subsidies after completing the activity may find it costly or impossible to obtain the funds needed for purchase before their B's and E's are officially verified. Of course, it is the consumptions of this very category of agents which the mechanisms were designed to assure.[25]

The most striking difference between the ex-ante and ex-post mechanisms lies in their basic information structures. If agents are uncertain about their own signals before being screened and completing the activity, then they cannot correctly forecast their subsidies forthcoming from either program. However, the costs to the agent of misjudging his signal are substantially different in the two cases. In the ex-ante scheme, the agent can, at worst, lose the testing fee if the test disqualifies him from the special price. But in the ex-post version, the agents who would not buy without a subsidy risk the testing fee plus the difference between the market price and their B. To induce the participation of such desirable agents, the market price may have to be lower than would otherwise be optimal, causing misallocations that could be avoided under an ex-ante program.

In general, imprecise self-knowledge as well as noisy tests impair the power of self-selection and discriminatory pricing. In our model, based on self-knowledgeable agents and accurate cheat-proof screening procedures, we have constructed and studied self-selection allocation mechanisms designed to optimize the allocation of a good with externalities, in the face of costly information. We think that the simplicity of our assumptions has enabled new analytic insights which, we hope, may contribute to the design of better social programs.

APPENDIX

Proof of Theorem 10:

It suffices to show that the collection and features of the allocative mechanisms that are Π optimal do not coincide with those that are optimal for W. We prove this by deriving a contradiction from the assumption that for Π and W maximization, the market price, the existence of cppd, and the α programs and their special prices are all the same. The argument considers several cases. Suppose, first, that $p^* < c$. Theorem 7 immediately implies that $\hat{p} > p^*$, which is the requisite contradiction.

Suppose, instead, that $p^* = c$. Then, evaluated at $p = c$, $\partial W/\partial p \geq 0$. Equation (11) implies that, then, $\int_0^\infty g(c,E)dE = 0$. Thus, almost all consumers with $B = p^* = c$ obtain the input in α programs which screen on, say, $\alpha_1,\ldots,\alpha_k$. Assume that $s_i > 0$ for $i = 1,\ldots,k$. Then, $p_i^* < p^*$, $i = 1,\ldots,k$, for the programs to be desirable. Consequently, there would exist a reduction in the market price that would increase W because the savings in information costs would outweigh any concomitant misallocation. This contradicts the assumption that $s_i > 0$, $i = 1,\ldots,k$. So, let $s_1 = 0$. Thus, by the hypothesis of the theorem, α_1 cannot reveal the private benefits, B, of any consumers. It follows that the α_1 program cannot be perfectly discriminatory and that there must be an effective special price, $p_1^* < p^* = c$. Finally, by the arguments underlying Theorem 7, $\hat{p}_1$ cannot equal p_1^*. This is the needed contradiction.

Q.E.D.

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FOOTNOTES

1. It is well known that with perfect information, the government can utilize perfect price discrimination, see Pigou [1932], or complete internalization to effect a Pareto optimal allocation in the presence of externalities. Further, as Coase [1960] demonstrated, with perfect information and costless transactions, Pareto optimality can also be achieved without government intervention if property rights are clearly assigned. However, because of the costliness of the personalized information required for these classical solutions to the externality problem, much attention has been devoted to the design of governmental tax and subsidy schemes which can partially correct for externalities without the need for any personalized information, see Diamond [1973], Baumol and Oats [1975], Roberts and Spence [1974].

2. Although many of our results hold otherwise, we make this assumption for the sake of concreteness.

3. Perhaps this is the argument for free public education.

4. For example, it may be argued that in the sphere of health care, $B + m(E|B)$ is relatively high for the poor and for the rich, while B, the reservation price, is positively correlated with income. These features would yield a U-shaped $B + m(E|B)$ function. Then, the only candidates for optimality would be a zero price which would permit inefficient consumption by groups with $B + m(E|B) < c$, and a price satisfying (3) and (4). Although this price would eliminate inefficient use, it would also prevent the poor from purchasing socially desirable medical services.

5. Medicaid, Medicare, venereal disease treatment centers and drug treatment clinics are examples of programs which offer special prices to selected groups: the poor, the aged, and the specially afflicted.

6. More generally, we need only assume that the existence of the allocation programs discussed does not provide strong enough motivation for agents to alter these characteristics.

7. This restriction may rule out some socially advantageous possibilities. For example, costly verification of an α signal is completely unnecessary, given that agents perfectly select themselves into the α-testing program believing that they will be so tested. Perhaps a less costly placebo test should be sometimes substituted for the true test. Another possibility is randomly administering the true test

and finding (with a prior announcement) those agents who are discovered to have misrepresented possession of the requisite characteristics. Under the assumption of perfectly accurate tests, a severe enough penalty could be threatened to dissuade all cheating, even with a small probability of test administration. If, however, the tests are perceived to be subject to error, or if agents are uncertain about their own characteristics, then such a system would also dissuade from participation some of the agents who do have the desired signal.

8. Many social programs can be usefully viewed as realizations of the abstract mechanism developed in the text. For example, special student subscription rates for technical journals may result from viewing the group of agents with the easily verified signal "student" as being located in E,B space much like the subset α in Figure 3.

The Medicaid program offers a discriminatory price for health care to "welfare recipients." The government may view this group as having low B's and high E's, on average, thus meeting our criteria for special treatment. Furthermore, (7) would stipulate an optimal group-specific price close to 0.

People with the signal "venereal disease symptoms" may be viewed as having both extremely high spillover benefits from treatment and a wide range of reservation prices. Then, the existence of free treatment centers into which those with the publicized symptoms can self-select is consistent with the theory developed in Section II. There it was shown that the optimal price is zero if the gross average social value is everywhere above the relevant marginal cost.

9. Elements of such an allocative mechanism can be found in the dispensation of health care by some public-minded hospitals and clinics which examine patients who present themselves, treat those who are diagnosed to need care, and offer a sliding fee schedule based on ability to pay. These organizations screen B's with means tests and only offer special subsidized prices to patients who demonstrably cannot afford care at the regular market price. Further, they screen agents' E's by means of medical diagnoses, and refuse to offer special prices to patients, no matter how poor, whose medical needs are not serious enough by common standards. Prospective patients who are aware of these standards will only present themselves to the facilities if they think that they will qualify for special treatment. This process of self-selection keeps manageable the medical and financial screening chores of the hospitals and clinics.

10. Some effects of dropping this assumption are discussed in the concluding section.

11. If there are agents with more than one of the screened characteristics, h^{α_i} will depend on the special prices offered, although not on the market price.

12. It is somewhat more detailed, but still valid, if A_{m+1} intersects the sets of agents participating in other allocative programs.

13. Here, a denotes an agent and B(a) and E(a) denote his B and E.

14. Note that this is not the optimal value for p_{m+1}, into whose calculation goes the additional effect of the savings in information costs due to the movement of agents from the cppd into the α_{m+1} program.

15. In particular, this assumption requires that it is costly to ascertain agents' private benefits.

16. Theorems formally demonstrating this tendency for a particular class of screening programs appear in the Appendix to Ordover and Willig [1976].

17. No welfare gain is achieved by allowing $p^*(E)$ to be negative.

18. The government uses elements of cpi to allocate funds to various R&D projects. The anticipated <u>private</u> benefits of some of these projects may be prohibitively costly or impossible for the government to estimate, thus rendering cppd ineffective. Nevertheless, cpi may be effectively implemented because the expected spillover benefits can be feasibly estimated via familiar appraisals of research proposals, facilities, and staff.

The $p^*(E) = 0$ portion of the optimal cpi price schedule can be identified with cost-plus R&D grants. (14) indicates that cost-plus financing should only be rationally offered when the expected unappropriable part of the social benefit is, by itself, large enough to cover the total cost. If, however, $E < c + e$, (14) stipulates that the R&D project be supported with a flat-fee grant. The optimal fee implied by (14), E + [cost of inputs at market prices] - (c+e), leaves part of the project costs to be defrayed from private benefits.

19. Because both Π and W are additively separable functions of p and p_i, it suffices to examine each price separately. Let p^* and $\hat{p}$ maximize W and Π respectively. By assumption, $p^* < c$. At p^*, $0 = \partial W/\partial p < \partial \Pi/\partial p$. Thus (20) implies that $\hat{p} \neq p^*$ and that $\Pi - W$ is an increasing function of p. Suppose $\hat{p} < p^*$. By definition, $\Pi(\hat{p}) > \Pi(p^*)$ and $W(\hat{p}) < W(p^*)$. Thus, $\Pi(\hat{p}) - W(\hat{p}) > \Pi(p^*) - W(p^*)$, which is a contradiction The same argument applies to p_i. Differentiation shows that (20) also holds for α programs that draw consumers from cppd.

20. Equation (18) shows that at $p = c$, $\partial\Pi/\partial p$ is above and bounded away from 0 as t goes to 0. Further, because all second derivatives of Π with respect to p and t are bounded, there exist ε and $\bar{t}$ such that $\partial\Pi/\partial p$ is positive for $0 \leq t < \bar{t}$ and $c - \varepsilon < p \leq c$. For $t = 0$, $\hat{p} = c$, and so, by continuity of $\hat{p}$ as a function of t, there is some t^* such that $c \geq \hat{p} > c - \varepsilon$ for $0 \leq t < t^*$. Let $\hat{t} = \min(\bar{t}, t^*)$. For $0 \leq t < \hat{t}$, $c \geq \hat{p} > c - \varepsilon$ and, were $\hat{p} < c$, $\partial\Pi/\partial p > 0$ would contradict optimality.

21. More formally, if there exists an $\varepsilon > 0$ such that it is costly to validate all signals which perfectly reveal the private benefits of all consumers with $c - \varepsilon \leq B \leq c$, then the Π and W optimal allocations differ.

22. The possibility of resale would also render ineffective the classical price discrimination mechanism.

23. However, the experience of the food stamp program shows the difficulty in enforcing rules against such resale.

24. For example, ex-ante R&D subsidy decisions can only be based on such signals as the perceived quality of research staffs while, ex-post, the quality of the research itself becomes an available signal.

25. Consider the ineffectiveness of an ex-post program of college scholarships. Potential students having high ability but few funds would find it little easier to borrow tuition with such a program than without it.

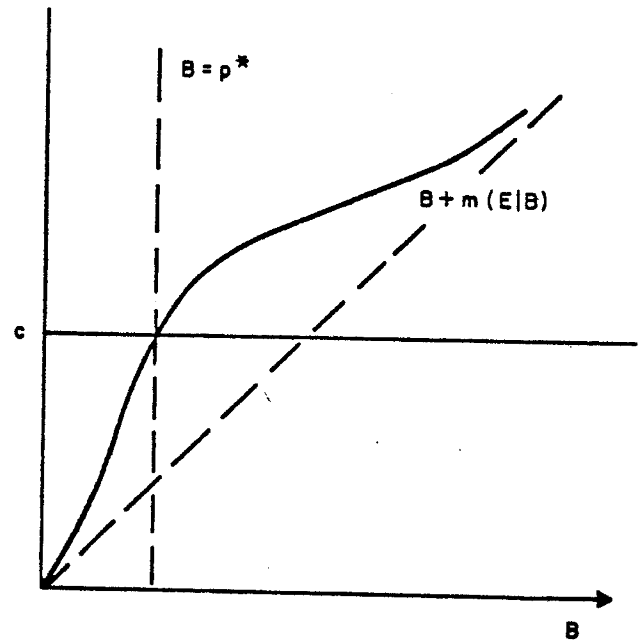


Figure 1

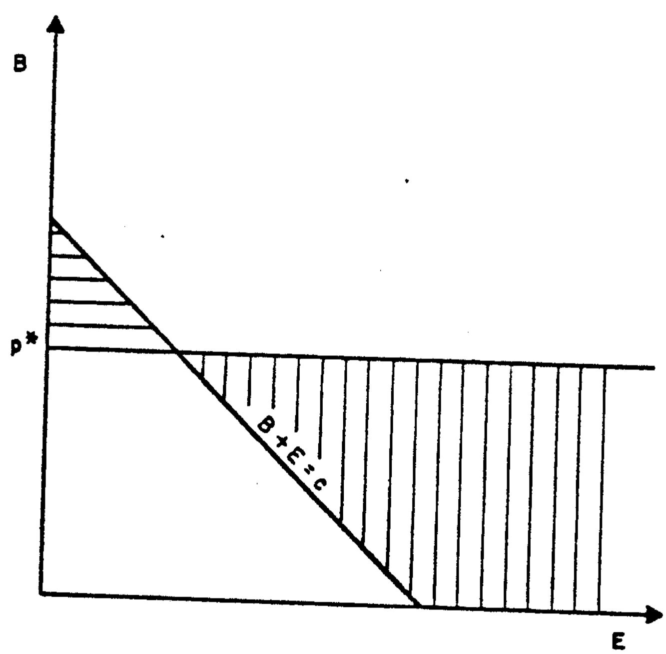


Figure 2

Figure 3

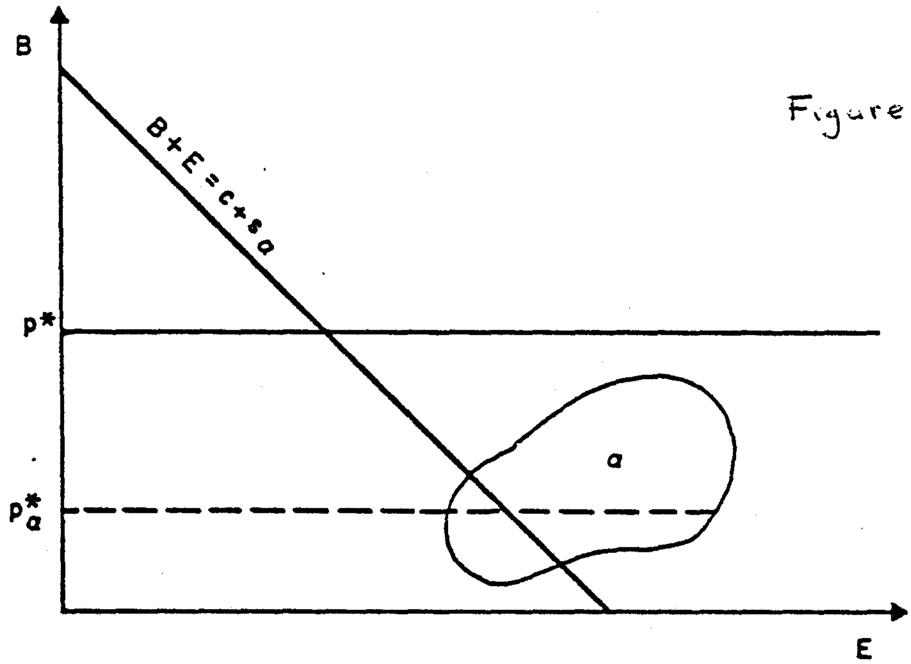
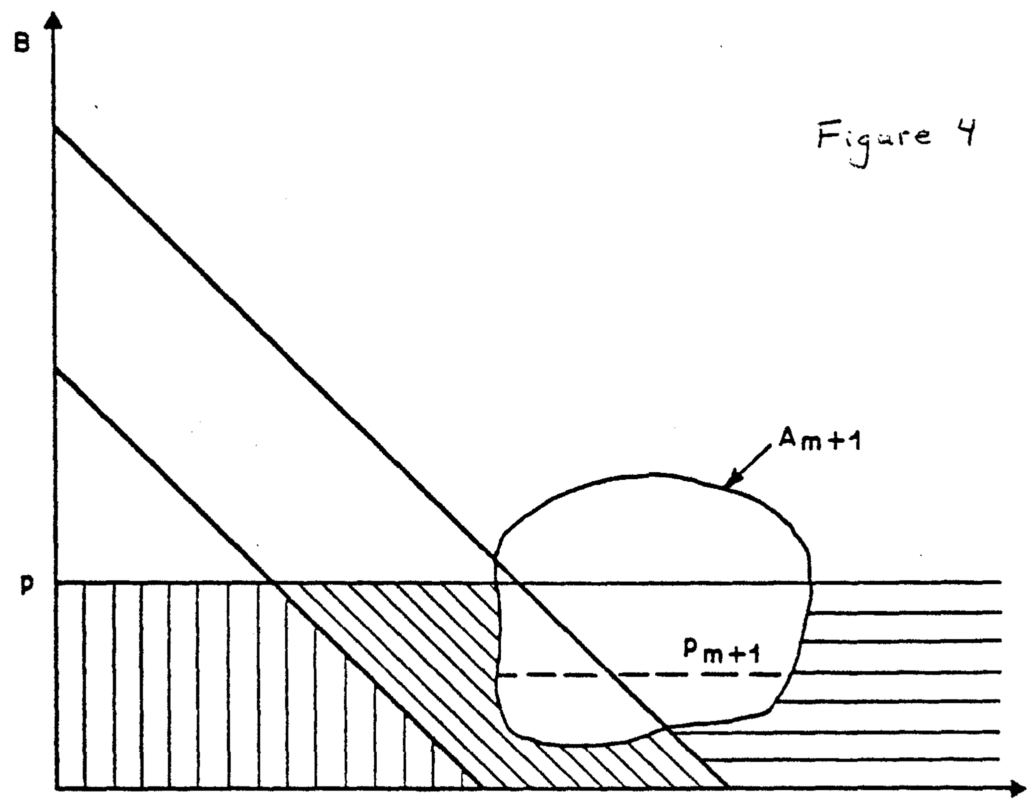
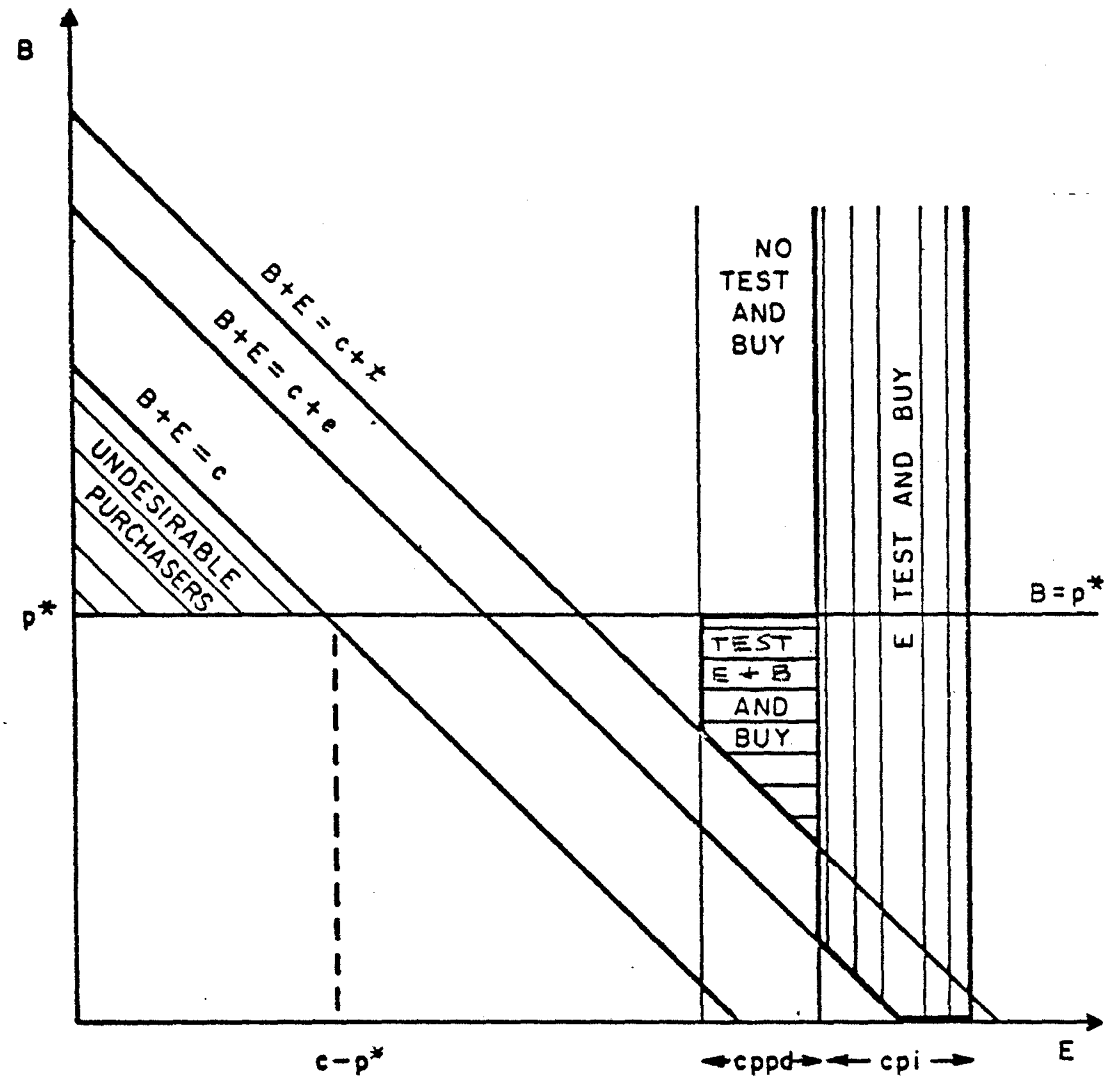


Figure 4





B
p*
B+E = c+t
B+E = c+e
B+E = c
UNDESIRABLE
PURCHASERS
NO
TEST
AND
BUY
E TEST AND BUY
B = p*
TEST
E + B
AND
BUY
c−p*
cppd
cpi
E

Figure 5