THE ECONOMETRIC APPROACH TO BUSINESS-CYCLE ANALYSIS RECONSIDERED

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Abstract

The research performed and reported in this article represents an econometric analysis of the model generated by John Blatt which was discussed in this journal in 1978. The major point made by Blatt was that the standard stabitical procedures used by economists would have led to egregious errors in inference; namely estimating a stable linear relationship in a world generated by an unstable non-linear model. The main point of this paper is that the use of specification error tests pioneered by one of the authors leads to immediate and overwhelming rejection of the incorrect model. That empirical economists must learn to test their models before attempting to make inferences is the major policy conclusion.

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In an interesting article in 1978 in the <u>Oxford Economic Papers</u> by John Blatt, the author demonstrates vividly the dangers of making casual inferences from linear models when used to approximate a non-linear model. The sample is most useful, not only in highlighting sources of incorrect inferences, but also in demonstrating the all-important role of specification error tests.

Blatt (1978) specifies a Hicksian type of business-cycle model with no random disturbance terms, but with an unstable accelerator coefficient coupled with floors and ceilings. The model of three equations has a period of 15.

The data generated by this model are used in a simple linear autoregressive "reduced" form version of the model to provide by means of the ordinary least squares technique estimates of the reduced form coefficients. The regression equation used regresses income in period t on income lagged one and two periods respectively.

The regression results are dramatic and highly instructive. A non-linear model without stochastic components and involving an explosive value for the accelerator (2.0) is estimated as a linear stochastic model with a stable value for the accelerator; the highest value obtained for the accelerator coefficient in a series of regressions was 0.947 and the lowest 0.793.

Blatt claims that the traditional measures of evaluating a regression model, goodness of fit, tests for first order auto-correlation, coefficient signs, plausible values and t ratios are all most acceptable in terms of the conventional wisdom. In short, a very serious error of inference has been made which, indeed, is compounded by increasing the sample size and the corresponding values for the degrees of freedom.

From the perspective of the authors, this article provides a classic example of the great importance of specification error tests in any regression analysis. As is clearly illustrated by the article itself, the inferential error induced by the model misspecification is insiduous in that it will not be detected, even with infinite sample sizes, unless one is trying to discover the presence of specification errors. The second aspect and an item of striking importance in this sample is that, if one does look for specification errors, the test results are very clear in their implication. The linear model is seriously misspecified and the "disturbances" have a period of fifteen. These conclusions continue to hold even when the model is "fudged" by adding observational error terms in order to provide a more realistic test.

The Specification Error Tests Used and the Empirical Results

The various specification error tests to be used on the Blatt regression model have already been discussed at length in a number of previous publications. A brief description of the form and role of each test follows, but for details the reader is referred to Box and Jenkins (1976), Hoaglin and Welsch (1978), Ramsey (1970, 1974, 1976), Thursby (1979), and Wu (1973).[1]

Four basic specification error tests were used in this study: RESET, BAMSET, WSET, and the Box and Jenkins version of the Q-sum test. The formulation and use of each of the tests is briefly described below.

The basic linear regression model is:

(1) $$Y = X\beta + U$$

where under the null hypothesis of no specification error, the disturbances are assumed to be distributed as:

$$U \sim N(\emptyset, \sigma^2 I).$$

Y is an $T \times 1$ regressand vector, X is an $T \times K$ regressor matrix of rank K, and β is a $K \times 1$ vector of coefficients.

The various specification errors to be handled by the four tests can be separated into three groups:

(a) omitted variables, incorrect functional form, simultaneous equation problems;

(b) heteroscedasticity;

(c) nonnormality.

For each group there corresponds an alternative hypothesis H_i to the null hypothesis H_0:

(a') H_1: $U \sim N(\eta, \sigma^2 I)$ where η is a nonstochastic vector

(b') H_2: $U \sim N(\emptyset, \Omega)$ where Ω is a diagonal matrix with unequal elements

(c') H_3: $U \sim F(\eta, v)$ where F is not the normal distribution and has mean vector η and covariance matrix V

In contrast, the ordinary least squares residuals, $\hat{U} = Y - \hat{Y}$, are distributed as follows:

(a") H_1: $\hat{U} \sim N(M\eta, \sigma^2, M)$

(b") H_2: $\hat{U} \sim N(\emptyset, M\Omega M)$

(c") H_3: $\hat{U} \sim F^*(\eta^*, V^*)$

where $M = I - X(X'X)^{-1}X'$ and $F^*(\cdot), \eta^*, V^*$ are the corresponding transforms induced by $\hat{U} = My$.

RESET (Regression Specification Error Test) is designed to test H_0 against H_1. Assume that η (from a') can be approximated by a polynomial in powers of $\hat{Y}$ with parameter vector $\underline{\gamma}$. Ramsey and Schmidt (1976) show that under H_0 the test that $\underline{\gamma} = \emptyset$ yields a central F statistic for the following augmented regression:

$$\text{(iii)} \quad Y = X\beta + Q\gamma + e$$

where r is a 3 × 1 vector of coefficients

Q is a T × 3 matrix of powers of $\hat{Y}$

e is the disturbance vector, $e \sim N(\emptyset, \sigma^2 I)$

Under H_0 the F statistic has 3 and T - K - 3 degrees of freedom. Under H_1 the F statistic is distributed approximately as noncentral F.

RESET is robust to non-normality as well as heteroscedasticity.

Thursby (1979) provides evidence that RESET is robust against autocorrelation as well.

BAMSET (Barlett's M Specification Error Test) is designed to test H_0 against H_2. The test statistic is distributed as a Chi-square with 2 degrees of freedom. The residuals are partitioned into three groups, for each of which the sum of squared residuals is $s_i^2 = \frac{1}{v_i} \sum_{j=1}^{v_i} U_{ij}^2$,

where v_i is the degrees of freedom for each group i, i = 1,2,3 and where $\sum_{i=1}^{3} v_i = T - K$. The BAMSET statistic, B, is computed as follows:

$$B = -2\ln \prod_{i=1}^{3} \left(\frac{s_i^2}{s^2}\right)^{v_i/2} ,$$

where

$$s^2 = \frac{1}{T-K} \sum_{j=1}^{T-K} U_j^2 .$$

A significant value for B indicates heteroscedasticity, see pages 367-369 in Ramsey (1974) for further discussion.

WSET (Shapiro-Wilk Specification Error Test for Normality) is designed to test H_0 against H_3. WSET is scale and origin invariant. The derivation will not be presented here, see Shapiro and Wilk, 1965.

Small values of the WSET statistic, W, are significant and indicate nonnormality. The percentage points for the distribution of W are to be found in Table 6 on page 605 in Shapiro and Wilk, 1965.

The Box-Jenkins Q-sum test is a Chi-square test on the first R autocorrelations that are calculated using the estimated residuals. Under the null hypothesis of no autocorrelation Q is distributed as approximately Chi-square with R - p - q degrees of freedom. p is the order of the autoregressive process in the residuals; q is the order of the moving average process. See Box and Jenkins for details and discussion of the test.[2] Thursby (1979) provides evidence that Q is the best test for autocorrelation in a variety of models.

For a discussion of strategies for discrimination among alternative hypotheses, see Thursby (1979) and Ramsey (1974).

Three different definitions of residuals can be used in the tests. First, the ordinary least squares (OLS) residuals are defined by

$\hat{U} = Y - \hat{Y}$, where $\hat{Y}$ is the ordinary least squares forecast of Y.

Second, the Studentized (STU) residuals are defined by:

$$U_i^S = U_i/[S(i)(1 - h_i)^{\frac{1}{2}}],$$

where $(T - K - 1)s(i) = (T - K)s^2 - [\hat{U}_i^2/(1 - h_i)]$, $(T - K)s^2 = \Sigma(\hat{U} - \bar{U})^2$, $T\bar{U} = \Sigma\hat{U}_i$, and h_i is the ith diagonal element of $X(X'X)^{-1}X'$. See Hoaglin and Welsch (1978) for further discussion.

Third, the recursive (REC) residuals are identically and independently distributed as $N(\emptyset,\sigma^2 I_{T-K})$ under H_0.

The elements in the T - K vector of REC residuals are given by:

$$U_j^R = \frac{y_j - x_j' b_{j-1}}{[1 + x_j(X_{j-1}'X_{j-1})^{-1}x_j]^{\frac{1}{2}}} \qquad j = k + 1,\ldots,T$$

$$b_{j-1} = (X_{j-1}'X_{j-1})^{-1}X_{j-1}'Y_{j-1}$$

where $(y_j x_j)'$ is the jth row of (Y,X)

$(Y_{j-1}X_{j-1})$ is the first j - 1 rows of (Y,X)

See Phillips and Harvey (1978) for further discussion.

BAMSET can utilize any one of the three definitions of residual vectors. Residuals are used in WSET and the Q-sum test. RESET implicitly utilizes the OLS residuals.

We are now in a position to discuss the results of our tests.[3] Blatt indicates on page 299 that there is a clearly observed cycle in the residual plots, which observation alone should indicate that the regression model is wrong. Consequently, any inferences based on such obvious nonrandom behavior in the disturbances cannot be valid under the usual assumptions about the distribution of residuals. In order to provide a greater challenge to the specification error tests we introduced measurement error in Y(t) in order to obfuscate any obvious patterns in the residuals.

While no plots are presented in this paper in the interests of conservation of space, the residual plots examined did not show any clear patterns so that formal tests are needed.

The model which generated the Blatt data was:

$$C(t) = a + mY(t - 1)$$

(2) $$I(t) = \max I_f, I_1 + v[Y(t - 1) - Y(t - 2)]$$

$$Y(t) = \min[Y_c, C(t) + I(t)]$$

with equilibrium income level $\bar{Y} = (a + I_1)/(1 - m)$. In this model income has a ceiling Y_c and investment has a floor value I_f and an equilibrium (steady state) value of I_1. The chosen values for the parameters were:

$a = 0.2$	$m = 0.86$	$v = 2.0$
$I_f = 0$	$I_1 = 0.3$	$Y_c = 4.0$

The Blatt regression model was:

(3) $$Y(t) = A + BY(t - 1) + CY(t - 2) + U(t)$$

and in comparison the solution of Y(t) in terms of its lagged values for solutions not on the boundaries I_f, Y_c is:

(4) $$Y(t) = (a + I_1) + (m + v)Y(t - 1) - vY(t - 2)$$

We can relate the coefficients easily enough by noting that $A = a$, $B = m + v$ and $C = -v$. The regression model which we ran was:

(5) $$Y(t) = A + BY(t - 1) + CY(t - 2) + v(t) ,$$

where a measurement error term is added for each value of t; the error terms are drawn independently from an $N(0,\sigma^2)$ distribution with σ^2 equal to 0.5% of the variance of Y(t) as defined in equation (3).

Two equations were run and two sets of specification error tests were implemented, one for the original Blatt model with 60 observations (equation (3)) and one for the model with added errors of observation (equation (5)). The results are presented in Tables I to III.

The Q statistic indicates misspecification in both regressions. In each case the 15 period cycle of the true model is clearly indicated. As one increases the sample size, the results become even more significant, despite the insignificant H and DW statistics; these results are not cited.[4]

The RESET test statistic is highly significant and indicates a non-null mean vector for the OLS residuals in both regressions. The RESET statistics are significant in the 0.1% critical area.

The BAMSET test statistic was not found to be significant at any reasonable test level in either regression.

The WSET statistic is significant in the 1% critical area for the error free model, but not for the error added model. The addition of the "measurement" error means that the "observed" error term is composed of two parts, a normally distributed part and a systematic portion due to specification error. If the former part dominates with respect to the normality test, one will not reject at any reasonable size of test at more than the corresponding probabilities of Type I error.

The overall conclusion is that even with a measurement error term added, there is obvious and highly significant evidence of misspecification, in particular of the functional form. Indeed, this is the vitally important second part of the lessons to be learnt from this instructive example.

Proper statistical procedures should include the use of a variety of specification error tests in order to test for the presence of serious modelling or sampling errors. When such errors are discovered, one knows to treat all inferences from such models with great suspicion especially when one recognizes, as is demonstrated in this sample, the egregious nature of the errors which can be made.[5]

Another aspect clearly illustrated by this example is that obtaining a good fit, as measured by R^2 values, or significant t ratios with plausible signs is in no way a guarantor of the use of appropriate statistical procedures, nor is it a guarantor of having reached even approximately reasonable inferences. Specification error tests, therefore, are an indispensable tool in our efforts to obtain <u>useful</u> inferences and to have some confidence in the robustness of our results over time.[6]

Table I

Regression Results on Blatt's Model
with No Measurement Error

		Coefficient Estimates	t statistics
$\hat{A}$	=	0.651	(5.6)*
$\hat{B}$	=	1.65	(20.8)*
$\hat{C}$	=	- 0.866	(-11.1)*
$\hat{a}$	=	0.651	
$\hat{m}$	=	0.786	(20.9)*
$\hat{v}$	=	0.866	

R^2 = 0.92

D.W. statistic = 1.91 $H(\sim \chi^2(1)) = 0.16$[a]

60 observations

$F_{2,56}$ = 340*

[a]Based on T = 61 in the computation.

*Significant in 0.1% critical area.

Table II

Regression Results on Blatt's Model with Variables Measured with Error[a]

		Coefficient Estimates	t statistics
$\hat{A}$	=	0.639	(5.3)*
$\hat{B}$	=	1.64	(19.9)*
$\hat{C}$	=	- 0.852	(- 10.6)*
$\hat{a}$	=	0.639	
$\hat{m}$	=	0.791	(20.2)*
$\hat{v}$	=	0.852	
R^2	= 0.92		
D.W. statistic	= 1.94	$H(\sim \chi^2(1)) = 0.08$[b]	
60 observations			
$F_{2,56}$	= 314*		

[a]Measurement error $\varepsilon(t)$ is distributed as $N(0,\sigma^2)$, where σ^2 is 0.5% of the variance of Y(t).

[b]Based on T = 61 observations in the computation.

*"Significant" in .01% critical area.

Table III

Specification Error Test Results for the Models Presented in Tables I and II

					with measurement error	without measurement error
RESET Test $F_{3,53}$				=	26.08*	133.91*
BAMSET Test						
OLS residuals		χ^2		=	1.13	0.35
Studentized residuals		χ^2		=	1.20	0.30
Recursive residuals		χ^2		=	1.21	0.63
Shapiro-Wilk Test (using the first 50 recursive residuals)						
		W		=	0.9578	0.878§
Box-Jenkins Q sum test						
number of autocovariances	Degrees of freedom					
10	10		Q	=	4.61	6.76
14	14		Q	=	13.32	17.85
15	15		Q	=	13.50	61.78†
16	16		Q	=	39.59†	63.90†
20	20		Q	=	48.98†	67.22†
30	30		Q	=	61.92†	115.48†
40	40		Q	=	104.69†	127.31†

*Significant in 0.1% critical area.

†Significant in 0.5% critical area.

‡Significant in 10% critical area.

§Significant in 5% critical area.

Notes

1. In this short note we will not consider De-Min Wu tests of independence between stochastic regressors and disturbances. His tests require instrumental variables. Since we would have to augment our data set and possibly alter Blatt's original example, De-Min tests would only lengthen our exercise without providing more clarity on points raised in this note.

2. See pages 393 to 394 in Box and Jenkins (1976) for the discussion of the Q test. However, we estimate the autocovariances as suggested on page 58 by Nerlove, Grether and Caravalho (1979). We adjust the estimates with the degrees of freedom unlike Box and Jenkins who use (1/T).

3. H is Durbin's (1970) H statistic which is distributed as a Chi-square with one degree of freedom. H is used to test the null hypothesis that the first order serial correlation equals zero when one of the regressors is a lagged dependent variable.

DW is the Durbin-Watson statistic for testing the null hypothesis that the first order serial correlation equals zero. See Theil (1971) pages 199 to 201 for details.

4. Significant specification error test results are only a necessary condition for establishing misspecification. Insignificant results imply that misspecification cannot be detected. It is possible that if different kinds of specification error are present in a model, the power of the respective tests is reduced.

5. For examples of applying our specification error tests and for the finite properties of the tests, see Ramsey and Gilbert (1972), Ramsey and Zarembka (1971), Thursby (1979) and Thursby and Schmidt (1977).

6. As a final aside it should be noted that for a different choice of initial condition used to generate equation (3) Blatt could have derived a chaotic time path of solutions for Y'(t).

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