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ON-THE JOB SEARCH WITH INFORMATION OBSOLESCENCE

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ON-THE-JOB SEARCH WITH INFORMATION OBSOLESCENCE*

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Abstract

In the model of on-the-job search proposed here, fixed wage contracts are offered to job applicants. The contract offered evolves according to a Markov chain; by sampling the process job applicants both learn the current state of demand and purchase the right to lock themselves into the current contract. Therefore, upon sampling the process the agent decides (1) whether to change jobs and (2) when to sample the process again, if ever. The model nests the original Burdett (1978) formulation of the on-the-job search problem and is a private-information analog to the Lippman and McCall (1976) and Lippman and Mamer (1989) models of employment search in nonstationary environments when the state of demand is public information.

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1. Introduction

There exist a number of models of on-the-job search in the labor market; perhaps the most well-known statement of the basic model is contained in Burdett (1978). In his model, infinitely-lived employees currently paid a wage of w make a decision of whether or not to search for a new job. His model is cast in a discrete-time framework in which an employee does not have to "quit" to search, and where one wage offer is generated in a period if the direct cost of search c is paid. Then given a fixed wage offer distribution F, discount factor β, and cost of sampling c, the employee with a current wage of ω will search if $\omega < w^*(c,\beta,F)$ and will accept a new job offer of $\tilde{w}$ if $\tilde{w} > \omega$. This simple model is broadly consistent with several features of life-cycle wage profiles. In particular, it implies that wages are monotone increasing, as they are observed to be in most individual level cross-sectional or panel data sets, and that there exists a positive association between turnover and wage growth.[1]

In this paper we attempt to embed the basic structure of the Burdett model in the context of a small labor market in which the wage offer distribution evolves stochastically over time and in which the state of the demand is not public information. Throughout we will deal with the simple case in which all individuals who apply for jobs in period t are offered a wage of w_t.[2] Furthermore, the wage offered at time t is a function of the wage offered at t-1. Let the set of wage offers be denumberable, with the total number of potential offers given by S. Let there be stationary stochastic process generating the wage offers which is given by $\mathcal{P}$. The wage

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[1]This relationship is found in most empirical studies on the subject, especially when job changes are separated into "voluntary" and "involuntary" ones. In general, young labor market members who voluntarily change jobs experience the largest wage changes, followed by those who do not change employers, and finally by those who change employers involuntarily. See Flinn (1993) for a further discussion of this evidence.

[2]The assumption that the wage offer distribution at each point in time is degenerate is different from that imposed in Lippman and McCall (1976) and Lippman and Mamer (1989). These papers analyze the case of search when the current state of the world is public information and nondegenerate wage offer distributions are indexed by a "superparameter" ϑ which represents the state of demand. If the wage offer distribution were degenerate in their models, the search problem would be trivial.

only observable in period t *if* the individual pays the sampling cost c. Payment of c entitles the individual both to an observation on the process *and* the right to a job offer at the current wage. In deciding on whether or not to search in period t, an individual will utilize her history of past observations of the process, denoted $\mathcal{H}_t$. We will characterize the decision rule of whether to search implicitly by $\Lambda(\omega, c, \beta, P, \mathcal{H}_t, t)$. As in Burdett's model, *given* search in period t an individual will change jobs if $w_t > \omega$.

The model has several noteworthy features. Perhaps the most interesting is the dependence of the search decision on a time-varying state variable $\mathcal{H}_t$. In the Burdett model, an individual earning wage ω who searches in period t but receives a wage offer $w_t < \omega$ will search again in period t+1. In the model developed here, an individual who searches in period t but receives a wage offer $w_t < \omega$ also does not change jobs. However, her decision of whether or not to search in period t+1 will be a function of $\mathcal{H}_{t+1} = \mathcal{H}_t \cup w_t$. Not only will it generally be the case that the individual will *not* search again in period t+1, the number of periods the individual waits until searching again will be a function of $\mathcal{H}_t$, w_t and ω. Thus in characterizing job search behavior, the individual's current wage w will not be a sufficient statistic; instead sufficient statistics will consist of the pair $(\omega, \mathcal{H}_t)$.

Related to the fact that the current wage is not sufficient to describe an agent's behavior, our model is consistent with the observation that an individual with a current wage of ω' searches in period t while an individual with a current wage of ω $[\omega' > \omega]$ does not. Thus we provide a behavioral interpretation for a source of heterogeneity in search behavior when the agent's information set is not observable by the analyst.

Besides Burdett's model, the model developed here is most closely linked to the search and learning models of DeGroot (1970), Rothschild (1974), and Morgan (1985), among others, and the limited literature on search over the "business cycle" [*e.g.*, Lippman and McCall (1976) and Lippman and Mamer (1989)]. We briefly consider the connections.

Cast in terms of the notation used to describe Burdett's model, learning models consider the case in which the wage offer distribution, F, is unknown. In these models F is taken to be fixed over time. Let a labor market entrant's original prior on F be denoted F^0. Then given a history $\mathcal{H}_t$ and a Bayesian learning rule, the agent's posterior estimate of the wage offer distribution is $\hat{F}(F^0,\mathcal{H}_t)$. The state variables in an on-the-job search context become $(\omega,\hat{F})$. In this case, history matters in determining search behavior, though all the dynamics come through the process of individual learning about a static labor market.[3]

In the search over the cycle models of Lippman and McCall (1976) and Lippman and Mamer (1989), the wage offer distribution at time t is given by $F(\cdot|\vartheta_t)$, where ϑ_t is a scalar random variable representing the state of demand at time t. The state of demand evolves according to a first order Markov chain. By ordering the distributions in terms of first order stochastic dominance, so that $F(w|\vartheta') \geq F(w|\vartheta)$ for all w and $\vartheta' > \vartheta$, the authors are able to derive results on quiting behavior and the duration of unemployment over the cycle. In both papers, search can only take place in the unemployed state [so that currently employed individuals must quit into unemployment if the wish to take a chance on getting a higher-paying job] and the state of the demand is public information. These are the two principal differences between the setup in these papers and the one utilized below.

The structure of the paper is as follows. In Section 2 we describe and analyze the problem formally. In Section 2.1 we prove that the state valuation functions are unique and present some comparative statics results. We also indicate the manner in which Burdett's model can be embedded in our framework. In Section 2.2 we briefly consider the decision to initially enter the labor market. In Section 3 we present some simulations to illustrate how the model works in practice. Besides working out the decision rules and state valuations for example labor markets, we look at the behavior of aggregate statistics in the context of an OLG model as we vary the key parameter of the model, the cost of observing the demand process. Section 4 contains a brief conclusion.

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[3] The dependence of search behavior on the history differs in the model developed here and the ones analyzed in the "static" search literature. In those models, wage offers are i.i.d. draws from F and thus have equal weight in the estimator $\hat{F}$. In the model here, under the first-order Markov assumption on the wage offer process only the most recent wage offer will affect search behavior.

2. Model

Individuals in the labor market act so as to maximize the expected present value of wage payments minus costs of search. Search in our case refers to the act of ascertaining the state of the labor market at some given point in time [time is discrete throughout]. By paying a sampling fee, which may be best thought of as costs associated with applying for jobs, the individual simultaneously (i) learns the state of demand in the labor market at that moment in time and (ii) gains the option to accept employment at the going wage. Individuals are expected wealth maximizers and face a constant risk of death $1-\lambda$, $\lambda \in (0,1]$.

In the model the evolution of the "demand" for labor is taken as totally exogenous. Demand for labor is reflected in the wage contract offered to individuals who have applied for jobs in period τ. Contracts specify a wage, $w(\tau)$, are fixed in real terms and are for life. Only workers have the option of breaking the contract, and may leave the employer for one offering a higher wage at any time.

The labor market is in one of S states each period, where $2 \le S \le \infty$. The time homogeneous stochastic process $\mathcal{P}$ describes the movements between these states. We have made the homogeneity assumption since our desire is to focus on stationary strategies. We will not require P to satisfy other conditions, such as ergodicity, for labor market participants' decision rules to be well-defined.[4]

Attached to each labor market state is a wage function, which specifies the wage contract offered in that state of the world. The wage function will be denoted $w(s)$, where $s \in S$. Without loss of generality, we index that states so that $w(s) > w(s-1)$, $s = 2,\ldots,S$. Furthermore we have $w(S) < \infty$ and $w(1) \ge 0$ so that current period utility is bounded and nonnegative under all possible contracts.

A strategy of the individual will be a function of the individual's history of sampling the labor market. When an individual initially enters the labor market, at time τ say, she observes the labor market in state s_τ and

[4]While ergodicity will not be required to examine the behavior of individuals once they have enterred the labor market, it will when we analyze the initial labor market entry decision in Section 2.2.

accepts the wage offer associated with this state, $w(s_\tau)$. Upon observing the state, she decides on the length of time she will wait until resampling the market. The minimum wait is one period; if she samples the market after 1 period, say, she would observe the state of demand at time $\tau+1$ and have the opportunity to switch jobs at that point. In general, she will wait ℓ periods, $\ell \in \mathbb{Z}^+$, so that her next resampling of the market would be in period $\tau+\ell$. The maximum wait is ∞; in this case she will never resample the labor market and thus will retain her initial wage $w(s_\tau)$ throughout her entire labor market career.

Consider the decisions made by an individual who has a wage rate of ω and history of observations of the market $\mathcal{H}_{t-1}$ prior to sampling the market at time t. Upon observing the market in state s_t, the agent makes two choices.

Decision A:

Accept employment with new employer at t iff $\omega < w(s_t)$.

This simple "reservation wage" rule is exactly of the same form as appears in Burdett's model of on-the-job search in which wage offers are drawn from a fixed [but non-degenerate] wage offer distribution. If a new job is accepted at time t, then $\omega = w(s_t)$ in period t.

Decision B:

Set the next sampling time of the process, which will be given by $\ell(\omega,\mathcal{H}_t) \in \mathbb{Z}^+$, where ω is the current wage after observing $w(s_t)$ and $\mathcal{H}_t = \mathcal{H}_{t-1} \cup s_t$.

Because all information concerning the state of demand is private so that no revision of the history is possible between sampling times, decision times are synonymous with sampling times. In some sense then, the decision period is determined endogenously.

We can formally describe the problems the individuals face strictly in terms of the sampling history $\mathcal{H}_t$. For our purposes, we will describe the history as follows. Say an individual has just sampled the process at t, and that this was the Q^{th} time she has sampled the process, $Q \in \mathbb{Z}^+$. Then

[2.1] $$\mathcal{H}_t = \{\xi_1,\ldots,\xi_Q, s_{\xi_1},\ldots,s_{\xi_Q}\},$$

where ξ_q denotes the calendar date of the q^{th} observation of the process and s_{ξ_q} denotes the state of demand in period ξ_q. Note that the date of her original entry into the labor market is ξ_1.

Her current wage is

[2.2] $$\omega = \max\{w(s_{\xi_q})\}_{q=1}^{Q}.$$

Furthermore, she will be observed to change employers in period t if

[2.3] $$w(s_t) > \max\{w(s_{\xi_q})\}_{q=1}^{Q-1} .$$

Finally, consider the determination of the next sampling time. This sampling time is determined within the following functional equation.

[2.4] $$V(\mathcal{H}_t) = \sup_{\ell\in\mathbb{Z}^+} \{ \sum_{t=1}^{\ell} \beta^t \max\{w(s_{\xi_q})\}_{q=1}^{Q} - \beta^{\ell}c + \beta^{\ell} E[V(\mathcal{H}_{t+\ell})|\mathcal{H}_t,\ell] \} ,$$

where $\beta \in [0,1)$ is the discount factor. The first term in the sup operator is the present value of the current contract ω which by definition will be received until the next sampling time, ℓ periods away. At that time, the sampling cost c will be paid, so that the discounted resampling cost is $\beta^{\ell}c$. At time $t+\ell$, the history will be expanded to include the value of the process at that time, $s_{t+\ell}$. The distribution of $s_{t+\ell}$ is a function of <u>both</u> the history $\mathcal{H}_t$ and the length of time elapsed until the next observation is made.

In the next subsection we show that under the assumption that the state of demand is Markov the problem is quite tractably characterized.

2.1 Formal Analysis of the Model

In the analysis presented here we will further restrict the process $\mathcal{P}$ to be a Markov chain with stochastic matrix

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1S} \\ p_{21} & p_{22} & \cdots & p_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ p_{S1} & p_{S2} & \cdots & p_{SS} \end{bmatrix},$$

where p_{ij} is the probability that the labor market state changes from i to j in one period, and where $\sum_j p_{ij} = 1 \ \forall\ i$.

From inspection of [2.4], we can easily determine the sufficient statistics for the agent's resampling problem. Let ω denote the individual's wage after Q labor market observations, so $\omega = \max\{w(s_{\xi_q})\}_{q=1}^{Q}$. Then ω is the only "utility" relevant characteristic of $\mathcal{H}_Q$.[5] Furthermore, in terms of forecasting future states of the labor market only the last state observed is relevant[6] given that the labor market process is Markovian. Let the last state of the market observed be denoted $\sigma = s_{\xi_Q}$. Then we can rewrite [2.4] as

$$[2.5] \qquad V(\omega,\sigma) = \sup_{\ell\in\mathbb{Z}^+} \{ \sum_{t=1}^{\ell} \beta^t \omega - \beta^{\ell} c + \beta^{\ell} E[V(\max(\omega,w(\sigma')),\sigma') | \sigma,\ell] \},$$

where σ' denotes the state of the process at the next sampling time.

The expectation operator is defined with respect to the conditional probability distribution associated with the state variable σ and the policy variable ℓ. The probability of going from state σ to state σ' after ℓ periods is given by $P^{\ell}_{\sigma,\sigma'}$, which is the element in row σ column σ' of the ℓ^{th} power of P.

We now turn to defining the state space of the problem. While ω and σ can take any of S values in principle, the structure of the decision problem limits the combinations of ω and σ potentially observable for any stochastic matrix P. For example, an individual who samples the process in state S will never resample since she has obtained the highest wage rate possible w(S) and sampling is costly. Thus when $\omega = w(S)$, it must be the case that $\sigma = S$.

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[5]That is, it is the only function of the history that produces positive utility flows until the next resampling occurs.

[6]That is, conditional on the last state observed all previous states observed are uninformative in a predictive sense.

Consider some other wage ω associated with a state s [that is, $\omega = w(s)$] where $s < S$. Then since the current wage must be associated with the highest-valued state ever sampled, it must be the case that $\sigma \leq s$. Then the state space of the model is the set

$$\Xi = \{(w(1),1),(w(2),1),(w(2),2),\ldots,(w(s),1),\ldots,(w(s),s),\ldots,\text{For} \\ (w(S-1),1),\ldots,(w(S-1),S-1),(w(S),S)\}.$$

For any **P**, β, and c the set Ξ has at most $(S-1)S/2 + 1$ elements.

To investigate the properties of the value functions associated with the states we consider the specific form of [2.5] for various elements in the state space. The simplest case is for $(w(S),S)$, where

$$V(w(S),S) = \frac{\beta}{1-\beta} w(S).$$

This state is the preferred one in that its valuation is the largest over the set of all state valuations.

Now turn to a description of the general problem. It will prove useful to define the state associated with a wage of ω as $s(\omega)$. Then the value of state (ω,σ) can be written as

$$[2.6] \qquad V(\omega,\sigma) = \sup_{\ell} \{ \sum_{t=1}^{\ell} \beta^t\omega - \beta^{\ell}c$$

$$+ \beta^{\ell}\Big(P^{\ell}_{\sigma,1}V(\omega,1) + P^{\ell}_{\sigma,2}V(\omega,2) + \ldots + P^{\ell}_{\sigma,s(\omega)}V(\omega,s(\omega))$$

$$+ P^{\ell}_{\sigma,s(\omega)+1}V(w(s(\omega)+1),s(\omega)+1) + \ldots + P^{\ell}_{\sigma,S}V(w(S),S)\Big).$$

By stacking the state valuations of all the elements of the state space we can succinctly write the problem as

$$[2.7] \qquad V = \sup_{\ell \in \mathbb{Z}^+_{\mathcal{D}}} A(\ell) + B(\ell)\,V \text{ , where}$$

$$V = \begin{bmatrix} V(w(1),1) \\ V(w(2),1) \\ V(w(2),2) \\ \vdots \\ V(w(s),1) \\ \vdots \\ V(w(s),s) \\ \vdots \\ V(w(S-1),1) \\ \vdots \\ V(w(S-1),S-1) \\ V(w(S),S) \end{bmatrix} ;$$

$$A(\ell) = \begin{bmatrix} \sum_{t=1}^{\ell(w(1),1)} \beta^t \, w(1) - \beta^{\ell(w(1),1)} \, c \\ \sum_{t=1}^{\ell(w(2),1)} \beta^t \, w(2) - \beta^{\ell(w(2),1)} \, c \\ \sum_{t=1}^{\ell(w(2),2)} \beta^t \, w(2) - \beta^{\ell(w(2),2)} \, c \\ \vdots \\ \sum_{t=1}^{\ell(w(s),1)} \beta^t \, w(s) - \beta^{\ell(w(s),1)} \, c \\ \vdots \\ \sum_{t=1}^{\ell(w(s),s)} \beta^t \, w(s) - \beta^{\ell(w(s),s)} \, c \\ \vdots \\ \sum_{t=1}^{\ell(w(S-1),1)} \beta^t \, w(S-1) - \beta^{\ell(w(S-1),1)} \, c \\ \vdots \\ \sum_{t=1}^{\ell(w(S-1),S-1)} \beta^t \, w(S-1) - \beta^{\ell(w(S-1),S-1)} \, c \\ \beta \, w(S)/(1-\beta) \end{bmatrix}$$

and the matrix $B(\ell)$ is

$$
B(\ell) = \begin{bmatrix}
\beta^{\ell(1,1)} & \Big(P^{\ell(1,1)}_{1,1} & 0 & P^{\ell(1,1)}_{1,2} & \cdots & 0 & \cdots & P^{\ell(1,1)}_{1,s} & \cdots & 0 & \cdots & P^{\ell(1,1)}_{1,S-1} & P^{\ell(1,1)}_{1,S} \Big) \\
\beta^{\ell(2,1)} & \Big(0 & P^{\ell(2,1)}_{1,1} & P^{\ell(2,1)}_{1,2} & \cdots & 0 & \cdots & P^{\ell(2,1)}_{1,s} & \cdots & 0 & \cdots & P^{\ell(2,1)}_{1,S-1} & P^{\ell(2,1)}_{1,S} \Big) \\
\beta^{\ell(2,2)} & \Big(0 & P^{\ell(2,2)}_{2,1} & P^{\ell(2,2)}_{2,2} & \cdots & 0 & \cdots & P^{\ell(2,2)}_{2,s} & \cdots & 0 & \cdots & P^{\ell(2,2)}_{2,S-1} & P^{\ell(2,2)}_{1,S} \Big) \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\
\beta^{\ell(s,1)} & \Big(0 & 0 & 0 & \cdots & P^{\ell(s,1)}_{1,1} & \cdots & P^{\ell(s,1)}_{1,s} & \cdots & 0 & \cdots & P^{\ell(s,1)}_{1,S-1} & P^{\ell(s,1)}_{1,S} \Big) \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\
\beta^{\ell(s,s)} & \Big(0 & 0 & 0 & \cdots & P^{\ell(s,s)}_{s,1} & \cdots & P^{\ell(s,s)}_{s,s} & \cdots & 0 & \cdots & P^{\ell(s,s)}_{s,S-1} & P^{\ell(s,s)}_{s,S} \Big) \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\
\beta^{\ell(S-1,1)} & \Big(0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & P^{\ell(S-1,1)}_{1,1} & \cdots & P^{\ell(S-1,1)}_{1,S-1} & P^{\ell(S-1,1)}_{1,S} \Big) \\
\vdots & \vdots & \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\
\beta^{\ell(S-1,S-1)} & \Big(0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & P^{\ell(S-1,S-1)}_{S-1,1} & \cdots & P^{\ell(S-1,S-1)}_{S-1,S-1} & P^{\ell(S-1,S-1)}_{S-1,S} \Big) \\
0 & \Big(0 & 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & 0 \Big)
\end{bmatrix}
$$

We can rewrite the problem as

$$[2.8] \qquad V = TV.$$

The uniqueness of the state valuation functions is demonstrated in the following.

Proposition 2.1: There exists a unique valuation $V(\omega,\sigma)$ for all labor market states $(\omega,\sigma) \in \Xi$.

Proof: We proceed by demonstrating that the mapping T satisfies Blackwell's sufficient conditions for it to be a contraction.

(i) Monotonicity.

Write the policy function which chooses a value or values of ℓ for each V as $\ell(V)$. Then rewrite [2.7] as

$$V = A(\ell(V)) + B(\ell(V))\, V.$$

The rule $\ell(V)$ exists since each element of the vector V is bounded from above by $\beta\, w(S)/(1-\beta)$ which is finite by construction. Now consider two vectors $V' \geq V$. Then since

$$\begin{aligned} TV' &= A(\ell(V')) + B(\ell(V'))\, V' \\ &\geq A(\ell(V)) + B(\ell(V))\, V' \\ &\geq A(\ell(V)) + B(\ell(V))\, V \\ &= TV \,, \end{aligned}$$

the operator T is monotone increasing.

(ii) Discounting.

To the vector V add the row vector $\Delta = (\nu \ldots \nu)'$, where $\nu > 0$. Then we must establish

$$T(V+\Delta) \le TV + \delta\Delta,$$

where $\delta \in [0,1)$. Now

$$T(V+\Delta) = A(\ell(V+\Delta)) + B(\ell(V+\Delta))(V+\Delta)$$

$$= \left[A(\ell(V+\Delta)) + B(\ell(V+\Delta))\right] V + B(\ell(V+\Delta))^{*}\Delta .$$

Since

$$TV \ge \left[A(\ell(V+\Delta)) + B(\ell(V+\Delta))\right] V ,$$

it is sufficient to demonstrate that $B(\ell(V+\Delta))\ \Delta \le \delta\Delta$ for some $\delta \in [0,1)$. Consider the product of the row in $B(\ell)$ corresponding to labor market state (i,j) and the row vector Δ, which is given by

$$\nu\ \beta^{\ell(i,j)} \sum_{k=1}^{S} P_{j,k}^{\ell(i,j)} = \nu\ \beta^{\ell(i,j)}$$

since

$$\sum_{k=1}^{S} P_{j,k}^{\ell(i,j)} = 1$$

because $\ell(i,j) \in \mathbb{Z}^{+}$. Then

$$B(\ell(V+\Delta))\ \Delta \le \beta\ \Delta < \Delta. \qquad \blacksquare$$

We now proceed to characterize some properties of the value function and sampling rules associated with this search problem. In the next subsection, we will turn to an analysis of the initial labor market entry decision. In the characterization of the decision to enter the labor market the following intuitive result will be needed.

Proposition 2.2: $V(\omega,\sigma)$ is nonincreasing in c for all $(\omega,\sigma) \in \Xi$.

Proof: Write V_c to denote the value of the problem when the cost of sampling is c. Then

$$V_c = A(\ell(V_c),c) + B(\ell(V_c))\ V_c \ ,$$

but by the definition of A,

$$A(\ell(V_c),c') + B(\ell(V_c))\ V_c \geq TV_c = V_c \quad \text{for } c' < c,$$

or

$$(I_{\mathcal{D}} - B(\ell(V_c)))^{-}A(\ell(V_c),c') \geq V_c \ ,$$

where X^{-} denotes the generalized inverse of X and I_m is the identity matrix of dimension m. Similarly,

$$A(\ell(V_c),c') + B(\ell(V_c))\ V_{c'} \leq V_{c'} \ ,$$

so that

$$V_{c'} \geq (I_{\mathcal{D}} - B(\ell(V_c)))^{-}A(\ell(V_c),c'),$$

and thus $V_{c'} \geq V_c$. ■

Increasing the cost of sampling will strictly decrease the value of being in a state (ω,σ) when resampling occurs at some positive rate in that state. When resampling has ceased in some state [so that from a behavioral point of view it is an absorbing state] increasing the cost of sampling results in no welfare reduction.

In proposition 2.3 we consider some properties of the value function and sampling rules as we vary the state variables. We can obtain an obvious monotonicity result concerning the current wage, but not on the information set. The value of the sampling/turnover problem is not generally monotone in the information set [holding the current wage fixed] because the transition matrix is left unstructured. Monotonicity results on the information set would require strong assumptions on P.

Proposition 2.3: $V(\omega',\sigma) \geq V(\omega,\sigma)$ *and* $\ell(\omega',\sigma) \geq \ell(\omega,\sigma)$ *for all* $\omega' > \omega$ *and* $\sigma \leq s(\omega)$.

Proof (sketch): The first part of the proposition is trivial since and individual in state (ω',σ) can mimic the sampling <u>and</u> turnover policies of an individual in state (ω,σ) and obtain a higher level of welfare than the (ω,σ) individual. I have not yet proved the second part concerning the sampling times.

Finally, we consider the relationship between our model and the one originally exposited by Burdett (1978). Recall that in his model of on-the-job search, individuals searched every period from a fixed wage offer distribution until such time that they drew a wage in some subset of offers. This subset of offers formed a connected set - in the continuous distribution case, the subset consists of $[\omega^*,\bar{w}]$, where $\bar{w}$ is the upper limit of the support of the wage offer distribution F. In the result stated below, we show that when the offer draws are i.i.d., it is in fact optimal to either sample every period or never again, and we go on to establish the existence of a critical value ω^* which defines the set of "absorbing" wages. We have shown how the relaxation of the i.i.d. assumption makes this form of sampling suboptimal.

Proposition 2.4: Let the state of demand be i.i.d. [$\Rightarrow$ rows of P *are identical]. Then*

[i] $V(\omega,\sigma) = V(\omega)$ *and* $\ell(\omega,\sigma) = \ell(\omega)$ *for all* $\sigma \le s(\omega)$.

[ii] $\ell(\omega) \in \{1,\infty\}$ *for all* ω, *with*

$$\ell(\omega) = \begin{cases} 1 & \text{iff} \quad \omega < \omega^*(\beta,c,P) \\ \infty & \text{iff} \quad \omega \ge \omega^*(\beta,c,P) \end{cases}$$

for some $\omega^*(\beta,c,P)$.

Proof (sketch):

[i] Let $A(\ell;\omega,\sigma)$ and $B(\ell;\omega,\sigma)$ denote the relevant rows of the the $A(\ell)$ and $B(\ell)$ matrices for the state (ω,σ). Now $A(\ell;\omega,\sigma)$ can be written $A(\ell;\omega)$ since σ does not appear as an argument. When the rows of P are identical, inspection of the matrix $B(\ell;\omega,\sigma)$ reveals that $B(\ell;\omega,\sigma) = B(\ell;\omega,\sigma')$ for all $\sigma,\sigma' \le s(\omega)$. Then $\arg\sup_{\ell(\omega,\sigma)} A(\ell;\omega) + B(\ell;\omega)\, V = \ell(\omega)$, and it follows that $V = TV \Rightarrow V(\omega,\sigma) = V(\omega,\sigma') = V(\omega)$.

[ii] Define $R(\omega) \equiv \sum_{j=1}^{S} p(j) \max \{V(\omega);V(w(j))\}$, and let $\mu(\omega) \equiv \omega - c + R(\omega)$. First consider the case in which $\beta\omega + \beta^2\mu(\omega) > \beta\mu(\omega)$, so sampling in two periods yields higher value than sampling in 1 for fixed $V(\omega)$. Now compare $\beta\omega + \beta^2\omega + \beta^3\mu(\omega)$ and $\beta\omega + \beta^2\mu(\omega)$. Since

$$\begin{aligned}&\text{sign}\{[\beta\omega + \beta^2\omega + \beta^3\mu(\omega)] - [\beta\omega + \beta^2\mu(\omega)]\} \\ &= \text{sign}\{\omega + \beta^2\mu(\omega) - \beta\mu(\omega)\} \\ &> 0 ,\end{aligned}$$

so sampling in 3 periods yields a higher return then sampling in 2. By induction, if $\beta\omega + \beta^2\mu(\omega) > \beta\mu(\omega)$, $\ell(\omega) = \infty$. It is equally straightforward to show that when the inequality is reversed, $\ell(\omega) = 1$.

Now consider any ω for which

$$\begin{aligned}&\beta\omega + \beta^2\mu(\omega) > \beta\mu(\omega) \\ &\Rightarrow \omega > (1-\beta)\mu(\omega) \\ &\Rightarrow \omega > \frac{1-\beta}{\beta}[R(\omega) - c].\end{aligned}$$

Now consider another $\omega' > \omega$. Show that the $\max(R(\omega')-R(\omega)) \le \beta/(1-\beta)(\omega'-w)$. Then $\omega > (1-\beta)/\beta[R(\omega)-c] \Rightarrow \omega' > (1-\beta)/\beta[R(\omega')-c]$. Define $\omega^*(\beta,c,P)$ as the smallest value in the set of wage offers for which $\omega > (1-\beta)/\beta[R(\omega)-c]$. ∎

2.2 Initial Entry Decision

While the cost of search is an important parameter in the turnover decision, it is also critical in determining the initial labor market entry decision. As in the standard job search model [*e.g.*, Flinn and Heckman (1982)], it may be interesting to investigate the decision to initially enter the labor market as well as the search problem conditional on entry. We will particularly be interested in the relationship between the sampling cost and the participation decision. To link the two problems, we will assume that the initial cost of sampling the market is the same as the sampling cost for employed individuals. This, coupled with the assumptions that the value of any labor market state is bounded from below by zero and the value of nonparticipation is zero produce the implication that once individuals enter the market they never leave.

In Proposition 2.1 we proved that unique valuation functions and well-defined sampling rules exist <u>for</u> <u>any</u> <u>stochastic</u> <u>matrix</u> P and cost of search $c \geq 0$. In order to analyze the initial entry decision, however, we require some additional assumptions regarding the nature of the Markov chain.

Assumption 2.2.1 Labor market conditions evolve according to an irreducible ergodic chain.

In this case there exists an unique distribution π^* which satisfies the "equilibrium equations" [see, *e.g.*, Cox and Miller (1965)]

$$\pi_k^* = \sum_{j=1}^{S} \pi_j^* p_{jk} \quad , \quad k = 1,2,\ldots,S \ .$$

We also need an assumption about the value of being out of the labor market, or nonparticipation.

Assumption 2.2.2 The value of nonparticipation in the labor market is 0.

Now consider the decision of whether to enter the market. An agent outside the market has never sampled the labor market process. Given knowledge of the labor market process as summarized by the stochastic matrix **P**, under A2.2.1, an unique stationary distribution over the labor market states exists $[\pi^*]$. The agent knows that initial entry will result in her initially occupying a state in the set $\{(w(1),1),\ldots,(w(S),S)\}$. In terms of timing conventions for payoffs, we let the initial entry decision result in a job offer at the end of the period. Then the expected value of entering the labor market is

$$[2.9] \qquad V_E = \beta\ [\pi_1^*\ V(w(1),1) + \ldots + \pi_S^*\ V(w(S),S)] - \beta\ c.$$

Proposition 2.5: There is an unique value $c^*(\beta,P)$ such that for all $c \geq c^*(\beta,P)$ labor market entry will take place while for $c < c^*(\beta,P)$ it will not.

Proof: Entry will take place when $V_E \geq 0$. The RHS of [2.9] is a sum of continuous, monotone decreasing functions of c. At $c = 0$, $\beta c = 0$. Since $w(s) >$

0 for s = 2,3,...,S, and since $V(w(s),s) \geq \beta w(s)/(1-\beta)$, and if $\pi_1^* < 1$, then the RHS[2.9] > 0 at c = 0. At c = ∞, RHS[2.9] = −∞ since the V(w(s),s) is bounded from above for all c. Then there exists an unique value c* for which $V_E(c^*) = 0$. ■

The modest goal of this subsection was to develop conditions under which there will exist some $c^* \in (0,\infty)$ such that labor market entry will occur. If one wishes to analyze an employed search model consistent with initial labor market entry, the results indicate that the restriction that **P** be an irreducible ergodic Markov chain seems essential. If **P** is taken not to be ergodic, then a model of labor market entry will apparently require a specification of agents' beliefs about the state of the world determined outside of the model.

The existence of a $c^* < \infty$ also necessitates imposing the condition that $c \leq c^*$ when performing simulation exercises or as a constraint on the parameter space if this model were used as a basis for an estimation exercise. These conditions could only be ignored if the initial entry and on-the-job search were thought of as "independent" decisions, a seemingly unattractive viewpoint.

3. Simulation Exercises

Because it is difficult to derive many interesting comparative statics results except for certain special cases of **P**, in this section we present optimal sampling times, state valuations, and sample paths of aggregate wages for several labor market examples. We will be particularly interested in illustrating the relationship between the costs of sampling and these outcome measures for various choices of the Markov chain **P**.

3.1 Constructing the Examples

Throughout all the examples, we restrict the number of states (S) to 4. This is large enough to illustrate the heterogeneity in behavior for individuals with the same current wage ω. Each **P** utilized below also is irreducible and ergodic and thus has an unique stationary distribution π^*. [This distribution is used to generate the initial conditions for each of the sample paths examined.] We define the function mapping the states of the labor market {1,2,3,4} into the wage offers by $w(s) = s - 1$, so that the wage states are {0,1,2,3}. We consider the case in which the discount factor (β) = .8 and the period-to-period survival probability (λ) = .9.[7] This implies that the "effective" discount factor used by agents is $\tilde{\beta} = \beta \cdot \lambda = .72$. In each labor market [where by a labor market we are referring to a particular value of P] we will look at behavioral outcomes for four different values of c: 0, .6, 1.2, and ∞. In the case of $c = \infty$, there will be no job change after initial entry into the market. In the case of $c = 0$, observations of the process are "free," so all individuals who have a wage less than w(S) will sample the process each period. These two cases obviously represent opposite ends of a continuum in terms of labor market adaptation to the realization of the Markov chain.

For any given labor market $\{\beta,\lambda,c,P\}$ we solve for the state valuation functions and the optimal sampling times using the method of successive approximation. At iteration k, when the current guess is V^k, we find

$$V^k = \sup_{\ell^k} \; A(\ell^k) + B(\ell^k)\, V^k$$

by evaluating $A(x) + B(x)\, V^k$ for $x \in \{1,2,\ldots,100\}$. If any element of ℓ^k = 100, we assume this corresponds to an infinite waiting time. The successive approximations terminated when $\max_{j\in\Xi} |V_j^k - V_j^{k-1}|$ was less than .000001. We refer to ℓ^k as the optimal sampling time vector.

The aggregate labor market experiments were conducted in an OLG context. We began by generating a sample path 5000 periods long for each of the four P used. In the first period, the invariant distribution π^* is used to map a draw from the uniform distribution on [0,1] into the state s_1. In other periods $t > 2$, the state realized in period t-1, s_{t-1}, implies a probability distribution $P(s_{t-1},\cdot)'$ for the state s_t. This probability distribution along with an i.i.d. draw from the uniform distribution on [0,1] determines s_t.

Using the sample path s, we generate a wage path for each cohort. A

[7] The survival probability must be strictly less than 1 for the OLG economies defined below to be well-behaved asymptotically.

cohort entering the labor market at date t initially samples it in state s_t. Their wage remains at $w(s_t)$ until their next observation of the process, which occurs at $t+\ell(w(s_t),s_t)$. At that time, their wage becomes $w(\max\{s_t, s_{t+\ell(w(s_t),s_t)}\})$, and a new sampling time is computed. The process was repeated until a history 100 periods in length had been constructed for each entry cohort.[8] These cohort histories were then used to construct aggregate measures of labor market outcomes in the following way.

In each period a continuum of individuals enter the labor market, with the measure of the set of entrants being normalized to 1. Given the constant survival probability of λ, the measure of the set of agents alive at each point in time is $N = 1 + \lambda + \lambda^2 + \ldots = (1-\lambda)^{-1}$. Now the *proportion* of agents alive in any period who have been in the labor market m periods is given by

$$\alpha(m) = \lambda^{(m-1)}/N = \lambda^{(m-1)}(1-\lambda).$$

[Note that entrants in the labor market in a period are considered to have a labor market age of 1.] Now let the period t wage of individuals from cohort r [$r \le t$] be given by $w(r,t)$. Then the mean wage at time t is

$$\bar{w}_t = \sum_{r=1}^{t} \alpha(t-r+1)\, w(r,t)$$

when t is sufficiently large for the population to be stationary. We will also compute the within period standard deviation in wages, which is

$$SD(w_t) = \left[\sum_{r=1}^{t} \alpha(t-r+1)\, w(r,t)^2 - \bar{w}_t^2\right]^{.5}.$$

Finally, we compute sample means, standard deviations, and autocorrelations for the aggregate wage realization $\{\bar{w}_t\}$. All of the graphs and statistics computed refer to observations over the periods 3000 through 3300. These periods lie approximately in the middle of the history of the labor market and thus are free of initial conditions effects. We limited attention to 301 periods to facilitate inspection of the graphs.

[8] Given the survival rate of .9, the probability of an individual surviving 100 periods is less than .00003.

3.2 Results

Table 3.1 contains the state valuations and optimal sampling times for the first example labor market. The stationary distribution associated with P indicates that the process spends approximately the same amount of time in each of the four states. Turning to the panel containing the sampling times, note that when the cost of search is 0 the optimal response is to sample in any state in which the wage is less than w(4) [= 3 in this case]. Thus individuals will search every period until finding the economy in state 4, at which time they will end their search. In terms of life cycle earnings profiles, the implication of such a model is that any infinitely-lived cohort member will find a job with a wage of w(4) with probabilty 1 as long as P is irreducible. The optimal sampling times will be independent of P whenever c = 0, as is indicated in the tables which follow.

When the cost of search is increased to .6, the sampling times in all the states increase. As one expects, given that the last state observed was 1, the sampling time increases from 2 to 3 to 10 as the current wage increases from 0 to 1 to 2. However, when c = .6, individuals with wages less than w(4) continue to sample for all information sets. Thus infinitely-lived individuals will attain a wage of 3 in the limit with probability 1.

This is not the case when c = 1.2. In this case, all individuals with a current wage of 2 will not search on-the-job, no matter what their information set. Individuals with an information set equal to 1 will search in 3 periods if their current wage is 0 and will search in 5 periods if their current wage is 1. Individuals with a current wage of 1 and an information set of 2 will resample more quickly than individuals with the same current wage but an information set of 1, loosely speaking because the transition rates into states 3 and 4 are "higher" from state 2 than from state 1. Because individuals with a current wage of 2 do not search when c = 1.2, infinitely-lived cohort members will in the limit occupy either the wage state 2 or the wage state 3, depending upon which they initially entered from the states 1 or 2.

It is also interesting to note the implication that certain states cannot be reached given optimizing behavior. When the cost of search is 1.2, individuals in state (3,3) never resample the process. But the only way an individual can ever be observed with a wage of 2 [2 = w(3) recall] is for her to switch from a state 1 or 2 wage to a state 3 wage, which implies all

TABLE 3.1

State Valuations and Optimal Sampling Times

Discount factor $[\beta] = .8$

Survival rate $[\lambda] = .9$

$$P = \begin{bmatrix} .700 & .200 & .075 & .025 \\ .200 & .500 & .200 & .100 \\ .100 & .200 & .500 & .200 \\ .025 & .075 & .200 & .700 \end{bmatrix} \Rightarrow \pi^* = \begin{bmatrix} .261 \\ .239 \\ .239 \\ .261 \end{bmatrix}$$

State $(\omega+1,\sigma)$	Optimal Sampling Time			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	1	2	3	∞
(2,1)	1	3	5	∞
(2,2)	1	2	3	∞
(3,1)	1	10	∞	∞
(3,2)	1	6	∞	∞
(3,3)	1	3	∞	∞
(4,4)	∞	∞	∞	∞

	State Valuation			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	2.115	1.044	0.591	0.000
(2,1)	3.567	2.842	2.641	2.571
(2,2)	4.118	3.159	2.769	2.571
(3,1)	5.469	5.145	5.143	5.143
(3,2)	5.671	5.149	5.143	5.143
(3,3)	5.877	5.192	5.143	5.143
(4,4)	7.714	7.714	7.714	7.714

individuals with a wage of 2 start in the state (3,3). Since there is no resampling from this state, the states (3,1) and (3,2) can never be reached.

The state valuations are given in the bottom panel. As was demonstrated analytically, the state valuations are nonincreasing in the costs of search.

The second example labor market is described in Table 3.2. The results are rather similar to those displayed in connection with the first labor market example. Looking at the sampling times, the most noteworthy feature is the fact that individuals in information state 2 have the shortest resampling times. This is primarily due to the fact that the probability of moving from state 2 to state 4 is .3, whereas the probabilty of moving from state 3 to 4, for example, in one period is 0.

The third example labor market which is described in Table 3.3 illustrates the manner in which the Burdett model is nested in ours. Note that all the rows of **P** are identical [and equal π^* of course], so that the state of demand in the market is i.i.d. As was shown in Proposition 2.4, this implies that there is in fact only one state variable, the current wage, since the history has no predictive content. Furthermore, for a given cost of search c individuals with a current wage of ω will either search each period or will never search again. We see that when the cost of search is .6, individuals with a current wage of 0 or 1 will continue to search each period. When the cost of search is increased to 1.2, only individuals with a wage of 0 continue to search. The state valuations reflect the fact that σ is not a state variable in the i.i.d. case.

The results for the final sample labor market are presented in Table 3.4. The main feature to note concerning **P** in this case is that the probability of a transition from state 3 to state 4 is quite high [.500]. Also notable is the fact that the <u>only</u> way to get to state 4 is through state 3. These characteristics of **P** translate into a high sampling rate when the individual has last observed the process in state 3; in this case the agent will resample the process immediately both when c = .6 and when c = 1.2. On the other hand, when c = .6 agents with a wage of 1 who last observed the process in state 1 wait a full 12 periods before resampling, while individuals with the same wage but who last observed the process in state 2 only wait 2 periods. Perhaps the most interesting results are associated with the case of c = 1.2. In this case, all agents with a wage of 1 never resample, while those of a wage of 0 resample after 8 periods. Of the agents with a wage of 2 [2 = w(3) recall],

TABLE 3.2

State Valuations and Optimal Sampling Times

Discount factor [β] =.8

Survival rate [λ] = .9

$$P = \begin{bmatrix} .800 & .150 & .050 & .000 \\ .100 & .600 & .000 & .300 \\ .050 & .200 & .750 & .000 \\ .100 & .200 & .000 & .700 \end{bmatrix} \Rightarrow \pi^* = \begin{bmatrix} .323 \\ .306 \\ .065 \\ .306 \end{bmatrix}$$

State $(\omega+1,\sigma)$	Optimal Sampling Time			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	1	2	4	∞
(2,1)	1	4	6	∞
(2,2)	1	1	2	∞
(3,1)	1	8	∞	∞
(3,2)	1	2	∞	∞
(3,3)	1	6	∞	∞
(4,4)	∞	∞	∞	∞

	State Valuation			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	1.644	0.772	0.392	0.000
(2,1)	3.342	2.797	2.616	2.571
(2,2)	4.625	3.795	3.124	2.571
(3,1)	5.430	5.153	5.143	5.143
(3,2)	6.157	5.414	5.143	5.143
(3,3)	5.483	5.161	5.143	5.143
(4,4)	7.714	7.714	7.714	7.714

TABLE 3.3

State Valuations and Optimal Sampling Times

Discount factor $[\beta] = .8$

Survival rate $[\lambda] = .9$

$$P = \begin{bmatrix} .300 & .400 & .200 & .100 \\ .300 & .400 & .200 & .100 \\ .300 & .400 & .200 & .100 \\ .300 & .400 & .200 & .100 \end{bmatrix} \Rightarrow \pi^* = \begin{bmatrix} .300 \\ .400 \\ .200 \\ .100 \end{bmatrix}$$

State $(\omega+1,\sigma)$	Optimal Sampling Time			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	1	1	1	∞
(2,1)	1	1	∞	∞
(2,2)	1	1	∞	∞
(3,1)	1	∞	∞	∞
(3,2)	1	∞	∞	∞
(3,3)	1	∞	∞	∞
(4,4)	∞	∞	∞	∞

	State Valuation			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	3.299	2.275	1.496	0.000
(2,1)	4.217	3.194	2.571	2.571
(2,2)	4.217	3.194	2.571	2.571
(3,1)	5.669	5.143	5.143	5.143
(3,2)	5.669	5.143	5.143	5.143
(3,3)	5.669	5.193	5.143	5.143
(4,4)	7.714	7.714	7.714	7.714

TABLE 3.4

State Valuations and Optimal Sampling Times

Discount factor $[\beta] = .8$

Survival rate $[\lambda] = .9$

$$P = \begin{bmatrix} .900 & .100 & .000 & .000 \\ .200 & .600 & .200 & .000 \\ .000 & .100 & .400 & .500 \\ .300 & .400 & .200 & .100 \end{bmatrix} \Rightarrow \pi^* = \begin{bmatrix} .621 \\ .231 \\ .095 \\ .053 \end{bmatrix}$$

State $(\omega+1,\sigma)$	Optimal Sampling Time			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	1	4	8	∞
(2,1)	1	12	∞	∞
(2,2)	1	2	∞	∞
(3,1)	1	∞	∞	∞
(3,2)	1	∞	∞	∞
(3,3)	1	1	1	∞
(4,4)	∞	∞	∞	∞

	State Valuation			
	$c = 0$	$c = .6$	$c = 1.2$	$c = \infty$
(1,1)	0.740	0.141	0.009	0.000
(2,1)	2.785	2.572	2.571	2.571
(2,2)	3.616	2.943	2.571	2.571
(3,1)	5.216	5.143	5.143	5.143
(3,2)	5.500	5.143	5.143	5.143
(3,3)	6.479	5.836	5.230	5.143
(4,4)	7.714	7.714	7.714	7.714

only those who last observed the process in state 3 will resample, and they will do so after 1 period. Thus an individual who takes a new job in state 3 will immediately resample after 1 period. She has a .5 probability of getting the high wage of 3, while she has a .4 probability of again finding the process in state 3 [in which case she will immediately resample]. She has a .1 probability of finding the process in state 2, in which case she will <u>never</u> resample. Finally note that since the probability of going from state 3 to state 1 is 0 the state (3,1) will never be reached by any agent.

We turn to the description of the simulation exercises. The process of generating the sample paths was described in Section 3.1. The portion of the four sample paths covering the periods 3000-3300 are presented in Figures 3.0.A through 3.0.D. The sample path for the first example market exhibits a relatively substantial amount of persistence in the low and high states. Sample path 2 exhibits some persistence in all states and one can notice that all entries into state 4 are from state 2. The sample path for the third labor market shows the least persistence, which is due to fact that it is generated by i.i.d. draws. The fourth sample path exhibits a large degree of persistence in state 1, some in state 2, and little in states 3 and 4. Note that all entries into state 4 come from state 3.

The remaining figures present the sample paths of aggregate wages and the within period standard deviation of wages for the four costs of search and for the four sample labor markets. The behavior of the sample paths are somewhat similar across the four labor markets so we will comment only on those generated in the first labor market [Figures 3.1.A through 3.1.H].

In Figure 3.1.A the average wage is plotted for the case of c = 0. In this case sampling occurs every period for all individuals with a wage less than 3. Comparing the mean wage process with the sample path for demand, it is not suprising to see that shifts upward in demand produce immediate shifts up in the aggregate wage [one for one], while shifts down are mediated by the fact that workers not new to the labor market retain their old higher wage jobs.

This tendency is exhibited to some extent for positive c, but the general tendency to follow demand increases is mediated by the fact that a number of workers will not observe higher wages because the wage increase is not in their information sets. A comparison of 3.0.A and 3.1.C [the case of c = .6] shows that brief periods in the high wage state tend to have little effect on average wages because many labor market participants will not happen to sample the process in the period the wage is high. Only when the state of demand is high for a number of periods does the average wage respond strongly.

When the cost of search is increased to 1.2 [Figure 3.1.E] the response of the average wage to shifts to the high demand state becomes even more attenuated. As we saw in Table 3.1, when c = 1.2 only individuals with a wage less than 2 searched at all, and these individuals searched only after 3 or 5 periods. Though the paths in 3.0.C and 3.0.E have similar shapes, it is apparent that path corresponding to the higher cost case exhibits less variability and has a lower mean than the one generated when the cost of search is .6.

Figure 3.1.G plots a benchmark case of $c = \infty$, when there is no resampling of the process after entry. Movements in the average wage reflect solely changes in demand conditions confronting successive cohorts of labor market entrants.

In Table 3.5 we summarize patterns exhibited by the sample paths for all the labor market examples. The mean aggregate wage is a decreasing function of the cost of search in all cases, as is to be expected. The differences in the means over the periods do not reflect welfare differences in general since the cost of sampling is not accounted for.[9] The standard deviation in the aggregate wage is not a monotone function of the sampling cost. The table also includes autocorrelations in the aggregate wage series. It is clear that changes in the pattern of temporal dependence as described by the auto-correlation process is sensitive to the sampling cost parameter in several of the labor market examples.

4. Conclusion

In this paper we have added an additional state variable [besides the current wage] to the on-the-job search framework which can account for the following observations: (1) the probability of turnover may not be a decreasing function of the current wage; (2) the probability of turnover will be

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[9] Only for the case in which the cost of sampling is 0 do mean differences represent a measure of average welfare of individuals from randomly-selected cohorts.

TABLE 3.5

Descriptive Statistics for Aggregate Wage Series in Simulated Labor Markets

(Periods 3000 through 3300)

Cost	*Mean*	*SD*	*Autocorrelation (Lag)* 1	2	3	4	5
			Labor Market 1				
0.0	2.354	0.638	0.853	0.694	0.554	0.422	0.299
0.6	2.156	0.625	0.921	0.779	0.634	0.495	0.366
1.2	1.977	0.583	0.936	0.802	0.642	0.489	0.345
∞	1.410	0.500	0.972	0.908	0.821	0.718	0.606
			Labor Market 2				
0.0	2.277	0.694	0.890	0.780	0.680	0.589	0.519
0.6	2.237	0.708	0.902	0.789	0.685	0.579	0.515
1.2	2.121	0.709	0.940	0.832	0.726	0.624	0.546
∞	1.381	0.521	0.977	0.931	0.874	0.814	0.755
			Labor Market 3				
0.0	2.234	0.474	0.766	0.581	0.464	0.404	0.320
0.6	1.988	0.359	0.721	0.516	0.392	0.338	0.290
1.2	1.452	0.241	0.780	0.639	0.559	0.506	0.427
∞	1.057	0.248	0.922	0.828	0.749	0.692	0.638
			Labor Market 4				
0.0	1.624	0.812	0.928	0.818	0.699	0.584	0.482
0.6	1.463	0.806	0.938	0.834	0.720	0.614	0.528
1.2	0.981	0.574	0.963	0.897	0.816	0.729	0.648
∞	0.677	0.432	0.979	0.935	0.878	0.817	0.755

different for individuals with the same wage; and (3) the probability of turnover exhibits state dependence even when conditioning on the current wage ω <u>and</u> the information set σ.

The modelling strategy relies heavily on the assumption that all information concerning the state of the labor market is private, which is what distinguishes the model from those of Lippman and McCall (1976) and Lippman and Mamer (1989). It would be interesting to construct a hybrid example which had search occurring in an environment with both private and public information. We would also be interested in relaxing the assumption that wage contracts are guaranteed indefinitely. For example, Lippman and Mamer (1989) consider the case in which the probability of a wage contract being terminated is a function of the state of demand.

We are most interested in potentially applying the model developed here to account for job turnover in micro-data. The model gives a "structural" interpretation to heterogeneity in the turnover process which should be capable of generating empirically falsifiable predictions. The present paper should be seen as a first step in the process of estimating an on-the-job search model with endogenous information sets.

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FIGURE 3.0.A
Sample Path 1

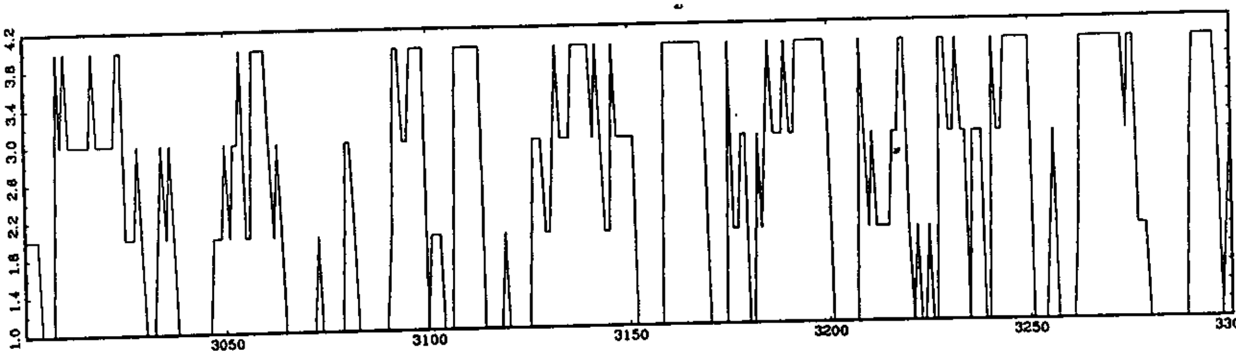

FIGURE 3.0.B
Sample Path 2

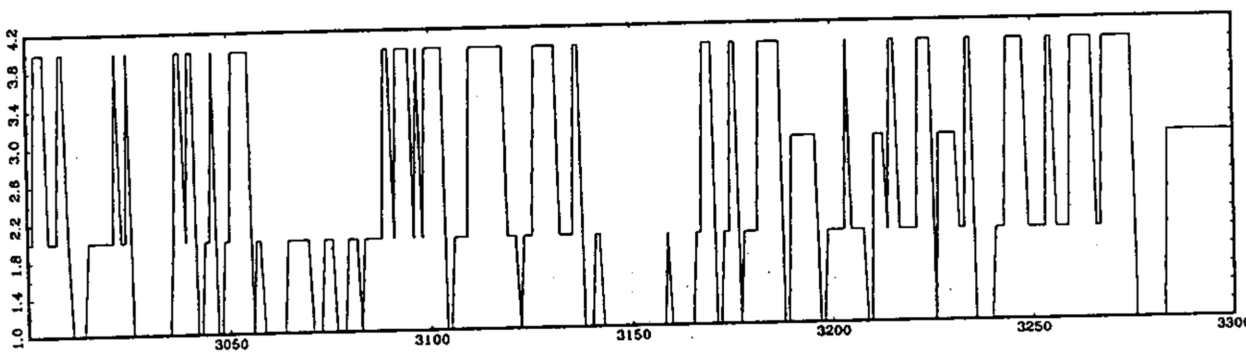

FIGURE 3.0.C
Sample Path 3

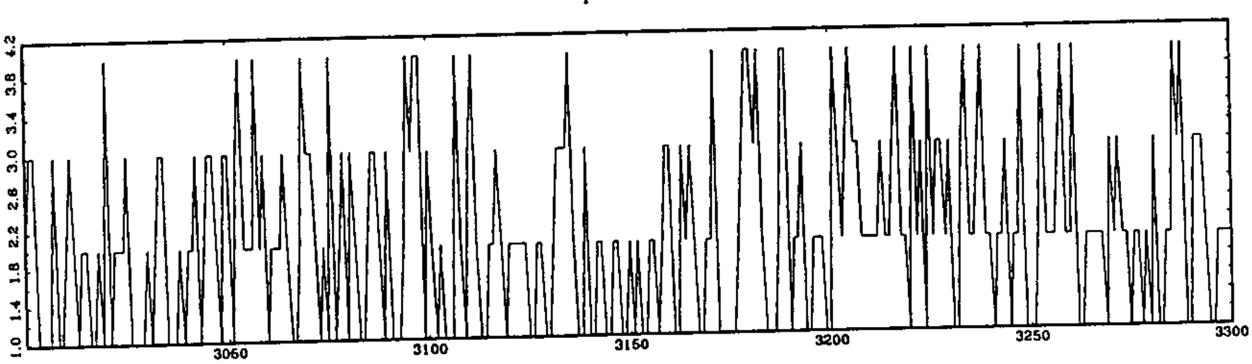

FIGURE 3.0.D
Sample Path 4

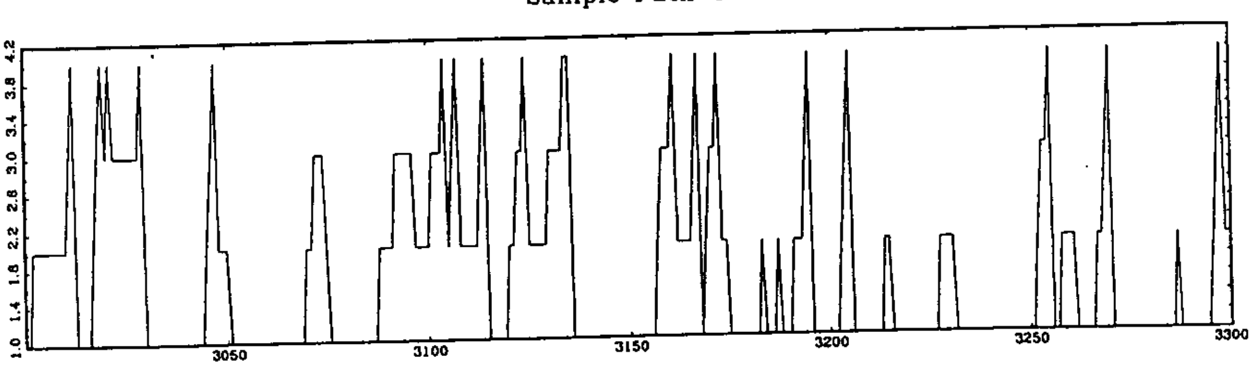

FIGURE 3.1.A
Mean Wage: C = 0

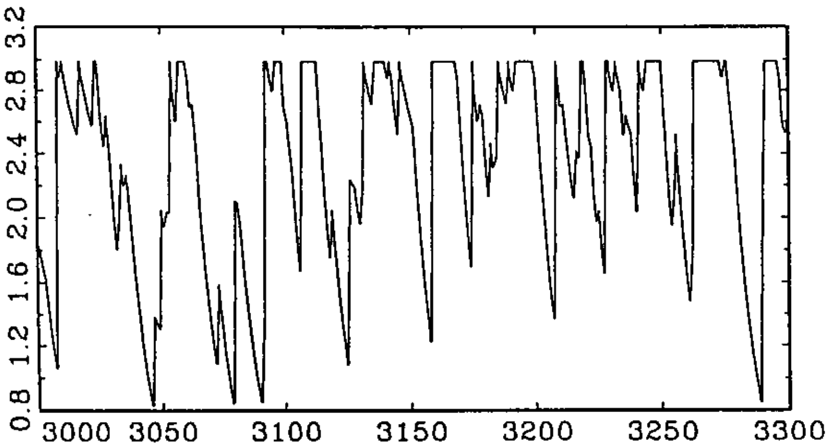

FIGURE 3.1.B
SD Wage: C = 0

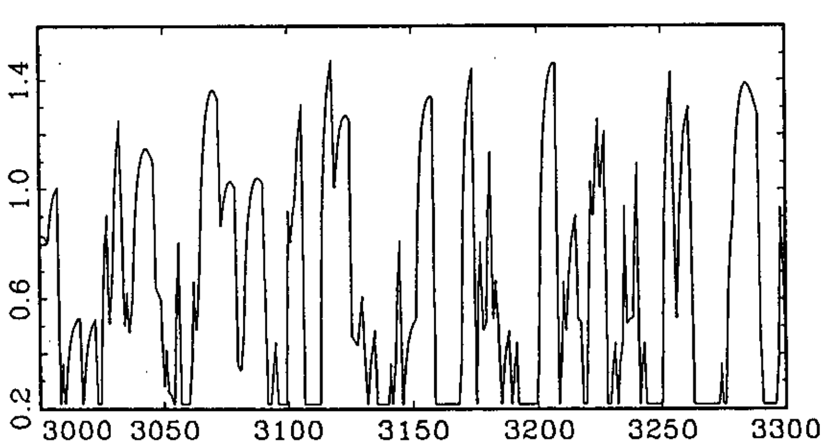

FIGURE 3.1.C
Mean Wage: C = .6

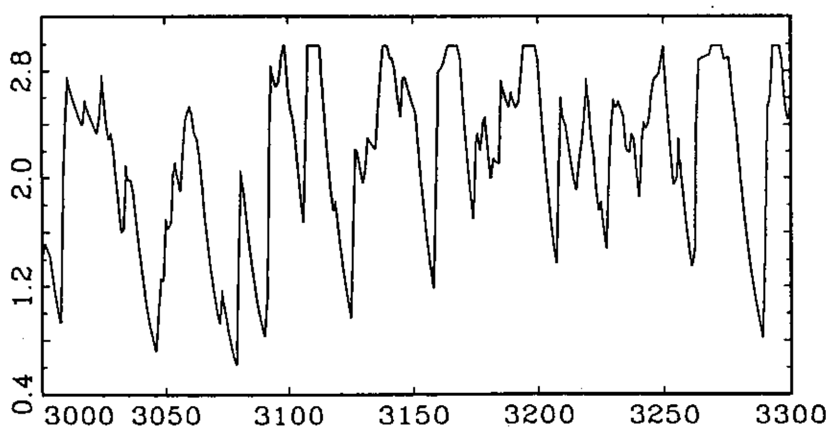

FIGURE 3.1.D
SD Wage: C = .6

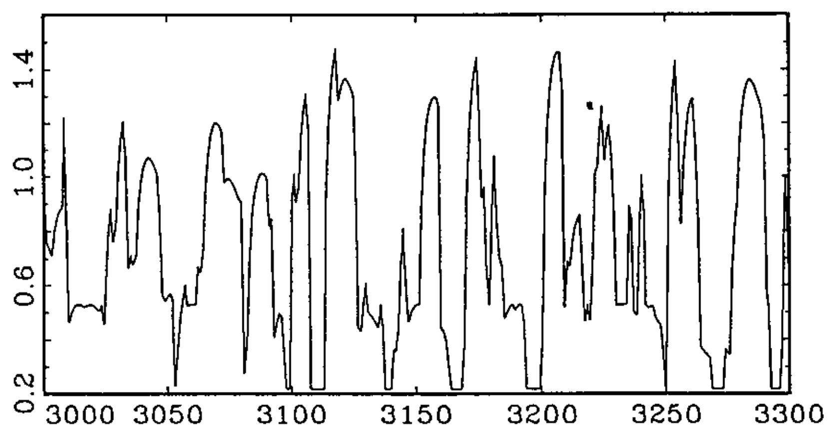

FIGURE 3.1.E
Mean Wage: C = 1.2

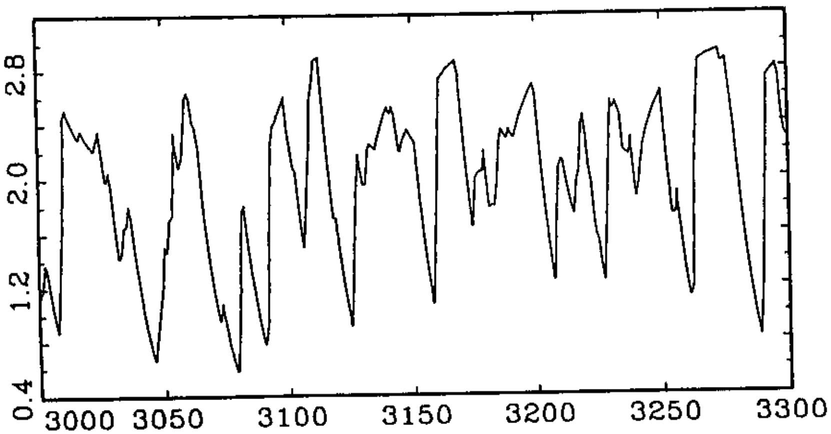

FIGURE 3.1.F
SD Wage: C = 1.2

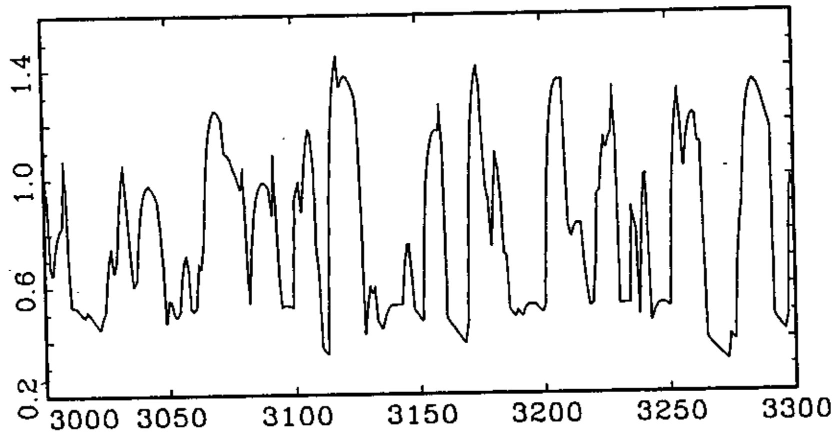

FIGURE 3.1.G
Mean Wage: C = ∞

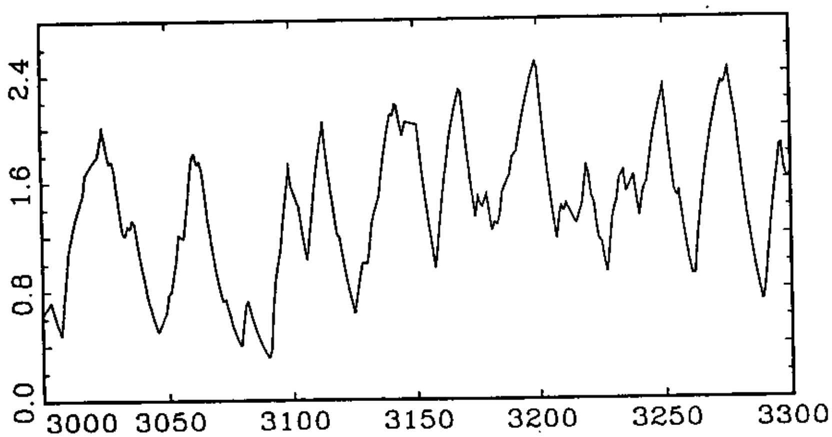

FIGURE 3.1.H
SD Wage: C = ∞

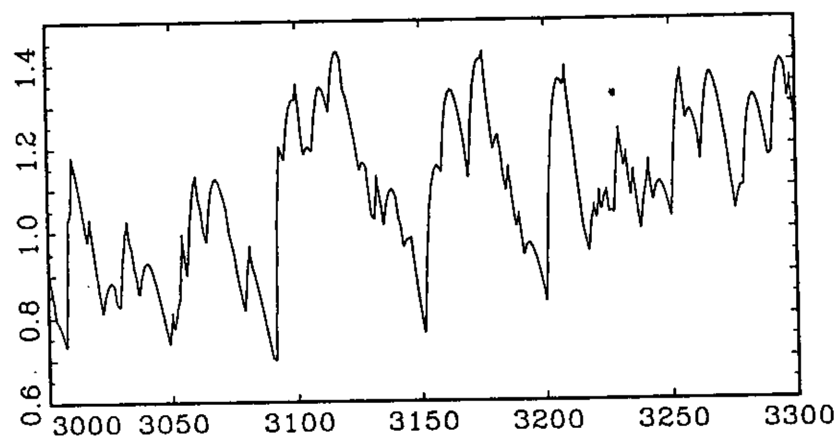

FIGURE 3.2.A
Mean Wage: C = 0

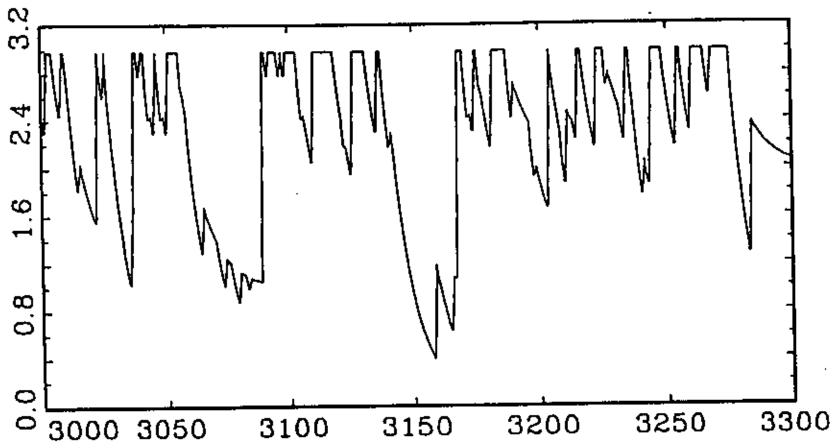

FIGURE 3.2.B
SD Wage: C = 0

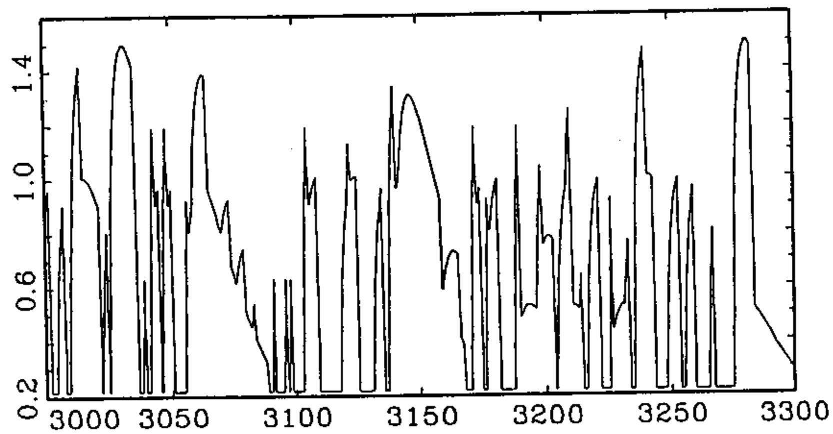

FIGURE 3.2.C
Mean Wage: C = .6

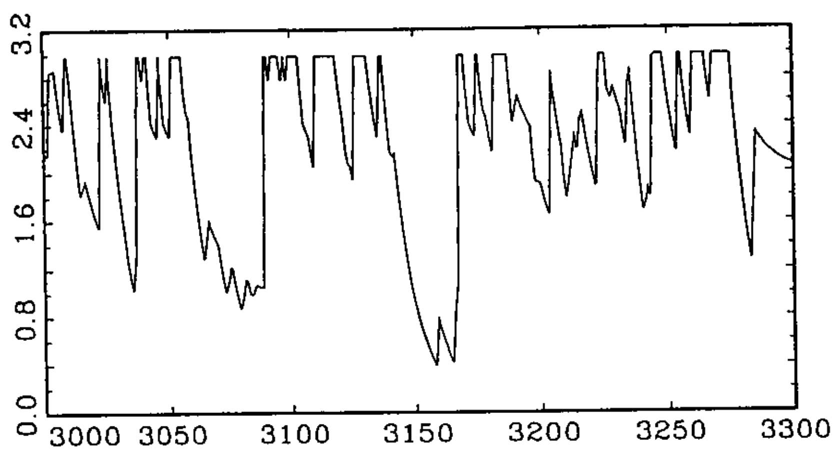

FIGURE 3.2.D
SD Wage: C = .6

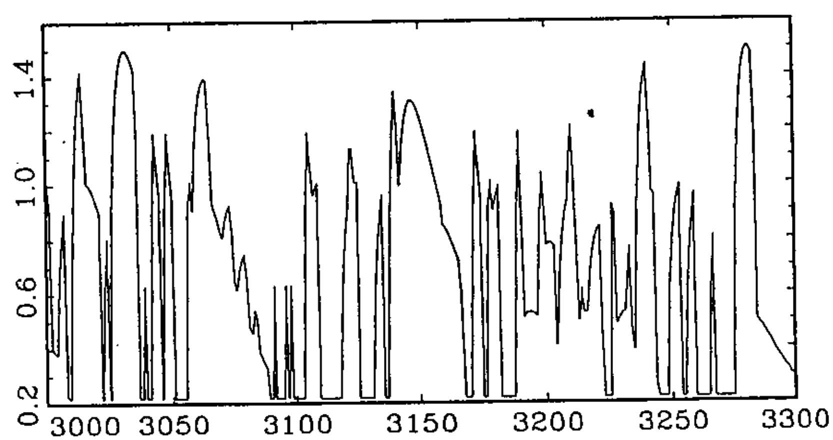

FIGURE 3.2.E
Mean Wage: C = 1.2

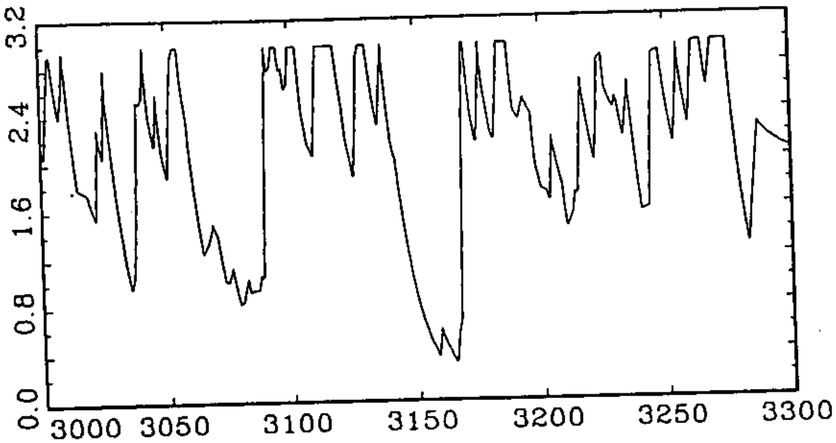

FIGURE 3.2.F
SD Wage: C = 1.2

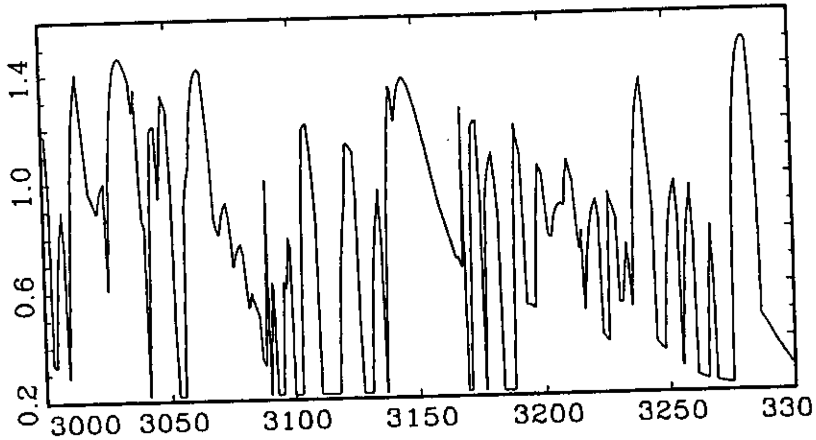

FIGURE 3.2.G
Mean Wage: C = ∞

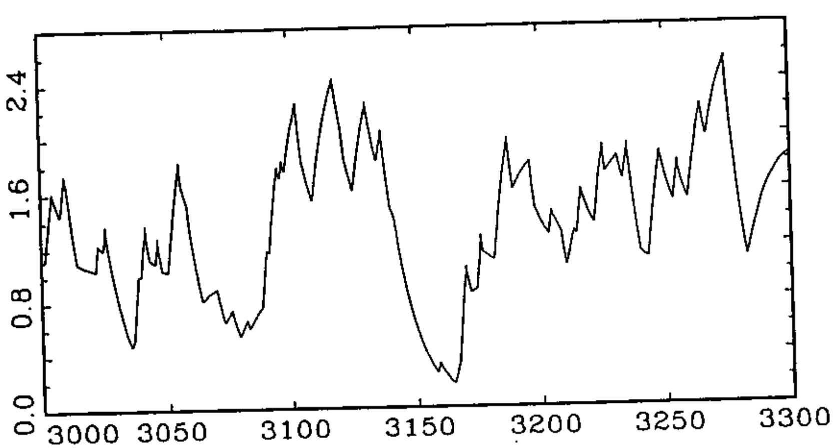

FIGURE 3.2.H
SD Wage: C = ∞

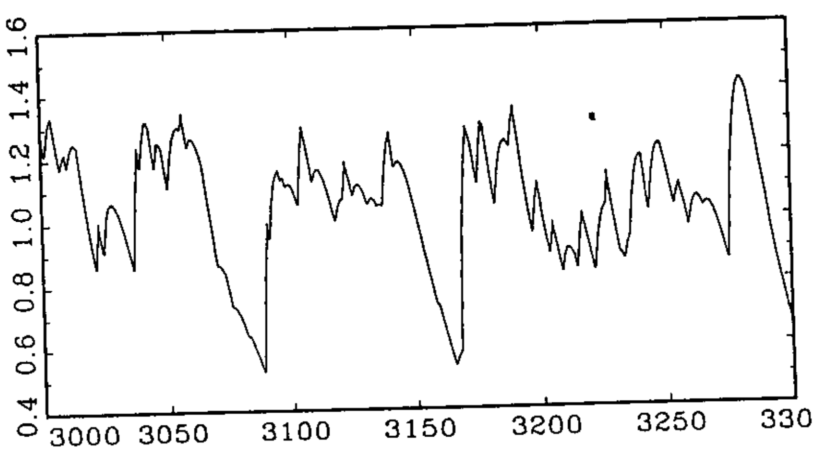

FIGURE 3.3.A
Mean Wage: C = 0

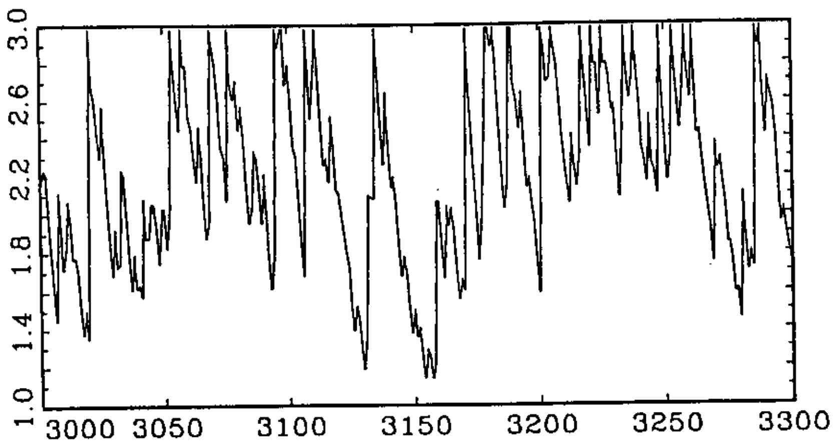

FIGURE 3.3.B
SD Wage: C = 0

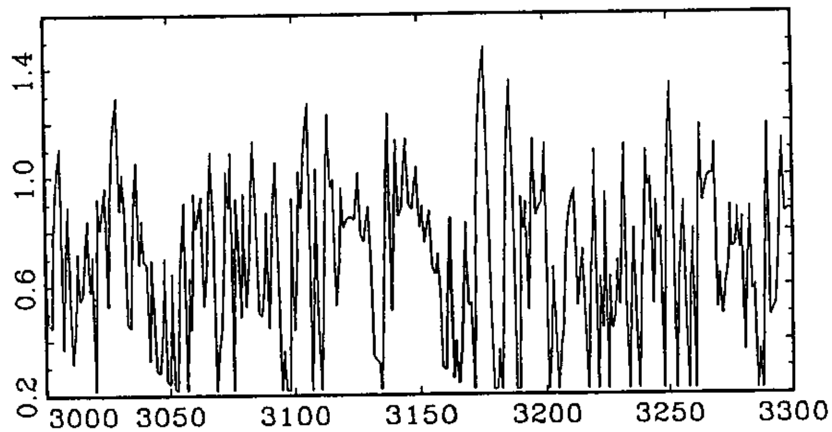

FIGURE 3.3.C
Mean Wage: C = .6

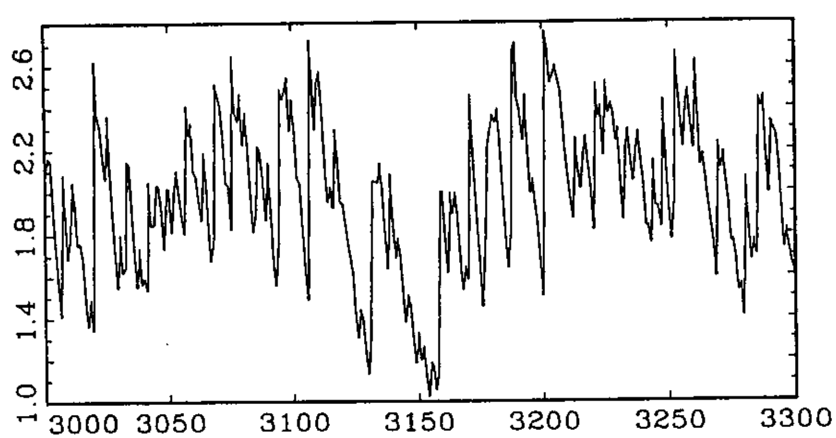

FIGURE 3.3.D
SD Wage: C = .6

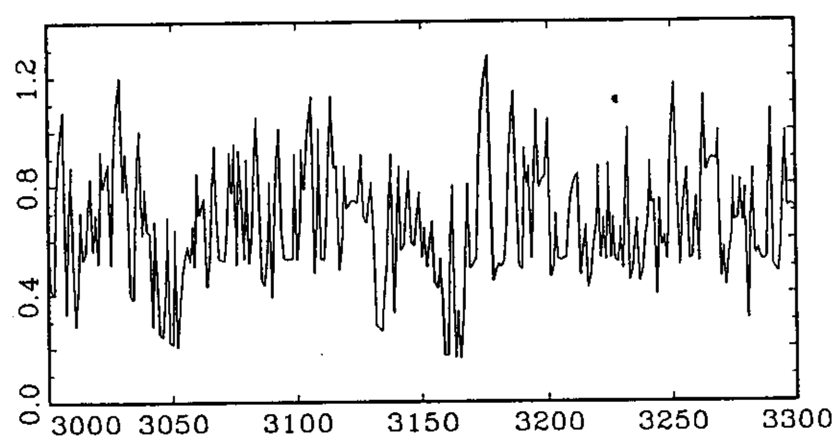

FIGURE 3.3.E
Mean Wage: C = 1.2

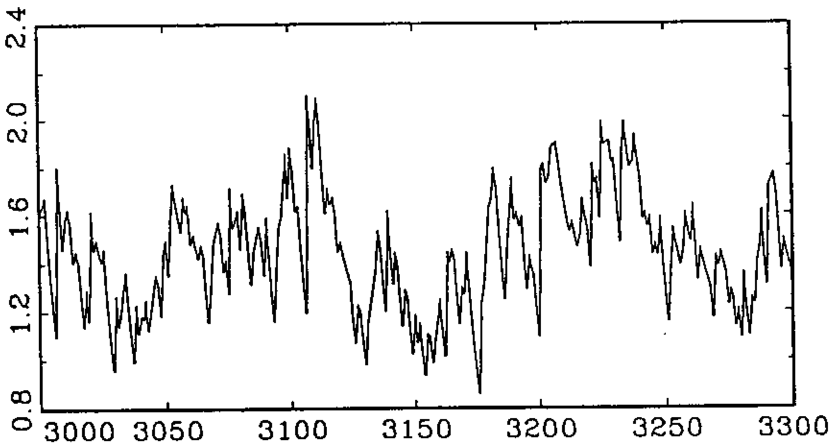

FIGURE 3.3.F
SD Wage: C = 1.2

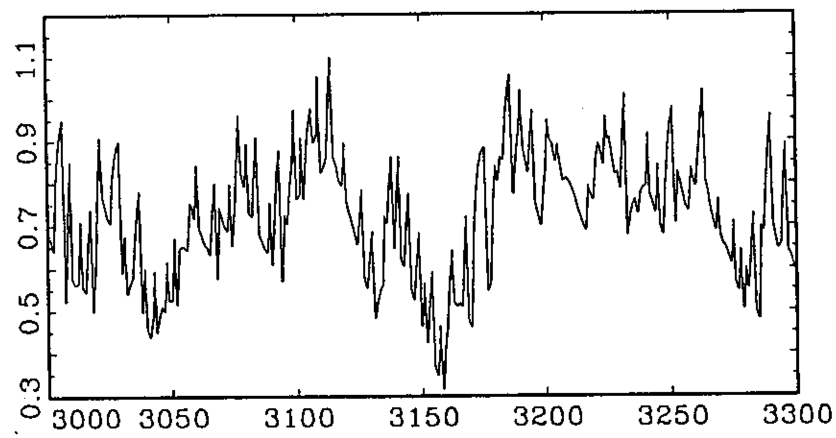

FIGURE 3.3.G
Mean Wage: C = ∞

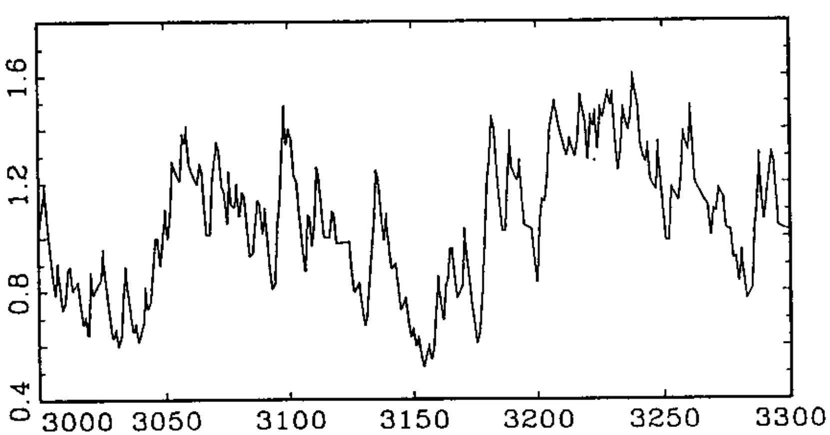

FIGURE 3.3.H
SD Wage: C = ∞

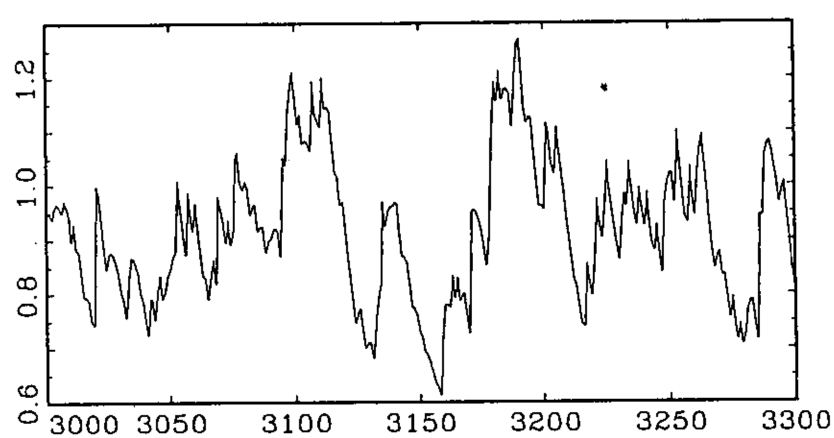

FIGURE 3.4.A
Mean Wage: C = 0

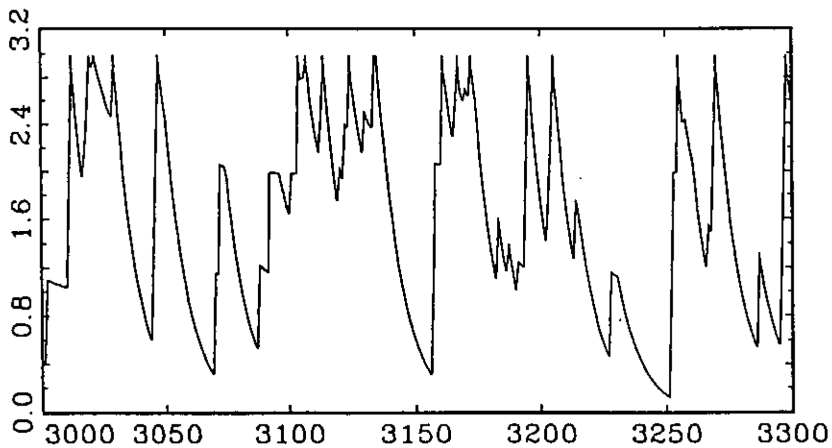

FIGURE 3.4.B
SD Wage: C = 0

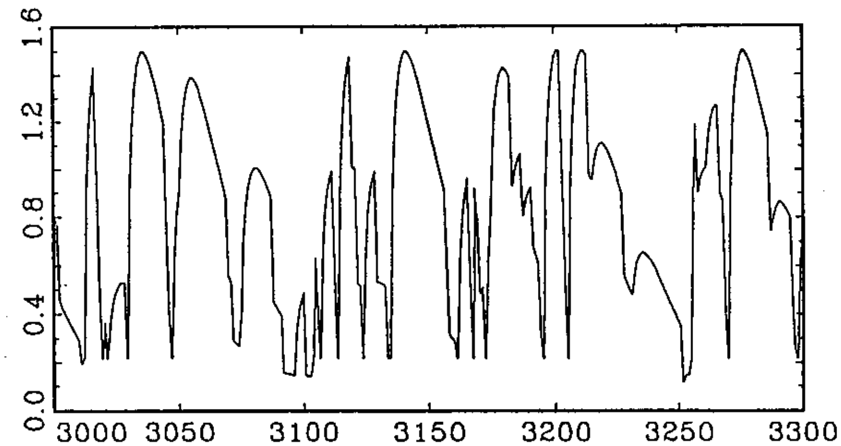

FIGURE 3.4.C
Mean Wage: C = .6

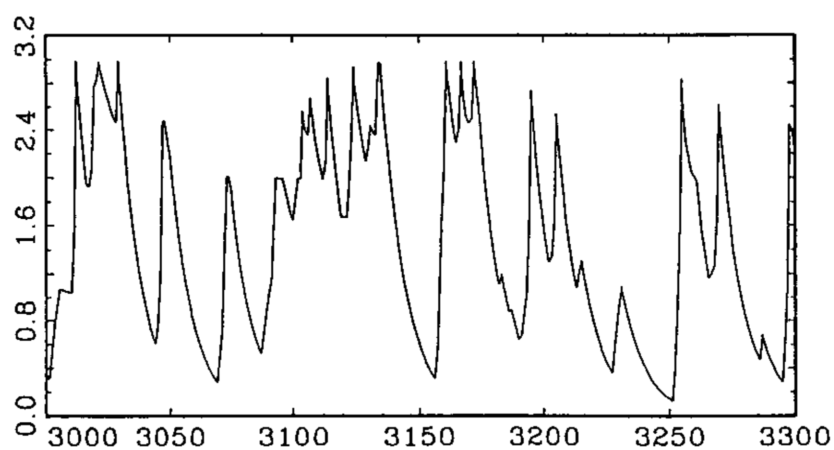

FIGURE 3.4.D
SD Wage: C = .6

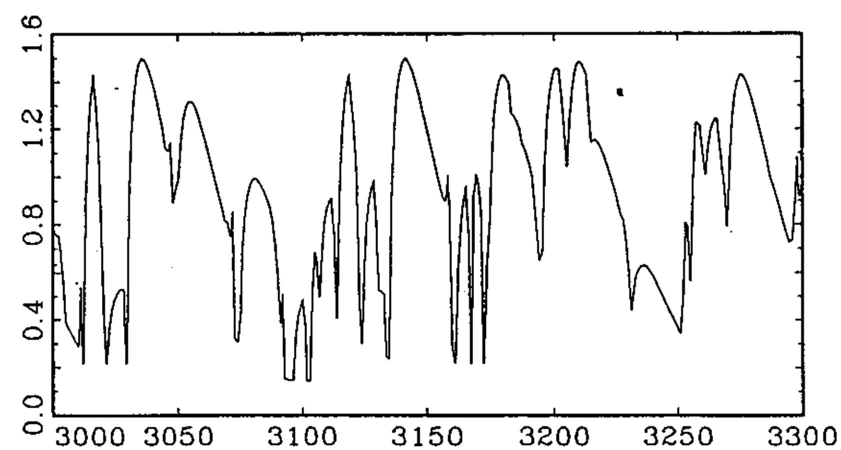

FIGURE 3.4.E
Mean Wage: C = 1.2

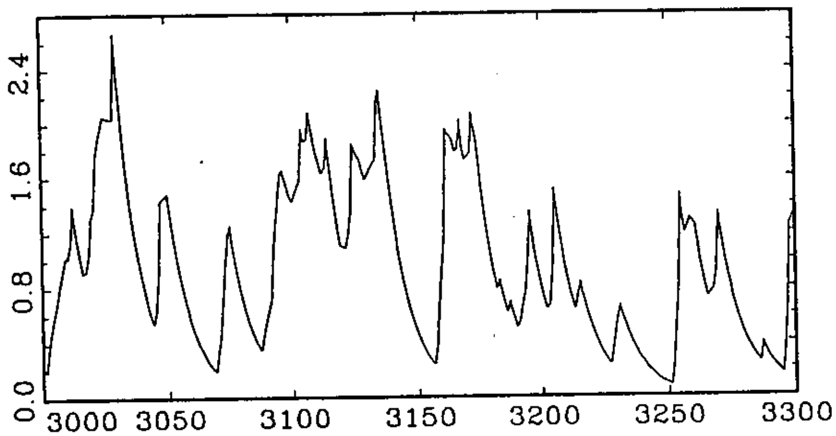

FIGURE 3.4.F
SD Wage: C = 1.2

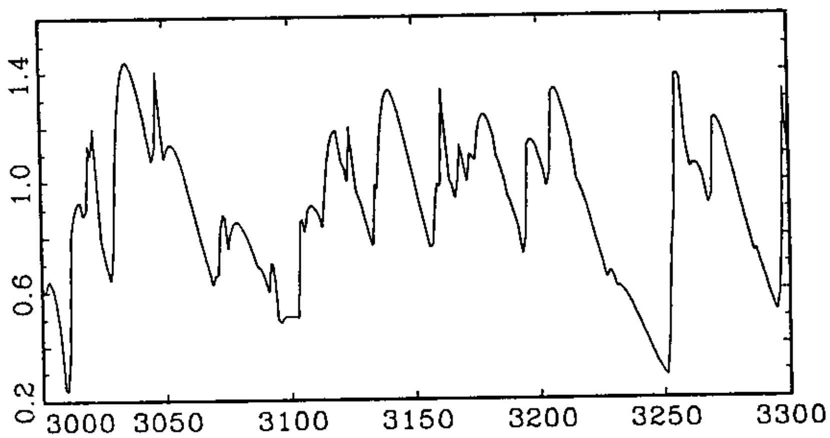

FIGURE 3.4.G
Mean Wage: C = ∞

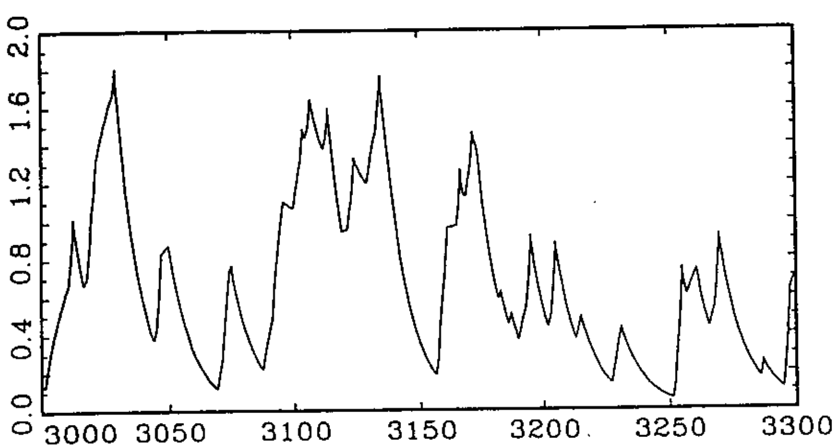

FIGURE 3.4.H
SD Wage: C = ∞