ECONOMIC RESEARCH REPORTS

FOLLOW THE LEADER: ON GROWTH AND DIFFUSION

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RR # 93-03 *February, 1993*

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January 1993

VERY PRELIMINARY

Acknowledgements: We would like to thank Boyan Jovanovic for helpful and incisive comments. We also would like to thank the C.V. Starr Center for Applied Economics at New York University for technical assistance.

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Abstract

Do countries whose firms and industries have a technological lead over others have an "advantage"? Or, in a competitive environment, is it better to be a laggard that takes advantage of technology diffusion while the leaders are expending resources pushing at the frontiers of knowledge? We will present an equilibrium model of growth and diffusion, with competitive firms and free trade, which suggests that consumers in the follower country can indeed be strictly better off when the diffusion rate of technology is above some threshold.

JEL Classification numbers: O10, C73
Key Words: technology diffusion, first-comer, late-comer.

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1. Introduction.

Do countries whose firms and industries have a technological lead over others have an "advantage"? Or, in a competitive environment, is it better to be a laggard that takes advantage of technology diffusion while the leaders are expending resources pushing at the frontiers of knowledge? We will present an equilibrium model of growth and diffusion, with competitive firms and free trade, which suggests that consumers in the follower country can indeed be strictly better off when the diffusion rate of technology is above some threshold.

We should point out that our argument is quite distinct from that of "leap-frogging", based on the idea that there is a penalty for being a leader. In such models the leader may lock-in early to a fixed technology only to find it costly and difficult to incorporate new innovations, while a late-comer has the advantage of a fresh start. Alternatively, the late-comer may be able to avoid the early mistakes of the leader and eventually overtake it.[1] By contrast, in our model the follower is better off if he chooses to stay behind and to live off the diffusion of technology, with more of its resources allocated to producing goods than to expanding the frontiers of knowledge.

We model technological diffusion as a gradual process. Even though blueprints for the general outlines of industrial and manufacturing processes are readily available, the importation and implementation of new technologies presents many challenges and obstacles: Local engineering and management know-how must be developed at all levels of the operation. Production and management techniques must be tailored and adapted to specific local conditions and supplies. A network of local suppliers must be trained and cultivated to deliver

[1] For a critical review of leap-frogging see Ames and Rosenberg [1963]. For a modern treatment see Brezis, Krugman and Tsiddon [1991].

products and services that meet specification and quality standards. All this takes time and experience. It is not surprising then to find that technology diffusion often starts with simple assembly plants and the production of simple parts: components that require advanced technological expertise are, in the beginning, imported from countries that are the technology leaders. Domestic content in production rises gradually, irrespective of whether the initiative for the operation comes from local entrepreneurs that copy foreign technology, from firms in the technologically advanced countries by way of investment or licensing arrangements, or from some combination of both.[2]

In the next section we present a very simple model, that nevertheless captures all the essence of our argument, in which labor can be allocated either to produce goods or to increase productivity. Furthermore, the growth of total factor productivity (TFP) is driven by labor and, in the follower countries, by technology diffusion. In section 3 we generalize this model to allow for capital and human capital inputs to produce goods. Human capital growth depends on its existing levels as well as on the quantity of labor allocated to its production. The growth of TFP therefore depends on human capital through domestic innovations, and in the follower country also on technological diffusion. Comparisons of welfare, prices and wages under free trade and autarky are studied in sections 2.2-2.3. Section 3.2 discusses some relevant empirical literature and we conclude in section 4 with some consideration of strategic elements.

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[2] The product-cycle literature (see Vernon [1966]) has also stressed the role of gradual diffusion: Technologies for new products diffuse to technologically less advanced countries only as they become increasingly standardized. See also Grossman and Helpman [1992].

2. A Decentralized Model.

2.1 Firms, Consumers and Markets.

In the simple version of our model without physical and human capital, firms hire labor to produce goods, as well as to improve their technology for producing goods. Inside a country, we allow for productivity spillovers from one firm to another in the production of goods. The production function for goods is given by:

$$q = A^{\epsilon}a^{\mu}f(n) \quad , \qquad \epsilon \geq 0, \quad \mu \in (0,1) \tag{2.1}$$

where q is output, n is labor employed in the production of goods, f is a standard neoclassical production function, a is the productivity parameter reflecting "knowledge" that is specific to the firm, and A represents aggregate knowledge in the economy. The parameter ϵ measures the strength of the spillover effects.

The growth of "knowledge" depends on the amount of labor allocated to its production, represented by u, and also on the rate of technology diffusion. We assume that new technology, once widely in use, diffuses to firms that are technologically lagging; the rate of diffusion is proportional to the distance between the technological knowledge of the firm and the aggregate level of technological knowledge in the advanced country. Thus the growth of productivity for any firm, independent of its location, is given by

$$\dot{a} = A^{1-\eta}a^{\eta}u^{\gamma} + m(A^{*}(t) - a) \qquad 0 \leq \eta \leq 1 \tag{2.2}$$

where $A^*(t)$ is the aggregate level of knowledge in the advanced country at time t, and m is the diffusion parameter. The component $A^{(1-\eta)}$ is external to the firm and represents spillovers from aggregate knowledge to the production of firm-specific knowledge. We require $\eta + \gamma \leq 1$ to assure that the firm's optimization problem is concave. Note that it is possible to set $\eta = 0$, $\gamma = 1$.

For simplicity we assume that firms within a country start with the same level of technological know-how. Therefore in a symmetric equilibrium they make identical decisions and have identical levels of know-how given by $a(t)$. That is, if firms within each country are indexed by s and distributed over the unit interval, then

$$A(t) = \int_0^1 a_s(t)ds = a(t)\int_0^1 ds = a(t) . \tag{2.3}$$

It follows that in the technologically advanced country, the growth of aggregate technological knowledge, given by $A^*(t)$ simply reduces to

$$\dot{A}^* = A^* u^\gamma \tag{2.4}$$

because $a^* = A^*$. In contrast, for a follower country whose A lags behind A^*, the growth of aggregate productivity is

$$\dot{A} = Au^{\gamma} + m(A^{*}(t) - A) , \tag{2.5}$$

where (2.5) follows from integrating (2.3) with respect to s.

The typical firm maximizes discounted profits:

$$\text{Max} \int_{0}^{\infty} (pA^{\epsilon}a^{\mu}f(n) - wn - wu)e^{-Rt}dt \tag{2.6}$$

subject to:

$$\begin{aligned} \dot{a} &= A^{1-\eta}a^{\eta}u^{\gamma} \\ a(0) &= a \end{aligned} \tag{2.7}$$

where p is the price of the good, R is the discount rate and w is the market wage. For the firm's problem to be well-behaved, we also assume the joint concavity of the function $a^{\mu}f(n)$ in (a, n). First order conditions for the firm are given by:

$$pA^{\epsilon}a^{\mu}f'(n) = w = \lambda\gamma A^{(1-\eta)}a^{\eta}u^{\gamma-1} \tag{2.8}$$

$$\dot{\lambda} = (R + M - \eta A^{(1-\eta)}a^{\eta-1}u^{\gamma})\lambda - \mu pA^{\epsilon}a^{\mu-1}f(n) \tag{2.9}$$

where λ is the usual multiplier associated with the constraint in (2.7).

Labor supply is assumed to be inelastic, so market clearing requires

$$n + u = 1 . \tag{2.10}$$

The consumer maximizes discounted utility subject to a budget constraint:

$$\text{Max} \int_0^\infty U(c(t))e^{-Rt}dt$$

$$\text{subject to} \tag{2.11}$$

$$\int_0^\infty (pc(t) - w(t) - \pi(t))e^{-Rt}dt = 0$$

where U is a concave utility function and $\pi(t)$ is the profit of the firms at time t, received by the consumer. The first order condition for the consumer requires

$$U'(c(t)) = \sigma p(t) \tag{2.12}$$

where σ is a time-invariant multiplier for the intertemporal budget constraint. Market clearing implies that

$$A(t)^{\epsilon}a(t)^{\mu}f(n(t)) = A(t)^{\epsilon+\mu}f(n(t)) = p(t)c(t) = w(t) + \pi(t) , \tag{2.13}$$

for all t, where the first equality follows from (2.3). Without loss of generality, we may set $\sigma = 1$ since, given (2.13), the budget constraint is always satisfied in a market-clearing equilibrium.

Integrating (2.6)-(2.9) and using the first order and market clearing conditions to simplify, we obtain the following system:

$$\begin{aligned}\dot{A} &= (1-n)^{\gamma}A + m(A^{*}(t) - A) \\ \dot{\lambda} &= (R - \eta(1-n)^{\gamma} + m)\lambda - \mu A^{\epsilon+\mu-1}U'f(n) \\ \lambda\gamma A(1-n)^{\gamma-1} &= U'A^{\epsilon}a^{\mu}f'(n)\ .\end{aligned} \tag{2.14}$$

To simplify the analysis we assume that production is Cobb-Douglas and utility is logarithmic:

$$\begin{aligned}f(n) &= n^{\beta} \qquad 0 < \beta < 1 \\ U(c) &= \ln(c)\ ,\end{aligned} \tag{2.15}$$

where $\ln(c)$ is the natural logarithm of c.

Under these assumptions, the dynamics of (2.14) simplifies to:

$$\begin{aligned}\dot{A} &= (1-n)^{\gamma}A + m(A^{*}(t) - A) \\ \dot{\lambda} &= (R - \eta(1-n)^{\gamma} + m)\lambda + \mu A^{-1} \\ n(1-n)^{\gamma-1} &= \beta(\gamma\lambda A)^{-1}\ .\end{aligned} \tag{2.16}$$

Equations (2.16) describe the dynamics of the economy in both the leader and follower countries. To equations (2.16) we must also add the transversality condition

$$\lim_{t\to\infty} e^{-Rt}\lambda(t)A(t) = 0 . \tag{2.17}$$

This condition follows from the fact that a similar condition, with A replaced by a, must hold for each firm as a necessary and sufficient condition in the problem defined in equation (2.6) above.

In the leading country, firms take $A^*(t)$ as given but, provided they all start at the same level of technology so that $a_s(0) = \bar{a}$, their identical equilibrium choices of u and n make $a_s(t) = A^*(t)$ for all t and s. Therefore the second term on the right-hand side of the first expression in (2.17) disappears. In the follower country where $a_s(0) = a(0) < A^*(0)$ for all s, firms will also have identical technology levels for the same reasons as the leading country, so that $a_s(t) = A(t)$, but $A(t)$ will remain below $A^*(t)$, as will be shown in the next section.

Notice that because of the externalities of aggregate knowledge A, the equilibrium described by the equations (2.10)-(2.13) might not immediately correspond to the solution of an optimization problem. However in the case we are considering, because of the special form of the technology and preferences, the reduction is possible. Indeed it is easy to check that the equilibrium described above corresponds to the solution of

$$\max \int_0^\infty e^{-Rt} B \ln(A^{\epsilon+\mu} n^{\beta'}) dt$$

$$\text{subject to} \tag{2.18}$$

$$\dot{a} = (1 - n)^{\gamma} A(t)^{1-\eta} a^{\eta} + m(A^* - a)$$

$$a(0) = A(0) = \bar{a}$$

where $B = \mu(\mu + \epsilon)^{-1}$, $\beta' = \beta(\mu + \epsilon)\mu^{-1}$ and $A(t)$ is taken as given and set equal to $a(t)$ in the solution. This observation will be useful later, to deal in a simple way with the more complex model of section 3.

2.2 Dynamics.

For the country which is the technological leader we have $A^* \equiv A$, so the solution to (2.16) is easy to characterize. Let $z = \lambda A$. Then we have, if n were a constant, using (2.16) we can derive:

$$\dot{z} = (R + (1 - \eta)(1 - n)^{\gamma})z + \mu \tag{2.19}$$

and

$$z(t) = (z(0) - \mu((1-\eta)(1-n)^{\gamma} + R)^{-1})e^{(R+(1-\eta)(1-n)^{\gamma})t} + [(1-\eta)(1-n)^{\gamma} + R]^{-1}\mu. \tag{2.20}$$

However, transversality conditions require that $\lim_{t\to\infty} e^{-Rt}z(t) = 0$, so that $z(t)$ must be constant:

$$\lambda(t)A(t) = z(t) = \mu[(1 - \eta)(1 + n)^{\gamma} + R]^{-1}. \tag{2.21}$$

It follows, therefore, from the third equation in (2.16), that n is indeed a constant. In fact, substituting (2.21) into the third equation of (2.16), it is easily checked that there exists a unique solution, n^*, with $n^* \in (0, 1)$. Furthermore the solution with the constant n^* must be the unique solution that satisfies the transversality conditions: this follows from the control problem (2.18) which gives rise to first-order conditions identical to (2.16), and must satisfy the same transversality conditions. Since (2.18) satisfies strict concavity requirements for a unique solution, the solutions to (2.16) which satisfy transversality must also be unique. The growth of technology in the leader country is then given by (since $\lambda^* A^*$ is constant),

$$A^*(t) = A^*(0)\, e^{(1-n^*)\gamma t}, \quad \lambda^*(t) = \lambda^*(0)\, e^{-(1-n^*)\gamma t}. \tag{2.22}$$

We now turn to an analysis of the follower country, for which $A(t) < A^*(t)$. This is most easily analyzed in the case where the production of firm-specific knowledge involves only labor but the stock of aggregate domestic knowledge provides an externality, that is we set $\eta = 0$, $\gamma = 1$. This is also the specification used by Azariadis and Drazen [1990]. In this particular case we can solve for n in the third equation of (2.16) in terms of λ and A, and substitute into the first two equations of (2.16). Defining new variables $z(t) = \lambda(t)A(t)$ and $x(t) = A^*(t)/A(t)$ we can express these two equations as:

$$\begin{aligned} \dot{z} &= (R + 1 + mx)z - \mu - \beta \\ \dot{x} &= ((1 - n^*) + m(1 - x) - 1 + \beta/z)x \,. \end{aligned} \tag{2.23}$$

Noting that $n^* = \beta(R+1)/(\beta+\mu)$ when $\eta = 0$, $\gamma = 1$, we can derive the steady state values of x and z:

$$\bar{x} = (1 + \beta\mu^{-1})(1 + m^{-1}) > 1$$
$$\bar{z} = (\beta + \mu)(R + 1 + mx)^{-1} . \qquad (2.24)$$

Since $\bar{x} > 1$ at the steady state, we see that $A^*(\infty) > A(\infty)$: the technology leader at the start remains the leader at the steady state. The phase diagram associated with equations (2.23) is given below in figure 1 and the equilibrium trajectory is represented by the saddle-path that converges to the steady state.

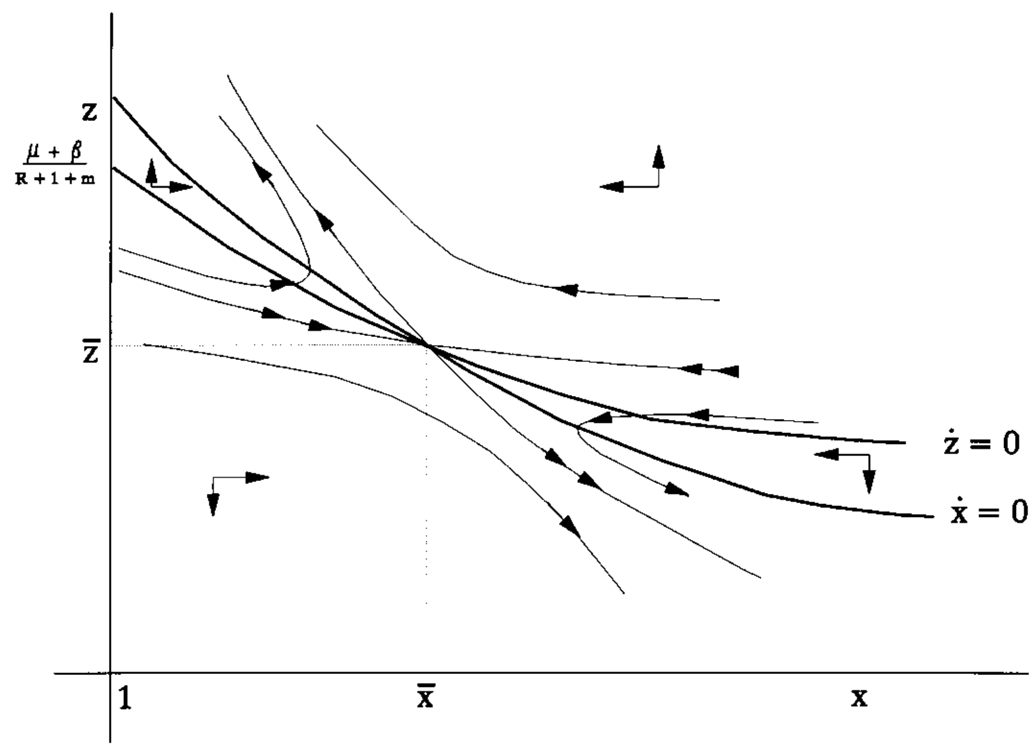

Figure 1

Since the country with the technological lead is better at producing knowledge, it is not surprising that the steady state level of employment in the goods-producing sector is smaller in that country. Computing the employment levels $\hat{n}$ and n^* for the follower and leader countries respectively from (2.17) and (2.24) we obtain:

$$\hat{n} = \beta(R + 1 + m\bar{x})(\beta + \mu)^{-1} > n^* = \beta(R + 1)(\beta + \mu)^{-1} . \qquad (2.25)$$

In the next section we compare the welfare levels of the technological leader and follower countries.

2.3 Welfare Comparisons.

We can easily compare welfare levels in the leader and a follower country for the case where $\eta = 0$, $\gamma = 1$, as in the previous section. Consider the case where the follower country starts on its balanced growth path (BGP). The ratio of BGP consumption levels at each point in time is then given by

$$Q \equiv \frac{A(t)(\hat{n})^\beta}{A^*(t)(n^*)^\beta} = \bar{x}^{-1}(1 + m(1 + R)^{-1}\bar{x})^\beta . \qquad (2.26)$$

It is clear that as the rate of diffusion increased, that is m gets large, Q becomes larger than unity. In this case the follower country is better off, even though it starts at a technology level $A(0)$ below the leader. This conclusion would be reinforced if $A(0)$ were closer or equal to $A^*(0)$, and A only asymptotically converged to its steady state level growth path.

2.4 Trade, Prices and Wages.

In this model, much like open economy models of capital accumulation, consumers in the countries with low levels of technological know-how (or human capital, as will be discussed later) will attempt to smooth their consumption streams by borrowing at first and paying off debt at later stages of development. If, however, initial conditions put the firms of each country at their steady state paths of technology described in the previous sections, and there are no prior accumulated debts or credits, there will not be any trade flows. We can show that the equilibria for the leader and follower countries discussed in the previous sections continue to be equilibria at steady states if, because of potential trade the price of the consumption good is the same in the two countries. Then at each date, consumers in each country will consume exactly the amount that is produced by its domestic firms.

To see this consider equation (2.12) and let $\sigma = 1$ in the leader country. Then, if the price of goods is the same in both countries, that is if $p(t) = p^*(t)$, (2.12) implies that

$$\frac{U'(c(t))}{U'(c^*(t))} = \frac{1}{\sigma}. \tag{2.27}$$

The ratio of marginal utilities has to be constant on this path because the multiplier σ is constant. In both countries n and $\bar{x} = A^*(t)/A(t)$ are constants at the BGP. If the utility function is homothetic, (2.27) will be satisfied for a proper choice of the multiplier σ for the follower country. Furthermore, all other first-order and market-clearing conditions remain unaffected.

What if countries do not start at their steady state level of technologies associated with the autarkic equilibrium? Some simple algebra shows that, again much like small open economies accumulating capital, the BGP levels of technology A, and the labor allocation n of the autarkic equilibrium are also steady state equilibria in the trading equilibrium. However, while equation (2.27) still holds along the equilibrium path, σ, and therefore the steady state ratio of the consumptions that countries converge to has to be adjusted to balance the budgets given by (2.11): countries that borrow at the beginning must adjust their asymptotic consumption levels and repay their debts. (The algebra underlying this section is available from the authors upon request.)

The above discussion shows that once trade incentives arising from intertemporal smoothing considerations are removed by the steady-state assumptions, technological differences alone will not lead to trade flows. This conclusion depends however on the sustained wage differentials between the leader and follower countries that may exist due to barriers to migration. Consider for example the case where consumption levels are higher in the follower country: this will be the case in the autarkic equilibrium when the constant Q, given by equation (2.26) is greater than unity. From (2.27) this also implies that $\sigma > 1$. Since $A^* > A$ and $n^* < \hat{n}^*$ at the BGP, and the price of goods is the same in both countries, it follows from the first equality of (2.8) (because $A = a$) that real wages will be higher in the leader country, creating pressures for immigration. The BGP picture that emerges is one where the leader country uses more of its resources to advance technology, produces fewer goods, has a higher level of real wages than the follower country, and, if $Q > 1$ $(Q < 1)$, has a lower (higher) level of welfare . Of course, our analysis has no doubt left out some economic and non-economic advantages that may arise from technological leadership.

3. Diffusion with Physical and Human Capital.

3.1 The Model.

The model presented in section 2 is probably the simplest one can use to study the diffusion of technology. It can be criticized, however, because the productivity parameter A which it represents the general knowledge of a country cannot be thought as produced by direct labor. In this section we present and study a richer model, in which knowledge is produced by application of human capital. In addition, we allow for the produced good to be accumulated as a productive good, capital, subject to a rate of depreciation θ. Similarly, the human capital h can be increased by use of human capital and direct labor, subject to a depreciation factor δ.

As in the simpler model, the technology which is relevant from the point of view of the firm is Cobb-Douglas; so the firms are facing a concave intertemporal problem. The firm maximization problem is defined by:

$$\max \int_0^{+\infty} (pq - wn - wu)e^{-Rt}dt \qquad (3.1a)$$

subject to

$$\dot{a} = A^{\alpha}a^{1-\alpha}h^{n} + m(B - a)x_{(B\geq a)} \qquad (3.1b)$$

$$\dot{h} = h^{\gamma}n^{1-\gamma} - \delta h \qquad (3.1c)$$

$$\dot{k} = A^{\beta_1} a^{\beta_2} u^{\beta_3} k^{\beta_4} - \theta k - q \tag{3.1d}$$

$$a(0), h(0), k(0)) = (a_0, h_0, k_0) \tag{3.1e}$$

where $\alpha, \gamma \in [0, 1]$; $\beta_2 + \beta_3 + \beta_4 \leq 1$.

We have written the problem for the firms in the country which is a follower. Again the term $x_{(B \geq a)}$ reminds us that the diffusion operates only when the country has indeed a lower level of a.

The problem for the firm in the leading country is similar; indeed it is the one described by (3.1a) to (3.1d), with $m = 0$. The model is closed with the consumer's choice and budget constraint, and market clearing conditions:

$$n + u = 1, \quad q = c .$$

For the consumer, $U(c) = \ln c$; and the budget constraint is as in section 2.

A balanced growth path for this model is an equilibrium path $(A^*, h^*, k^*, u^*; \hat{A}, \hat{h}, \hat{k}, \hat{u})$ for the two countries where each variable has an exponential growth. In the appendix we prove that:

1. A balanced growth path exists and is unique; the ratio between A^* and $\hat{A}$ is constant and different from one.

2. On the balanced growth path, $h^*, u^*, \hat{h}, \hat{u}$ are constant with $h^* > \hat{h}$, $\hat{u} > u^*$.

So, on the BGP the leading country has a permanently higher A and h, a higher proportion of the labor force employed in the accumulation of human capital, and correspondingly a lower proportion of labor employed in the goods producing sector.

It is also easy to check that along the balanced growth path, the ratio of the consumption in the follower country is less than one for large enough values of the diffusion parameter m.

3.2 Related Empirical Work.

The empirical results of Benhabib and Spiegel [1992] lend support to the augmented model discussed above. Benhabib and Spiegel estimate the production function $q = Ak^{\alpha}n^{\beta}$ using log differences over twenty years (1960-1980):

$$\begin{aligned} \ln(q(t)) - \ln(q(0)) &= (\ln(A(t)) - \ln(A(0))) + \alpha(\ln(k(t)) - \ln(k(0))) \\ &\quad + \beta(\ln(n(t)) - \ln(n(0))) + (\ln(\epsilon(t)) - \ln(\epsilon(0))) \end{aligned} \tag{3.2}$$

using the Summer-Heston data set and with human capital data representing average years of education in the labor force, based on the work of Psacharoupulos and Arriagada [1986] and Kyriacou [1991].[3] The diffusion of technology is represented by a discrete-time analogue of equation (3.1b) above, so that the log differences in $A(t)$ are given by:

$$\ln(A(t)) - \ln(A(0)) = c + g\tilde{h} + m\tilde{h}(y^{*}(0) - y(0)) , \tag{3.3}$$

[3] Using literacy or other stock measures of human capital does not affect their results.

where $\tilde{h}$ is the average level of h over the period. $(y^* - y(0))$ proxies for $A^*(0) - A(0)$ and represents a "catch-up" term associated with technological diffusion that interacts with $\tilde{h}$. The empirical results show that g, representing the effect of human capital through domestic innovation, is insignificant. The estimate of m however is significant at the 1% confidence level and positive, indicating an important role for the "catch-up" effect. When the sample is broken into three equal parts however, g is significant and positive and m is insignificant and positive for the richest tier, while the converse holds for the poorest tier. These results indicate that human capital, as measured by average years of schooling in the labor force, enhances growth through domestic innovation in the leading countries while technology-diffusion and catch-up seems to be important as an engine of growth for the poorer nations. Benhabib and Spiegel [1993] show that similar results hold for an analysis with cross-state data within the U.S.

4. Strategic Considerations.

So far we have considered a competitive equilibrium model. It may be objected that the real behavior of the two countries over such a fundamental issues as research and diffusion, is strategic rather than competitive. The possibility that a technology leader may be worse off in a competitive environment with technology diffusion raises serious policy issues about the role of government in funding, channelling and regulating R&D, with particular concern for the ratio of D to R.[4]

[4] For example, the most recent NSF "plan for the year" now calls for increases in funding to areas that "are of key interest to industry" while recommending cutbacks to many "core programs" which support "curiosity-driven" research in the traditional disciplines (Cordes [1993]).

A strategic model is different in a fundamental way from the competitive model. In particular, the leading country "knows" that by consciously shifting the effort from research (which benefits the competitors) to direct production it can eventually take advantage of the spillovers from the other country.

A detailed analysis of this issue is beyond at least the present paper. Nevertheless, we point out below a simple case where an outcome very similar to the equilibria presented in the previous sections can also be the equilibrium outcome of a game between two countries. To be more precise, consider the game where the payoff to player i is given by:

$$\max \int_0^{+\infty} U(A_i n_i^{\beta})\,dt$$

$$\text{subject to} \qquad\qquad (P_i)$$

$$\dot{A}_i = (1 - n_i)A_i + m(A_j - A_i)\chi_{(A_j \geq A_i)}$$

$$n_i \in [0,1], \quad A_i(0) = A_0^i \; ;$$

where $j \neq i$; strategies are defined as usual, and subgame perfect equilibria are considered.

In particular, a stationary strategy for player i is a function σ from the pair of states (A_1, A_2) to n_i, the amount of labor employed in production. Consider now the case where m is large enough; more precisely assume that

$$\beta(R + m) \geq 1 > \beta R .$$

Let, now say, $A_0^1 \geq A_0^2$, an define the strategy of the leader as the optimal strategy of problem (P_1) above, where in the case of the leader we must set $m = 0$. It is easy to compute that optimal policy is given by:[5]

$$n^* = \beta R . \tag{4.1}$$

So, the leader is following the optimal strategy of an isolated country. Then define the follower's problem as the problem (P_2) above, with the additional equation

$$\dot{A}_2 = (1 - \beta R)A_2$$
$$A_2(0) = A_0^2 .$$

This is in fact a stationary dynamic programming problem, with an optimal policy which is a function of the two states. In fact one can prove that this optimal policy is to set $n \equiv 0$, as long as $A_1 \geq A_2$. An easy argument then shows: The leader and follower strategies described above define an equilibrium in stationary strategies of the game.

The equilibrium outcome has a balanced growth path. Both countries have, in the long run, the same growth rate $(1 - \beta R)$, but persistent differences in levels. In fact, the limit value for the ratio A_1/A_2 and so the ratio in the

[5] A technical remark: the Hamiltonian of the corresponding problem is not jointly concave in the state and control variable; but the reduced Hamiltonian, obtained by substituting the optimal control expressed as a function of the state and costate variable, is concave in the state. See, for example, Van Long and Vousden [1977].

balanced growth path, is equal to $1/(1-\beta)$.

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Appendix 1:

The optimal solution for a firm is characterized as the solution to the system defined by (3. a) to (3. d) above, plus

$$\dot{\lambda} = R\lambda - (1-\alpha)\lambda A^{\alpha}a^{-\alpha}h^{\alpha} + \lambda m - \beta_2\nu A^{\beta_1}a^{\beta_2-1}u^{\beta_3}k^{\beta_4} \qquad (A.1)$$

$$\dot{\mu} = R\mu - \alpha\lambda A^{\alpha}a^{1-\alpha}h^{\alpha-1} - \mu\gamma h^{\gamma-1}n^{1-\gamma} + \mu\delta \qquad (A.2)$$

$$\dot{\nu} = R\nu - \beta_4\nu A^{\beta_1}a^{\beta_2}u^{\beta_3}k^{\beta_4-1} + \nu\theta \qquad (A.3)$$

$$p = \nu \qquad (A.4)$$

$$w = (1-\gamma)\mu h^{\gamma}n^{\gamma} = \beta_3\nu A^{\beta_1}a^{\beta_2}u^{\beta_3-1}k^{\beta_4} \qquad (A.5)$$

$$\lim_{t\to\infty} (\lambda a + \mu h + \nu k)e^{-Rt} = 0 . \qquad (A.6)$$

At equilibrium

$$a = A, \quad n = 1 - u, \quad q = c . \qquad (A.7)$$

We also let $\beta \equiv \beta_1 + \beta_2$. Then (A.1) to (A.6) give:

$$\dot{\lambda} = (R - (1-\alpha)h^{\alpha} + m)\lambda - \beta_2\nu A^{\beta-1}u^{\beta_3}k^{\beta_4} \qquad (A.8)$$

$$\dot{\mu} = (R - \gamma(n/h)^{1-\gamma} + \delta)\mu - \alpha\lambda A h^{\alpha-1} \qquad (A.9)$$

$$\dot{\nu} = (R + \theta - \beta_4 A^{\beta} u^{\beta_3} k^{\beta_4 - 1})\nu \tag{A.10}$$

$$\mu(1-\gamma) = \nu\beta_3 A^{\beta} u^{\beta_3 - 1} k^{\beta_4} (n/h)^{\gamma} \tag{A.11}$$

Let us now focus on a balanced growth path (BGP) equilibrium. On this path, the ratio of the values of A for the leader and the follower is a constant, again denoted by x.

We now construct a BGP which satisfies the necessary and sufficient conditions for an equilibrium. We begin with the leading country; we shall show that in the BGP that state and dual variables have the form

$$(A, \lambda, h, \mu, k, \nu) = \left(A_0^* e^{(h^*)^{\alpha} t}, \lambda_0 e^{-(h^*)^{\alpha} t}, h^*, \mu^*, k_0^* e^{\frac{\beta (h^*)^{\alpha}}{1-\beta_4} t}, \nu_0^* e^{-\frac{\beta (h^*)^{\alpha}}{1-\beta_4} t}\right) \tag{A.12}$$

Clearly with these variables the transversality condition is satisfied. The product λA is constant if and only if

$$\lambda_0^* A_0^* = \frac{\beta_2 \nu_0^* A_0^{\beta - 1} {k_0^*}^{\beta_4} {u^*}^{\beta_3}}{R + \alpha h^{*\alpha}} \tag{A.13}$$

Since at the BGP that $\dot{h} = 0$, we have

$$\frac{1-u^*}{h^*} = \frac{n^*}{h^*} = \delta^{\frac{1}{1-\gamma}} \tag{A.14}$$

and substituting in (A.9) and (A.11) the equations (A.13), (A.14) above we have

$$\mu_0^* = \frac{\alpha \beta_2 \nu_0^* A_0^* {A_0^*}^{\beta} {k_0^*}^{\beta_4} {u^*}^{\beta_3} h^{*\alpha - 1}}{(R + \alpha h^{*\alpha})(R + \delta(1-\gamma))} \tag{A.15}$$

$$\mu_0^* = \frac{1}{1-\gamma} \beta_3 \nu_0^* {A_0^*}^{\beta} {k_0^*}^{\beta_4} {u^*}^{\beta_3 - 1} \delta^{\frac{\gamma}{1-\gamma}} \tag{A.16}$$

Eliminating μ_0^* from these we get

$$(R + \alpha h^{*\cdot})(R + \delta(1-\gamma))\delta^{\frac{\gamma}{1-\gamma}} = \frac{\beta_2}{\beta_3}(1-\gamma)h^{*\cdot -1} \tag{A.19}$$

It is easy to check that (A.19) determines in fact a unique value h^*; from (A.14) we now determine u^*.

For the following country we proceed similarly. Elimination of $\hat{\mu}_0$ gives

$$(R + mx + \alpha\hat{h}^{\alpha})(R + \delta(1-\gamma))\delta^{\frac{\gamma}{1-\gamma}} = \frac{\beta_2}{\beta_3}(1-\gamma)\alpha\hat{h}^{\alpha-1}. \tag{A.20}$$

For x to be finite, the growth rate of A in the two countries must be the same, so

$$h^{*\cdot} = \hat{h}^{\alpha} + m(x - 1) \tag{A.21}$$

and finally

$$\frac{1 - \hat{\mu}}{\hat{h}} = \delta^{\frac{1}{1-\gamma}}. \tag{A.22}$$

Equation (A.21) determines $\hat{h}$ as a function of x. If we substitute this into (A.19), we obtain an equilibrium in x; it is easy to show that this equation also has a unique solution $\hat{x} > 1$. This completely determines the equilibrium values of $(h^*, u^*, \hat{h}, \hat{u}, x)$.

The equation

$$\frac{\dot{\nu}}{\nu} = R + \theta - \beta_4 A_0^{\beta} u^{\beta_3} k_0^{\beta_4 - 1} = -\frac{\beta}{1 - \beta_4}(h^*)^{\alpha} \tag{A.23}$$

determines now the value of k_0 (it is clear that a value of k_0 which solves (A.23) exists).

From the first-order conditions for the consumer, $U'(c) = p$, and market clearing conditions we get: $1/q = p$; then, from (A.4) for the firm:

$$p = \nu = q^{-1} \tag{A.29}$$

Since ν grows at an exponential rate: $-\beta(h^*)^\alpha/(1-\beta_4)$, so do p and q^{-1}. It follows that:

$$\frac{q}{k} = \frac{q_0}{k_0} . \tag{A.25}$$

We substitute this into the equation (3.1d) to get; (after substituting the other equilibrium condition $A = a$):

$$\frac{\dot{k}}{k} = A_0^\beta u^{*\beta_3} k_0^{\beta_4 - 1} - \theta - \frac{q_0}{k_0} = \frac{\beta(h^*)^\alpha}{1 - \beta_4} . \tag{A.26}$$

This additional equation determines the value of q_0, and therefore p_0 and ν_0. Now all the variables describing the balanced growth path have been determined.