ON THE NONEXISTENCE OF PARETO SUPERIOR OUTLAY SCHEDULES

Janusz A. Ordover
New York University

and

John C. Panzar
Bell Telephone Laboratories, Inc.

by

Janusz A. Ordover*
New York University
New York, New York

and

John C. Panzar*
Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey 07974

1. Introduction

In an important recent paper, R. D. Willig demonstrated that a change from a uniform price to a nonlinear outlay schedule can not only benefit consumers in the aggregate,[1] but also make each and every economic agent strictly better off; i.e., nonlinear pricing can always achieve a Pareto superior allocation of resources. Although the framework of Willig's analysis placed few restrictions on the preferences of the customers of the monopolistic purveyor of the service in question, his implicit assumption of independent user demands limits the applicability of his results.

Allowing for the possibility of underlying market equilibrium relationships between firms employing the monopolist's product as an input leads to their demands being interdependent. In this note we demonstrate that this complication can lead to situations in which the algorithm used by Willig to construct a Pareto superior nonlinear outlay schedule breaks down. We also show that it is impossible to construct any Pareto superior tariff in such cases.

2. Competitive Firms as Consumers

We assume that the monopolist has two types of customers, both of which are competitive firms active in the same final product market. Type 1 ("small") firms employ a freely available technology which requires one unit of x, the input sold by the monopolist, for each unit of q, the final product.[2] The cost function of a type 1 firm is then given by

$$(1) \qquad C^1(q,w,r) = wq + V^1(q,r)$$

where w is the monopolist's unit price and r is a vector denoting the prices of other inputs.[3] When facing a uniform price for the monopoly input the average cost of a type 1 firm is given by

$$(2) \qquad AC^1(q,w) = w + V^1(q)/q.$$

Let $\bar{q}_1$ denote the level of output which minimizes average cost; i.e.,

$$\bar{q}_1 = \min_q w + V^1(q)/q.$$

In addition to small, type 1 firms operating with a freely available technology, we assume that the competitive final product market also contains a <u>fixed number</u>, $\bar{n}_2$, of larger, more efficient firms which have access to some specialized factor in inelastic supply, and thus earn economic rents. The costs of **a representative type 2 firm are given by**

$$(3) \qquad C^2(q,w) = wq + V^2(q).$$

where it is assumed that

$$(4) \quad V^2(q) < V^1(q); \; \frac{\partial V^2}{\partial q} \equiv V_q^2 < V_q^1 \equiv \frac{\partial V^1}{\partial q}; \; \forall \; q > 0.$$

It is now possible to characterize equilibrium in the final product market for any uniform price w set by the monopolist. Since the type 1 technology is freely available, the equilibrium final product price p must be given by the level of the minimum point of the small firms' average cost curve. That is,

$$(5) \qquad p = w + V^1(\bar{q}_1)/\bar{q}_1.$$

With price parametric, the optimality conditions for the large firms are given by

$$(6) \qquad p - \frac{\partial C^2}{\partial q} = p - (w + V_q^2(q_2)) = 0; \; V_{qq}^2 > 0.$$

Given (4), the scale of output of the large firms will always exceed $\bar{q}_1$. Equating industry supply to market demand, Q(p), determines the number, n_1, of type 1 firms. That is,

$$(7) \qquad n_1\bar{q}_1 + \bar{n}_2 q_2 - Q(p) = 0.$$

This is the framework in which we wish to examine the possibility of introducing a Pareto superior nonlinear outlay schedule for the monopolist's product.

3. Failure of the Willig Algorithm

Willig constructed a nonlinear outlay schedule Pareto superior to a uniform price greater than marginal cost by offering the largest consumer type a slight discount on the price of any units purchased in excess of those demanded at the uniform price. That is, the uniform price was replaced by a declining block tariff whose first block, equal in length to the initial demand of the largest user, had a price equal to the initial uniform price. The price of the second or trailing block was reduced to slightly below that level. Figure 1 (essentially the same as Willig's Fig. 3) illustrates why this move benefits both the monopolist and the largest user when, as Willig assumed, the demand curves of different users are independent. A user purchasing x_2^o units at an initial uniform price of w^o achieves a gain in surplus equal to area abe when given the opportunity to purchase additional units at a price $w^o - t$. The monopolist's profits also increase because incremental revenues (area $x_2^o bex_2'$) exceed the cost of producing the additional units (area $x_2^o fgx_2'$). Willig then shows that some of this gain in profit can be used to lower the price of the first block below the initial uniform price. Hence, all agents are better off.

We demonstrate that this algorithm may fail when the demands of the users are interrelated due to downstream market forces. We begin by characterizing equilibrium in the downstream industry when the firms face the declining block tariff described above. The cost function of the type 2 firms is now given by:

$$\hat{C}^2(q_2,w^o,t) = w^o q_2^o + (w^o-t)(q_2-q_2^o) + V^2(q_2), \qquad q_2 \geq q_2^o$$

where, by our fixed proportions assumption, $q_2^o = x_2^o$, the quantity purchased at the uniform price w^o. Since the market price in the downstream industry is determined by the average costs of type 1 firms (which are too small to take advantage of the trailing block price),[4] it will not be affected by the introduction of the declining block tariff. Thus equilibrium downstream can be characterized by

$$(8)\quad p^o - \partial\hat{C}^2/\partial q_2 = p^o - (w^o-t) - V_q^2(q_2) = 0;\ V_{qq}^2 > 0$$

and

$$(9)\qquad n_1\bar{q}_1 + \bar{n}_2 q_2 - Q(p^o) = 0.$$

These equations determine the endogenous variables q_2 and n_1 as functions of the discount t. Standard comparative statics analysis yields

$$\frac{\partial q_2}{\partial t} = \frac{1}{V_{qq}^2} > 0. \tag{10}$$

$$\frac{\partial n_1}{\partial t} = \frac{-\bar{n}_2}{V_{qq}^2 \bar{q}_1} < 0. \tag{11}$$

As one would expect, the output of type 2 firms increases and the number of type 1 firms declines as t is increased.

In order to examine the change in the monopolist's profit, we make use of the fact that price and quantity are constant in the final product market. Therefore the total quantity sold by the monopolist and hence his total cost remains unchanged and the change in profits (π) is **exactly** equal to the change in revenues:

$$R_m(w^o,t) = w^o n_1 \bar{q}_1 + w^o \bar{n}_2 q_2^o + (w^o - t)(q_2 - q_2^o)\bar{n}_2$$

$$\frac{dR_m}{dt} = w^o \bar{q}_1 \frac{\partial n_1}{\partial t} + \bar{n}_2 (w^o - t) \frac{\partial q_2}{\partial t} - \bar{n}_2 (q_2 - q_2^o).$$

Using (10) and (11), we obtain

$$= -n_2[t/V_{qq}^2 + (q_2 - q_2^o)] \leq 0 \tag{12}$$

with the inequality strict for $t > 0$. Thus any discount made available to the large, type 2 firms will, after equilibrium is re-established downstream, result in losses for the monopolist. The first, profit-making step of the Willig algorithm breaks down.[5]

Intuitively, the reason for this result is quite clear. Since price, and thereby total quantity, downstream is pegged by the type 1 firms which are given no incentive to alter their behavior, the total number of units sold by the monopolist remains unchanged. Therefore the increased revenues resulting from additional purchases by type 2 firms at price $w^o - t$ are more than offset by the decrease in revenues resulting from the <u>exit</u> of some of the type 1 firms. In other words, offering the discount merely converts some high price sales into low price sales.

4. An Impossibility Result

The analysis thus far has demonstrated that the declining block tariff constructed by Willig is not necessarily Pareto superior to a uniform price when user demands are interdependant. The natural question, then, is whether or not some other artfully constructed nonlinear tariff will accomplish the task. Unfortunately, our simple model can be used to show that, in general, no such nonlinear outlay schedule exists.

Proposition: When user demands are interdependent, there does not, in general, exist a nonlinear outlay schedule which is Pareto superior to a uniform price.

Proof: We extend the model of the previous section in order to provide a class of counter examples to any general Pareto dominance claim. As above, we assume that large and small users are competitive firms in the same final product market, and that they employ one unit of the monopolist's output for each unit of the final product produced.[6] In addition, we assume that the demand for the final product has an elasticity less than unity over the relevant range.[7] In this context, there are clearly three conditions which any Pareto superior nonlinear outlay schedule, $R(q)$, must satisfy: (1) The profits of the monopolist must not decline; (2) The economic rents of the large (type 2) competitive firms must not decline; and (3) The price facing final consumers must not increase; with at least one of the above classes of agents made strictly better off.

Suppose there exists such an $R(q)$. Formally, we have for the monopolist

$$(13)\quad \Delta\pi = n_1' R(q_1') + \bar{n}_2 R(q_2') - C(n_1' q_1' + \bar{n}_2 q_2') - w^o(n_1^o q_1^o + \bar{n}_2 q_2^o) + C(n_1^o q_1^o + \bar{n}_2 q_2^o) \geq 0.$$

where $C(\cdot)$ is the cost function of the monopolist. Primes indicate the equilibrium values of endogenous variables determined by the outlay schedule $R(\cdot)$ and naughts refer to the values of said variables under a uniform price of w^o. The change in the economic rents earned by type 2 firms is given by

$$(14)\quad \bar{n}_2\{[p'q_2'-R(q_2')-V^2(q_2')] - [(p^o-w^o)q_2^o-V^2(q_2^o)]\} \geq 0.$$

Since they operate with a freely available technology, type 1 firms will earn zero economic profits in both cases:

$$(15)\quad n_1'[p'q_1'-R(q_1')-V^1(q_1')] = -n_1^o[(p^o-w^o)q_1^o-V^1(q_1^o)] = 0.$$

Adding (13), (14), and (15), we obtain

$$(16)\qquad \Delta E - \Delta SC \geq 0,$$

where ΔE is the change in the expenditure of final consumers and ΔSC is the change in the total social costs of producing the final product. That is,

$$\Delta E = p'(n_1'q_1'+\bar{n}_2q_2') - p^o(n_1^oq_1^o+\bar{n}_2q_2^o) = p'Q(p') - p^oQ(p^o)$$

$$\Delta SC = [n_1'V^1(q_1')+\bar{n}_2V_2(q_2')+C(n_1'q_1'+\bar{n}_2q_2')] - [n_1^oV^1(q_1^o)+\bar{n}_2V^2(q_2^o)+C(n_1^oq_1^o+\bar{n}_2q_2^o)]$$

We now make use of the condition that $p' \leq p^o$. Since the demand for the final product is assumed to be downward sloping but inelastic, we must have

$$(17)\qquad Q(p') \geq Q(p^o)$$

and

(18) $$\Delta E = p'Q(p') - p^{o}Q(p^{o}) \leq 0.$$

Consider next the change in the total social costs of production, ΔSC. Because of our fixed proportions assumption[8] and the competitive structure of the downstream industry, it should be clear that the initial quantity produced, $Q(p^{o})$, was produced in the most efficient way, i.e., at minimum social cost. Hence to produce a level of output at least as large, $Q(p')$, requires at least as much resources. That is,

(19) $$\Delta SC \geq 0.$$

Equations (18) and (19) can be consistant with (16) only if $p' = p^{o}$. But in that case, either (13) or (14) and thus (16) must be a strict inequality if any economic agent is to be made better off. Thus, in the present example, the minimal conditions which must be satisfied by a Pareto superior tariff are mutually inconsistent. Therefore, no such non-linear outlay schedule necessarily exists when user demands are interdependent.

The framework which we have developed also serves to indicate the limited applicability of another of Willig's major results, allowing us to state the following extension of our main result.

Corollary: When user demands are interdependent, an outlay schedule which offers the largest user a price unequal to marginal cost cannot always be Pareto dominated.

This is obviously a direct consequence of the impossibility of dominating even a simple uniform price in the above scenario.

5. Sustainability Implications

Willig's results also have implications for the theory of monopoly sustainability, as discussed by Panzar and Willig and Baumol, Bailey and Willig. Those papers examined the vulnerability of a regulated monopolist to competitive entry under the assumption that both the monopolist and potential entrant's could set only simple linear prices for the goods and services offered. Since regulated utilities can easily engage in nonlinear pricing, it is important to extend the discussion of sustainability to situations in which both the monopolist and potential entrant may set nonlinear outlay schedules.

Although Willig does not directly discuss this issue, his results imply that a necessary condition for monopoly sustainability is that the largest user face a marginal price equal to marginal cost. Put another way, no linear price unequal to marginal cost can be sustainable. Thus, when there are economies of scale, a monopoly that does not engage in nonlinear pricing cannot be sustainable. To demonstrate this assertion, suppose the monopolist was initially earning zero profits with a uniform price equal to $w°$. By Willig's theorem, a potential entrant can offer an outlay schedule everywhere strictly lower than the monopolist's (thereby capturing all its customers), and earn positive profits. This necessary condition, while an important insight, also depends upon the assumption of independent user demands. When user demands are interdependent, a uniform price may in fact be sustainable even if potential entrants are allowed to offer nonlinear outlay schedules.

We demonstrate this proposition by establishing, in the context of our simple model, that any nonlinear outlay schedule, $R^e(g)$, which would make the uniform price $w°$ unsustainable must satisfy the three mutually exclusive inequalities: (13), (14), and (17) of Section 4. We begin by assuming that the monopolist faces a declining average cost curve and, initially, earns zero economic profits selling at the undominated uniform price $w°$. Since any potential entrant must anticipate making non-negative profits, this means that R^e must satisfy (13). Because the inefficient technology is freely available, the equilibrium downstream price resulting from the entrant's offering cannot rise. If it did, firms would enter that market by purchasing the input from the monopolist at a $w°$ price and force the equilibrium final product price back down to $p°$, driving the entrant's customers (and the entrant) out of the market. Therefore, given monotonicity of Q, (17) must hold. The same argument applies with respect to the rents earned by type 2 firms. If these were actually lower under R^e, they would choose to continue purchasing from the monopolist. Thus (14) must also hold under R^e. But we have already seen that, given the identity (15), equations (13), (14), and (17) cannot hold simultaneously. Therefore, no such (nontrivial) R^e exists and $w°$ is sustainable.[9]

6. Conclusions

Our results should not be interpreted as an attack on the general desirability of nonlinear pricing policies when circumstances permit their use.[10] The recent theoretical literature[11] has amply demonstrated their usefulness in increasing aggregate scalar welfare measures such as the sum of producers' and consumers' surplus and other types of Bergsonian welfare functions. We merely wish to point out that they do not necessarily make possible a Pareto improvement. In the public utility pricing context, for example, this means that simple average cost pricing may well be Pareto efficient, given available policy instruments. Our method of modelling interdependent user demands constitutes a special, though not implausible, set of circumstances. The same kind of analysis can also be constructed when the monopolist's output is used as an input by two or more industries whose final products are substitutes.[12] Other plausible scenarios resulting in plausible user demand interdependencies can surely be constructed.

The literature on nonlinear pricing has paralleled the optimal taxation literature. In particular, the analog of Willig's result has been presented in a recent paper by Seade. He found that any Pareto efficient nonlinear tax schedule must involve a zero marginal tax rate for individuals with the highest income level. Our results lead one to suspect that the conclusion may also have to be altered if placed in a context in which recognition of underlying equilibrium conditions results in the labor supply functions of economic agents being interdependent. This is clearly an important topic for future research.

Notes

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[1]As indicated by the level of consumers' plus producers' surplus, or other type of Bergsonian social welfare function.

[2]This "fixed proportions" assumption greatly simplifies the analysis and, at the same time, removes any incentive for the monopolist to vertically integrate into the downstream industry.

[3]Henceforth, these (constant) prices will be suppressed when writing the cost functions.

[4]As Willig demonstrates, it is always possible to make the "perturbation" small enough so that only the largest user is affected.

[5]It is impossible to remedy the situation by lowering $w°$, because, following Willig, we assume that $w°$

is <u>undominated</u>. That is, no <u>lower</u> uniform price generates as must profit for the monopolist as w°.

[6]For notational convenience, we shall express both input and output quantities by the variable q.

[7]This, of course, rules out the possibility that w° was the <u>profit-maximizing</u> uniform price. However, much of the nonlinear pricing literature, as well as the thrust of Willig's results are directed toward regulated utilities; and average cost pricing may well result in operation at a point of inelastic demand.

[8]This guarantees that downstream production decisions cannot be distorted by the monopolist charging a price greater than marginal cost.

[9]We ignore the possibility that the entrant could duplicate the monopolist's uniform price and make zero profits if it "replaced" the monopolist. This possibility is usually ruled out in sustainability discussions (see Panzar and Willig). The issue is slightly more complicated in the present context because with only two consumer types there are an infinite number of outlay schedules which deviate from w° only outside the relevant range. Offering such schedules would not affect any agent's behavior or welfare. Therefore, we assume that

they would not allow the entrant to attract any customers from the monopolist. Viewed another way, the requirement that the entrant's offering must make some consumer strictly better off, means that an R^e weakly satisfying (13), (14) and (17) does not render $w°$ unsustainable.

[10]That is, when technological or legal conditions prevent the resale of the good or service in question.

[11]See the references cited by Willig.

[12]We are indebted to Joe Stiglitz for pointing this out.

References

Baumol, W.J., E.E. Bailey and R.D. Willig. "Weak Invisible Hand Theorems on the Sustainability of Prices in a Multiproduct Monopoly." American Economic Review. Vol. 67 (1977), 350-65.

Panzar, J.C. and R.D. Willig. "Free Entry and the Sustainability of Natural Monopoly." Bell Journal of Economics, Vol. 8, No. 1 (Spring 1977), 1-22.

Seade, J.E. "On the Shape of Optimal Tax Schedules." Journal of Public Economics. Vol. 7 (1977), 203-235.

Willig, R.D. "Pareto-Superior Nonlinear Outlay Schedules." Bell Journal of Economics, Vol. 9, No. 1 (Spring 1978), 56-69.

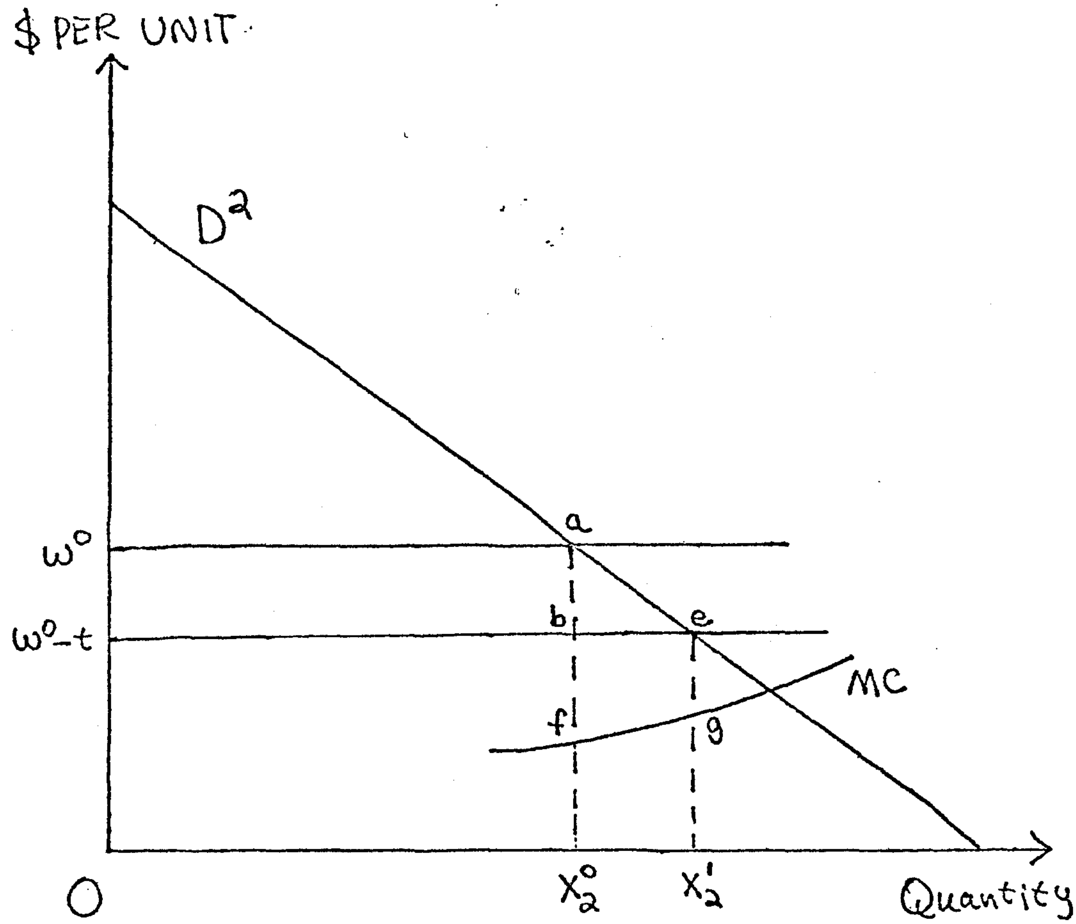

FIGURE 1