ECONOMIC RESEARCH REPORTS

FINAL-OFFER ARBITRATION WITH A BONUS

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R.R. #88-03 February 1988

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ABSTRACT

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Final-offer arbitration (FOA) is an arbitration procedure, used in about a dozen states and some professional sports, under which the arbitrator is restricted to choosing one or the other of two "final offers" proposed by two parties to a dispute. Modeled as a two-person, zero-sum game of imperfect information, in which the parties are assumed to know the probability distribution of the arbitrator's fair settlements and to make final offers to maximize their expected payoffs, FOA does not induce the two parties to converge but rather to make final offers usually two or more standard deviations apart. However, if either or both parties attach added value to winning <u>per se</u> (i.e., by having their offer chosen)--independent of the value of the settlement--then the Nash equilibrium final offers will tend to be drawn together and, in some cases, converge.

Such added value is called an <u>internal bonus</u>. If only one party receives the bonus, its optimal offer is generally less favorable to itself, and the other party's optimal offer more favorable to itself, than in the absence of a bonus. Arbitration data from New Jersey indicate that unions tend to have larger internal bonuses than management and, accordingly, make more conservative offers.

When a bonus that is a function of the gap separating the final offers is paid to the winner by the loser, thereby making it an <u>external bonus</u> that changes FOA into a new procedure (FOAB), equilibrium final offers will be drawn together by as much as two-thirds. By comparison, an earlier revision proposed in FOA, called combined arbitration, gives the parties an incentive, under rather general conditions, to converge completely and so is preferred to FOAB.

FINAL-OFFER ARBITRATION WITH A BONUS

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1. Introduction

Final-offer arbitration (FOA) is an arbitration procedure, used in about a dozen states and some professional sports, under which the arbitrator is restricted to choosing one or the other of two "final offers" proposed by two parties to a dispute. Unlike conventional arbitration, whereby the arbitrator can impose a settlement that is usually a compromise between what each party offers, the parties under FOA would seem well advised to be reasonable in their final offers. For if one side makes an outrageous final offer, the arbitrator will choose the (more reasonable) final offer of the other side.

Unfortunately, the apparent incentive that each side has to be reasonable in its demands under FOA does not generally promote complete convergence of the final offers--either in theory or in practice--as we shall see. [However, changing the incentive structure to a mix of FOA and conventional arbitration--what we have called "combined arbitration" (Brams and Merrill, 1986)--does, under rather general conditions, produce such convergence; we shall briefly compare this procedure with a modification in FOA that we shall introduce later in this paper.]

But now suppose that one or both players under FOA derives value solely from winning (i.e., having its offer chosen), independent of the settlement that each proposes. To model the situation in which winning *per se* adds value to the settlement under FOA, we assume that the final offer selected by the arbitrator is more valuable to the player who proposed it than were the same final offer proposed by the other party and selected by the arbitrator.

Whether the parties to a dispute gain value from winning under FOA is an empirical question. Some data from New Jersey public-employee disputes involving the police indicate that there may be a differential benefit that the parties attach to winning, with the unions--perhaps to impress or assuage their members--assigning greater weight to getting their offers accepted than the local governments in these cases. Political candidates in a two-party race may also derive differential benefits from winning, which, as we will show, may have consequences for the positions they take in order to attract the median voter.

As one might expect, the recognition by one party to a dispute that its opponent inordinately values winning <u>per se</u> enables the first party to obtain a better settlement under FOA than if both parties considered winning immaterial. Auspiciously, when both parties value winning equally, and by a great enough amount above and beyond the actual settlement, their optimal final offers will converge under FOA.

In situations in which winning is valued for its own sake under FOA, we call the added value that a party attaches to winning its <u>internal</u> bonus. But there is no reason in principle why a bonus cannot be built into the reward structure of FOA. In fact, we shall analyze a modified form of FOA in which a bonus is given to the winner that is paid by the loser, which we call FOAB.

Under FOAB, internal bonuses in effect become external by being explicitly incorporated into FOA as rewards to the winner. More specifically, we assume that an external bonus is awarded to the winner and paid by the loser that is a function of the distance between the two (nonconvergent) final offers.

Given that neither party attaches an internal bonus to winning, does FOAB induce the two sides to converge? Unfortunately, the answer is no, whatever the distance function, but awarding an external bonus to the winner equivalent in value to the gap between the final offers can help, reducing by two-thirds the distance between the optimal (nonconvergent) final offers. (Of course, internal bonuses can induce the two sides to approach each other further.)

We would give qualified support to the use of external bonuses under FOAB but still find it wanting. Although an improvement over FOA in getting two parties to approach each other on a settlement, combined arbitration still seems to us the better procedure for inducing convergent final offers, for reasons we discuss in the concluding section.

2. FOA with a Bonus

In Brams and Merrill (1983), we gave necessary and sufficient conditions for existence of a Nash equilibrium under FOA, where this procedure was modeled as a two-person, zero-sum game of imperfect information. The parties in our model are assumed to know the probability distribution of the arbitrator's fair settlements and to make final offers in an infinite strategy space (modeled by the real line) that maximize their expected payoffs.

Nash equilibria, when they exist in pure strategies, represent offers which are (1) symmetric about the median of the common probability distribution for the arbitrator's notion of a fair settlement, and (2) separated from one another by a quantity equal to the reciprocal of the density function of this distribution evaluated at the median. For most common distributions, this separation represents two or more standard deviations (for the normal distribution, the gap is $\sqrt{2\pi} \approx 2.51$ standard deviations); the separation also depends on the risk aversion of the parties (Wittman, 1986).

Our model assumes that the parties realize utility only from the numerical value of the settlement. In their expected-utility calculations, they trade off that value against the probability of being chosen by the arbitrator.

It seems reasonable to assume that some bargainers receive, in addition, utility for winning the award <u>per se</u>, irrespective of the value of the settlement. A labor negotiator in a wage dispute, for example, may be more satisfied with, say, an 8 percent raise in the pay rate--if that settlement is his own final offer--than if the same settlement is achieved but represents management's final offer. Such bonuses for winning above and beyond the value of the settlement may accrue to both parties in the dispute and need not be equal.

There are several reasons why labor's bonus may exceed that of management, although our subsequent conclusions in no way depend on such a rationale. The rank and file, on whose votes labor leaders depend, tend to identify with the euphoria of winning and the frustration of losing a settlement. A string of losses may be fatal to a labor negotiator's position as a union leader. Management negotiators are likely to have less personal stake in the appearance of victory and focus more on the possible long-run consequences of appearing weak. Labor negotiation is only one, albeit an important one, of the concerns of management.

Assume that two bargainers, A (the "low" bidder, e.g., management, whose final offer is a) and B (the "high" bidder, e.g., labor, whose final offer is b) perceive themselves to attain bonuses S_A and S_B, respectively, for winning a settlement per se. Moreover, assume that these bonuses are known to each player--they are common knowledge.

As in Brams and Merrill (1983), we assume that the arbitrator's notion of a fair settlement has a continuous distribution with probability density function f and distribution F with F' = f (or at least one-sided derivatives of F exist for each x), and that the median of this distribution is, without loss of generality, 0. We also assume that player A seeks to minimize the payoff function

$$\begin{aligned} G_A(a,b) &= (a - S_A)F[(a+b)/2] + b\{1 - F[(a+b)/2]\} \\ &= b - (b - a + S_A)\ F[(a+b)/2]. \qquad (1) \end{aligned}$$

Note that the bonus S_A is subtracted from A's offer when management wins--it wants a lower wage scale--which will occur with probability F[(a+b)/2]. That is, with this probability the arbitrator's choice will be closer to A's offer than B's; similarly, with complementary probability B's offer will win and A will receive no bonus. Player B, on the other hand, seeks to maximize

$$G_B(a,b) = aF[(a+b)/2] + \{1 - F[(a+b)/2]\}(b + S_B)$$
$$= (b + S_B) - (b - a + S_B)F[(a+b)/2]. \tag{2}$$

Taking partial derivatives, we obtain, for a critical point (a,b),

$$\frac{\partial G_A}{\partial a}(a,b) = -[(b - a + S_A)/2]f[(a+b)/2] + F[(a+b)/2] = 0, \tag{3}$$

and

$$\frac{\partial G_B}{\partial b}(a,b) = [1 - (b - a + S_B)/2]f[(a+b)/2] - F[(a+b)/2] = 0. \tag{4}$$

Adding and subtracting (3) and (4) yields

$$b - a = 1/f[(a+b)/2] - (S_A + S_B)/2, \tag{5}$$

$$F[(a+b)/2] = 1/2 - [(S_B - S_A)/4]f[(a+b)/2]. \tag{6}$$

Writing the distance or gap between the two final offers as

$$d = b - a \tag{7}$$

and their midpoint as

$$m = (a + b)/2, \tag{8}$$

we may rephrase (5) and (6) as

$$d = 1/f(m) - (S_A + S_B)/2 \tag{9}$$

and

$$F(m) = 1/2 - [(S_B - S_A)/4]f(m). \tag{10}$$

A local equilibrium occurs at a solution (a_1, b_1) of (5–6), or equivalently, at the corresponding solution of (9–10), if G_B has a local maximum and G_A a local minimum, i.e., if

$$\frac{\partial^2 G_B}{\partial^2 b}(a_1, b_1) < 0$$

and

$$\frac{\partial^2 G_A}{\partial^2 a}(a_1, b_1) > 0.$$

Applying (9) and (10) to (3) and (4), the inequalities above hold if

$$\frac{-2f^2(m_1)}{1-F(m_1)} < f'(m_1) < \frac{2f^2(m_1)}{F(m_1)} \quad (11)$$

where $m_1 = (a_1 + b_1)/2$ is the midpoint between the equilibrium offers a_1 and b_1. Note that when there are no bonuses ($S_A = S_B = 0$), $F(m_1) = 1/2$ and $m_1 = 0$, so that condition (11) becomes $|f'(0)| < 4f^2(0)$, as given in Brams and Merrill (1983, p. 930).

Convergence of the final offers is obtained if $d = 0$, i.e., according to (9), if

$$1/f(m) = (S_A + S_B)/2. \quad (12)$$

Denote by (a_0, b_0) the equilibrium strategy in the absense of bonuses. If f has its maximum at 0 (as is the case for any symmetric, unimodal distribution), then (12) implies that

$$S_A + S_B \geq 2/f(0) = 2(b_0 - a_0)$$

(see Brams and Merrill, 1983, p. 929), i.e., the sum of the values of the bonuses, to induce convergence, must be equal to or greater than twice the gap between the equilibrium offers that occurred before the bonuses.[1] Thus, if the bonuses are equal, each must be equal to at least the gap in order for the parties to be motivated to close it.

We consider now some special cases:

Case I. Suppose that the perceived bonuses for each bargainer are the same, i.e., $S_A = S_B = S$. Then, by (10), $F(m) = 1/2$, $m = 0$, and

$$b - a = 1/f(0) - S.$$

Condition (11) becomes $|f'(0)| < 4f^2(0)$, as before the bonus. In fact the Nash equilibrium[2] is

$$(a_1, b_1) = (-1/2f(0) + S/2,\ 1/2f(0) - S/2),$$

which is symmetric about the origin and converges if $S = 1/f(0)$, i.e., if the bonus for each party equals in value the entire gap between equilibrium strategies without the bonus, as we indicated earlier.

Case II. Party B perceives a positive bonus $S_B = S$, whereas A does not ($S_A = 0$). As before, assume B is labor and A is management. Conditions (9) and (10) become

$$d = 1/f(m) - S/2 \tag{13}$$

$$F(m) = 1/2 - (S/4)f(m) \tag{14}$$

where, again, $m = (a + b)/2$. For convergence to occur, we would need $S = 2/f(m)$, in which case $F(m) = 1/2 - 1/2 = 0$, i.e., $m = F^{-1}(0)$. Clearly, if $F^{-1}(0) = -\infty$, no convergence occurs.

If $m = F^{-1}(0) \neq -\infty$ and $f(m) > 0$, then convergence occurs at $a_1 = b_1 = F^{-1}(0)$, i.e., at the left-most point of the support of the probability density f, a point decidedly in favor of management. For example, if f has an exponential distribution $f(x) = e^{-x}$, $x \geq 0$, an equilibrium without a bonus occurs at $(a_0, b_0) = (\ln 2 - 1, \ln 2 + 1) = (-0.307, 1.693)$. If, however, B (labor) perceives a bonus equal to the gap between equilibrium strategies ($d = 2$), then convergence occurs at $(a_1, b_1) = (0, 0)$, the left-hand end-point most favoring management. Note that both strategies are affected by the knowledge of a bonus to one party.

The convergent equilibrium for a uniform distribution also occurs at the left-hand end-point, which is, coincidentally, A's equilibrium strategy when there is no bonus (B's equilibrium strategy without a bonus is the right-hand end-point). For a symmetric triangular distribution, $F^{-1}(0)$ is the left-most point of the support of the density f. Since $f = 0$ at this point, however, there is no convergence.

For a variety of well-known distributions and $S = 2$ (twice the gap d that occurs without a bonus), we obtain the values in Table 1 for m, d (with a bonus), a_1, and b_1. For easy comparison, each density is normalized so that $f(0) = 1$.[3] Under this normalization and without a bonus ($S = 0$), the equilibrium pair of strategies is $(-0.5, 0.5)$, $m = 0$, and $d = 1$, independently of the distribution (see Brams and Merrill, 1983, p. 930). In the presence of a bonus, however, the equilibrium strategies

(a_1, b_1) depend on the distribution, as indicated in Table 1. It follows from (14) that $m \leq 0$ for any positive value of S, i.e., the mean equilibrium offer cannot increase in the presence of a bonus. The exponential distribution is omitted from the table because no solution exists if S exceeds the gap (convergence occurs when S equals the gap).

Convergence of the equilibrium offers is achieved for the exponential distribution (for S = 1) and the uniform distribution (for S = 2), but not for the other distributions considered. Computer calculations suggest that all these local equilibria for FOA with a bonus are in fact global. In each case depicted in Table 1, a bonus for labor causes the gap between equilibrium offers to decrease or stay the same.[4] Furthermore, both offers decrease or remain the same, i.e., labor's equilibrium offer is less favorable for labor and management's more favorable for management.

3. Application to New Jersey Police Arbitration Cases, 1978-80

Ashenfelter and Bloom (1984) studied 423 arbitration cases in New Jersey involving salary disputes between the police and local governments over the period 1978-80. Of these, 324 were conducted under FOA and 99 under conventional arbitration. Of the 324 FOA settlements, more than two-thirds (69 percent) were awarded to labor. After presenting convincing evidence that the FOA "decisions were generated by a set of impartial arbitrators who were systematically applying the same standards used in conventional arbitration cases," Ashenfelter and Bloom (1984, p. 123) further indicate that under FOA

> the parties may not typically position themselves equally distant from, and on opposite sides of, the arbitrator's preferred award. This might

> happen either because unions have a more conservative view of what arbitrators will allow, or because unions may be more fearful of taking a risk of loss than are employers. (Ashenfelter and Bloom 1984, p. 123)

We suggest that one way to account for this conservative behavior by labor is to postulate that labor, but not management, perceived itself to receive a bonus for winning. As indicated in Table 2, final offers (expressed as percent increases in total compensation) averaged 7.9% for labor and 5.7% for management. Conventional arbitration awards averaged 7.7%, much closer to the labor than the management mean under FOA.

If management and labor final offers are denoted by a and b, respectively, then the probability that management wins the award, for an impartial arbitrator, is $F[(a + b)/2]$, where F is the distribution function for the arbitrator's preference. Ashenfelter and Bloom assume F is a normal distribution. Although the arbitrator's true preference is not revealed, the two parameters, μ and σ, of this normal distribution may be estimated by probit analysis from a series of observed arbitrator choices from union and management final offers. Based on the outcome of the 324 FOA cases in New Jersey, Ashenfelter and Bloom estimate the mean of this distribution to be 8.0%, only slightly higher than the mean conventional-arbitration award and approximately equal to the mean union offer under FOA.

To determine the equilibrium strategies, either with or without a bonus, we must also estimate the standard deviation of the arbitrator's preferences. This standard deviation, estimated to be 2.3% by Ashenfelter and Bloom, incorporates between-arbitrator and between-case variation, as well as uncertainty about the arbitrator's preference in the eyes of labor and management. Ashenfelter and Bloom (1984, p. 122) attempt to eliminate between-arbitrator variability by incorporating arbitrator indicator functions into their regression model. The residual standard deviation varies from 1 to 3% (for different years) but the confidence intervals are wide because only a fraction of the data, involving frequently used arbitrators, is retained. Bloom (1986, p. 583), in a separate study in which arbitrators were presented with simulated cases based on the New Jersey police arbitration experience, found that separating out the between-case variation reduced the standard deviation from 1.82% to 1.52%.

Suppose, as a working hypothesis, that the standard deviation of arbitrator uncertainty, after accounting for between-arbitrator and between-case variation, is 1.5% and that the mean arbitrator position is 8.0% (as estimated by Ashenfelter and Bloom). We may compute equilibrium final offers both with and without a labor bonus. Assuming a normal distribution and no bonus, equilibrium management and labor offers are 6.1% and 9.9%, respectively, both much higher than the observed mean offers (5.7% and 7.9%).

Were labor to perceive a bonus for winning of 3.8% (equal to the gap between the equilibrium offers without bonus), then the equilibrium offers become 5.9% for management and 8.4% for labor, closer to the offers actually observed (see Figure 1). Decreasing the estimated standard deviation from 1.5% to 1.0% narrows the equilibrium offers both with and without a bonus but similarly yields an equilibrium with a bonus that is consistent with the observations of Ashenfelter and Bloom. Note, as Ashenfelter and Bloom do, that labor pays dearly for enhancing its probability of winning: the expected value of the settlement decreases significantly in value.

4. Application to Spatial Models of Electoral Competition

The FOA model has an interesting interpretation in political science. In a two-candidate election using a unidimensional spatial model (Downs, 1957; Enelow and Hinich, 1984), candidates and voters are placed on a real-line scale from liberal (left) to conservative (right). In the simplest form of the model, voters vote for the candidate whose position is closer to their own.

The classical median-voter theorem for two-candidate races (McKelvey, 1975; Enelow and Hinich, 1984) states that, under reasonable conditions, the optimal strategy for purely winning-oriented candidates is to place themselves at the median position. In practice, however, competing candidates seldom declare identical platforms. We suggest that a more realistic model may be patterned on FOA with a bonus.

Suppose that each candidate has an ideal political position, one to the left and one to the right of center, which each would like to see implemented. Each has a declining utility for other positions as they recede toward the center from the ideal. Suppose, for the moment, that a candidate reaps no utility from winning *per se*, attaching the same utility to an implemented platform regardless of who implements it.

This setting simulates FOA without a bonus. The declared positions of the candidates can be interpreted as final offers to the voters. The median voter operates as an arbitrator: the candidate whose declared position is closer to that of the median voter receives a majority of the votes and wins. Other things being equal, the candidates can be expected to choose equilibrium strategies (declared platforms) equidistant from the median voter position, just as under FOA without a bonus.

If, however, candidates seek to win as well as wish to see desirable policies implemented (Wittman, 1983), their behavior can be modeled as FOA with a bonus. The bonuses for winning, sought presumably by both sides, will act not only to narrow the gap between the declared platforms but also to skew both of these adopted positions away from the candidate who is more willing to yield position in order to win.

The utility, however, of a bonus sufficient to induce convergence of the platforms may need to be quite high indeed. In most two-candidate races, therefore, the candidates tend to be distinguishable in terms of some underlying dimension.

5. External Bonus under FOAB

So far we have assumed that the bonuses of the two parties for winning, whether in a dispute or an election, are internal: they accrue to the winner of FOA, or, in an election, from the satisfaction or pride of winning. But now suppose that the rules of FOA are modified and, under what we earlier called FOAB, the bonus is made external.

In particular, assume that the winner is awarded a bonus equal in value to the gap between the final offers, to be paid for by the loser. In other words, given final offers a and b, if labor wins, labor is awarded not b but b + (b − a), and management receives not a but a − (b − a). The situation is reversed if management wins.

This escalation of the stakes narrows the gap between equilibrium final offers by a factor of 3, i.e., the gap (b − a) is lowered from 1/f(0) to 1/3f(0).[5] Unlike internal bonuses, each equal to the gap $b_0 - a_0$ between equilibrium offers, which induce convergence, external bonuses of the same size do not. This occurs because, should either party move toward the center, the external bonus diminishes, reducing the expected payoff relative to that from an internal bonus, which remains fixed.

The equilibrium under FOAB occurs at precisely the set of final offers such that, under the modified rules of FOAB, the settlement will be one or the other of the equilibrium strategies that would have occurred under ordinary rules. For example, if equilibrium strategies under ordinary FOA are ± 1, the equilibrium strategies under the modified rules are ± 1/3, defining a gap of 2/3 which, when awarded to the winner and paid by the loser, yields a settlement at either +1 or −1. Hence, FOAB has no effect on the one-sidedness of outcomes under FOA but does have the virtue of moving the two parties much closer together, from where they may be able to close the gap on their own (given the rules permit them to negotiate before one or the other offers is selected by the arbitrator). From a normative viewpoint, therefore, it is more appealing than FOA.

More generally, if the winner wins (and the loser loses) an amount proportional to the gap, say $p(b - a)$, then equilibrium strategies are again symmetric, with the equilibrium gap equal to

$$b - a = \frac{1}{(2p+1)f(0)} . \qquad (15)$$

Convergence of the final offers is obtained only asymptotically as the proportion p approaches infinity.

Note that the proportion $p = 0$ corresponds to ordinary FOA; $p = 1$ corresponds to FOAB. A settlement defined by (exactly) "splitting the difference" is equivalent to a negative bonus of $(b - a)/2$, i.e., to $p = -1/2$.

Unfortunately for the parties, as p approaches $-1/2$, $(b - a)$ approaches infinity, i.e., divergence of the offers occurs. This fact helps explain why an arbitrator, likely to split the difference under conventional arbitration, simply encourages extreme posturing and outrageous demands by each side.

Although FOAB does not in theory produce complete convergence, the near convergence it induces may, in practical terms, be sufficient to persuade the two parties to come together on their own--either before FOAB is implemented or before the arbitrator announces a decision. By converting the arbitration process into a high-stakes gamble, FOAB has the potential to draw the two parties together. However, it may render more difficult the task of convincing the disputants to accept FOAB in the first place, especially if they are risk-averse.

6. Conclusions

It is not uncommon that one side is perceived to win in a dispute settled by conventional arbitration--even when the arbitrator splits the difference in some fashion--but it is far more common that the settlement is seen as a compromise for both sides, with neither completely happy about the outcome. Under FOA, by contrast, there is never any question about who won, but to win by "giving the house away" vitiates, of course, the very meaning of winning.

The New Jersey dispute data suggest that the unions, because they probably value winning more than management, are more conservative in their final offers. If they attach an internal bonus to winning that the local governments do not--and the governments as well as the unions know this--the governments can use this knowledge to choose equilibrium strategies that exploit this information. But the unions also have equilibrium strategies that take into account the fact that their bonuses will enable them to win more often.

In general, when one side but not the other sees a bonus in winning, the side that places the added value on winning will be hurt in the settlement, suggesting that it should try to hide this fact. When the two sides value winning equally, however, their equilibrium strategies will approach each other, converging when the bonus of each is exactly the gap between their final offers. Similarly, political candidates probably approach each other to the extent that they both value winning and do so more or less equally.

This result led us to incorporate an external bonus into FOA, giving FOAB, to try to induce two parties to resolve their differences on their own by creating a greater incentive to converge in their final offers. The most promising revision in FOA is to add the gap between the two final offers to the winner's settlement, which the loser would have to pay. Thus, the penalty for the loser is increased, and the winning settlement is sweetened, because the loser must pay not only the winner's final offer but also the difference between this offer and his or her own. Yet such a "double" penalty for losing is still not sufficient to induce convergent final offers (without internal bonuses as well), which combined arbitration (at least in principle for symmetric, unimodal distributions) accomplishes (Brams and Merrill, 1986).

Briefly, under combined arbitration, FOA is used if the arbitrator's choice falls between the two final offers; if they converge or criss-cross, the average of these offers is taken as the settlement. On the other hand, if the arbitrator prefers a settlement outside the two final offers, more extreme than either one, this becomes the settlement. In the latter case, combined arbitration robs the parties of the incentive to be extreme themselves--the arbitrator "protects" each on its side, enabling each to "afford" to compromise.

A rule that shifts somewhat the onus back to the arbitrator, as does combined arbitration when the arbitrator's choice of a settlement prevails, seems necessary in order to induce the two sides to converge completely. Paradoxically, the need for the arbitrator actually to make a choice under combined arbitration is obviated, at least in theory, because the incentives are such that the two parties will converge on their own. Thereby, the threat of using combined arbitration may lead to its own demise.[6]

FOAB would certainly increase the pressures on the parties to be more accommodating by making it more costly for them to lose. But these pressures are never, at least in theory, strong enough to induce convergence. Moreover, the equilibrium settlement will be the same one-sided outcome as under FOA, leading us to favor combined arbitration, which promotes complete convergence of the disputants' final offers.

Notes

[1] Intuitively, the bonuses must equal at least <u>twice</u> the gap because, in a particular settlement, only one of the bonuses can be won, making the expected value of each only half its "face value."

[2] To prove that (a_1, b_1) is a Nash equilibrium, it suffices to show that $G_B(a_1, b) \leq S/2$ for all b [since $G_B(a_1, b_1) = S/2$ by (2)]. The proof is almost identical to that of Theorem 1, part II, in Brams and Merrill (1983, pp. 930–931).

[3] This normalization is possible for any distribution satisfying the necessary condition, $f(0) \neq 0$, for the existence of a local equilibrium without a bonus.

[4] Distributions can be constructed for which the gap increases in the presence of a bonus. For example, let $f(x) = 1$ for $|x| \leq 0.25$ and 1/3 for $0.25 < |x| \leq 1$. When $S = 3$, $d = 1.5$, $a_1 = -1$, and $b_1 = 0.5$, so that a sufficiently large bonus can induce the offers to diverge for this distribution. If, however, f is symmetric and exceeds $e^{-2|x|}$ on the interval $[m, 0]$, then $d < 1$ and $b_1 < 0.5$, i.e., the parties are drawn together and the equilibrium offer for labor is lower than without a bonus.

[5] Specifying $S_B = (b - a)$ and $S_A = (b - a)$ in formulas (1) and (2), the modified game becomes zero-sum, yielding a single payoff function $G(a,b) = [a - (b - a)]F(m) + [b + (b - a)][1 - F(m)] = -3(b - a)F(m) + 2b - a$, where m denotes $(a+b)/2$. Taking partial derivatives as in section 2, we obtain a critical point for the payoff

function at ± $1/3f(0)$, which is at least a local equilibrium whenever $|f'(0)| < 12[f(0)]^2$. Note that one cannot substitute directly in (9) and (10) because the bonus involves the variables of differentiation.

6 Even under FOA, the threat of its use seems to put the disputants in a bargaining mood. Thus, in 1987 only 26 of 109 professional baseball players who initially filed for arbitration in salary disputes actually decided to use FOA, as prescribed in such disputes, in the end (Chass, 1988).

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Table 1. Equilibrium strategies with and without a bonus for player B (labor) for distributions normalized so that f(0) = 1.

Model	midpoint (m)	gap (d)	a_1	b_1
Without a bonus (S = 0) (distribution free)	0.000	1.000	−0.500	0.500
With a bonus (S = 2)				
Double exponential	−0.347	1.000	−0.847	0.153
Normal	−0.371	0.543	−0.643	−0.100
Logistic	−0.361	0.618	−0.670	−0.052
Triangular (symmetric)	−0.382	0.618	−0.691	−0.073
Uniform	−0.500	0.000	−0.500	−0.500

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Table 2. New Jersey Police Arbitration Cases, 1978-80

	Mean	Standard Deviation
Union final offers	7.9%	1.4%
Management final offers	5.7%	1.9%
Predicted arbitrator position from FOA awards (n = 324)	8.0%	2.3%
Conventional arbitration awards (n = 99)	7.6%	2.2%

Source: Ashenfelter and Bloom (1984; adapted from Table 1, p. 117).

Figure 1. Equilibrium offers with and without a bonus.