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by

Gerhard O. Orosel

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C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

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Abstract

According to the standard approach of the stock market, stock prices are determined by expected future dividends. This article shows that with heterogeneous information the standard approach breaks down and gives way to a Keynesian beauty contest view of the stock market. Further, it is shown that whereas the stock market is efficient with respect to public information, only agents with private information influence the price, i.e., agents with respect to whose information the market is efficient do not influence the price, and for agents who influence the price the market is not efficient.

1. Introduction and Results

There are two views of the stock market. According to the first one stock prices are determined by "market fundamentals" or, more exactly, by available information about market fundamentals. More specifically, this view is represented by the hypothesis that stock prices equal the present value of the stream of expected future dividends, i.e.,

$$(1.1) \qquad P_t = \sum_{k=1}^{\infty} R^k E_t(D_{t+k})$$

where P_t is the stock's price in period t, R is the (constant) discount factor, D_{t+k} are the dividends paid out in period t + k, and E_t denotes the expected value operator conditional on information available in period t. This view, which is dominant in economic theory, will be called the standard approach.

The other, contrasting view asserts that stock prices are not determined by market fundamentals but by speculative processes which have a life of their own. The most famous example of this view is Keynes' beauty contest model (Keynes 1936), according to which stock prices are determined not by expectations about market fundamentals but by "what average opinion expects average opinion to be" (Keynes 1936, p. 156). However, in spite of the *intuitive* appeal of Keynes' beauty contest parable,the standard approach seems to have the greater *theoretical* appeal to the majority of economic theorists. The reason for this probably lies in the fact that Keynes' beauty contest model is just a parable and, contrary to the standard approach, not a model of the stock market. In particular, it lacks the time structure of the stock market, where, if at all, economic agents have to guess not today's but tomorrow's average opinion.

In this article we will analyze the stock market for the case of informational heterogeneity.[1] In Section 2 we will show that in this case it is not expectations about future dividends which are most relevant, but expectations about others' expectations.[2] This result is more close to Keynes' view of the stock market than to the standard approach. Note also that Keynes repeatedly stressed that expectations are heterogeneous.

In Section 3 we will analyze the stock market equilibrium under certain assumptions about expectation formation. We will show that the equilibrium is efficient in the sense that in each period agents who have no private information will take the equilibrium price as their best estimate of next period's discounted value of the stock. But we will also show that only agents who have private information influence the price (which does not fully reveal their information).[3] I.e., the equilibrium price is not determined by public information; rather, only agents who have private, non-public information influence the price and all other agents can ignore whatever (public) information they have and base their estimate of the stock's value tomorrow only on the stock's price today. Further, if the economic agents' predictions are unbiased and have errors which are uncorrelated with predetermined variables, data generated by our model will pass tests of market efficiency.

Our results do not, however, place bounds on the variance of stock prices as does the standard approach. Therefore, our model might be useful in explaining the seeming contradiction between tests of market efficiency on the one hand and tests of variance bounds on the other (see, e.g., LeRoy 1984, LeRoy and Porter 1981, Shiller 1981).[4]

2. Implications of Informational Heterogeneity

In this section we show that informational heterogeneity in the stock market leads to the "Keynesian" implication that it is not expectations about future dividends which determine the stock price but rather expectations about other agents' expectations. We will illustrate this implication for a simple model but it should be clear that the basic result is not restricted to this simple model chosen for expository purposes.

Let $t = 1,2,\ldots$ denote time, measured discretely, P_t the price in period t of a given stock, and D_t the dividends per share paid in period t to the person who owned the stock in period $t - 1$. There are n economic agents $i = 1,\ldots,n$ in the market. We will assume demand functions of these agents which are admittedly ad hoc but convenient in the sense that they allow an easy comparison of our results with the standard approach, according to which stock prices are given by discounted expected future dividends. Since with heterogeneous information we cannot assume risk neutrality, as this would create existence problems for the competitive equilibrium, with a general von Neumann-Morgenstern utility function the results would not be as readily comparable with the standard approach to the stock market, which, except for special cases, depends on the assumption of risk neutrality. Let r_t be the interest rate for a riskless asset in period t and $R'_t = \frac{1}{1 + r_t}$ the corresponding discount factor. By $e_i(x_{t+k}/I_{ti})$ we denote the estimate of agent i of the expected value of a stochastic variable x_{t+k} given information I_{ti}. If the probability distribution of x_{t+k} is known, e_i is the expected value of x_{t+k} conditional on information I_{ti}. Demand of agent i for the given stock, denoted by Z_{ti}, is assumed to be proportional to the expected rate of return, taking account of interest costs $(1 + r_t)P_t$:

$$(2.1) \qquad Z_{ti} = a_{ti} \frac{R'_t e_i(P_{t+1} + D_{t+1}/I_{ti}) - P_t}{P_t}, \; i = 1,\ldots,n; \; t = 1,2,\ldots$$

where $a_{ti} > 0$ is a positive parameter which depends on the wealth of agent i (in period t), on his information about the risk associated with the stock, and on his degree of risk aversion (for risk neutrality a_{ti} would be infinite).[5] For simplicity we assume that a_{ti} does not depend on P_t, hence all a_{ti} are exogenous. The demand function (2.1) gives the stock of shares agent i wants to hold in his portfolio in period t; it is not his "flow demand." A negative value of Z_{ti} means that agent i has outstanding obligations from selling short.

We do not assume that the economic agents know the relevant (objective) probability distributions. Rather we look at their estimates e_i as something similar to estimating the mean of an unknown distribution. Therefore, we do not need to assume that the relevant probability distributions are stationary. Rather they will change in time exactly because the economic agents try to learn more about the stock market and thereby change their own behavior, which in turn influences the relevant probability distributions. In general, this process will not converge (especially if the stochastic process of the underlying "fundamentals" is not stationary).

We assume that in each period t there is a fixed positive supply of shares, denoted by S_t. Then, in equilibrium $S_t = \sum_{i=1}^{n} Z_{ti}$ or

$$P_t S_t = \sum_{i=1}^{n} a_{ti}[R_t' e_i(P_{t+1} + D_{t+1}/I_{ti}) - P_t] \tag{2.2}$$

$$P_t = \frac{R_t' a_t}{a_t + S_t} \sum_{i=1}^{n} w_{ti} e_i(P_{t+1} + D_{t+1}/I_{ti}) \tag{2.3}$$

where

$$a_t := \sum_{i=1}^{n} a_{ti} \tag{2.4}$$

$$w_{ti} := a_{ti}/a_t \tag{2.5}$$

With the notation

$$R_t := \frac{R_t' a_t}{a_t + S_t} < 1 \tag{2.6}$$

where R_t is to be interpreted as the risk-adjusted discount factor, we get

$$P_t = R_t \sum_{i=1}^{n} w_{ti} e_i(P_{t+1} + D_{t+1}/I_{ti}), \quad t = 1,2,\ldots \tag{2.7}$$

i.e., the price is a discounted weighted average of the expectations about tomorrow's value of the share.

The standard approach to the stock market is a special case of (2.7). Assume that R_t is constant in time, i.e., $R_t = R$ for all t, and that expectations about prices and dividends are identical for all agents i for all periods t and are equal to the (objectively given) expected values conditional on public information of period t, i.e., $e_i(P_{t+k} + D_{t+k}/I_{ti}) = E_t(P_{t+k} + D_{t+k})$ for all positive t and k. Then (2.7) implies

$$(2.8) \qquad P_t = R[E_t(P_{t+1}) + E_t(D_{t+1})], \quad t = 1,2,...$$

and under the additional assumption $\lim_{k\to\infty} R^k E_t(P_{t+k}) = 0$ we get

$$(2.9) \qquad P_t = \sum_{k=1}^{\infty} R^k E_t(D_{t+k}), \quad t = 1,2,...$$

which is the standard approach.

A naive "generalization" of (2.9) to heterogeneous expectations (but constant R_t) would be

$$(2.10) \qquad P_t = \sum_{k=1}^{\infty} R^k \sum_{i=1}^{n} w_{ti} e_i(D_{t+k}/I_{ti}), \quad t = 1,2,...$$

according to which P_t is given by average expectations about future dividends. If (2.10) were in fact true, the standard approach would generalize nicely to the case of heterogeneous information. It is, however, the main conclusion of this section that this is not the case.

Intuitively it is easy to see why (2.10) does not hold. If it were true, economic agents would make use of it in forming their expectations; hence they would try to estimate tomorrow's average expectations $\sum_{i=1}^{n} w_{t+1i}\, e_i(D_{t+1+k}/I_{t+1i})$ in order to form their expectations about P_{t+1} in period t. But (2.10) does not contain any expectations about expectations. Hence it can be true at most for special cases but not generally.

In the standard approach expectations about the (endogenous) future price could be eliminated by assuming that economic agents use (2.8) for their predictions. What do we get if we apply this procedure to the general case, i.e., to (2.7)? Substituting (2.7) into itself gives

$$P_t = R_t \sum_{i=1}^{n} w_{ti} e_i (D_{t+1}/I_{ti}) + R_t \sum_{i=1}^{n} w_{ti} e_i [R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i (D_{t+2}/I_{t+1i})/I_{ti}] +$$
$$+ R_t \sum_{i=1}^{n} w_{ti} e_i [R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i (P_{t+2}/I_{t+1i})/I_{ti}]$$

and, after substituting K - 1 times,

$$(2.11) \qquad P_t = R_t \sum_{i=1}^{n} w_{ti} e_i (D_{t+1}/I_{ti}) +$$
$$R_t \sum_{i=1}^{n} w_{ti} e_i [R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i (D_{t+2}/I_{t+1i})/I_{ti}] +$$
$$R_t \sum_{i=1}^{n} w_{ti} e_i \{R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i [R_{t+2} \sum_{i=1}^{n} w_{t+2i}\, e_i (D_{t+3}/I_{t+2i})/ /I_{t+1i}]/I_{ti}\} +$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$R_t \sum_{i=1}^{n} w_{ti} e_i \{R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i [\ldots R_{t+K-1} \sum_{i=1}^{n} w_{t+K-1i} \cdot$$
$$\cdot\; e_i (D_{t+K}/I_{t+K-1i})/I_{t+K-2i}]/\ldots/I_{ti}\} +$$
$$R_t \sum_{i=1}^{n} w_{ti} e_i \{R_{t+1} \sum_{i=1}^{n} w_{t+1i} e_i [\ldots R_{t+K-1} \sum_{i=1}^{n} w_{t+K-1i} e_i (P_{t+K}/I_{t+K-1i})/\ldots$$
$$\ldots/I_{ti}\}$$

Let us, for the sake of simplicity and ease of comparability, look at the special case where R_t is known and constant, $R_t = R$ for all t, and assume, in analogy to the standard approach, that the last term of (2.11) vanishes for $K\to\infty$. Then we get

$$(2.12) \qquad P_t = \sum_{k=1}^{\infty} R^k \sum_{i=1}^{n} w_{ti} e_i \{ \sum_{i=1}^{n} w_{t+1i} e_i [\sum_{i=1}^{n} w_{t+2i} e_i (\ldots \sum_{i=1}^{n} w_{t+k-1i} \cdot$$

$$\cdot\ e_i (D_{t+k}/I_{t+k-1i})/I_{t+k-2i}]/\ldots/I_{ti}\},$$

which differs dramatically from the "naive generalization" (2.10). The price of the stock is not given by the agents' expectations about future dividends but by the agents' expectations about expectations of others. Clearly this reminds us of Keynes' beauty contest parable, according to which "what average opinion expects average opinion to be" (Keynes 1936, p. 156) is of central importance.

According to (2.11) and (2.12) the agents' own expectations about future dividends play only a very minor role since they appear only in the first term (apart from their negligible influence on <u>average</u> expectations). All other terms are average expectations about average expectations. Average expectations about dividends come up finally at the end of the chain

$$\sum_{i=1}^{n} w_{ti} e_i \{ \sum_{i=1}^{n} w_{t+1i} e_i [\ldots \sum_{i=1}^{n} w_{t+k-1i} e_i (D_{t+k}/I_{t+k-1i})/\ldots/I_{ti}\}$$

and this in some sense anchors the expectations in market fundamentals. Therefore, our result differs from Keynes' approach as well as from the standard approach. It differs from Keynes' approach because Keynes' beauty contest parable did not take account of the time structure of the stock market. In the beauty contest parable agents guess at other agents' simultaneous guesses, whereas in the stock market agents guess at other agents' next period's guesses. Therefore, finally each chain of expectations about others' expectations ends up in a term giving expectations about dividends. On the other hand, our result differs even more dramatically from the standard approach, according to which only expectations about future dividends matter. Clearly, with heterogeneous expectations it is much more important to find out what other agents will expect tomorrow than to find out what dividends are going to be. Also, (2.12) does not imply the variance bounds on stock prices which are implied by (2.9), the standard approach, and which seem to be violated empirically (LeRoy and Porter 1981, Shiller 1981; for other explanations see LeRoy 1984).[6] Note that even with stable expectations about future dividends P_t can fluctuate a lot due to volatile expectations about average expectations.

Finally, it is doubtful whether (2.12) is of much help to the economic agent trying to predict P_{t+1}. These doubts arise from two facts. First, of all the terms appearing in (2.12) only the dividends are observable, and therefore the predictions of average expectations cannot be compared with the corresponding realizations. Second, the terms of (2.12) seem to be very formidable even for rational agents. Adherents of a theory of bounded rationality in particular will have doubts about whether human beings are able to make reasonable predictions of average expectations of average expectations of average expectations of average expectations . . . of average expectations of dividends.

We can summarize the results of this section as follows. With informational heterogeneity we get, within a neoclassical framework, results which differ dramatically from the standard approach. They are Keynesian in the sense that it is expectations about average expectations which matter, not expectations about future dividends. It is fairly obvious that this general conclusion does not depend on the special demand functions chosen for expositional simplicity.

3. Speculation, Equilibrium, and Market Efficiency

In this section we will show how, under certain assumptions, the price P_t can be used as a source of information although it is not fully revealing the private information of the other agents. It will be shown that agents who do not have private information will use the present price of the stock as their estimate of the stock's value tomorrow and therefore have a demand which is independent of the stock's price. Further, only agents who have private information influence the price. Therefore, the market is efficient in the sense that it incorporates all public information. In fact, it will in general incorporate more information and will make the public information (other than P_t) obsolete. Whatever the public information (other than P_t) may be and whatever the predictions it leads to, agents who only have public information will base their expectations exclusively on P_t. Therefore, one cannot say, for our model, that the price P_t is determined by public information (as is the case in the standard approach). Rather, it is the other way around: for the uninformed agents the price P_t _is_ all the relevant public information in the sense that it makes all other public information obsolete. At the end of this section we will also show that the data generated by our model would pass certain empirical tests of market efficiency.

In each period t an agent will have information about predetermined data, etc., and about the price P_t. It is useful to distinguish conceptually P_t from other pieces of information because P_t is endogenous. Given their information, the agents have to make an estimate of next period's dividends and stock price. We do not specify or derive analytically how the agents arrive at such an estimate, e.g., whether they use (2.11) when they make their estimate. If they are rational, they will use the model's solution for the equilibrium price for their predictions, which implies that they will try to predict the other agents' expectations. Our model is perfectly consistent with such a "rational expectations" hypothesis; in fact, such a hypothesis will enter the analysis of how the present price is used as a source of information. However, it will be sufficient to make rather general and quite innocent assumptions about the formation of expectations.

Let Q_t be the set of possible information, in period t, except the price P_t of this period, and let $J_{ti} \subset Q_t$ be the set of information (apart from P_t) agent i has in period t. Hence, the information set I_{ti} of agent i in period t is $I_{ti} = J_{ti} \cup \{P_t\}$. Information may be information about future dividends or, more importantly, information about other agents' future expectations. In particular, prior knowledge of future public information may be of great importance. A good example of such prior knowledge of future public information is given by the fact, reported in the New York Times of May 18, 1984, that five persons made a profit of \$909,000 between October 17, 1983 and February 27, 1984 by (illegally) exploiting their prior knowledge of the Wall Street Journal's column "Heard on the Street".

The vector $I_t := (I_{t1},\ldots,I_{tn})$ will be called a state of information. For notational convenience we define

$$V_t := P_t + D_t, \quad t = 1,2,\ldots \tag{3.1}$$

We keep the assumption that the demand functions are given by (2.1). In general, economic agents will know R_t' but not R_t because they don't know a_t. However, for short periods and large a_t the risk-adjusted discount factor R_t will be close to one and will not play an important role. Therefore, it seems not worthwhile to formulate the model for R_t unknown. Rather, we make the convenient simplifying assumption that all agents know R_t.

Assumption 1: The demand functions are given by (2.1) and all economic agents know R_t in period t (t = 1,2,...).

We have to make two further assumptions. The first one states that agents with identical information form identical expectations $e_i(V_{t+1}/I_{ti})$.

Assumption 2: For any subset $G \subset Q_t \cup \{P_t\}$ the estimate $e_i(V_{t+1}/G)$ is given by a function $g_t(G)$ for all i, i.e., $e_i(V_{t+1}/G) = g_t(G)$, $i = 1,\ldots,n$.

The next and final assumption pertains to the properties of the estimate g_t. Define $C_t(I_t)$ as the set of all convex combinations of the estimates $e_i(V_{t+1}/I_{ti}) = g_t(I_{ti})$, $i = 1,\ldots,n$, given a state of information I_t. We make the following assumptions about g_t which will be stated formally:

(a) Assume an agent i starts out with an estimate $g_t(J_{ti})$, $J_{ti} \subset Q_t$, and then gets to know a number b about which he has the information that it is a convex combination of his original estimate $g_t(J_{ti})$ and an estimate $g_t(J)$ which is based on more information, i.e., $J_{ti} \subset J$, and therefore better than $g_t(J_{ti})$. Assume further that he knows neither the set J nor the weights of the convex combination, i.e., his information set is given by the set $J_{ti} \cup \{b, b = \lambda g_t(J_{ti}) + (1 - \lambda) g_t(J), J_{ti} \subset J, \lambda \in [0,1]\}$. Therefore, he cannot calculate $g_t(J)$. We assume that in such a case b is the agent's estimate.

(b) Assume an agent i has information $J_{ti} \subset Q_t$ and on top of this gets a number c of which he knows that it is a convex combination of all the agents' estimates $g_t(I_{ti})$, $i = 1,\ldots,n$, but he knows neither the other agents' information sets nor the weights of the convex combination, i.e., his information set is given by the set $J_{ti} \cup \{c, c \in C_t(I_t)\}$. Then we assume that his estimate $g_t(J_{ti} \cup \{c, c \in C_t(I_t)\})$ lies in the closed interval defined by $g_t(J_{ti})$ and c. This means that the additional information of c either does not influence his estimate g_t at all or moves it in the direction of c, but not further than to c. We also assume that g_t is continuous with respect to c.

(c) We assume that in each period the estimate $g_t(J)$ is bounded for $J \subset Q_t$.

<u>Assumption 3</u>:

(a) For all $i = 1,\ldots,n$ it holds that

$$g_t(J_{ti} \cup \{b, b = \lambda g_t(J_{ti}) + (1 - \lambda) g_t(J), J_{ti} \subset J, \lambda \in [0,1]\}) = b.$$

(b) For all $i = 1,\dots,n$ it holds that

$g_t(J_{ti} \cup \{c, c \in C_t(I_t)\}) = v(J_{ti},c)g_t(J_{ti}) + [1 - v(J_{ti},c)]c$

for some function $v(J_{ti},c)$ with $v(J_{ti},c) \in [0,1]$ and $v(J_{ti},c)$ continuous with respect to c.

(c) In each period t, $t = 1,2,\dots,$ there exists a real number $\bar{g}_t$ such that $g_t(J) \in [0,\bar{g}_t]$ for all $J \subset Q_t$.

Next we have to define the equilibrium. A price P_t is an equilibrium if it clears the market, given the state of information I_t. Note that the price P_t enters I_{ti} because $I_{ti} = J_{ti} \cup \{P_t\}$. The price P_t is an equilibrium if and only if

$$(3.2) \qquad P_t = R_t \sum_{i=1}^{n} w_{ti} g_t(J_{ti} \cup \{P_t\})$$

Rational economic agents will know (3.2) and use it when they make inferences from P_t. First we prove that an equilibrium exists.

Theorem 1: There exists an equilibrium and every equilibrium is of the form

$$(3.3) \qquad P_t = R_t \sum_{i=1}^{n} \lambda^*_{ti}(J_{ti}, P_t/R_t) g_t(J_{ti})$$

where

$$(3.4) \qquad \lambda^*_{ti} = \frac{w_{ti} v(J_{ti}, P_t/R_t)}{\sum_{i=1}^{n} w_{ti} v(J_{ti}, P_t/R_t)}, \qquad i = 1,\dots,n$$

if $v(J_{ti},P_t/R_t) > 0$ for at least one i. If, for $P_t = P^*_t$, $v(J_{ti},P^*_t/R_t) = 0$ for all $i = 1,\dots,n$, then P^*_t is an equilibrium.

Proof: Assume P_t is an equilibrium. If $v(J_{ti},P_t/R_t) > 0$ for at least one i, $\sum_{i=1}^{n} w_{ti}v(J_{ti},P_t/R_t) > 0$. Because of (3.2), $P_t/R_t \in C_t(I_t)$, hence Assumption 3(b) implies

$$g_t(I_{ti}) = v(J_{ti},P_t/R_t)g_t(J_{ti}) + [1 - v(J_{ti},P_t/R_t)]P_t/R_t. \tag{3.5}$$

Inserting (3.5) into (3.2) gives (3.3). If $v(J_{ti},P_t^*/R_t) = 0$ for all i, $g_t(I_{ti}) = P_t^*/R_t$ for all i and P_t^* fulfills (3.2) and is, therefore, an equilibrium. It remains to prove existence. If there exists a $c^* \in [0,\bar{g}_t]$ such that $v(J_{ti},c^*) = 0$ for all i, then $P_t^* = R_t c^*$ is an equilibrium which therefore exists. Hence, assume that such a c^* does not exist. Then $\sum_{i=1}^{n} w_{ti}v(J_{ti},c) > 0$ for all $c \in [0,\bar{g}_t]$. Define the function F(c) by

$$F(c) := \sum_{i=1}^{n} \frac{w_{ti}v(J_{ti},c)}{\sum_{i=1}^{n} w_{ti}v(J_{ti},c)} g_t(J_{ti}), \quad c \in [0,\bar{g}_t]$$

where $\bar{g}_t$ is the upper bound of g_t, which exists according to Assumption 3(c). Because of Assumption 3(b), F(c) is continuous, and because of Assumption 3(c), $F(c) \in [0,\bar{g}_t]$. Therefore, by Brouwer's fixed point theorem there exists a c^* such that $F(c^*) = c^*$. It is easy to see that $P_t = R_t c^*$ is in fact an equilibrium (and $c^* = P_t/R_t \in C_t(I_t)$). Q.E.D.

The standard approach is a special case of (3.3) because for homogeneous information $J_{ti} = J_{t0}$, $i = 1,\dots,n$, $P_t = R_t g_t(J_{t0})$ is an equilibrium. Assuming $R_t = R$ for all t, $g_t(J_{t0}) = E_t(V_{t+1})$ and $\lim_{k\to\infty} R^k E_t(P_{t+k}) = 0$ we get (1.1).

The next theorem shows that agents who have only public information and know this take P_t/R_t as their prediction of V_{t+1} (i.e., $g_t(I_{ti}) = P_t/R_t$), disregarding their own (public) information J_{ti}. Denote public information other than P_t by J_{t0}. Thus an agent who has only public information and knows this has an information set $I_{tk} = J_{t0} \cup \{J_{t0} \subset J_{ti}, i = 1,\ldots,n\} \cup \{P_t\}$.

Further, the next theorem shows that agents who have only public information and know this do not influence the price because for such agents v and therefore the weight λ^*_{ti} in (3.3) is zero. This implies that such an agent demands $Z_{ti} = a_{ti}(R'_t - R_t)/R_t = w_{ti}S_t$, which is independent of the price; the reason for this is that with $g_t(I_{ti}) = P_t/R_t$ the expected rate of return of the stock (net of interest costs) is independent of P_t because the estimate of V_{t+1} changes proportionally with P_t. Correspondingly, only agents who have private information have a positive weight in (3.3) and influence the price P_t.

<u>Theorem 2</u>: If P_t is an equilibrium,

(a) $g_t(J_{t0} \cup \{J_{t0} \subset J_{ti}, i = 1,\ldots,n\} \cup \{P_t\}) = P_t/R_t$

(b) $v(J_{t0} \cup \{J_{t0} \subset J_{ti}, i = 1,\ldots,n\}, P_t/R_t) = 0$

<u>Proof</u>: See Appendix.

The final question is whether the data generated by our model would pass certain tests of market efficiency.[7] In order to analyze this question we look at

(3.6) $$u_t := R_t V_{t+1} - P_t$$

The tests of market efficiency we consider will be passed if the average $\frac{1}{T}\sum_{t=1}^{T} u_t$ does not differ significantly from zero and if u_t is not correlated with any predetermined variable, especially not correlated with past prices. Whether this is the case will, of course, depend on the estimates g_t. However, in a rational expectations equilibrium these estimates will be unbiased and the errors $V_{t+1} - g_t(I_{ti})$ will be uncorrelated with all predetermined variables. The following theorem shows that this is sufficient for $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} u_t = 0$ and for u_t to be uncorrelated with all predetermined variables, i.e., the data generated by our model will pass the tests of market efficiency considered (except, of course, in cases where the test gives the wrong answer, which, for a finite sample, is always possible).

Theorem 3: If $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} [V_{t+1} - g_t(I_{ti})] = 0$ for all $i = 1,\ldots,n$ and if $V_{t+1} - g_t(I_{ti})$ is uncorrelated with all predetermined variables for all $i = 1,\ldots,n$, then $\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} u_t = 0$ and u_t will be uncorrelated with all predetermined variables.

Proof: The proof follows immediately from $u_t = R_t \sum_{i=1}^{n} w_{ti}[V_{t+1} - g_t(I_{ti})]$, which follows from (3.2) and $I_{ti} = J_{ti} \cup \{P_t\}$.

Theorem 3 shows that the data generated by our model will pass the tests of market efficiency considered. Our model, however, does not imply any bounds on the variance of the stock prices P_t. It can thus be seen as an explanation of the fact that empirical studies have confirmed the market efficiency hypothesis on the one hand and rejected the empirical validity of the variance bounds on the other. Further, Theorem 2 implies that market efficiency does not mean that the price P_t is determined by public information. Rather, causality goes in the other direction: the price P_t determines what agents who have only public information expect V_{t+1} to be.

4. Conclusions

The general conclusion of this article is that the assumption of the standard approach, according to which all agents have identical expectations, is decisive for the result that stock prices are determined by expected future dividends. If expectations are heterogeneous, it is not expectations about dividends but expectations about others' expectations which matter. This resembles Keynes' important insights which he explained by means of his well-known beauty contest model. The main difference of our approach to the beauty contest model is that the beauty contest model does not capture the fact that the relevant expectations are not about others' present expectations but about others' future expectations.

The second result is that empirical validation of market efficiency does not necessarily validate the standard approach to the stock market. In our model the market is efficient with respect to public information although it does not imply that stock prices are determined by expected future dividends. Further, one cannot interpret market efficiency in the way that public information determines the (informationally) efficient equilibrium price. Rather, the efficient equilibrium price generates public information. In our model economic agents with respect to whose information the market is efficient do not influence the price; and for agents who influence the price the market is not efficient in the sense that they believe that they have a better prediction than the "market". These results can help clarify the seemingly contradictory evidence of stock market efficiency on the one hand and the high volatility of stock prices on the other.

APPENDIX

Proof of Theorem 2

Define $\bar{J}_{t0} := J_{t0} \cup \{J_{t0} \subset J_{ti},\ i = 1,\ldots,n\}$. Because of Assumption 1, $R_t \in J_{t0}$, hence $\bar{J}_{t0} \cup \{P_t\} = \bar{J}_{t0} \cup \{P_t/R_t\}$ and because of (3.2) $P_t/R_t \in C_t(I_t)$. From Assumption 3(b) it follows that (a) and (b) of Theorem 2 are equivalent, because obviously (b) implies (a), and (a) implies (b) for $g_t(\bar{J}_{t0}) \neq P_t/R_t$; since v is continuous, (a) implies (b) for $g_t(\bar{J}_{t0}) = P_t/R_t$ as well, hence (a) and (b) are equivalent. We will prove Theorem 2 by proving

$$g_t(\bar{J}_{t0} \cup \{P_t/R_t\}) = P_t/R_t \tag{A.1}$$

Define $C_t^K(I_t)$ as the set of convex combinations of $g_t(I_{tk})$, $k = 1,\ldots,K$:

$$C_t^K(I_t) := \{c_K / c_K = \sum_{k=1}^{K} \lambda_k g_t(I_{tk}),\ \lambda_k \geq 0,\ \sum_{k=1}^{K} \lambda_k = 1\} \tag{A.2}$$

For $K = n$, $C_t^K(I_t) = C_t^n(I_t) = C_t(I_t)$. We prove by induction that

$$g_t(\bar{J}_{t0} \cup \{c_K, c_K \in C_t^K(I_t)\}) = c_K \quad \text{for } K = 1,\ldots,n \tag{A.3}$$

which implies (A.1) and therefore Theorem 2.

For $K = 1$, $c_K = c_1 = g_t(I_{t1})$ and (A.3) holds because, given $J_{t0} \subset I_{t1}$, c_1 is the best estimate agent i with $J_{ti} = \bar{J}_{t0} \cup \{c_1\}$ can make. Assume (A.3) holds for $K = m - 1$, $2 \leq m \leq n$. We show that then (A.3) holds for $K = m$ as well. If $\sum_{k=1}^{m-1} \lambda_k = 0$, i.e., $\lambda_m = 1$, this is obviously true because this case is equivalent to $K = 1$; hence without loss of generality we assume $\lambda_m < 1$. Then we can rearrange

$$g_t(\bar{J}_{t0} \cup \{c_m = \sum_{k=1}^{m} \lambda_k g_t(I_{tk}),\ c_m \in C_t^m(I_t)\}) =$$

$$= g_t(\bar{J}_{t0} \cup \{c_m = (1 - \lambda_m) \sum_{k=1}^{m-1} \frac{\lambda_k}{1 - \lambda_m} g_t(I_{tk}) + \lambda_m g_t(I_{tm}),\ c_m \in C_t^m(I_t)\}) =$$

$$= g_t(\bar{J}_{t0} \cup \{c_m = (1 - \lambda_m) g_t(\bar{J}_{t0} \cup \{c_{m-1} = \sum_{k=1}^{m-1} \frac{\lambda_k}{1 - \lambda_m} g_t(I_{tk}),\ c_{m-1} \in C_t^{m-1}(I_t)\}) +$$

$$+ \lambda_m g_t(I_{tm}),\ c_m \in C_t^m(I_t)\})$$

because (A.3) holds for K = m - 1 by assumption, i.e.,

$$g_t(\bar{J}_{t0} \cup \{c_{m-1} = \sum_{k=1}^{m-1} \frac{\lambda_k}{1 - \lambda_m} g_t(I_{tk}),\ c_{m-1} \in C_t^{m-1}(I_t)\}) =$$

$$= c_{m-1} = \sum_{k=1}^{m-1} \frac{\lambda_k}{1 - \lambda_m} g_t(I_{tk}).$$

Define $I' := \bar{J}_{t0} \cup \{c_{m-1} = \sum_{k=1}^{m-1} \frac{\lambda_k}{1 - \lambda_m} g_t(I_{tk}),\ c_{m-1} \in C_t^{m-1}(I_t)\}$. Since $c_m = (1 - \lambda_m) g_t(I') + \lambda_m g_t(I_{tm})$ is a convex combination of $g_t(I')$ and $g_t(I_{tm})$, it is a convex combination either (i) of $g_t(I')$ and $g_t(I' \cup I_{tm})$ or (ii) of $g_t(I_{tm})$ and $g_t(I' \cup I_{tm})$, whatever the value of $g_t(I' \cup I_{tm})$ is, because c_m must lie either in between of $g_t(I')$ and $g_t(I' \cup I_{tm})$ or in between of $g_t(I_{tm})$ and $g_t(I' \cup I_{tm})$. This is illustrated in Figure 1.

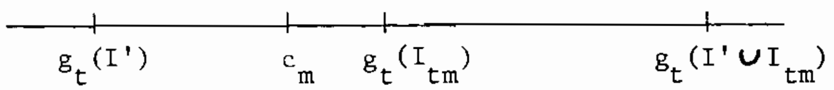

Figure 1

Assume that case (i) holds, i.e., that c_m lies between $g_t(I')$ and $g_t(I' \cup I_{tm})$. Then there exists a $\phi \in [0,1]$ such that $c_m = \phi g_t(I') +$ $+ (1 - \phi) g_t(I' \cup I_{tm})$ and, by Assumption 3(a),

$$(A.4) \qquad c_m = g_t(I' \cup \{\phi g_t(I') + (1 - \phi) g_t(I' \cup I_{tm}), I' \subset I' \cup I_{tm}, \phi \in [0,1]\})$$

Hence, $g_t(\bar{J}_{t0} \cup \{c_m = (1 - \lambda_m) g_t(I') + \lambda_m g_t(I_{tm}), c_m \in C_t^m(I_t)\}) =$ $= g_t(\bar{J}_{t0} \cup \{c_m = g_t(I' \cup \{\phi g_t(I') + (1 - \phi) g_t(I' \cup I_{tm}), I' \subset I' \cup I_{tm},$ $\phi \in [0,1]\}), c_m \in C_t^m(I_t)\}) = c_m$ because $\bar{J}_{t0} \subset I'$ and, since this follows by logic, $\{\bar{J}_{t0} \subset I'\} \subset \bar{J}_{t0}$ (i.e., $\bar{J}_{t0} \subset I'$ is known); therefore, c_m is a more informed estimate that $g_t(\bar{J}_{t0})$ and will therefore replace $g_t(\bar{J}_{t0})$. For case (ii), where c_m lies between $g_t(I_{tm})$ and $g_t(I' \cup I_{tm})$ the same result follows analogously. This completes the proof of (A.3), which in turn implies Theorem 2.

NOTES

1. For related but different approaches, see, e.g., Harrison and Kreps (1978) and the references, given in note 3, on the literature concerned with the problem of how prices aggregate and reveal heterogeneous information.

2. Expectations about others' expectations also play a role in the recent literature on rational expectations in macroeconomics. See, e.g., Di Tatta (1983), Evans (1983), Frydman (1982, 1983), Phelps (1983). For a related approach within a partial equilibrium framework see Townsend (1983a, 1983b).

3. There is an expanding literature on information revealing prices. See, e.g., Figlewski (1982, 1984), Grossman (1976, 1978, 1981), Grossman and Stiglitz (1976, 1980), Hellwig (1980, 1982), Jordan (1983), Kihlstrom and Mirman (1975), Verrecchia (1982); for a review of rational expectations in microeconomics see Radner (1982).

4. A different empirically oriented critique of the standard approach is Mehra and Prescott (1984).

5. We do not try to justify these demand functions by special assumptions about the agents' utility functions and about the probability distribution of the stochastic variable $P_{t+1} + D_{t+1}$. The basic results of Section 2 hold for any demand functions which depend on heterogeneous expectations about P_{t+1}. In Section 3 the implication of (2.1), that the equilibrium price is uniquely related to average expectations, is used in the analysis. As long as this holds, the special form of the demand functions does not matter.

6. Although one can argue that each term of the series (2.12) should fluctuate less than dividends, the series as a whole can be more volatile than dividends due to co-movements of the terms.

7. The meaning of "market efficiency" is not clear-cut (cf., e.g., LeRoy 1982, pp. 205-208). In the present context we assume that the test of market efficiency consists in testing (a) whether the present price P_t is an unbiased predictor of $R_t V_{t+1}$, the stock's next period's discounted value, i.e., whether $R_t V_{t+1} - P_t$ is zero on the average; and (b) whether the "errors," i.e., the differences $R_t V_{t+1} - P_t$ are uncorrelated with all predetermined variables, especially with past prices (because if they were correlated with a predetermined variable, this correlation could be used for predicting $R_t V_{t+1}$ better than P_t with public information only).

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