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WELFARE, WAGES AND VACANCY RATES

by

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0: INTRODUCTION

In recent years considerable attention has been given to turnover in the labor market. See, Jovanovic, 1979 and 1984. This reflects a growing awareness that the labor market is characterized by incomplete information and so search and matching. Only incomplete information can explain the large flows of labor between employers observed in the U.S. labor market. In particular, only imperfect information can explain the extremely large amount of search conducted by new entrants to the labor force. See, Topel and Ward, 1985. In such markets where information is costly one would expect middlemen or intermediaries to arise to enable traders to economize on information gathering. However, all the formal models of turnover and search by new entrants that have been developed exclude the possibility of such intermediaries. This is becoming an empirical embarrassment because of the rapid growth in the U.S. of the proportion of job changes that are mediated by such intermediaries.

This growth in the use of intermediaries has not gone unremarked by those involved in the market. It is, for instance, a complaint among personnel managers that the growth in use of intermediaries has forced up wages for the groups of workers concerned and generally to have caused the labor market to have tightened.

The principal aim of this paper is to show what role labor market intermediaries, or as we will call them, search firms, play and what impact their presence will have on the competitive equilibrium wage and vacancy rates. We find that the introduction of search firms will indeed raise the equilibrium wage rate and also, for reasonable parameter values, lower the equilibrium vacancy rate. However, this "tightening" of the labor market

reflects a rise in the efficiency with which labor is allocated. A secondary, aim of the paper was to model the hiring strategy of the firm in the face of random quits by employees. We find that in a labor market with incomplete information an optimal hiring policy may involve hiring before the incumbent worker has quit, i.e. labor hoarding. Formally, labor hoarding by the hiring firm represents a situation in which the firm chooses, at a cost, to search with perfect recall. The cost of acquiring this right is that if the firm has to pay a wage greater than the new hire's marginal product of labor. When, in addition, the quits are stochastic, the firm may also advertise vacancies that do not exist, in the sense that even if the firm finds a suitable worker it may not hire him.

The paper is organized as follows. Section 1 describes briefly the labor market intermediary industry and sets the stage for the analysis of the subsequent sections. Section 2 analyses the firm's optimal hiring policy when it faces nonstochastic quits. Section 3 extends the analysis to stochastic quits and identifies which hiring strategies are caused by incomplete information in the labor market and which by the stochastic nature of quits. Using the results of these sections, section 4 shows that in equilibrium all firms will hire via search firms even when the search technology of the latter is the same as that of the employers. In section 5 the steady state competitive equilibrium of the labor market is analysed with and without search firms and the impact of the latter on welfare, wages and the vacancy rate derived. Section 6 contains some concluding comments.

1: THE GROWTH AND STRUCTURE OF THE INDUSTRY

The personnel supply services industry (SIC 736) is both large and

Table 1

Growth of Private Employment Agencies: 1954-1982

	1939*	1948*	1954	1958	1963	1967	1972	1977	1982
PRIVATE AGENCIES (SIC 7361)									
Number of Establishments	1,400	2,200	1,993	2,746	3,490	4,471	6,378	7,123	9,608
% Increase Over Previous Census Count	-----	57%	-9%	38%	27%	28%	43%	12%	35%
Number of Employees	-----		10,028	16,783	17,932	26,540	38,123	55,095	96,890
% Increase Over Previous Census Count	-----		-----	67%	7%	48%	44%	45%	76%

* 1939 and 1948 data not fully comparable to figures in later years as Census of Business' counting method revised.

Sources: '39 - '77 data from U.S. Census of Business '82 data from 1982 Census of Service Industries.

growing. In 1982 it included over 16,000 establishments, employed 600,000 people and had total revenues of \$9 billion. The major parts of the personnel supply services industry are employment services (SIC 7361) and help supply services (SIC 7362). This paper concentrates on the former.

The private placement "business" has been almost entirely ignored by both labor economists and industrial relations specialists. See Ornati and Eisen, 1981. In fact this business is made up of different sets of firms with distinct but overlapping characteristics. These are the private employment agencies that provide placements to employers and are not generally involved with executives; the "search" firms that specialize in the placement of executives; the casting bureaux that primarily serve the theatrical and movie world; and the temporary services supply firms that differ from the others because the employees that they place remain on the placing agencies' payrolls.

In 1982 there were 9,600 employment agencies with 97,000 employees and a total revenue of \$1.9 billion. In terms of establishments, there are roughly as many employment agencies as automobile rental establishments and coin operated laundry and dry cleaning establishments and more agencies than movie theaters or hotels. On the other hand, in terms of revenue, the employment agency industry is only as large as the National Can Co., the 188th. largest company in the Fortune 500 listing.

Not only is the industry large but it is growing fast. Table 1 shows the rapid growth of employment agencies since World War II. With an increase in payroll of 126% since the 1977 Census, employment agencies have grown more rapidly than any other business service industry except for computer programming services, management consulting and interior designing firms. Geographically, like most business services, employment agencies are

Table 2

Employment Agency Industry Structure

	ALL FIRMS	MEAN	MODE	SMALL CAT.			LARGE CAT.		
				Class	Mean Size	No.	Class	Mean Size	No.
EMPLOYMENT	90,289	11.32	30.43	< 5 emps.	1.9	4588	500+emps.	807	10
RECEIPTS	\$1,846,314,000	\$231,513	\$682,291	< \$100,000	\$45,588	4,088	\$10 mill+	\$16.6 mill.	8
PAYROLL	\$1,000,696,000	\$125,479	-----	-----	-----	-----	-----	-----	-----
ESTABLISHMENTS	8,980	1.13	1	Single Unit	1	8031	5 estabs.+	11.2	60
RECEIPT/PAYROLL RATIO	1.845	1.845	-----	-----	-----	-----	-----	-----	-----
RECEIPT/ESTAB. RATIO	\$205,602 per est.	\$204,877	682,291	-----	-----	-----	-----	-----	-----

Source: 1982 Census of Service Industries.

concentrated in California, New York, Texas and Illinois. From an industrial organization point of view the industry is characterized by easy entry, high labor intensity and low fixed costs. The result of this cost structure is that the industry is primarily made up of small, single establishment firms with low per employee payrolls and with a large proportion of individual proprietorships. See Table 2. Some firms are, however, large. Only 5% of employment agencies are multi-unit establishments, but this 5% accounts for almost one quarter of the industry's revenues.

No exact figure can be placed on the impact of these agencies on the labor market because data for the volume of placements made by these agencies are not available. Given that the product being sold by the agencies is the availability and quality of certain workers then one would expect them to become a more important influence on the labor market when the demand for this product rises. Several developments suggest that this demand has risen substantially in the last twenty years. Demographically, the low birth rate in the 1930's resulted in a shortage of people of executive age in the 1970's and so increasing pressures to find people with the appropriate talents and backgrounds. Geographically, the movement of corporate headquarters out of city cores to the suburbs and away from the north east and to the south west has raised the demand for the type of information provided by employment agencies and search firms. Finally, the dramatic increase in the professionalism of management has resulted in increases in the size and specialization of managements and so increased demand for the product of the search firms.

2: DETERMINISTIC QUIT: SUCCESSION PLANNING

The forces at work in determining a firm's decisions about searching for and hiring new workers can be demonstrated most clearly in a model in which the date at which the job's incumbent will quit is known in advance, e.g. a retirement, and where the replacement worker is presumed to stay in the future. In the human resources literature a strategy for filling the vacancy created by such a quit is known as succession planning. Succession planning is contrasted in that literature with the strategies used when filling a vacancy that has occurred unexpectedly. These latter strategies are called replacement planning. As will be shown below there are good economic reasons for adopting quite different strategies in the two cases. Analysing the firm's succession planning, apart from being interesting in its own right also has the advantage of demonstrating which features of a firm's behavior stem from incomplete information in the labor market and which from the stochastic nature of quits.

Consider, then, a firm that has one job. Workers outside the firm are of two kinds, suitable (well-matched) or unsuitable for this job at this particular firm.¹ The proportion of workers that are suitable is $(1-p)$, $0 < p < 1$. If a suitable worker is in the job the firm receives and can observe a per period profit of $\pi > 0$ while if the job is vacant the firm incurs, because of fixed costs, a per period loss of $-k$, $k \geq 0$. If an unsuitable worker is in the job the firm incurs a loss that is sufficiently great, in our notation greater than $V(1)$, that it will never hire such a worker. In order to hire a suitable worker the firm must first find one. Neither it nor the workers

¹ Note that "suitable" means well-matched rather than highly productive to all possible employers. Match quality can be thought of as being continuous and employers as choosing a cut-off minimum level of match quality below which they will not hire a worker. Suitable workers are then those workers whose match quality exceeds this cut-off level.

know, ex ante, whether a particular worker is or is not suitable for the job. However, the firm can discover the suitability of a worker by testing the worker at a cost $c > 0$. This test can be thought of as an interview, a written or practical test or one period of observing the worker on some trial job. The firm can test as many workers as it likes in any period though each worker can go through only one test per period. The labor market is, therefore, characterized by incomplete information about workers' characteristics.

The job is currently occupied but at some known date T in the future the incumbent will quit and the job fall vacant. We wish to know how the firm will go about finding a replacement worker. This is best done by considering a set of nested strategies that the firm might follow. Consider first the myopic strategy of waiting until the job falls vacant in period T and then searching optimally for a replacement. Denote by $V(1)$ the value to the firm of having a replacement. Given that the replacement is not expected to quit in the future², $V(1)$ is given by

$$V(1) = \pi / (1 - \beta) \quad (2.1)$$

where β , $0 < \beta < 1$, is the firm's discount factor. Denote by $V(0, n)$ the value to the firm of having the job vacant at or after T and sampling $n \geq 0$ workers in such a period. Thus

$$V(0, n) = -k - cn + \beta V(1) - p^n \beta [V(1) - V(0, n)] \quad (2.2)$$

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All of the results in this section can be extended to the case in which the replacement worker will quit after a known, finite period of time in the job.

The firm must then choose an optimal sample size, n^* . Rearranging (2.2) this choice can be written as

$$\text{Max}_n V(0,n) = \frac{-k - cn + \beta(1-p^n)V(1)}{1 - \beta p^n} \quad (2.3)$$

This problem is concave and, ignoring the integer nature of n , yields a unique value of n^* which will be one or greater provided, as we will assume to be the case, $\pi + k \geq c(1-\beta)/(1-p)\beta$, that is provided it is optimal to search at all.³ Denote the maximized value of $V(0,n)$ by $V(0)$. As one would expect, the intensity of search for a replacement, or testing of workers, is increasing in $\pi+k$, the cost of having the job vacant, and decreasing in the cost of testing, c .⁴

This strategy for finding a replacement is myopic because it ignores the fact that at $T-1$ the firm knows that in the following period the job will be vacant. This suggests that the firm should begin to search at least at $T-1$. Denote the value to the firm of searching for a replacement at $T-1$ by $V_{-1}(1, s_{-1})$ where s_{-1} is the sample size chosen. This can be written as

$$V_{-1}(1, s_{-1}) = \pi - cs_{-1} + \beta V(1) - p^{s_{-1}-1} \beta [V(1) - V(0)] \quad (2.4)$$

A comparison of (2.2) and (2.4) shows that the optimizing value of s_{-1} , \hat{s}_{-1} , is just n^* . Moreover, this strategy will dominate waiting until T to search provided it would pay the firm to search if the job were vacant, i.e. if

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See Appendix 1 for a proof of this.

⁴ See Benhabib and Bull (1984) and Morgan (1984) for further analysis of optimal sample size search strategies in a sequential search setting.

$$V(1)\beta(1-p) \geq c.$$

So far we have established that if the firm is to search at all for a replacement then it should begin searching at least at $T-1$ and use a search intensity of n^* . However, it may be optimal to start searching even earlier than this. In order to insure itself against incurring a loss of $\pi+k$ at and after T the firm may decide to search prior to $T-1$. This will depend in part on the inventory cost of carrying a suitable worker, if one is found, from the date at which he is found to T . Denote by $w > 0$ the per period inventory cost of carrying such a worker. Notice that if the firm were given the ability to search with perfect recall it need never incur this inventory cost. However, it seems more reasonable to assume that perfect recall must be purchased by the firm. Indeed, we will assume that in order to recall the replacement at T , the firm must actually hire the worker despite the fact that his productivity will be lower than his wage up until the departure of the incumbent. Indeed, if his productivity were zero then the cost of keeping the replacement worker would simply be his wage. In effect the firm can always choose to search with perfect recall rather than without recall if it is willing to hoard labor, i.e. employ workers at a wage above their marginal product.⁵

The conditions under which some searching will occur prior to $T-1$ can be established by looking at the choice of whether to search in period $T-2$. If the firm does not search in period $T-2$ and behaves optimally thereafter the present value of its profits, $V_{-2}(1, s_{-2}=0)$, are

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The usual model of labor hoarding relies on there being a fixed cost of hiring a worker and a positive probability that if the worker is laid off he will quit the firm and not respond to a recall notice. See Bull and McCarthy, 1985. The same forces are at work here except that the hiring costs are now explicit.

$$V_{-2}(1, s_{-2}=0) = \pi + \beta\pi + \beta V(0) \quad (2.5)$$

If, alternatively, it searches in T-2 its expected profits are

$$V_{-2}(1, s_{-2}) = \pi - s_{-2}c + \beta V_{-1}(2) - p^{s_{-2}-2}\beta[V_{-1}(2) - V_{-1}(1)] \quad (2.6)$$

where $V_{-1}(x)$, $x=1,2$, is the expected profit of the firm in period T-1 conditional on it having x workers employed. Note that from (2.4) we have⁶

$$V_{-1}(2) = V(1) - w \quad \text{and} \quad V_{-1}(1) = V(0) + \pi + k \quad (2.7)$$

(2.6) is concave in s_{-2} and so unless $V_{-2}(1, s_{-2}=1) - V_{-2}(1, s_{-2}=0) \geq 0$ it will not be optimal for the firm to search at T-2. Substituting (2.7) into (2.6), evaluating the latter at $s_{-2}=1$ and then subtracting (2.5) gives the condition

$$-c + (1-p)\beta[V(1)-V(0)] - (1-p)\beta[\pi+k+w] \geq 0 \quad (2.8)$$

This condition for the optimality of searching at T-2 and so, probabilistically, hoarding labor shows that such search and hoarding is more likely when the opportunity cost of not being able to search with perfect recall is high, i.e. π and k are high, and when the cost of buying the ability to recall perfectly is low, w low. More interesting is the effect of an increase in p , that is a reduction in the success rate for search, on (2.8).

⁶ The expression for $V_{-1}(2)$ uses the fact that once a replacement worker has been hired no further search will take place, while $V_{-1}(1)$ is obtained by substituting (2.2) into (2.4) and using the fact that $s_{-1} = \frac{\pi}{\pi}$.

Raising p has two effects. First of all it lowers $V(0)$ and so raises the penalty for reaching T without a replacement worker. By itself this raises the incentive to search prior to $T-1$. However, the rise in p also has the effect of lowering the success rate of searching prior to $T-1$ which by itself would cause the firm to delay search until $T-1$. The size of this latter effect depends on the size of w because the marginal benefits of search prior to $T-1$ are inversely proportional to w . Thus if w is high the reduction in the efficiency of search prior to $T-1$ has little effect on the marginal benefit of search at that time.⁷ If w is high then firms with highly specialized jobs for which only a low proportion of workers were suitable, $(1-p)$ small, will be more likely to search with recall and to hoard workers than firms with less specialized jobs.

So far we have restricted attention to searching from period $T-2$ onwards. In fact the firm may choose to begin search even earlier. However, the costs of labor hoarding and so searching with recall go up linearly with the length of the period of hoarding. The effect of this is captured in the following proposition.

Proposition 1

Let the optimal sample sizes at $(T-\tau)$ be denoted by $\hat{s}_{-\tau}$, $\tau \in \{1, 2, \dots\}$, and be constrained to be non-negative integers. Assume that it is profitable to search at T . Then in the model above,

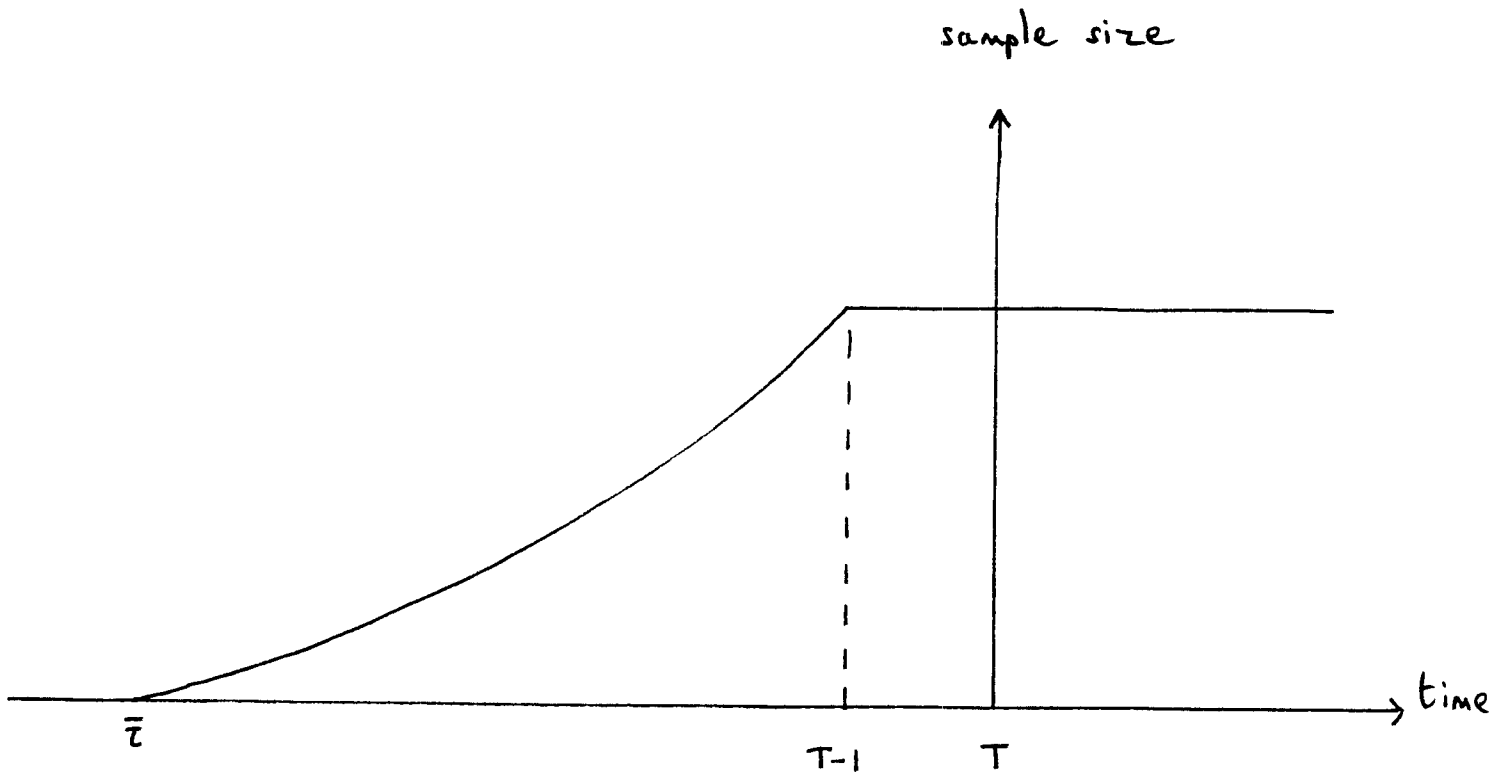
$$\hat{s}_{-(\tau+1)} \leq \hat{s}_{-\tau} \text{ for all } \tau$$

and there exists a finite $\bar{\tau}$, greater than 1 such that for all $\tau \geq \bar{\tau}$, $\hat{s}_{-\tau} = 0$.

Proof: See Appendix 1.

⁷ For proofs of these statements see Appendix 1. The results for w and k hold π constant and so refer to changes in the composition of total costs.

Figure 1



The weak inequality in this proposition comes solely from the integer nature of the sample size. The behavior of the optimal sample size over time is shown in Figure 1. Notice that the proposition implies that the probability of hiring in a period, conditional on no hire already having been made, rises after \bar{t} and reaches a plateau at $T-1$.

We have seen that, when faced by incomplete information in the labor market and an anticipated quit, a firm will choose to begin looking for a replacement worker at least one period prior to the quit and may even begin search earlier, though at a lower intensity, and hire a replacement with some positive (but not unit) probability prior to the quit, i.e. the firm may choose to hoard labor. We turn in the next section to what additional behavior might be observed if the quit were random.

3: STOCHASTIC QUILTS: REPLACEMENT PLANNING

Consider a firm like that of the previous section except that rather than a quit occurring once at a fixed date, each worker at the firm quits with an exogenous probability $(1-\gamma)$, $0 < \gamma < 1$, each period. This probability is independent over time and across workers. If the firm has one worker it will be faced by the prospect of the job being unoccupied at some random date in the future. In this case, as before, the simplest strategy for the firm to follow is to search for a replacement worker only when the job falls vacant. Letting n be the number of workers sampled per period of search we obtain

$$V(1,0) = \pi + \beta\gamma V(1,0) + \beta(1-\gamma)V(0,n) \quad (3.1)$$

$$V(0,n) = -cn - k + \beta(1-p^n)V(1,0) + \beta p^n V(0,n) \quad (3.2)$$

Solving (3.1) for $V(1,0)$ and substituting into (3.2) shows the firm's problem to be

$$\text{Max}_n V(0,n) = \frac{\pi\beta(1-p^n) - (1-\beta\gamma)(cn+k)}{(1-\beta)(1+\beta-\beta\gamma-p^n\beta)} \quad (3.3)$$

This problem is concave and yields a unique maximizing value of n , n^* . It can readily be shown that n^* is nondecreasing in π and k and nonincreasing in c .

If the firm follows the strategy above, then whenever an incumbent quits it will have the job empty for at least one period. Recalling the previous section it is natural to ask whether this strategy can be dominated by one in which the firm hires a replacement in anticipation of a quit, i.e. carries an inventory of workers. However, unlike the previous section there is now an alternative to this route. Rather than hiring workers to put in inventory the firm might choose to search for a replacement worker but only hire one if the incumbent quits. This strategy could not have been optimal in the set-up of the previous section because the date of the quit was known in advance and so the firm knew in advance that it either would or would not hire a suitable worker if one was found. Given this knowledge it would only search if it knew it would in fact hire that period.

In order for the strategy of searching for a replacement worker in all periods but hiring only when the incumbent quits to be optimal, it must be the case that deviating for one period from the strategy of only searching when the job is vacant and sampling one worker while the job is occupied will raise

the firm's discounted expected profits. Denote the expected profits from such a deviation by V^d .

$$V^d = -c + \pi + \beta[1-(1-\gamma)p]V(1,0) + \beta(1-\gamma)pV(0,n^*) \quad (3.4)$$

Using (3.1) we see that

$$V^d - V(1,0) = \beta(1-\gamma)(1-p)[V(1,0)-V(0,n^*)] - c > 0 \quad (3.5)$$

is necessary for searching while the job is occupied to be optimal. Note that for any hiring to be profitable $V(1,0) > V(0,n^*)$. As one would expect, the lower is the probability that the incumbent will quit, $(1-\gamma)$, the less likely it is that the firm will find it optimal to search when the job is occupied. Conversely, the higher is π or k , the higher is $[V(1,0)-V(0,n^*)]$ and so the higher is the cost of having the job empty. This makes (3.5) more likely to be fulfilled and so makes it more likely that the strategy of searching but not hiring while the job is filled will be optimal.

If we assume that (3.5) is fulfilled and denote the firm's optimal choice of sample sizes when the job is empty or occupied by η^* and s^* respectively, the following propositions can be shown (See Appendix 2 for the proof),

Proposition 2

The optimal sample size when the job is empty is greater than or equal to the optimal sample size when the job is occupied, $\eta^* \geq s^*$.

Proposition 3

The optimal sample size when the job is vacant is smaller for a given

firm if it searches when the job is occupied than if it does not, $\eta^* < n^*$.

The first of these propositions is unsurprising. The marginal benefit of increasing the sample size when the job is occupied is discounted by the probability that the job will become vacant next period and is thus lower than the marginal benefit when the job is vacant. As the marginal cost of search is the same in both cases one would expect the optimal level of search to be lower when the job is occupied. Indeed, the only reason why the inequality in Proposition 2 is weak rather than strong is the integer nature of n .

The second proposition is more surprising because one might think that the choice of intensity of search when the job was empty would be independent of the intensity of search when the job was occupied. However, searching when the job is occupied lowers the value of having the job occupied by causing the firm to incur search costs. Thus setting $s^* > 0$ lowers the marginal benefit from increasing the sample size when the job is empty and so lowers the optimal sample size in that state.⁸

One could object that the strategy of searching but not hiring when the job is occupied is that it will always be dominated by one which involved hoarding a worker. This is not the case. w does not enter condition (3.5) but if w is large enough the firm will find it prohibitive to hoard a worker.

While moving to stochastic quits opens up a new, potentially optimal,

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This strategy of searching but not hiring when the job is occupied has implications for the interpretation of vacancy statistics. A firm following this strategy would advertize a vacancy, i.e. search for a suitable worker, even when with probability γ it would not hire a successful candidate. This suggests that taking the number of vacancies advertized as a measure of the number of workers demanded at the advertized wage may overestimate the number of jobs actually available. However, if the sample size is correlated with the number or size of job advertizements then changes in the latter will be positively correlated with changes in the number of jobs available.

search strategy it also opens up the possibility of new optimal hoarding strategies. In the model of the previous section because hoarded workers never quit it never paid to hire more than one replacement worker. Here, because of the randomness of the quits, it may be optimal to hoard more than one worker. Certainly if w were zero then it would be optimal to hire every suitable worker found while searching irrespective of the number of workers already employed. With $w > 0$ the firm will have a maximum number of workers it will be willing to employ at any time but that number could be considerably greater than two especially if the probability of quitting were high. Unfortunately, analysing the optimal hoarding strategy is computationally difficult because of the combinatorial expressions that must be evaluated in order to find the probability distribution over the labor force given any particular search and hiring strategy. However, as we shall see in the next section, this does not pose a problem for our analysis of search firms.

4: SEARCH FIRMS

There are two ways of modeling intermediaries. One is to assume that they have some comparative advantage in search compared with the firms. The alternative taken in this paper is to find some gains from trade between the firms and postulate that the intermediaries arise to facilitate this trade. To this end we assume that the intermediaries, whom we call search firms, sample workers at random and have the same testing costs as the firms.

The role of search firms in the market can either be viewed as providing a market in which firms can trade tested workers⁹ or as the provision of

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In practice, the workers with which search firms deal are professional and managerial.

insurance against sampling variance. To understand this role consider an industry in which all firms find the same subset of workers to be suitable. Within the industry each firm is identical to that of the previous section and, in the absence of search firms, each finds it optimal to search for workers only when there is a vacancy and to choose a sample size of two. Each individual firm faces a sampling risk because with probability p^2 it will not find a suitable worker. However, if there are a continuum of such firms with positive measure μ , then we know that with probability one $2(1-p)\mu$ suitable workers will be found. Thus the sampling risk is perfectly diversifiable in the sense that if each firm took a sample size $(1-p)^{-1}$ then in aggregate the number of suitable workers found would equal in probability the number of jobs available. A search firm can sell contracts to provide a suitable worker with probability one to each firm and then sample $\mu(1-p)^{-1}$ workers and fulfill its contracts.

At first one might suspect that because the firms are risk neutral the provision of such insurance by search firms might not be strictly more efficient than letting each firm search in isolation. However, this not the case. In the example above, each firm will find two suitable workers with probability $(1-p)^2$ but will hire only one, chosen at random, the other being returned to the pool. This means that the information that the firm has gained, at a cost, about the worker who is returned to the pool is, from the social point of view, wasted. It is this waste of information concerning the suitability of the worker returned to the pool that can be avoided by the use of the search firm. This efficiency gain ensures that an equilibrium with search firms will be strictly more efficient than one without search firms.

In addition to the efficiency gain described above, there is a second gain due to the timing of the search. Again this efficiency gain can be

attributed to the provision of insurance or the conservation of information. In the example above the firms chose to search for workers only after a vacancy had arisen. However, let the measure of all firms in the industry be one. Then in any period exactly $1-\gamma$ of the firms will suffer the loss of a worker. Thus, in aggregate there is no uncertainty about the number of vacancies each period and we are, in a sense, back in the situation studied in Section 2. A search firm could offer a contract to a set of firms with positive measure, μ_1 , that promised to provide a suitable worker in the period after the vacancy occurred thereby removing the one period of lost profits that would otherwise accompany the loss of an incumbent worker. The search firm would know that exactly $\mu_1(1-\gamma)$ workers would be demanded by its customers each period and so could sample $\mu_1(1-\gamma)/(1-p)$ workers each period and deliver them at the time of the vacancy. Any lost profits are a social as well as private loss in this industry, and so this contract will, in a competitive equilibrium, dominate one that provides a worker one period after the incumbent quit.

So far it has been tacitly assumed that in a competitive equilibrium¹⁰ the firms would choose to fully insure themselves with the search firms against lost profits. This is in fact the case as stated in the following proposition.

Proposition 4

In a competitive equilibrium the firms of the previous section will contract with a search firm for a suitable worker to be delivered at the time the incumbent quits.

Proof: See Appendix 3.

¹⁰ By competitive equilibrium we mean that the search firms earn zero profits.

We turn next to the observable impacts of the introduction of search firms into the labor market.

5: THE EFFECT OF SEARCH FIRMS ON STEADY STATE VACANCIES AND WAGES

In this section we embed the analysis of the previous sections in a steady state model of the labor market. This enables us to answer questions about the effect of search firms on labor market variables notably the degree of "tightness" in the labor market as measured by the vacancy rate and the wage rate. With regard to the latter, it is occasionally suggested that the advent of search firms has driven up wage rates. Our model will enable us to see whether such an effect is possible.

We deal first with the steady state prior to the advent of search firms. Let the number of suitable workers entering the market¹¹ each period be an exogenously given constant $T > 0$. Denote by E and V , respectively, the number of workers employed in the industry and the number of vacancies, i.e. unfilled jobs. We assume that the costs of search are such that the optimal sample size for a firm to take if it is searching is one and that it will only search when its job is vacant.¹² Thus the number of vacancies in any period t is given by

$$V_t = (1-\gamma)E_{t-1} + pV_{t-1} \quad (5.1)$$

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Recall that suitability refers to match quality and so the workers who are unsuitable in this market will be suitable in some other market.

¹² This is not restrictive. All that is required is that some search take place.

At the steady state, $V_t = V_{t-1} \equiv V^S$ and $(1-\gamma)E^S = T$. Using this and (5.1) shows that the steady state values of employment, E^S , vacancies and jobs, J^S are

$$V^S = T/(1-p), \quad E^S = T/(1-\gamma), \quad J^S = E^S + V^S = T(2-\gamma-p)/(1-p)(1-\gamma) \quad (5.2)$$

If we define the vacancy rate as $v \equiv V^S/J^S$, then

$$v = (1-\gamma)/(2-p-\gamma) \quad (5.3)$$

Now consider the market after the entry of a competitive search firm industry. In this case each job is filled at the instant the vacancy occurs and so the number of jobs and the level of employment coincide. Denoting the steady state values of the variables by a superscript c , we find that,

$$V^c = (1-\gamma)E^c = T, \quad E^c = T/(1-\gamma), \quad J^c = E^c \quad (5.4)$$

This implies that

$$v^c = (1-\gamma) \quad (5.5)$$

Thus we see that the introduction of search firms leaves the steady state level of employment unaltered but reduces both the number of vacancies and the number of jobs. Because these two latter effects tend to be off-setting, the direction in which the vacancy rate moves is ambiguous. If $(p+\gamma) > 1$, then the vacancy rate will fall but if, as seems more reasonable, $(p+\gamma) < 1$, then the vacancy rate will rise. These results are summarized in the following proposition.

Proposition 5

Let the cost of search be such that in the market without search firms employers find it optimal to search for a new worker after a vacancy has occurred and to use a sample size of one. If a competitive search industry enters this market steady state employment is unaffected, the steady state level of jobs and vacancies will fall and the steady state vacancy rate will rise or fall according to $(p+\gamma) < 1$ or $(p+\gamma) > 1$.

The economics of this is as follows. Competition among firms for labor means that, at the original number of jobs, wages were just high enough to ensure zero expected profits for the firms. In that situation, if each firm that had a vacancy contracted with a search firm for immediate delivery of a replacement worker there would not be enough new suitable workers to satisfy the demand. Wages would, therefore, be driven up, profits down and exit would take place. This would continue until a point was reached at which the firms in the industry were earning zero expected profits and the supply and demand for new workers was equal. The impact of this process is summarized in the following proposition.

Proposition 6

Moving from a steady state equilibrium in which there are no search firms to one in which there is a competitive search industry will raise steady state equilibrium wages.

Proof:

Let $V^S(0)$ and $V^S(1)$ denote the steady state equilibrium values of having one and zero workers, respectively, when there was no search firms. The steady state probabilities of being in these states are $(1-\gamma)/(2-p-\gamma)$ and $(1-p)/(2-p-\gamma)$ and are denoted by ρ^S and $(1-\rho^S)$ respectively. In this equilibrium wages will be just high enough that

$$\rho^S V^S(0) + (1-\rho^S) V^S(1) = 0$$

In the steady state with search firms, the steady state probability that the incumbent will quit at the end of a period is $(1-\gamma)$ and that he will not is γ .

In either case the job is never left vacant. Thus the value of a firm, V^C , in this equilibrium is

$$V^C = \pi + \beta V^C - (1-\gamma)\beta c / (1-p) \quad \text{or,}$$

$$V^C = \pi / (1-\beta) - (1-\gamma)\beta c / (1-\beta)(1-p)$$

By using (2.1) and (2.2) for the case where $n=1$ we obtain

$$V^S(1) = \frac{\pi(1-p\beta)}{(1-\beta)(1+\beta-\gamma\beta-p\beta)} - \frac{(k+c)(1-\gamma)\beta}{(1-\beta)(1+\beta-\gamma\beta-p\beta)} < V^C$$

Given that $V^S(0) < V^S(1)$, this implies that at any given wage rate expected profits are higher with than without the search firms and so in the steady state containing the search firms wages must be higher in order for expected profits to be zero.

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These two propositions show that the complaints of firms about the impact of search firms on labor market "tightness" may have some observational foundation. The model predicts that wages will rise after the advent of search firms and that if $(p+\gamma) < 1$, which seems reasonable, the vacancy rate will rise. However, these impacts should not be taken as indicating that welfare falls after search firms enter the market. On the contrary, the reason why wages rise is because the increase in allocative efficiency caused by the search firms is imputed back to the factor supplied inelastically, which in our case is the set of suitable workers.

6: CONCLUSIONS

This paper has shown that in a labor market characterized by incomplete information, labor market intermediaries can increase efficiency despite being no more adept at searching for workers than the employers. They raise efficiency because they enable the sampling variance that each firm would face

when searching to be diversified over a large number of firms. This means, in turn, that valuable jobs are not left vacant for want of a suitable worker. This gain in efficiency shows up in an increase in the steady state equilibrium wage paid to suitable workers and fewer vacant jobs. In this sense the labor market intermediaries make the labor market "tighter".

Some aspects of the behavior of actual search firms have not been captured in this model. Perhaps the most important of these is the role of search firms in reallocating workers across employers. In the model of this paper such reallocation does not occur because the match between the worker and the firm is revealed accurately by the testing procedure. One extension of the model would be to make testing imprecise and embed the search firms in a more complete matching and turnover model.

Finally, although our analysis has been cast in terms of the labor market and labor market intermediaries it is of more general interest. An alternative example is the role of realtors in filling vacant apartments. Treating the number of apartments as fixed the model predicts that the advent of realtors in the tenant-landlord market in a city would raise rents, reduce the number of renters coming to the city to look for housing and, probably, lower the vacancy rate.

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APPENDIX 1

Existence of a unique maximum for (2.3)

From (2.3) we have $\partial V(0,n)/\partial n > (<) 0$ iff,

$$\pi + k > (<) c \frac{1 - \beta p^n (1 - n \ln p)}{-\beta p^n \ln p} \quad (A1.1)$$

The r.h.s. of (A1.1) has a derivative with respect to n of $c(1 - \beta p^n)/\beta p^n > 0$ and the limit of the r.h.s. is ∞ . Thus (A1.1) is satisfied with equality at most once at a value of n denoted by n^0 . Ignoring the integer nature of n then n^0 maximizes (2.3). To insure that the optimal integer n is greater than or equal to one we require $V(0,1) > V(0,0)$ or

$$\pi + k > c(1-\beta)/(1-p)\beta$$

Effects of π, k and w on (2.8)

$$\frac{\partial V(1)}{\partial k} = \frac{\partial V(1)}{\partial w} = 0; \quad \frac{\partial V(1)}{\partial \pi} = \frac{1}{1-\beta}$$

$$\frac{\partial V(0)}{\partial k} = \frac{-1}{1-\beta p^n}; \quad \frac{\partial V(0)}{\partial w} = 0; \quad \frac{\partial V(0)}{\partial \pi} = \frac{\beta(1-p^n)}{1-\beta p^n} \cdot \frac{\partial V(1)}{\partial \pi}$$

Define $D \equiv [V(1) - V(0)]$, then

$$\frac{\partial D}{\partial k} = \frac{1}{1-\beta p^n}; \quad \frac{\partial D}{\partial w} = 0; \quad \frac{\partial D}{\partial \pi} = \frac{\partial V(1)}{\partial \pi} \cdot \left[1 - \frac{\beta(1-p^n)}{1-\beta p^n} \right] = \frac{1}{1-\beta p^n}$$

Dividing the lefthand side of (2.8) by $\beta(1-p)$ we see that the partial derivatives of the resulting expression are,

$$w: -1, \quad k: \frac{\beta p^n}{1-\beta p^n} > 0, \quad \pi: \frac{\beta p^n}{1-\beta p^n} > 0$$

Effect of p on (2.8)

The partial derivative of the lefthand side of (2.8) with respect to p is

$$-\beta[V(1)-V(0)] + \beta(\pi+k+w) - \frac{(1-p)\partial V(0)}{\partial p} \quad (A1.2)$$

$$[V(1)-V(0)] = \frac{(1-\beta)V(1) + k + cn}{1-\beta p^n} = \frac{\pi + k + cn}{1-\beta p^n} \quad (A1.3)$$

$$\frac{\partial V(0)}{\partial p} = \frac{-(1-p^n)\beta np^{n-1}\beta V(1) + np^{n-1}\beta[-cn-k+(1-p^n)\beta V(1)]}{(1-p^n\beta)^2}$$

$$= \frac{-np^{n-1}\beta[\pi + k + cn]}{(1-p^n\beta)^2} < 0 \quad (A1.4)$$

Substituting A1.3 and A1.4 into A1.2 we see that the sign of A1.2 is the same as the sign of

$$(1-p^n\beta)^2[\pi+k+w] - (1-p^n\beta)[\pi+k+cn] + np^{n-1}(1-p)[\pi+k+cn]$$

If w is sufficiently greater than cn, then the above expression is positive.

Proof of Proposition 1

For convenience we rewrite (2.4) here,

$$V_{-1}(1, s_{-1}) = \pi - cs_{-1} + \beta V(1) - p^{\beta-1}\beta[V(1)-V(0)] \quad (A1.5)$$

$$\text{Note that } V_{-\tau}(2) = V(1) + \sum_{t=1}^{\tau} \beta^{t-1} w \quad \tau \geq 1 \quad (A1.6)$$

and that $V_{-\tau-1}(1, s_{-\tau-1}) \geq V_{-\tau}(1, s_{-\tau})$. (A1.7)

Consider now the choice of s_{-2} .

$$V_{-2}(1, s_{-2}) = \pi - cs_{-2} + \beta V(1) - p^{S-2} \beta [V(1) - V(0)] \\ + p^{S-2} \beta [w + \pi + k] \quad (\text{A1.8})$$

Let \hat{s}_{-1} maximize (A1.5) and \hat{s}_{-2} maximize (A1.8). The additional positive fifth term on the right of (A1.8) ensures that $s_{-1} > s_{-2}$ if s were continuous. However, because s is discrete

$$s_{-1} \geq s_{-2} \quad (\text{A1.9})$$

Now consider an arbitrary period $T-r$, $r \geq 1$. Then

$$V_{-r}(1, s_{-r}) = \pi - cs_{-r} + \beta V(1) - p^{S-r} \beta [V(1) - V(0)] \\ + p^{S-r} \beta \left[\sum_{t=1}^{r-1} \beta^{t-1} w + V_{-r+1}(1, s_{-r+1}) - V(0) \right] \quad (\text{A1.10})$$

The maximizing value of s_{-r} is decreasing in the last term in (A1.10) and that term, by (A1.6) and (A1.7), is strictly decreasing in r . Thus, by induction, $s_{-\tau}$ is weakly decreasing in τ , $\tau \geq 1$.

Finally, note that $V_{-\tau}(1, s_{-\tau})$ has a finite limit as $\tau \rightarrow \infty$. Denote this limit by V_{∞} . Then,

$$V_{\emptyset} = \pi - cs + (1-p^S)\beta(\pi-w)/(1-\beta) + p^S\beta V_{\emptyset} \quad (\text{A1.11})$$

$$\text{or, } V_{\emptyset} = \{\pi - cs + (1-p^S)\beta(\pi-w)/(1-\beta)\}/(1-p^S\beta) \quad (\text{A1.12})$$

We wish to show that the value of s that maximizes (A1.12) is zero. Assume the contrary and consider deviating from this policy by not sampling workers for one period. Denote the value of this alternative policy by V_d ,

$$V_d = \pi + \beta V_{\emptyset} \quad (\text{A1.13})$$

Subtracting (A1.12) from (A1.13) yields the following expression divided by $(1-p^S\beta)$,

$$(1-p^S\beta)\pi - (1-\beta)\pi + (1-\beta)cs - (1-p^S)\beta(\pi-w)$$

or,

$$\beta(1-\beta)(1-p^S)\pi + (1-\beta)cs + w(1-p^S)\beta > 0, \text{ all } s > 0$$

Thus, by contradiction, we have shown that in the limit the optimal s must be zero. As s is an integer this implies that for some finite value of τ the optimal s must be zero.

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In order to prove Propositions 2 and 3 it is helpful to develop two inequalities. Let the value to the firm of having a worker and searching with a sample size s be $V(1,s)$ and the value of not having a worker and searching with a sample size n be $V(0,n)$. We normalize $\pi=1$. Then

$$V(1,s) = 1 - sc + \gamma \beta V(1,s) + (1-\gamma)p^s \beta V(0,n) + (1-\gamma)(1-p^s)\beta V(1,s) \quad (A2.1)$$

$$V(0,n) = -k - nc + (1-p^n)\beta V(1,s) + p^n \beta V(0,n) \quad (A2.2)$$

The usual manipulations lead to

$$V(1,s) = \frac{(1-\beta p^n)(1-sc) - \beta(1-\gamma)p^s(nc+k)}{(1-\beta)[1-\beta p^n + \beta(1-\gamma)p^s]} \equiv \frac{A(1,s)}{B(1,s)(1-\beta)} \quad (A2.3)$$

$$V(0,n) = \frac{\beta(1-p^n)(1-sc) - (1-\beta)(cn+k) - \beta(1-\gamma)p^s(nc+k)}{(1-\beta)[1-\beta p^n + \beta(1-\gamma)p^s]} \equiv \frac{A(0,n)}{B(0,n)(1-\beta)} \quad (A2.4)$$

For it to be optimal to raise s from any given level then, provided $V(1,s) > 0$,

$$[A(1,s+1) - A(1,s)]B(1,s) \geq [B(1,s+1) - B(1,s)]A(1,s) \quad (A2.5)$$

Analogously, for it to be optimal to raise n it is necessary that

$$[A(0,n+1) - A(0,n)]B(0,n) \geq [B(0,n+1) - B(0,n)]A(0,n) \quad (A2.6)$$

Computing the relevant differences shows,

$$A(1,s+1)-A(1,s) = -(1-\beta p^n)c + \beta(1-\gamma)(1-p)(nc+k)p^s$$

$$A(0,n+1)-A(0,n) = \beta p^n(1-sc)(1-p) - [(1-\beta)+\beta(1-\gamma)p^s]c$$

$$B(1,s+1)-B(1,s) = -\beta(1-\gamma)(1-p)p^n$$

$$B(0,n+1)-B(0,n) = \beta(1-p)p^n$$

Substituting these expressions into (A2.5) and (A2.6) we find that for it to be optimal to raise s ,

$$n \geq s - \frac{1}{c} - \frac{k}{c} + \frac{1}{1-p} + \frac{1-\beta p^n}{\beta(1-\gamma)(1-p)p^s} \quad (\text{A2.7})$$

For it to be optimal to raise n we need

$$n \geq s - \frac{1}{c} - \frac{k}{c} - \frac{1}{1-p} + \frac{1-\beta(1-\gamma)p^s}{\beta(1-p)p^n} \quad (\text{A2.8})$$

Proof of Proposition 2

We prove this proposition by showing that at any optimal value of n if $s=n$, then it is optimal to (weakly) lower s . Let (A2.8) hold with equality and set $n=s$. Let the RHS of (A2.7) and (A2.8) be, respectively, A and B . Then for it to be optimal to lower or leave s the same $A-B \geq 0$. $A-B$ is

$$\frac{2}{1-p} + \frac{1-\beta p^n}{\beta(1-\gamma)(1-p)p^n} - \frac{1-\beta(1-\gamma)p^n}{\beta(1-p)p^n} \quad \text{which takes the sign of}$$

$$\gamma - \beta p^n + \beta(1-\gamma)p^n(1+\gamma) = \gamma[1-\gamma\beta p^n] > 0$$

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APPENDIX 3

Proof of Proposition 4

In a competitive equilibrium each search firm must earn zero profits. The search firms sell contracts of the form (θ, q) where θ is the probability that the search firm will provide a worker in the current period and q is the price of the contract. Note that the cost to the search firm of providing a worker with probability θ is independent of whether the employer has a vacancy or not. This means that employers, if they choose to use a search firm at all will always contract for probable delivery of a replacement at the time the incumbent quits. Zero profits for the search firms require that

$$q = \theta c / (1-p) \quad (\text{A3.1})$$

We proceed by contradiction. Let the employer have optimally chosen a level of θ' in the interval $(0,1)$ and denote this level by θ_0 . This gives

$$V(1, \theta_0) = \pi + \gamma \beta V(1, \theta_0) + (1-\gamma)(1-\theta_0)\beta V(0) + (1-\gamma)\theta_0\beta V(1, \theta_0) - c\theta_0/(1-p)$$

By the assumption of the optimality of θ_0 we know that $V(1, \theta_0)$ is greater than the alternative strategy of deviating from θ_0 and choosing $\theta' = 0$ in one period. The value of this alternative strategy is

$$V(1, 0) = \pi + \gamma \beta V(1, \theta_0) + (1-\gamma) \beta V(0)$$

and

$$V(1, \theta_0) - V(1, 0) = (1-\gamma) \theta_0 \beta V(1, \theta_0) - (1-\gamma) \theta_0 \beta V(0) - (1-\gamma) \theta_0 c / (1-p) > 0$$

therefore,

$$\beta V(1, \theta_0) - \beta V(0) - c / (1-p) > 0$$

Now consider choosing $\theta' = 1$ for one period. This would yield

$$V(1, 1) = \pi + \beta V(1, \theta_0) - (1-\gamma) c / (1-p)$$

and gives

$$V(1, \theta_0) - V(1, 1) = (1-\gamma) \beta V(1, \theta_0) - (1-\gamma) (1-\theta_0) \beta V(0) - (1-\gamma) (1-\theta_0) c / (1-p) > 0$$

which contradicts the optimality of θ_0 . This shows that the optimal value for θ' is either zero or one and it is trivial to show that $\theta'=1$ dominates $\theta'=0$ provided it is optimal to search when there is a vacancy.

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