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TIME IRREVERSIBILITY AND BUSINESS CYCLE ASYMMETRY

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Abstract

The problem of business cycle symmetry is usefully addressed within the context of time reversibility. To this effect, we introduce a time domain test of time reversibility, the TR test. In an application we show that time irreversibility is the rule rather than the exception for two well-known representative macroeconomic datasets. This shows that business cycle fluctuations are asymmetric.

Keywords

Time Reversibility, Asymmetry, Nonlinearity, TR Test

JEL Categories

C22, C50, E32

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1. Introduction

If the probabilistic structure of a time series going forward in time is identical to that in reverse time, the series is time reversible. If the series is not time reversible, the series is said to be time irreversible. The importance of the issue of time reversibility for economics is easily seen from consideration of the standard modelling paradigm in macroeconomic analysis.

Wide classes of business cycle models are based on the interaction between impulse and propagation mechanisms. The usual idea is that independently and identically distributed shocks provide impulses which affect output through distributed lag relations. This modelling strategy stems from the early work of Frisch (1937) and Slutsky (1922) in which they show that a linear system of equations driven by random shocks could produce business cycle like behavior in the sample path of a random variable. Blanchard and Fischer (1989) stress that, while macroeconomists disagree both as to the main sources of these shocks and the exact nature of the propagation mechanism, the Frisch-type approach is currently the dominant one in both theoretical and empirical macroeconomics.

However, Blatt (1980) demonstrated that Frisch-type models are unable to capture cyclical asymmetries; asymmetries due to potential differences in the dynamic structure across business cycle expansions and contractions. If business cycle fluctuations are asymmetric, it is intuitively clear that the associated time series is time irreversible. For example, an asymmetric cycle in which the time series increases faster, but for a shorter time, than it decreases is time irreversible since in reverse time its expansions are longer but less sharp than its contractions. Correspondingly, a symmetric cycle is time reversible. Thus, the result that fluctuations in Frisch-type models are symmetric implies that these models are time reversible. In this light, the empirical question of business cycle asymmetry, studied in a line of work opened in a seminal article by Neftci (1984)[1], can be usefully restated as whether the dynamic behavior of key macroeconomic variables is time reversible. In this paper, then, we identify business cycle asymmetry with the concept of time irreversibility. This provides a unified framework for addressing the issue of business cycle asymmetry.

If the rate of adjustment toward long-run equilibrium differs across phases of the business cycle, then conventional forecasting techniques, such as Gaussian Autoregressive Moving Average (ARMA) models, will clearly be inappropriate. It can indeed be shown formally that stationary Gaussian ARMA models are time reversible[2]. Hence, detection of irreversibility in a particular time series implies that the conventional Gaussian ARMA approach is not an appropriate modelling strategy. Irreversible behavior would require consideration of alternative time series models capable of capturing this property. Evidence of time irreversible dynamics is also consistent with recent theoretical models of state-contingent adjustments[3].

The purpose of this paper is twofold. First, we introduce a time domain test, called the Time Reversibility (TR) test, to identify and characterize time irreversible stationary time series. We are not aware of any other test for time

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[1] Brock and Sayers (1988), DeLong and Summers (1986), Falk (1986), Neftci (1984), Rothman (1991), Sichel (1989, 1993), and Westlund and Öhlén (1991) use nonparametric techniques to study business cycle asymmetry. Parametric approaches include Boldin (1990), Brännäs and De Gooijer (1992), Brunner (1992, 1993), French and Sichel (1993), Goodwin (1993), Ham and Sayers (1990), Hamilton (1989), Hussey (1992), Luukkonen and Teräsvirta (1991), Mittnik (1991), Potter (1991), Rothman (1992), Stock (1989), and Tërasvirta and Anderson (1992).

[2] See Weiss (1975).

[3] See, for example, Ball and Mankiw (1992), Bertola and Caballero (1990), Caplin and Leahy (1991), and Tsiddon (1991). See also the recent paper by Evans and Honkapohja (1993) in which time irreversible behavior is generated in a model with multiple steady states due to a production externality.

irreversibility in the statistical time series literature[4,5]. In addition to providing a simple diagnostic to check the adequacy of a Gaussian ARMA modelling strategy for a particular time series, the TR test serves as a direct test of business cycle symmetry in the applied macroeconomics literature. Time irreversibility is a precise formalization of the concept of asymmetry in business cycle analysis. The TR test statistics also provide a concise characterization of the way in which the business cycle is asymmetric, i.e., either "fast up and slow down" or "slow up or fast down". Second, we apply the TR test to both the well-known Nelson and Plosser (1982) representative macroeconomic dataset recently extended by Schotman and Van Dijk (1991) and to the international dataset studied by Backus and Kehoe (1992). Our results provide ubiquitous evidence that business cycle fluctuations are time irreversible, showing that the business cycle indeed is asymmetric.

In Section 2 time reversibility is formally defined and a tool for identifying time irreversible stochastic processes, the symmetric-bicovariance function, is introduced. The TR test statistic is presented in Section 3. In Section 4 the TR test is applied to the business cycle indicators. Section 5 provides a characterization of the business cycle asymmetry detected by the TR test and Section 6 concludes the paper.

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[4] The work closest to ours is the bispectrum frequency domain test of Hinich (1982), since both we and he analyze sample bicovariances. A more direct analogue to our test would be to examine whether the imaginary part of the polyspectra is identically equal to zero. In personal communication Professor Hinich has informed us that he is developing such a frequency domain test of time reversibility for the bispectrum.

[5] The main ideas in this paper were originally formulated in Ramsey and Rothman (1988) and further refined in Rothman (1990).

2. Time Reversibility

A formal statistical definition of time reversibility for a stationary time series is:

Definition 2.1 A time series $\{X_t\}$ is time reversible if for every positive integer n, and every $t_1, t_2, \ldots, t_n \in \mathbb{R}$, the vectors $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ and $(X_{-t_1}, X_{-t_2}, \ldots, X_{-t_n})$ have the same joint probability distributions. A time series which is not time reversible is said to be time irreversible.

Note that the above definition does not impose stationarity on the time series $\{X_t\}$. This is in contrast to an alternative definition of time reversibility found, for example, in Tong (1990, p. 193).

Under Definition 2.1 it is possible to produce examples of nonstationary time reversible processes. Rothman (1990, p. 26) demonstrated the following:

Example 2.1: Let $\{X_t\}$ be the stochastic process defined by the sequence of independently, but not identically, distributed random variables where $F_t(x_t) = N(\mu \cdot t^2, \sigma^2)$, where $F_t(x_t)$ is the probability distribution function of X_t. Then $\{X_t\}$ is time reversible and clearly nonstationary.

The importance of this example is that there exists a nonstationary process that is time reversible, so that nonstationarity does not imply time irreversibility. As is well know, see Subba Rao and Gabr (1984) and Tong (1990), stationarity does not imply time reversibility. Hence, stationarity and a general definition of time reversibility are separate concepts and neither implies the other.

An alternative definition of time reversibility, due to Kelly (1979, p. 5) is:

Definition 2.2 A time series $\{X_t\}$ is time reversible if for every positive integer n, every $t_1, t_2, \ldots, t_n \in \mathbb{R}$, and all $m \in \mathbb{N}$, the vectors $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ and

$(X_{-t_1+m}, X_{-t_2+m}, \ldots, X_{-t_n+m})$ have the same joint probability distributions.

Under Definition 2.2 Kelly (1979) showed that time reversibility implies stationarity. Consider the special case in which the time indices $\{t_i\}$ are constructed as follows: $t_i = t_{i-1} + k$, $k \in \mathbf{R}$, $i = 2, \ldots, n$, i.e., the sequence $\{t_i\}$ is characterized by equal increments of time. Letting $m = t_1 + t_n$, it is seen that time reversibility under Definition 2.2 implies that the vectors $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ and $(X_{t_n}, X_{t_{n-1}}, \ldots, X_{t_1})$ have the same joint probability distributions. We shall use this stationary-restricted definition of time reversibility, so that time reversible processes are a subset of the class of stationary processes.

It is straightforward to show that $\{X_t\}$ is time reversible when $\{X_t\}$ is independently and identically distributed. The result that stationary Gaussian processes are time reversible appeared as Theorem 1 in Weiss (1975, p. 831). In the same paper Weiss proved the converse within the context of discrete-time ARMA models, i.e., if $\{X_t\}$ is a time reversible ARMA process, then the underlying innovations are normally distributed. This was the main contribution of his paper. Weiss conjectured, without proof, that this result holds when $\{X_t\}$ is a general linear process. This conjecture was shown to be true by Hallin, Lefevre and Puri (1988).

We next establish the equality between certain pairs of moments from the joint probability distributions for a time reversible stationary time series $\{X_t\}$. By the definition of mathematical expectation, and given the assumed uniqueness of the representation of the joint distributions by their moments, it is straightforward to prove the following[6]:

Theorem 2.1: Let $\{X_t\}$ be a stationary time series with mean zero and assume

[6] See Rothman (1990, pp. 30-31).

that the multivariate characteristic generating functions of (X_t, X_{t-k}) and (X_{t-k}, X_t) can be expanded as a convergent series in the moments and cross moments of the respective joint probability distributions; that is, it is assumed that the joint probability distributions are uniquely characterized by the respective sequence of moments and cross moments. Then, $\{X_t\}$ is time reversible only if:

$$E[X_t^i \cdot X_{t-k}^j] = E[X_t^j \cdot X_{t-k}^i] \tag{2.1}$$

for all i, j, k $\in$ **N**, where the expectation is taken with respect to each respective joint distribution.

By <u>Theorem 2.1</u>, for i = j = 1 we have:

$$E[X_t \cdot X_{t-k}] = E[X_t \cdot X_{t-k}] \tag{2.2}$$

for all positive integers k. Statement (2.2) is simply the tautology that the autocovariance of a stationary series at lag k is equal to itself. This is because the autocorrelation function is an even function of k. As such, we see that the autocovariance function can provide no relevant information with respect to the potential time irreversibility of any specific time series.

When at least one of i, j is greater than one, i,j $\in$ **N**, the two terms in (2.1) are called generalized autocovariances, following Welsh and Jernigan (1983). From <u>Theorem 2.1</u> it follows that if there exists a lag k for which these two moments do not equal one another, the series is time irreversible. While this is a sufficient condition for time irreversibility, it is not a necessary one, since (2.1) considers only a proper subset of moments from the joint distributions of $(X_{t_1}, X_{t_2}, \ldots, X_{t_n})$ and $(X_{t_n}, X_{t_{n-1}}, \ldots, X_{t_1})$. Further, we do not consider the case where $F_{t_1,t_2}(\cdot) = F_{t_2,t_1}(\cdot)$ but $F_{t_1,t_2,t_3}(\cdot) \neq F_{t_1,t_2,t_3}(\cdot)$ for some $t_1, t_2, t_3 \in \mathbf{N}$.

We propose to consider the difference between two bicovariances. The symmetric bicovariance function is defined as follows:

$$\gamma_{2,1}(k) = \{E[X_t^2 \cdot X_{t-k}] - E[X_t \cdot X_{t-k}^2]\} \tag{2.3}$$

for all integer values of k. If $\{X_t\}$ is time reversible, then $\gamma_{2,1}(k) = 0 \ \forall \ k \in \mathbf{N}$. Our reason for restricting attention to the differences in bicovariances is that the distributional properties will be more manageable than if one used higher-order moments and that as a practical matter the lower-order moments seem to be sufficiently informative in most cases that we have examined[7].

We next demonstrate the time irreversibility of two different time series models using the symmetric-bicovariance function, one linear and one nonlinear. First, consider the following non-Gaussian MA(1) model:

$$X_t = \epsilon_t - \theta \cdot \epsilon_{t-1}, \tag{2.4}$$

where $\{\epsilon_t\}$ is a sequence of independently and identically distributed random variables drawn from a non-symmetric probability distribution function. It is straightforward to see that for this stochastic process, $\gamma_{2,1}(1) = (\theta^2 + \theta)\mu_3^\epsilon$, where $\mu_3^\epsilon = E[\epsilon_t^3]$, showing that $\{X_t\}$ is time irreversible.

Next, consider the following simple bilinear model:

$$X_t = \alpha \cdot X_{t-1} + \beta \cdot X_{t-1} \cdot \epsilon_{t-1} + \epsilon_t, \tag{2.5}$$

where $\{\epsilon_t\}$ is a sequence of independently and identically distributed N(0,1) random variables. The third-order moment structure of this process has been

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[7] We are indebted to Pomeau (1982) for the suggestion of studying time reversibility through a similar but higher-order function, $\gamma_{3,1}(k)$. He did not, however, draw a direct connection between his proposed test and the formal statistical definition of reversibility. He also did not investigate the sampling distribution of any estimator of $\gamma_{3,1}(k)$.

computed by Subba Rao and Gabr (1984, pp. 53-57). From their calculations it follows that for this time series model, $\gamma_{2,1}(1) \neq 0$, showing that $\{X_t\}$ is time irreversible.

The two examples demonstrate that time irreversibility can stem from two sources: (1) the underlying model may be nonlinear even though the innovations are symmetrically (perhaps normally) distributed; or (2) the underlying innovations may be drawn from a non-Gaussian probability distribution while the model is linear. Linear models are inherently time irreversible, even though the special properties of the Gaussian distribution yield time reversibility in the context of a general linear process. We shall refer to the first nonlinear case as "Type 1" time irreversibility and to the latter as "Type 2" time irreversibility". Type 1 time irreversibility is consistent with a nonlinear process. Type 2 time irreversibility is consistent with a non-Gaussian linear process.

Nonlinearity does not imply Type 1 time irreversibility, however. That is, there exist stationary nonlinear processes which are time reversible[8]. A test for Type 1 time irreversibility, then, is not equivalent to a test for nonlinearity.

3. The TR Test

The TR Test statistics consist of a sample estimate of the symmetric-bicovariance function given by equation (2.3). The sample bicovariances for a mean zero stationary time series $\{X_t\}$ with T observations are:

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[8] For example, Lewis et. al. (1989) showed that the random-coefficient gamma MA(1) process is time reversible since its bivariate characteristic function is symmetric.

$$\hat{B}_{2,1}(k) = (T-k)^{-1} \cdot \sum_{t=k+1}^{T} X_t^2 \cdot X_{t-k}$$

and (3.1)

$$\hat{B}_{1,2}(k) = (T-k)^{-1} \cdot \sum_{t=k+1}^{T} X_t \cdot X_{t-k}^2$$

for various integer values of k.

With the bicovariance estimators from (3.1), the TR Test statistics are constructed as follows:

$$\hat{\gamma}_{2,1}(k) = \hat{B}_{2,1}(k) - \hat{B}_{1,2}(k) \tag{3.2}$$

for various integer values of k. It is straightforward to show that $\hat{\gamma}_{2,1}(k)$ is an unbiased and, under some additional restrictions, consistent estimator of $\gamma_{2,1}(k)$[9].

Under the null hypothesis that $\{X_t\}$ is time reversible, the expected value of $\hat{\gamma}_{2,1}(k)$ is zero for all k. Under some mixing conditions $\hat{\gamma}_{2,1}(k)$ has an asymptotic normal distribution. More specifically, assume $\{X_t^2 \cdot X_{t-k} - X_t \cdot X_{t-k}^2\}$ is a sequence of mixing random scalars such that either $\phi(m)$ or $\alpha(m) = O(m^{-\lambda})$ for $\lambda > r/(r-1)$, $r > 1$, where $\phi(m)$ and $\alpha(m)$ are the mixing coefficients as defined in Definition 3.42 of White (1984, p. 45). Then, by Theorem 5.19 of White (1984, p. 124), $\sqrt{T} \cdot [\hat{\gamma}_{2,1}(k) - \gamma_{2,1}(k)]/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$ is asymptotically distributed as $N(0,1)$[10]. In the more restrictive independently and identically distributed case, asymptotic normality follows directly from Theorem 4.3 of Welsh and Jernigan (1983, p.391).

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[9] See Theorem 1 of Rosenblatt and Van Ness (1965, p. 1125).

[10] Since we assume $\{X_t\}$ is stationary, the uniform convergence conditions required on $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ for the theorem to hold follow directly.

The exact small-sample expression for $Var(\hat{\gamma}_{2,1}(k))$ for $\{X_t\}$ independently and identically distributed is given next. Straightforward calculations show[11]:

Theorem 3.1: Let $\{X_t\}$ be a stationary sequence of independently and identically distributed random variables for which $E[X_t] = 0 \; \forall \; t$. Then:

$$Var[\hat{\gamma}_{2,1}(k) = 2(\mu_4\mu_2 - \mu_3)/(T-k) - 2\mu_2^3(T-2k)/(T-k)^2 \tag{3.3}$$

where $\mu_2 = E[X_t^2]$, $\mu_3 = E[X_t^3]$ and $\mu_4 = E[X_t^4]$.

If a given stationary series $\{X_t\}$ exhibits no serial correlation, $\hat{\gamma}_{2,1}(k)$ can be calculated on the raw data for a set of values of k. Since $\hat{\gamma}_{2,1}(k)$ is asymptotically normally distributed, rejection regions can be calculated using the expression for $Var(\hat{\gamma}_{2,1}(k))^{1/2}$ in Theorem 3.1.

We consider three procedures to test for time reversibility for the case in which the given series $\{X_t\}$ exhibits serial correlation. We adopt this testing strategy since the null hypothesis of time reversibility is rather complicated. The data generating mechanism for a time reversible series could be a linear process with Gaussian innovations or a nonlinear process with either Gaussian or non-Gaussian innovations. Our different procedures are designed to allow for these varied cases.

Each approach requires that an ARMA model first be fitted to the series. If the null hypothesis of time reversibility is restricted to the case of a Gaussian linear process, the following procedure is appropriate. $\hat{\gamma}_{2,1}(k)$ is calculated on the unfiltered series. An estimate of $Var(\hat{\gamma}_{2,1}(k))$ is then provided through a Monte Carlo simulation, with allowance for the ARMA structure and using Gaussian innovations.

In the second procedure we once again calculate $\hat{\gamma}_{2,1}(k)$ on the unfiltered

[11] See Rothman (1990, pp. 39-41).

raw data. But in this approach we estimate $Var(\hat{\gamma}_{2,1}(k))$ through a bootstrap simulation. We approximate a possibly time reversible nonlinear structure by fitting an ARMA model and drawing with replacement from the set of ARMA residuals[12]. By the Wold decomposition theorem the ARMA approximation will adequately facilitate the bootstrap simulation in so far as it produces uncorrelated, but not necessarily independent, residuals.

Rejection through these first two procedures is consistent with both Type 1 and Type 2 time irreversibility. The third procedure is designed to be consistent with Type 1 time irreversibility, i.e., time irreversibility due to a time irreversible nonlinear data generating mechanism. In this approach $\hat{\gamma}_{2,1}(k)$ is calculated on the ARMA residuals. $Var(\hat{\gamma}_{2,1}(k))$ is then calculated using the expression for the independently and identically distributed case given in Theorem 3.1.

If the null hypothesis is rejected in that $\hat{\gamma}_{2,1}(k)$ lies in the critical region as determined by the appropriate estimate of $Var(\hat{\gamma}_{2,1}(k))$ then the pattern of time irreversibility as characterized by $\hat{\gamma}_{2,1}(k)$ can be used as a diagnostic aide to the specification of an appropriate time series model. For example, through Monte Carlo simulations Rothman (1990) studied the power of the TR test against simple bilinear and threshold autoregressive models[13]. The TR test was shown to have excellent power against these time irreversible alternatives. Further, the characterization of time irreversibility produced in the TR test statistics for these two classes of models is quite revealing. For the bilinear

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[12] Computer programs to implement both the Monte Carlo and bootstrap variances are available from the authors. The reason why we estimate the variances in these cases is that the exact small sample expression for $Var[\hat{\gamma}_{2,1}(k)]$, when $\{X_t\}$ is ARMA, is an algebraically complicated function of the higher order moments whose computation is tedious.

[13] See Tong (1990) for detailed descriptions of these models.

models, the $\hat{\gamma}_{2,1}(k)$ values decline exponentially across k. For the threshold models, significant rejections occur only at lag k = 1.

4. Testing Business Cycle Symmetry with the TR Test

We first apply the TR test to the extended Nelson and Plosser (1982) dataset. In their seminal paper, Nelson and Plosser explored the stochastic trend null hypothesis for a representative set of fourteen annual real and nominal macroeconomic time series. This has been the most heavily analyzed macroeconomic dataset over the past decade. In a recent paper, Schotman and Van Dijk (1991) updated each time series in the dataset up to 1988.

For twelve of the fourteen series we analyze growth rates. For the other two series, the unemployment rate and bond yields, we work with raw first differences. The first step is selection of an ARMA model for the transformed series according to the Akaike Information Criterion (AIC); our results are not sensitive to model selection via the Schwartz Information Criterion (SIC)[14].

Table 1 presents standardized TR test statistics at lags k = 1,...,5, i.e., $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, where $\hat{\gamma}_{2,1}(k)$ is calculated directly on the transformed series and $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation using Gaussian innovations and estimated coefficients for the ARMA models selected by the AIC. The data are annual so that examination of the statistics at lags k = 1,...,5 provide evidence on the reversibility of the series at frequencies relevant for business cycle analysis. For twelve out of the fourteen series, the time reversibility null hypothesis is rejected at the 5% significance level for at least one lag k. Using the Bonferroni inequality, for each of these series the

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[14] $AIC = -2\cdot\ln(L) + k$, where L is the maximized value of the likelihood function and k is the number of parameters in the model. $SIC = -2\cdot\ln(L) + k\cdot\ln(n)$, where n is the number of observations.

first five standardized TR test statistics are jointly significantly different from zero at the 10% level. This provides evidence of time irreversibility, showing that the U.S. business cycle is asymmetric. The two series which fail to display any evidence of time irreversibility over these lags are the employment and nominal wage growth rates.

Table 2 presents standardized TR test statistics, for which $\hat{\gamma}_{2,1}(k)$ is once again calculated directly on the transformed series, but with $\text{Var}(\hat{\gamma}_{2,1}(k))$ estimated via bootstrap simulation using the residuals for the AIC-selected ARMA models for each series. For the same twelve series, the time reversibility null hypothesis is rejected with this approach at the 5% significance level for at least one lag. Once again, the employment and nominal wage growth rate series fail to exhibit any departure from time reversibility.

The rejections in Tables 1 and 2 are consistent with both Type 1 and Type 2 time irreversibility. The results in Table 3 provide a more direct check on Type 1 time irreversibility for these series.

Table 3 lists standardized TR test statistics, for which $\hat{\gamma}_{2,1}(k)$ is calculated on the ARMA residuals and the independently and identically distributed variance given in Theorem 2.1 is used. For twelve of the ARMA residual series, time reversibility is rejected at the 5% significance level for at least one lag, providing evidence of Type 1 time irreversibility. This suggests that nonlinearity is an important source of business cycle asymmetry in the U.S. economy. The employment growth rate residuals provide no evidence against time reversibility. But with this approach the nominal wage growth rate rejects while the money growth rate fails to reject.

We next turn to analysis of the Backus and Kehoe international dataset. We examined twenty four series, the same four indicators for six different countries. The four indicators chosen were real output, investment, price level, and money supply. These four series were selected for Australia, Canada, Italy, Sweden, the United Kingdom, and the United States. For all of the twenty four series we analyzed the growth rates.

Our TR test results for the Backus and Kehoe data are presented in Tables 4, 5, and 6. Table 4 reports standardized TR test statistics, where $\hat{\gamma}_{2,1}(k)$ is calculated directly on the transformed series and $Var(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation. For eighteen out of the twenty four series, the time reversibility null hypothesis can be rejected at the 5% significance level for at least one of the first five lags k. Results in Table 5 show that use of bootstrap estimates of $Var(\hat{\gamma}_{2,1}(k))$ slightly reduces the number of such rejections down to seventeen. The ARMA residual results in Table 6 show that time reversibility is rejected for twenty out of the twenty four series. The ARMA residual results therefore suggest that rejection is due to Type 1 time irreversibility, i.e., time irreversibility due to a nonlinear data generating mechanism.

5. Characterization of Business Cycle Asymmetry

The idea that the business cycle is asymmetric can be traced back over sixty years to the work of Mitchell (1927) and Keynes (1936). The conventional asymmetry hypothesis is that economic expansions are longer but less sharp than downturns. With respect to a counter-cyclical series such as the unemployment rate, this would imply that the unemployment rate increases quickly in recessions but declines relatively slowly during expansions.

If a time series were asymmetric in the Mitchell-Keynes sense, what type of behavior would we expect the TR test statistics to exhibit? Simulation shows that for an asymmetric series which expands more rapidly than it declines, the TR test values are positive at the lower lag lengths. Such simulation shows the opposite effect for an asymmetric cycle which expands more slowly than it decreases, i.e., the TR test values are negative at the lower lag lengths.

The most uniform set of results across both datasets is that the TR test statistics are positive at all lags examined for the inflation rate. This suggests that these series exhibit the "fast up and slow down" asymmetric behavior consistent with counter-cyclical dynamics. Such behavior would appear to be inconsistent with the traditional business cycle view of prices as being pro-cyclical. However, recent work by Cooley and Ohanian (1991) shows that the evidence in favor of the pro-cyclicality of prices is in fact extremely weak.

The TR test statistics are uniformly positive across the three testing procedures for the unemployment rate in the Nelson and Plosser dataset. This suggests that the unemployment rate indeed increases faster than it decreases, as predicted by the Mitchell-Keynes asymmetry hypothesis. The same is true for the Nelson and Plosser bond yields series.

The Nelson and Plosser real, nominal, and per capita GNP series, as well as the industrial production series, all generate negative TR test statistics. This suggests that these series generate "slow up and fast down" pro-cyclical Mitchell-Keynes asymmetric behavior. This is also the case for most of the Backus-Kehoe investment series.

6. Conclusions

In this paper we have explained the importance for macroeconomic analysis of the concept of time reversibility. We showed that the question of whether the business cycle is symmetric can be restated as whether business cycle time series are time reversible. Time irreversibility provides a precise formalization of the concept of temporal asymmetry in a time series.

We have introduced a test of time reversibility. In addition to serving as a simple diagnostic check for the adequacy of a Gaussian ARMA modelling approach, the TR test provides a direct test of business cycle asymmetry. The TR test also provides a characterization of the particular way in which a business cycle time series may by asymmetric. Distinct patterns in the TR test statistics are obtained for "fast up and slow down" counter-cyclical and "slow up and fast down" pro-cyclical asymmetric cycles.

In our application we showed that many key macroeconomic time series are time irreversible. Characterization of these series' time irreversibility through the TR test suggests asymmetric behavior indeed consistent with the Mitchell-Keynes asymmetry business cycle hypothesis. The following series exhibit "fast up and slow down" asymmetric behavior: the inflation rate, the unemployment rate, and bond yields. The following series exhibit "slow up and fast down" Mitchell-Keynes asymmetric cyclical dynamics: real, nominal, and per capita GNP, industrial production, and investment.

These asymmetry results should serve as a warning to those who analyze business cycle fluctuations with Frisch-type models. Asymmetry is an important property that these series possess which is not captured by such models. Our empirical results point more towards recent theoretical models of state-contingent and regime-switching time irreversible dynamics.

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Table 1

Standardized TR Test Statistics for Growth Rates of Extended Nelson and Plosser Data Using Monte Carlo Estimated Standard Errors

Series	k=1	k=2	k=3	k=4	k=5
Real GNP	-1.703	-1.226	-0.114	<u>-2.556</u>	-0.353
Nominal GNP	-0.179	<u>-3.894</u>	1.252	-0.490	-0.100
Real Per Capita GNP	-1.601	-1.093	-0.013	<u>-2.654</u>	-0.424
Industrial Production	<u>-5.267</u>	-1.274	-0.018	-1.944	-0.282
Employment	-1.649	-1.227	1.740	0.124	-1.085
Unemployment Rate *	<u>2.005</u>	<u>2.396</u>	0.709	<u>2.780</u>	<u>2.172</u>
GNP Price Deflator	<u>8.652</u>	0.314	<u>2.438</u>	<u>5.552</u>	<u>2.993</u>
CPI	<u>6.856</u>	<u>4.069</u>	<u>3.042</u>	<u>3.286</u>	<u>2.364</u>
Nominal Wage	0.753	0.248	1.236	0.924	-0.659
Real Wage	<u>-2.486</u>	1.643	0.762	1.244	1.054
Money	-1.039	<u>-5.199</u>	<u>-4.648</u>	-1.258	0.056
Velocity	0.203	<u>-2.496</u>	1.394	-0.371	-0.450
Bond Yields *	0.255	<u>6.016</u>	1.652	1.445	<u>8.606</u>
S&P500	<u>-4.278</u>	-0.387	<u>2.262</u>	-0.630	0.401

* Raw, not log, first differences used.

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, for extended Nelson and Plosser dataset, where $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation. Underlined statistics indicate rejection at the 5% significance level.

Table 2

Standardized TR Test Statistics for Growth Rates of Extended Nelson and Plosser Data Using Bootstrap Estimated Standard Errors

Series	k=1	k=2	k=3	k=4	k=5
Real GNP	-1.163	-0.983	-0.098	<u>-2.038</u>	-0.287
Nominal GNP	-0.064	<u>-2.151</u>	0.885	-0.366	-0.081
Real Per Capita GNP	-1.181	-0.900	-0.011	<u>-2.172</u>	-0.354
Industrial Production	<u>-4.291</u>	-1.136	-0.015	-1.643	-0.241
Employment	-1.153	-1.071	1.478	0.102	-0.941
Unemployment Rate *	1.518	1.860	0.522	1.811	<u>1.974</u>
GNP Price Deflator	1.944	0.115	1.297	<u>3.387</u>	2.026
CPI	<u>3.805</u>	<u>2.712</u>	<u>2.090</u>	<u>2.350</u>	1.694
Nominal Wage	0.370	0.167	0.939	0.743	-0.557
Real Wage	<u>-2.517</u>	1.639	0.751	1.189	0.988
Money	-0.680	<u>-3.589</u>	<u>-3.574</u>	-1.031	0.048
Velocity	0.198	<u>-2.423</u>	1.386	-0.366	-0.412
Bond Yields *	0.147	<u>4.000</u>	1.142	0.972	<u>5.599</u>
S&P500	<u>-3.196</u>	-0.334	1.910	-0.541	0.339

* Raw, not log, first differences used.

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, for extended Nelson and Plosser dataset, where $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via bootstrap simulation. Underlined statistics indicate rejection at the 5% significance level.

Table 3

Standardized TR Test Statistics for ARMA Residuals for Growth Rates of Extended Nelson and Plosser Data Using IID Standard Errors

Series	k=1	k=2	k=3	k=4	k=5
Real GNP	-1.553	-0.212	-0.511	<u>-2.011</u>	0.480
Nominal GNP	<u>-3.541</u>	-1.432	-0.189	-0.123	<u>2.890</u>
Real Per Capita GNP	-1.448	-0.201	-0.359	<u>-2.215</u>	0.279
Industrial Production	<u>-5.025</u>	-0.402	-0.747	-1.393	0.149
Employment	-1.137	-1.577	0.595	-0.608	0.387
Unemployment Rate *	<u>3.326</u>	0.791	1.230	1.037	-0.670
GNP Price Deflator	1.743	-1.533	0.589	<u>3.079</u>	<u>2.199</u>
CPI	<u>2.915</u>	<u>2.993</u>	<u>2.586</u>	1.846	<u>2.090</u>
Nominal Wage	-1.219	<u>-2.682</u>	1.442	0.773	1.836
Real Wage	<u>-2.608</u>	1.355	-0.568	0.213	0.517
Money	-1.729	-1.867	-0.223	0.124	0.695
Velocity	0.159	<u>-2.006</u>	1.539	-0.596	-0.018
Bond Yields *	0.567	1.891	1.143	0.995	<u>3.651</u>
S&P500	<u>-3.024</u>	-0.948	0.229	0.113	0.844

* Raw, not log, first differences used.

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, for extended Nelson and Plosser dataset, where $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via bootstrap simulation. Underlined statistics indicate rejection at the 5% significance level.

Table 4

Standardized TR Test Statistics for Growth Rates of Backus-Kehoe Data Using Monte Carlo Estimated Standard Errors

Country	Series	k=1	k=2	k=3	k=4	k=5
Australia	Real Output	-1.519	0.047	-0.621	-0.073	1.361
	Investment	-1.552	-1.597	-4.209	-6.346	-4.564
	Price Level	5.635	2.886	2.722	0.024	-0.309
	Money Supply	2.895	-0.807	0.350	-0.154	1.162
Canada	Real Output	-4.644	-0.861	-0.930	0.145	1.167
	Investment	0.216	-1.720	0.536	-0.493	-0.437
	Price Level	6.283	4.488	2.218	4.081	2.588
	Money Supply	1.101	1.395	0.590	1.642	0.068
Italy	Real Output	-14.804	-16.111	-9.987	-5.162	-4.072
	Investment	-3.889	-0.629	-5.305	-5.484	-1.864
	Price Level	2.652	0.328	0.282	3.271	2.192
	Money Supply	1.739	1.761	1.146	-1.119	-1.776
Sweden	Real Output	3.957	-2.980	-2.437	-0.222	-0.438
	Investment	-6.750	0.303	0.394	-4.631	-2.592
	Price Level	6.791	6.556	9.644	12.224	8.977
	Money Supply	0.387	0.433	0.796	1.424	1.430
U.K.	Real Output	-1.478	-1.095	-1.635	2.991	1.918
	Investment	-1.012	-5.722	-8.652	-7.815	-7.976
	Price Level	9.404	10.875	7.329	7.413	6.065
	Money Supply	1.562	2.426	3.239	3.588	3.085
U.S.	Real Output	-0.737	0.532	1.244	-1.432	-0.143
	Investment	1.503	-1.639	-2.675	-6.899	-1.700
	Price Level	6.600	2.601	1.752	4.748	3.707
	Money Supply	-1.849	-6.825	-5.198	-1.723	-0.422

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[Var(\hat{\gamma}_{2,1}(k))]^{1/2}$, for selected series in Backus and Kehoe international dataset, where $Var(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation. Underlined statistics indicate rejection at the 5% significance level.

Table 5

Standardized TR Test Statistics for Growth Rates of Backus-Kehoe Data Using Bootstrap Estimated Standard Errors

Country	Series	k=1	k=2	k=3	k=4	k=5
Australia	Real Output	-1.366	0.422	-0.525	-0.066	1.170
	Investment	-1.260	-1.234	<u>-3.265</u>	<u>-5.211</u>	<u>-3.810</u>
	Price Level	<u>5.223</u>	<u>2.531</u>	<u>2.358</u>	0.021	-0.286
	Money Supply	<u>2.198</u>	-0.564	0.266	-0.120	0.922
Canada	Real Output	<u>-4.336</u>	-0.817	-0.892	0.144	1.118
	Investment	0.106	-1.396	0.455	-0.424	-0.385
	Price Level	1.731	1.904	1.483	<u>2.926</u>	<u>1.979</u>
	Money Supply	0.523	0.889	0.471	1.386	0.059
Italy	Real Output	<u>-4.231</u>	<u>-7.200</u>	<u>-4.712</u>	<u>-2.496</u>	<u>-2.173</u>
	Investment	<u>-2.462</u>	-0.353	<u>-3.282</u>	<u>-3.254</u>	-1.171
	Price Level	0.280	0.062	0.081	1.259	1.032
	Money Supply	1.087	1.216	0.846	-0.863	-1.436
Sweden	Real Output	<u>2.302</u>	-1.916	-1.294	-0.117	-0.236
	Investment	<u>-4.303</u>	0.187	0.241	<u>-2.890</u>	-1.667
	Price Level	0.960	1.558	<u>3.481</u>	<u>5.607</u>	<u>4.910</u>
	Money Supply	0.378	0.423	0.778	1.391	1.397
U.K.	Real Output	-0.872	-0.780	-1.260	<u>2.280</u>	1.540
	Investment	-0.256	<u>-3.152</u>	<u>-4.522</u>	<u>-4.396</u>	<u>-4.332</u>
	Price Level	1.508	<u>2.576</u>	<u>2.504</u>	<u>3.214</u>	<u>3.192</u>
	Money Supply	1.417	<u>2.041</u>	<u>2.910</u>	<u>3.273</u>	<u>2.867</u>
U.S.	Real Output	-0.660	0.439	1.028	-1.169	-0.125
	Investment	0.825	-1.098	-1.310	<u>-4.937</u>	-1.356
	Price Level	<u>2.769</u>	1.638	1.189	<u>3.169</u>	<u>2.590</u>
	Money Supply	-1.311	<u>-4.873</u>	<u>-4.337</u>	-1.515	-0.386

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, for selected series in Backus and Kehoe international dataset, where $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation. Underlined statistics indicate rejection at the 5% significance level.

Table 6

Standardized TR Test Statistics for ARMA Residuals for Growth Rates of Backus-Kehoe Data Using IID Standard Errors

Country	Series	k=1	k=2	k=3	k=4	k=5
Australia	Real Output	-0.128	0.421	-0.701	-0.193	2.228
	Investment	-1.005	-0.953	-2.480	-4.648	-2.726
	Price Level	2.139	0.381	1.726	-0.266	-0.919
	Money Supply	2.069	0.073	0.791	0.276	0.495
Canada	Real Output	-2.907	-0.316	-1.421	-0.548	-0.217
	Investment	-0.277	-1.749	0.674	-0.229	0.268
	Price Level	1.879	-1.363	0.558	3.207	1.610
	Money Supply	-3.869	0.855	0.453	-0.702	-1.046
Italy	Real Output	-2.693	-4.610	-2.475	-1.247	-1.030
	Investment	-2.059	-0.616	-3.287	-3.598	-1.302
	Price Level	1.634	2.910	-0.139	4.752	2.544
	Money Supply	-1.044	0.0919	0.206	2.667	2.319
Sweden	Real Output	1.288	-1.180	-1.307	0.527	-0.467
	Investment	-4.834	0.329	-0.604	-2.705	-2.556
	Price Level	0.509	-3.088	0.394	3.351	1.421
	Money Supply	3.220	0.157	2.138	2.154	-1.630
U.K.	Real Output	-0.521	-0.345	-2.662	3.972	-0.311
	Investment	2.850	-2.049	-2.990	-1.982	-2.834
	Price Level	2.753	-0.423	-1.232	2.936	1.921
	Money Supply	-1.005	1.577	0.643	3.204	-0.031
U.S.	Real Output	0.097	0.115	0.682	-1.452	0.285
	Investment	0.272	-0.631	-0.841	-4.835	-0.282
	Price Level	0.177	2.786	0.290	2.408	2.376
	Money Supply	-1.211	-0.984	-0.083	-0.049	0.841

Table shows standardized TR test statistics, $\hat{\gamma}_{2,1}(k)/[\mathrm{Var}(\hat{\gamma}_{2,1}(k))]^{1/2}$, for selected series in Backus and Kehoe international dataset, where $\mathrm{Var}(\hat{\gamma}_{2,1}(k))$ is estimated via Monte Carlo simulation. Underlined statistics indicate rejection at the 5% significance level.