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HOW SHOULD VOTING ON RELATED PROPOSITIONS BE CONDUCTED?

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Abstract

Assume that voters choose between yes (Y) and no (N) on two related propositions, where YY, for example, signifies voting Y on the first and Y on the second. If a voter's preference order for the four possible combinations is, say, YY > NN > YN > NY, then this voter's preferences are "dependent," because whether he or she will prefer Y or N on the second proposition depends on whether Y or N is the outcome selected on the first. The fact that 67% of all preference orders are dependent when there are two propositions, and over 99% when there are three propositions, suggests that the simultaneous choices voters must make on referenda are problematic. Specifically, they may cause voters to experience regret, given that they do not have foreknowledge of outcomes on which to condition their choices.

The ability of voters to abstain on a proposition may provide some relief, but abstentions may be counted in different ways. Under "standard aggregation," they do not count—what combination is chosen depends only on which side (Y or N) gets the most votes on each proposition. Two alternative aggregation procedures, "approval aggregation" and "split aggregation," count abstentions as supportive of both sides, though in different ways. Each would have produced a different winning combination from that of standard aggregation on three related environmental propositions on the 1990 California general election ballot, based on the voting behavior of 1.7 million Los Angeles county voters.

Nevertheless, the alternative aggregation procedures do not "solve" the problem of voting on related propositions. Approval voting and the Borda count would enable voters better to express dependent preferences on them, but there remain practical difficulties in determining what "related" means and in limiting the number of combinatorial choices facing voters.

How Should Voting on Related Propositions Be Conducted?[1]

1. Introduction

Voting procedures are often binary, whereby voters are given only two choices. In most legislatures, for example, legislators are restricted to voting either yes or no on bills, or amendments to bills. Likewise, in most general elections in the United States, there are usually only two serious candidates—a Democrat and a Republican—so again voters can make only binary choices unless they abstain.

Our concern in this paper is with a different kind of binary election, which typically occurs in referenda with more than one proposition on the ballot. As in legislative and general elections, a voter is restricted to making binary choices on each proposition, but now he or she must make each choice without knowing the electoral outcomes on the others.

This is quite a different story from legislative voting, wherein the result of voting on, say, an amendment to a bill is known before the bill (with or without the amendment approved) is voted upon. The situation is also different from voting in sequential elections—such as a party primary followed by a general election, a plurality election followed by a runoff, or a sequence of primaries (as in presidential elections)—wherein earlier returns inform, if not structure, later choices.

To be sure, there are public elections in which a voter must make simultaneous binary choices among different sets of candidates for different offices. Thus, in about two-thirds of the states in a presidential election,

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[1]We thank Kenneth F. McCue for providing the Los Angeles county ballot data used in this study, and Gary King and Katri Saari for facilitating our use of these data. Hali Marko provided valuable research assistance, and Bob Ellsworth was unstinting in the computer assistance he gave us. Steven J. Brams thanks the C. V. Starr Center for Applied Economic Research for its support.

voters simultaneously select a candidate for president, for the Senate, and for the House, not to mention other candidates for state and local office.

Such elections have frequently resulted in "divided government" at the federal level (i.e., Congress, or at least the House, is controlled by one party, and the presidency is in the hands of the other party) as well as at the state level. Brams, Kilgour, and Zwicker (1994) showed that certain aggregation paradoxes can arise in simultaneous elections, which will be illustrated in a different context in section 5.

Our initial focus in this paper, however, is not on social-choice problems but, rather, on the individual-choice problem confronting the voter whose preferences are not "independent." In section 2 we illustrate such dependence by showing how a voter's foreknowledge of the voting result on one proposition may change the option he or she would choose on another proposition.

Because voters have no way of obtaining this foreknowledge—except, possibly, from polls, and this only imperfectly—they may vote differently from the way they would have had they possessed it. Likewise, they may vote differently if voting is sequential, in which their knowledge of earlier returns may affect their later choices.

In section 2, we illustrate with hypothetical examples the regret that voters may experience, after the fact, when they must vote on two propositions simultaneously.[2] In addition, we state more general results when there are three propositions (our later empirical case), indicating that independent preferences subsume more restrictive lexicographic and separable preferences.

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[2]Insofar as voters anticipate this problem, they may feel regret before the fact as well.

In section 3 we show how a third option that voters quite frequently exercise in voting on referenda—that of abstention on one but not all the propositions—may provide some relief from the simultaneity problem. However, abstentions may be counted in different ways. What we call "standard aggregation," which is the social-choice method used today, ignores abstentions: the outcome of voting on each proposition depends only on the numbers of yes votes for, compared with the number of no votes against, that proposition. To facilitate its comparison with other ways of aggregating votes, we show how standard aggregation can be conceptualized as a scoring system, in which the winning combination across all propositions maximizes a certain score.

In section 4 we analyze two alternative aggregation methods, which differ from standard aggregation in the support they give "compromise" outcomes (to be discussed later) and in the way they take account of abstentions. With respect to abstentions, "approval aggregation" counts an abstention as both a yes vote for, and a no vote against, a proposition—that is, abstention indicates approval of both sides of a proposition. "Split aggregation" assumes that each voter has one vote, which is split equally among all combinations of which he or she approves. We demonstrate how approval aggregation and split aggregation can lead to different outcomes from each other as well as from standard aggregation.

In section 5 we show that in the case of three ostensibly related propositions on the environment in the 1990 California general election, the two alternative aggregation procedures would have led to a different outcome, at least among the almost 1.7 million voters in Los Angeles county (we did not have individual-ballot data for other California voters in this election). We relate this empirical result to one of the aggregation paradoxes, showing that a mild kind of paradox occurred and a more severe—though not surprising—paradox occurred in voting on all 28 propositions in this election.

In section 6 we argue that binary voting, even with the possibility of abstention, is fundamentally inadequate to reflect voters' dependent preferences on related propositions. We suggest that approval voting and the Borda count would be superior systems for voting on combinations, or packages, of propositions, at least in the case in which there are only two or three related propositions that give rise to, respectively, four and eight combinations.

Although these other voting systems would not "solve" the problem of dependent preferences when voters must make simultaneous choices on related propositions, they would enable voters better to express themselves and possibly lead to more coherent social choices. However, practical problems in restructuring the ballot, including determining which propositions are related and therefore should be voted on as combinations, remain.

2. Dependent Preferences

When several propositions are on a ballot, there will typically be links among them. By "links" we mean that voting yes (Y) or no (N) on one, at least in the eyes of some voters, depends on the outcome of voting on others.

To illustrate such dependence, assume there are two propositions. Then an election may result in one of four possible outcomes, {YY, YN, NY, NN}, wherein the first letter of each ordered pair refers to the result of voting on the first proposition, the second letter the result of voting on the second proposition.

A voter has *independent preferences* over two propositions if foreknowledge of the voting result on one proposition does not affect the voter's preferred result on the other proposition (we will give a definition applicable to more than two propositions shortly). This is not true of a voter who has preference order

YY > NN > NY > YN,

because learning that the first proposition is going to win (i.e., a majority will vote Y) will motivate the voter to vote Y on the second proposition (because YY is preferred to YN), whereas learning that the first proposition is going to lose (i.e., a majority will vote N) will motivate the voter to vote N on the second proposition (because NN is preferred to NY). On the other hand, a voter whose preference order is

YY > NY > YN > NN

has independent preferences, because whether the first proposition wins or loses (i.e., is Y or N), the voter prefers Y on the second proposition.[3] Hence, he or she will never experience regret by voting for Y on this proposition.

In general, when there are *p* propositions, and 2^p combinations, a voter's preferences are *independent* if, for every proposition, the voter either always prefer that the proposition pass, or always prefers that it fail, whatever the results on the other p - 1 propositions. Thus, a voter with independent preferences always know for sure whether he or she wants each proposition to pass or fail, and therefore can vote confidently in a

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[3]Similarly, the voter's preference on the first proposition depends on the outcome of voting on the second—that is, dependence does not depend on the order in which the propositions are voted upon.

simultaneous referendum, without fear of regret for having supported, or not supported, any proposition.

Of the 4! = 24 possible strict preference orders over the four Y-N combinations, eight (33%) involve independent preferences (see Table 1).

Table 1 about here

When there are three propositions, there are eight possible outcomes,

{YYY, YYN, YNN, YNY, NYY, NYN, NNY, NNN},

and 8! = 40,320 strict preference orders. The number of independent preference order drops dramatically to less than 1%; more precisely, only 384 orders are independent, with this percentage negligible for four or more propositions.[4]

On a ballot containing several propositions, it would be surprising indeed not to find several propositions related, in terms of content, to each other. To illustrate such relatedness in the case of only two propositions (P1 and P2), suppose P1 is a proposition to lengthen prison sentences for certain crimes, and P2 is a proposition to build new prisons. A voter might support both P1 and P2, making YY his or her preferred combination, but—believing that jails are already overcrowded—think that the next-best

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[4]A method for determining the number of independent preferences, and identifying which they are, is given in Kilgour and Brams (1995). Independent preferences are equivalent to both separable and lexicographic preferences when $n = 2$; when $n > 2$, the latter categories are more restrictive. Thus, when $n = 3$, 96 preference orders are separable, and 48 are lexicographic; moreover, lexicographic preferences are a proper subset of separable preferences, and separable preferences are a proper subset of independent preferences. Lacy and Niou (1994) analyze referenda in which voters have nonseparable preferences, arguing that they may produce inefficient outcomes, whereas Benoit and Kornhauser (1994) show that the election of legislatures may lead to inefficient outcomes even when voters have separable preferences.

combination is for both propositions to fail (NN), and the third-best combination is that prisons be built before sentences are lengthened (NY), yielding

YY > NN > NY > YN,

which is a preference order we showed earlier to be dependent.

As a second example of dependent preferences, suppose that a voter supports several bond issues more or less equally but is concerned that if all are approved, the state's credit rating will fall and/or interest rates will rise.[5] If there are only two bond issues, the voter's preference order might be

YN > NY > NN > YY.

Knowing that the first proposition is going to win (Y), this voter would prefer N on the second, whereas knowing that the first proposition is going to lose (N), this voter would prefer Y on the second.

When Y and N votes are aggregated separately for each proposition to determine a winner, as is the case today, we call this *standard aggregation*. While voters with independent preferences have nothing to lose under standard aggregation, voters with dependent preferences may, without foreknowledge of the results of voting on the other propositions, find that they voted contrary to their interests once all the results are known. By contrast, if voting on propositions were done in sequence, and the results at each stage announced before the next stage commenced, the voter who, say, favored one bond issue but not two would be able to choose his or her preferred option in light of what had already happened.

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[5]Lacy and Niou (1994) give a real-life example of such a dilemma facing North Carolina voters in 1993.

To make more precise the problem of dependent preferences under standard aggregation, assume there are only two propositions. When a voter votes Y on the second proposition, this procedure credits him or her with preferring Y to N on this proposition, regardless of the result on the first proposition. In other words, the voter thinks both YY and NY are better than their "opposites" of NN and YN: YY > YN and NY > NN.

But this is exactly the rub: standard aggregation restricts the voter's ability to convey dependent preferences. For example, this voter may actually have dependent preferences

(1) YY > YN, ostensibly indicating that he or she prefers Y over N on the second proposition; and
(2) NN > NY, ostensibly indicating that he or she prefers N over Y on the second proposition.

Of course, this apparent contradiction is resolved by noting that this voter's preference for Y over N on the second proposition depends on Y's being the outcome on the first proposition, as give by (1); otherwise, his or her preference on the second proposition is the reverse, as given by (2).

How does one know, empirically, if preferences are linked across propositions? One prerequisite of dependence is the existence of a feature common to two propositions. Thus in our earlier example, stiffer prison sentences require more prisons, so prisons are common to building more of them and supporting lengthier incarceration in them.

Another problem that plagues voting on propositions simultaneously is that of reaching acceptable compromises. Suppose that voters are split about 50-50 between two reversed outcomes, YY and NN, as their top choices. Then it is quite possible that standard aggregation would lead to a "compromise" outcome, either YN or NY, that practically nobody likes. Is it reasonable to impose such an outcome simply because each group wins on one of the two propositions?

Such a combination may have very few supporters—in fact, *everybody* might rank it third or fourth. But because standard aggregation restricts voters to indicating only one preferred combination, it denies them the opportunity to register preferences for possible compromises that involve opposites.[6] In section 6, we will propose voting systems that enable voters to indicate their preferences over all combinations of related propositions.

3. Standard Aggregation: Formal Definition and Main Theorem

In referenda today, voters cannot vote directly on combinations of related propositions but only on each individual proposition. Although these separate votes come out as a vote for a single combination, it is a problematic choice when the separate votes are cast all at once, and voters' preferences are dependent, as we illustrated in section 2.

In fact, however, voting is not strictly binary, because voters need not vote on every proposition but can, instead, abstain. By allowing for abstentions, voters are given a third option, which makes voting ternary.

Of course, a voter who abstains on all propositions has stated no preference at all; hence, no account need be taken of such voters. However, we do wish to take account of voters who vote Y or N on some propositions

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[6]The same problem afflicts a voter's choosing among politicians running for different offices (Brams, Kilgour, and Zwicker, 1994). For instance, a voter today cannot indicate that his or her highest priority is that the same party control both the Senate and the House—that is, that the outcome be either DD (both houses Democratic) or RR (both houses Republican) over the mixed outcomes, DR or RD. This is because the voter who votes, say, DD is implicitly saying that the "compromise" of DR or RD is better than RR, as we will show quantitatively through our analysis of the standard-aggregation scoring method in section 3.

and abstain (A) on others, with the abstainers possibly different on each proposition.

Assume there are p propositions and n voters. Define variables v_i^j so that voter i's vote on proposition j is

$$v_i^j = \begin{cases} 1 \text{ if Y} \\ 0 \text{ if A} \\ -1 \text{ if N.} \end{cases}$$

Thus, voter i's p choices can be represented as a p-vector,

$$v_i = (v_i^1, v_i^2, \ldots, v_i^p) \in \{1, 0, -1\}^p,$$

which gives the combination of Ys, As, and Ns he or she casts over the p propositions. A *winner* (i.e., *winning combination*) is a p-vector,

$$c = (c_1, c_2, \ldots, c_p) \in \{1, -1\}^p,$$

that in some sense best represents the choices of all the voters $v_1, v_2, \ldots, v_n$ voting on the p propositions. Put another way, we wish the Y and N winners on each of the p propositions reasonably to reflect the choices, including A, of the n voters.

Under *standard aggregation*, the winner on proposition j is Y if the positive votes ($v_i^j = +1$) outnumber the negative votes ($v_i^j = -1$) across all voters i; it is N if the negative votes outnumber the positive votes, and both (i.e., a tie) if there are equal numbers of positive and negative votes. This aggregation method is embodied in

Definition 1. A winner under standard aggregation is a vector (combination) $c_{std} \in \{-1, 1\}^p$ that satisfies

$$c^{p}_{std} = \begin{cases} 1 \text{ if } \sum_{i}^{n} v_i^j \geq 0 \\ -1 \text{ if } \sum_{i}^{n} v_i^j \leq 0 \end{cases}$$

for every proposition $j = 1, 2, \ldots, p$. Note that both Y and N are considered to win if there is a tie; if there are ties on t propositions, then there are exactly 2^t winners.

We can now state our main characterization of standard aggregation:

Theorem 1. A vector $c_{std} \in \{1, -1\}^p$ is a winner under standard aggregation iff c_{std} maximizes the score

$$s_{std}(c) = \sum_{i=1}^{n} \sum_{j=1}^{p} v_i^j c^j \ .$$

Proof. Because

$$s_{std}(c) = \sum_{j=1}^{p} c^j \left[\sum_{i=1}^{n} v_i^j \right],$$

$s_{std}(c)$ is a maximum iff each term of the outer summation is maximized. Clearly, $c^j = 1$ maximizes the j^{th} term of the outer summation iff $\sum_{i=1}^{n} v_i^j$ is non-negative, whereas $c^j = -1$ maximizes the j^{th} term iff this summation is non-positive. Q.E.D.

Example 1. Suppose there are $p = 2$ propositions and $n = 7$ voters, who vote as follows on propositions P1 and P2:

$$(v_i^1, v_i^2) = \begin{cases} (1,1) \text{ by 3 voters} \\ (-1,0) \text{ by 2 voters} \\ (0,-1) \text{ by 2 voters.} \end{cases}$$

In other words, 3 voters vote Y on both P1 and P2, 2 voters vote N on P1 and A on P2, and 2 voters vote A on P1 and N on P2.

One consequence of Theorem 1 is that each voter can be considered to add a certain amount to the accumulated scores of each of the four possible non-abstention outcomes, (1,1), (1,-1), (-1,1), and (-1,-1).[7] These amounts are shown in Table 2. Thus, the winner is (1,1), because 2 is the largest

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Table 2 about here

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total. Of course, we know that (1,1) wins under standard aggregation, because each of P1 and P2 gets 3 Ys and 2 Ns.

The significance of Theorem 1 lies in its treatment of combinations. That is, it shows that an election conducted under standard aggregation can be thought of as being a contest among all possible combinations, with each voter's vote contributing a certain amount (which may be positive, negative, or zero) to the accumulated score of every combination. In the case of combination (1,1), each of the (1,1) voters contributes 1 vote based on his or her vote on P1—reflected in the fact that (1)(1) is the first term in the double summation of Theorem 1—and 1 vote based on his or her vote on P2, for a total of 2 votes. After all votes are tallied in this way, the combination with the highest score wins.

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[7]Note that, consistent with most election procedures, an outcome cannot be abstention, even if abstention gets the most votes.

The amounts that are added by each voter, as specified by Theorem 1, offer an interesting comparison. If there are two propositions and the voter votes for a specific combination, that combination receives an increment of 2 (i.e., +2 votes), the opposite combination receives a decrement of 2 (i.e., -2 votes), and the two remaining combinations, which match the voter's selection on one but not the other proposition, receive no increment or decrement (i.e., 0 votes) (see Table 2). Likewise, when a voter abstains on one proposition but expresses a preference on the other by voting Y or N, the two combinations that agree with the expressed preference receive +1 votes, whereas the two that disagree receive -1 votes, which is also illustrated in Table 2.

When there are three propositions, and the voter expresses a preference on all three (by not abstaining), then the combination that matches the voter's choice receives +3 votes, the combination that agrees on two propositions (and disagrees on one) receives +1 votes, the combination that agrees on one proposition (and disagrees on two) receives -1 votes, and the opposite of the voter's choice receives -3 votes. In general, the amount by which a voter increments or decrements the total for each combination equals the number of propositions on which the vote and the combination agree, minus the number on which they disagree.

This manner of scoring propositions and determining winners produces a sort of Borda count over all the combinations: the amounts added in the case of three propositions, as we just illustrated, are +3, +1, -1, or -3; in the case of two propositions (with the possibility of abstentions), the increments or decrements that each voter contributes to the non-abstention outcomes also differ by ±2 (see Table 2). Under standard aggregation, therefore, a voter effectively assigns a cardinal rating to all combinations—depending on the degree to which his or her votes across propositions agree with them—with abstention neither adding nor subtracting from the voter's level of agreement.[8]

4. Alternative Aggregation Methods

In this section, we define two aggregation methods that, like standard aggregation, allow for abstentions, which are rather common in real-world referenda. In the Los Angeles county referenda data we shall present in section 5, for example, the number of voters abstaining on a proposition was, on average, 11.4% (Dubin and Gerber, 1992).

Abstentions do not present a difficulty for standard aggregation. To determine the winner under this procedure, all one needs to do is count the numbers of Ys and Ns received by each proposition. Alternatively, Theorem 1 shows that the winning combination is that which has the greatest accumulated score, based on the scoring system for propositions discussed in section 3. This system takes account of abstentions by saying, in effect, that they neither add nor subtract points to a combination's score.

But there are other possible ways of aggregating votes—namely, to count votes directly for combinations while treating abstentions in different ways. In fact, we will propose two alternative scoring methods, "approval aggregation" and "split aggregation," each of which satisfies the following three conditions:

1. *Symmetry.* If the Y and N votes on some subset of propositions are interchanged, then the accumulated Y and N scores are also interchanged.

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[8]It is, of course, the ranking implicit in these ratings that may be an inaccurate reflection of a dependent voter's preferences. For example, the score of 0 that a (1,1) voter gives to outcome (1,-1) may not reflect the fact that this YY voter actually thinks that YN is the worst possible outcome rather than a middling outcome.

2. *Identity*. If no voter abstains on any proposition, then the total score for a combination is simply the number of people who vote for that combination.

3. *No difference*. Adding voters who abstain on every proposition does not change the relative scores of the combinations.

Our two alternative aggregation methods agree in assigning a vote for a specific Y-N combination, which contains no abstentions, as a vote only for that combination. They disagree in how they treat voters who abstain on some, but not all, the propositions.

Under *approval aggregation*, a voter supports a Y-N combination iff that combination does not disagree with the voter's Y or N choice on any of the *p* propositions. We assume that As do not cause disagreement, so YYA, for example, would be a vote for both YYY and YYN. Formally,

Definition 2. A winner under approval aggregation is a vector (combination) $c \in \{-1,1\}^p$ that maximizes the approval score

$$s_{app}(c) = |\{i: v_i^j c^j \neq -1 \ \forall \ j\}|.$$

This score is the number of voters who do not disagree with the combination in question, either because their Y-N votes are identical on each proposition or because they abstain on some propositions.

The more a voter abstains under approval aggregation, the more support he or she gives multiple combinations, whose number increases exponentially with the number of As that the voter casts. Thus, a voter who votes Y or N on every proposition casts one approval vote; a voter who abstains on one proposition casts two approval votes (i.e., for the two combinations that agree with the voter's Y-N votes on all propositions except the proposition in question, which may be either Y or N); and a voter who abstains on m propositions casts 2^m approval votes.

Approval aggregation is not the same as approval voting (Brams and Fishburn, 1983), which would allow voters to specify which combinations they consider acceptable.[9] For example, if there are two propositions, a voter might approve of the combinations YY and NN. By contrast, under approval aggregation, this voter could support these combinations only by abstaining on each, in which case he or she would be indicating approval of NY and YN as well—in other words, be approving of all the combinations, which would not distinguish among any of them.

The second aggregation method we propose is *split aggregation*. Under this method, each voter casts exactly one vote, which is split equally among all combinations of which the voter approves (as defined for approval aggregation). Formally,

Definition 3. A winner under split aggregation is a vector (combination) $c \in \{-1,1\}^p$ that maximizes the split score

$$s_{spl}(c) = \sum_{\{i:\, v_i^j c^j \neq -1\ \forall j\}} 2^{-m(i)},$$

where $m(i)$ is the number of propositions on which a voter i abstains. This score sums the split votes of the voters who do not disagree with the combination in question.

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[9]But it is the same as "yes-no voting" (Brams and Fishburn, 1993) when interpreted in the following way: a Y vote for a proposition means that any combination approved of must include Y for that proposition; an N vote against a proposition means that any combination approved of must include N against that proposition; and an A on a proposition means that any combination approved of may include either a Y or an N for that proposition.

Thus, if voter *i* abstains on exactly one proposition, he or she is considered to cast 1/2 a vote for each of the two combinations consistent with his or her A; if voter *i* abstains on *m*(*i*) propositions, he or she is considered to divide his or her support over the $2^{m(i)}$ combinations consistent this vote. Whereas approval aggregation gives an advantage—at least in terms of numbers of votes cast—to those voters who abstain on many propositions, split aggregation renders all voters equal in the sense that the more As a voter casts, the less each counts in the score for a combination and, hence, in making it a winner.

To clarify differences in approval aggregation and split aggregation, we return to

Example 1 (continued). The amounts that each of the three types of voters contribute to the four Y-N combinations under each method, and the accumulated scores for each of the four possible non-abstention outcomes, are shown in Table 3. Thus, approval aggregation elects NN with a score of

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Table 3 about here

4, whereas split aggregation elects its opposite, YY, with a score of 3, as did standard aggregation (see section 3). The reason that approval aggregation promotes NN to top place is that the 4 voters who abstain on either P1 or P2 each contribute a full vote to NN, which puts it over the top even though not a single one of the 7 voters voted for this combination.[10]

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[10]This is similar to what Brams, Kilgour, and Zwicker (1994) called a "paradox of vote aggregation": the winner under standard aggregation is the combination that receives the least direct support. We will illustrate this paradox, and milder variations, with the California referendum data in section 5.

We next give an example in which each of the three aggregation methods produces a different winner.

Example 2. Suppose there are $p = 2$ propositions and $n = 24$ voters, who vote as follows on propositions P1 and P2:

$$(v_i^1, v_i^2) = \begin{cases} (1, 1) \text{ by 2 voters} \\ (1, -1) \text{ by 9 voters} \\ (-1, 1) \text{ by 2 voters} \\ (1,0) \text{ by 1 voter} \\ (-1, 0) \text{ by 2 voters} \\ (0, 1) \text{ by 8 voters.} \end{cases}$$

Then it is not difficult to show that our three aggregation methods produce the following total scores for each of the four possible non-abstention outcomes:

	(1,1)	(1,-1)	(-1,1)	(-1,-1)
Standard aggregation	<u>11</u>	5	-5	-11
Approval aggregation	11	10	<u>12</u>	2
Split aggregation	6.5	<u>9.5</u>	7	1

Notice that we get a different winner (underscored), according to each of the three different aggregation methods, demonstrating the sensitivity of the outcome of propositional voting to the aggregation method used.

To be sure, voters may well vote differently under the different methods, so we do not wish to overstress these differences. In fact, there is good reason to believe that voters would adjust their voting strategies if a switch were made from, say, standard aggregation to approval aggregation, under which abstention on a proposition signifies approval of both Y and N.

We speculate that voters under approval aggregation would be less inclined to abstain, knowing that choosing A on a proposition gives a full vote to both sides. By contrast, this support is divided under split aggregation and nonexistent under standard aggregation, so abstention is less "dangerous" under the latter two aggregation methods.

In section 6 we will return to a comparison of these methods after analyzing the California data on related propositions. We will also consider alternative voting systems that may better enable voters to express their (dependent) preferences. But next we turn to the empirical data.

5. Voting on Related Propositions in Los Angeles County

On November 7, 1990, California voters were confronted with a dizzying array of choices on the election ballot: 21 state, county and municipal races, several local initiatives and referenda, and—most important from our standpoint—28 statewide propositions. These propositions concerned such issues as alcohol and drugs, child care, education, the environment, health care, law enforcement, transportation, and limitations on terms of office.

The data used in this investigation are the set of ballot images produced by 1,684,786 nonabstaining voters (i.e., who voted Y or N on at least one of the 28 propositions) in Los Angeles county in the 1990 general election. After extensive computer testing of voting on the propositions, we decided to focus on the following three related propositions on the environment:

P130. *Forest Acquisition Initiative*, which would ban clear-cutting in all forests, authorize a $742 million bond issue for the purchase of redwood forests, and place certain restrictions on the harvesting of timber, the burning of debris from logging, and the export of logs.

P148. *Water Resources Bond Act*, which would authorize $340 million for various water projects, including water storage and drought assistance, water treatment, and flood control.

P149. *Park, Recreation, and Wildlife Enhancement Act*, which would authorize $437 million in bonds to acquire, develop, and restore parks, beaches, and other recreational areas and pay for forest fire stations, museums, and zoos.

Not only do these propositions seem to be related, but they were all closely contested in Los Angeles county: P130 passed with 48.7% Ys, 43.9% Ns; P148 failed, with 42.2% Ys and 43.8% Ns; and P149 passed, with 45.7% Ys and 42.1% Ns (the remaining votes in each case were As).

Because all three propositions were pro-environment and involved the expenditure of substantial funds, there is good reason to believe that many voters saw them as related.[11] Additional evidence for this view is the fact that the propositions polarized the voters. Cross-tabulations of the numbers of voters choosing Y, N, and A for each pair of propositions show a remarkable grouping of voters. For example, on P148 and P149, more than 34% of the voters favored both propositions (YY) and more than 33% opposed both (NN). Of the remainder, the largest group (10% of the total) abstained on both (AA). Altogether, more than 3/4 of the voters agreed in their views by voting Y, N, or A on both propositions.

The situation for the other pairs was similar. On P130 and P148, more than 29% voted YY and more than 29% voted NN. On P130 and P149,

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[11]We will say shortly which of these voters probably had the most dependent preferences.

more than 33% voted YY and more than 29% voted NN. Not only did many voters take the same positions on all pairs, but large numbers took the same positions on all three propositions.

Figures for all triples of positions on the three propositions are shown in Table 4a. YYY is the largest vote-getter among the 27 triples with

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Tables 4a and 4b about here

430,807 votes, slightly outdistancing NNN with 422,916 votes. But the winner, according to standard aggregation, among the eight possible Y-N combinations is, as we noted earlier, YNY, which received only 99,176 votes. As shown in Table 4b, it maximizes $s_{std}(c)$, whereas the two other aggregation methods, approval aggregation and split aggregation, give the winner as YYY, which maximizes s_{app} and s_{spl}.[12]

In fact, YNY scores only fourth out of eight according to approval aggregation, and only fifth out of eight according to split aggregation, indicating a mild form of the paradox of vote aggregation (see note 11).[13] In both cases, YNY's total is barely 1/3 as great as the score of the winner,

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[12]Because approximately 0.02% of the ballots were spoiled, the figures in Table 4a do not sum to 1,684,786, the number of nonabstaining voters, but instead to 1,672,793.

[13]A genuine form of the paradox of vote aggregation occurred in voting on all 28 propositions by the Los Angeles county voters. The winning combination was NNNYNNYNYNNNNNNYNYNYNYYNNYNY on propositions 124 - 151 but, excluding all voters who abstained on any proposition, nobody made this winning choice. Hence, this combination received the fewest votes; however, more than 99% of all 2^{28} = 268.4 million possible Y-N combinations *must* have received 0 votes—even if each of the voters voted for a different combination—so there is nothing particularly surprising about this result. More surprising, and perhaps paradoxical, is that when there are as few as two propositions with abstention allowed, the winning combination can come in last of the $3^2 = 9$ combinations when there are only 15 voters; when there are three propositions and abstention is not allowed, the winning combination can come in last of the $2^3 = 8$ combinations when there are just 3 voters (Brams, Kilgour, and Zwicker, 1994).

YYY, which exceeds the second-place finisher, NNN, by 13% in the case of approval aggregation and 6% in the case of split aggregation.

Why is it that the standard aggregation winner, YNY, receives so many fewer votes under approval and split aggregation? Compared with YYY, these two combinations differ only in the middle term (P148), so each receives exactly the same support as does the other from voters who abstain only on P148. Moreover, there are not large absolute differences between the numbers of other abstention combinations that contribute to YNY versus YYY:

- 2,409 to YNY from ANY versus 11,260 to YYY from AYY;
- 4,543 to YNY from YNA versus 9,602 to YYY from YYA;
- 2,235 to YNY from ANA versus 4,442 to YYY from AYA.

In sum, YYY receives approval from 21,304 - 9,187 = 12,117 more partial abstainers than does YNY.

By far the biggest boost that the alternative aggregation methods give YYY comes from the more than 4:1 advantage in direct support that YYY (430,807) enjoys over YNY (99,176). The fact that 25.8% of the voters are pro-environment on all three propositions, and only 5.9% are pro-environment on P130 and P149 but not on P148, renders the YNY winner under standard aggregation, in our opinion, a dubious "compromise" choice.

Patently, the vote-aggregation method used on apparently related propositions can change the outcome. We suspect that even if voters had known that either approval or split aggregation were being used, they would not have changed their voting behavior so radically as to elect the unpopular YNY. Possibly the second-ranked NNN might have defeated YYY if more of the partial abstainers had switched to Ns than to Ys under one or both of the alternative aggregation methods.

If a compromise were sought, then instead of YNY, which less than 6% of voters supported, YNN would seem a better choice. Not only does it score higher than YNY under both the alternative aggregation methods, but it also receives more direct support than YNY (129,729 versus 99,176 votes).

So far we have discussed the effects of the alternative aggregation methods, showing that they in fact may have changed the outcome that voters selected on the three environmental propositions in the California election. But are these effects more severe when preferences are dependent—and how would we know if preferences were dependent?

On a probabilistic basis, as we noted in section 2, when there are three propositions (as in our empirical example), and all preference orders are equally likely (surely not the case in our example), the chances are greater than 99% that a voter's preferences will be dependent. We hypothesize, however, that for the 25.8% of voters who selected YYY, and the 25.3% who selected NNN, this dependence was not very great, in part because almost all probably preferred a "compromise" outcome (i.e., some Ys and some Ns) to the opposite of what they chose. For example, if a YYY voter preferred more Ys than Ns and, more specifically, had the (partial) preference order

YYY > YYN > YNN > NNN,

then whatever the outcome of the first or second elections, he or she would have a clear-cut preference for Ys on the subsequent elections and never experience regret.[14]

But what of the somewhat less pro-environment voter who wants two of the environmental propositions to pass but favors the first and third over the second and third, so has preference YNY > NYY? Then, depending on whether the first proposition passes or not, he or she will be against or for the second proposition; on the other hand, what happens to the second proposition does not change this voter's pro-feeling for the third.

In the case of the somewhat anti-environment voter, who wants only one of the three environmental propositions to pass but favors the first over the second (YNN > NYN), then his or her preference for the second proposition depends on what happens to the first. But what happens to the second does not change this voter's anti-feeling for the third.

We conclude that even when, as in the case of the three environmental propositions, a majority of voters favors "all or nothing" (i.e., YYY or NNN), there may well be significant numbers of voters who are moderately pro- or anti-environment and whose preferences are dependent. While no aggregation method can eliminate their problem when voting on propositions is simultaneous, there are voting systems, to which we turn in the final section, that may ameliorate their problem.

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[14]If Y is the outcome of the first election, this voter will unconditionally prefer that Y be the outcome of the second election; and if Y is the outcome of the second election, this voter will unconditionally prefer that Y be the outcome of the third election. In our empirical case, this says that a pro-environment voter wants more of the environmental propositions to pass, period. But insofar as this voter has intermediate preferences for which one or two propositions he or she would most prefer be passed, this voter's preferences will be dependent, as we illustrate next in the text.

6. Better Systems for Voting on Related Propositions?

It is worth noting that even voters with (largely) independent preferences—including probably most of the YYY and the NNN voters in our empirical case—may well have mixed feelings about switching to either approval or split aggregation. For although the YYY voters would have won big under both of these alternative aggregations methods—assuming that the behavior of voters, especially the abstainers, would not have changed significantly if these methods had been in place—voters in a close contest will not know this beforehand. Consequently, these methods might be viewed as risky, at least insofar as they tend to produce all-or-nothing outcomes instead of split verdicts (like the YNY outcome that actually occurred under standard aggregation).

As we indicated in section 5, however, the winning combination, YNY, which received direct support from only 5.9% of voters, is a dubious compromise. NYY, which received the direct support of 7.7% of the voters, is also a moderately pro-environment compromise that was not only preferred by 29% more voters than YNY but also does better under split aggregation (see Table 4b). Also, as we noted in section 5, YNN, a somewhat anti-environment compromise that received direct support from 7.8% of the voters, surpasses both YNY and NYY under approval and split aggregation.

Such figures for mixed outcomes, nevertheless, do not alter the fact that however a voter orders YNY, NYY, and YNN, his or her preferences are dependent in the following sense: what happens on the first proposition determines whether or not he or she prefers Y or N on the second or third proposition. Because a moderately pro- or anti-environment voter probably has more highly dependent preferences than the strongly pro- or strongly anti-environment YYY or NNN voter, his or her quandary in voting is greater. Furthermore, the simultaneity problem caused by dependent preferences does not go away after a winner is chosen; on the contrary, it becomes all the more apparent, because then a voter will know he or she had reason to be regretful.

Because 49% of voters on the environmental propositions did not choose YYY or NNN but one of the mixed outcomes (including those with one or two abstentions), and many of these voters probably had highly dependent preferences, we believe they would have benefited from a voting system that enabled them better to express their preferences. For example, approval voting, if applied to voting on combinations, would have enabled moderately pro-environment voters to approve simultaneously of YYN, YNY, and NYY—and perhaps YYY if this outcome were not viewed as too one-sided, or too expensive, compared to the mixed outcomes. Similarly, the Borda count would have permitted these voters to rank the aforementioned four outcomes at the top of their preference order.

To be sure, neither approval voting nor the Borda count eliminates the fact that almost any ordering of the eight possible outcomes reflects dependent preferences. Indeed, even the all-or-nothing voters will be seriously affected by the dependency problem if they order their two top outcomes either NNN > YYY or YYY > NNN, and they rank the six mixed outcomes lower.

It is precisely these voters, who believe that half-hearted measures are anathema, who are most in need of relief from the restriction of being able to vote for only one combination. Many would relish the opportunity, we believe, to vote for both YYY and NNN under approval voting, or rank them one and two under the Borda count, in order better to express their highly dependent preferences. We speculate, however, that relatively few voters would rank such opposite outcomes as their two most preferred.

In general, it seems, simultaneous voting on propositions will present the greatest difficulty for the voter whose preferences are most dependent. Although the alternative aggregation methods we have analyzed may produce different outcomes, as we showed empirically, they do not attack the dependency problem directly if voters still can vote for only one combination of related propositions.

Indeed, if these alternative methods have an effect, it is probably to induce more one-sided outcomes on related propositions. Because the all-or-nothing combinations are often the most popular, it is they that will probably benefit most under the alternative methods. But they will do so differentially, because each method counts the abstainers as supportive in different ways.

On philosophical grounds we favor split aggregation, because approval aggregation gives undue weight, in our opinion, to the influence of abstainers, whose support counts fully for both the Y and N positions on a proposition.[15] We believe that split aggregation would discourage abstentions, because the abstainers would have to give up two parts of their single vote for every part on which they take a Y or N position, which may induce them to become more informed so they would less often have to make this sacrifice.

When there are only two or three related propositions, we believe that voters should be able not only to choose among the four or eight

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[15]Thereby approval aggregation may help positions that have few or no direct supporters, leading to a combination about which nobody is enthusiastic (like the winning NNN combination in Example 1).

combinations directly, but they should also not be restricted to voting for just one. Rather, they should be allowed to vote for more than one, or rank them, using approval voting or the Borda count. Especially when voters' preferences are highly dependent—as is certainly true of the voter who most likes both YYY and NNN—they simply cannot express themselves well by casting only one vote.

Of course, allowing voters to vote directly for combinations using approval voting or the Borda count would radically alter the political landscape of referendum voting. It would require, among other things, some determination of what propositions are related and, therefore, which should be grouped as combinations on the ballot. It might also require that if there are more than three related propositions, the combinatorial alternatives be reduced to some manageable number, like eight. Although we think approval voting could be practically used to vote for more combinations, asking voters to rank more than eight alternatives under the Borda count is probably too demanding.

Grouping the 28 propositions on the 1990 California ballot into, say, 10 packages of two or three related propositions—each with 4 - 8 combinatorial choices—would, perhaps, double or triple the number of choices of voters to about 60 - 90. This is a large number, suggesting that more stringent criteria for putting propositions on the ballot might be introduced. More difficult to come up with would be principles for grouping propositions into a few packages that present voters with reasonable choices, but we do not think that this is an insurmountable problem.[16]

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[16]For a discussion of this and other proposed solutions to a special case of the dependency problem (preferences are not separable), see Lacy and Niou (1994).

These difficulties notwithstanding, we think our analysis demonstrates that, at least in the case of voting on the three environmental propositions, a questionable outcome was selected. Not only would another outcome probably have been selected under the alternative aggregation methods we discussed, but, arguably, it would have been more socially acceptable.

The real problem, however, is less with the aggregation method than with the voting system used to vote on related propositions. We suggested that two different voting systems, enabling voters either to approve or rank combinations of related propositions, would attenuate the problem of dependent preferences and facilitate more coherent social choices.

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Table 1

Independence of 24 Strict Preference Orders over {YY, YN, NY, NN}

Orders	*Independent?*
1. YY > YN > NY > NN	Yes
2. YY > YN > NN > NY	
3. YY > NY > YN > NN	Yes
4. YY > NY > NN > YN	
5. YY > NN > YN > NY	
6. YY > NN > NY > YN	
7. YN > YY > NY> NN	
8. YN > YY > NN > NY	Yes
9. YN > NY > YY > NN	
10. YN > NY > NN > YY	
11. YN > NN > YY > NY	Yes
12. YN > NN > NY > YY	
13. NY > YY > YN > NN	
14. NY > YY > NN > YN	Yes
15. NY > YN > YY > NN	
16. NY > YN > NN > YY	
17. NY > NN > YY > YN	Yes
18. NY > NN > YN > YY	
19. NN > YY > YN > NY	
20. NN > YY > NY > YN	
21. NN > YN > YY > NY	
22. NN > YN > NY > YY	Yes
23. NN > NY > NY > YN	
24. NN > NY > YN > YY	Yes

Table 2

Amounts Added to the Accumulated Scores under Standard Aggregation by 7 Voters in Example 1

Voters	*Non-Abstention Outcomes*			
	(1,1)	(1,-1)	(-1,1)	(-1,-1)
1st (1,1) voter	2	0	0	-2
2d (1,1) voter	2	0	0	-2
3d (1,1) voter	2	0	0	-2
1st (-1,0) voter	-1	-1	1	1
2d (-1,0) voter	-1	-1	1	1
1st (0,-1) voter	-1	1	-1	1
2d (0,-1) voter	-1	1	-1	1
Total score	2	0	0	-2

Table 3

Amounts Added to the Accumulated Scores under Approval and Split Aggregation by 7 Voters in Example 1

1. Approval Aggregation

Voters	*Non-Abstention Outcomes*			
	(1,1)	(1,-1)	(-1,1)	(-1,-1)
1st (1,1) voter	1	0	0	0
2d (1,1) voter	1	0	0	0
3d (1,1) voter	1	0	0	0
1st (-1,0) voter	0	0	1	1
2d (-1,0) voter	0	0	1	1
1st (0,-1) voter	0	1	0	1
2d (0,-1) voter	0	1	0	1
Total score	3	2	2	4

2. Split Aggregation

Voters	*Non-Abstention Outcomes*			
	(1,1)	(1,-1)	(-1,1)	(-1,-1)
1st (1,1) voter	1	0	0	0
2d (1,1) voter	1	0	0	0
3d (1,1) voter	1	0	0	0
1st (-1,0) voter	0	0	1/2	1/2
2d (-1,0) voter	0	0	1/2	1/2
1st (0,-1) voter	0	1/2	0	1/2
2d (0,-1) voter	0	1/2	0	1/2
Total score	3	1	1	2

Table 4

4a. Voting Totals on Three Environmental Propositions (n = 1,672,793)

	P149: Y	P149: N	P149: A
P130: Y			
P148: Y	430,807	49,604	9,602
P148: N	99,176	129,729	4,543
P148: A	32,504	5,439	52,698
P130: N			
P148: Y	128,153	64,102	4,767
P148: N	54,629	422,916	8,214
P148: A	8,937	10,569	31,927
P 130: A			
P148: Y	11,260	2,293	4,442
P148: N	2,409	8,367	2,235
P148: A	8,012	3,160	82,299

4b. Scores for Y-N Combinations under Three Aggregation Methods (Winning Combination Underscored)

	Standard	Approval	Split
YYY	132,408	<u>549,325</u>	<u>473,788</u>
YYN	-27,008	127,238	73,346
YNY	<u>186,784</u>	201,577	134,640.25
YNN	27,368	206,171	153,426.75
NYY	-27,368	197,498	151,730.25
NYN	-186,784	121,260	82,798.75
NNY	27,008	116,363	74,952.50
NNN	-132,408	487,388	445,821.50
Total score	0	2,006,820	1,590,494