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SOCIAL CHOICE IN A REPRESENTATIVE DEMOCRACY

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Abstract

In a representative democracy, citizens stand at two removes from legislation. First, they do not deliberate and vote directly on legislation. Rather they elect assemblies that enact such legislation in their stead. Second, and less commonly remarked, citizens do not vote directly for assemblies. Rather they vote for individual candidates, with the candidates receiving the most votes elected. This paper examines the efficiency properties of these voting systems. We show, first, that in general these procedures are inefficient. Second, we identify a condition on assembly preferences (called k-blockness) that insures the election of a pareto-optimal assembly. We then show that whatever neutral restriction is imposed on preferences, an "almost inefficient" assembly may be elected.

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1. Introduction.

In a representative democracy, citizens stand at two removes from legislation. First, they do not deliberate and vote directly on legislation. Rather they elect assemblies that enact such legislation in their stead. Second, and less commonly remarked, citizens do not vote directly for assemblies. Rather they vote for individual candidates. The winning candidates then constitute the assembly. Thus, in the elections of city councils, school boards, the United States Congress and the legislatures of the states, although in each case an assembly is being chosen, voters are called upon to vote for candidates and these candidates have their votes tallied as individuals.

Perhaps because these elections are candidate-based, analyses of them have often taken voters' preferences over candidates as their starting point. There is something peculiar in this

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approach, however. At least since Black (1948) and Arrow (1951) social choice theory has viewed society's decision problem as one of choosing an outcome based on the preferences of the individual members of society. That is, the theory assumes that each person has an ordering over the possible outcomes, and various functions from these orderings to an outcome are compared. In the present context, an outcome is an assembly. Therefore, preferences over assemblies, not candidates, are fundamental. Indeed, any ranking of the candidates must be understood relative to the ranking of the assemblies. An appreciation of this simple fact has important implications for the understanding of our political institutions.

Several authors (e.g., Downs 1957, Austen-Smith and Banks, 1988) have argued that voters have preferences first over legislative programs, and that these legislative preferences then induce preferences over assemblies. To the extent that there is a definite correspondence between assemblies and the programs they enact, this approach is not in conflict with the approach adopted here. Since preferences over legislative programs induce preferences over assemblies, authors who adopt this approach recognize the primacy of assembly orderings over candidate orderings. However, these authors have been primarily concerned with the fact that the induced assembly preferences may not be separable. As far as we know, no one has considered the difficulties which arise even when preferences are separable.[1]

Moreover, voters may evaluate assemblies only partially (or not at all) on the basis of the legislative programs they enact. A voter may prefer an assembly that "represents" all the relevant constituencies to one that does not. Or, because she does not know (and cannot imagine) what substantive issues may arise, she may have to evaluate the assembly on other grounds. In each of these cases, her preferences over assemblies will be more fundamental than her preferences over legislative outcomes or over candidates.

In this paper we show that unless preferences over assemblies satisfy strong conditions, typical procedures for electing assemblies are inefficient. We identify conditions that ensure the election of a Pareto Optimal assembly, but also show that whatever neutral restriction is imposed upon preferences, an "almost inefficient" assembly may be elected. We also briefly discuss proportional representation from an assembly-based perspective. All proofs are in the appendix.

2. Preliminaries.

2.1 Candidate-based and Assembly-based Procedures. As in Benoit and Kornhauser [1991], we classify procedures for electing assemblies along two dimensions: (a) whether candidates must

[1]Separable preferences exhibit a form of independence; separability is defined formally in section 3.

declare which post they contest and (b) the scope of the electorate that votes for each position in the assembly. We call assemblies in which candidates do not declare which post they contest at-large assemblies. An assembly in which candidates must declare the post they contest is a designated-post assembly. We divide designated-post assemblies into two types: if the entire electorate votes on each post, the assembly is a numbered-post one; if different portions of the electorate vote on specific posts, the assembly is districted.[2] Note that our definition of designated-post assemblies encompasses not only legislatures but also governments composed of separate houses and an executive.

Let $N = \{a_1, a_2, . . . a_n\}$ be the set of candidates. Let $Æ_m$ be the set of all possible assemblies. Let v be the number of voters. Let $\Gamma(N)$ be the set of all possible orderings of the elements of N;[3] let $\Gamma(Æ_m)$ be the set of all possible orderings over $Æ_m$. We may now define the following:

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[2]In the United States, school boards and the legislative bodies of many cities are elected at-large. The United States Congress as well as the legislatures of the states are elected using districted procedures. The election of the executive in many cities and states uses a numbered-post procedure: each candidate declares which executive post--governor, attorney general, or treasurer--she seeks and then the entire electorate votes on each post.

[3]When candidates are running for different posts, $\Gamma(N)$ is the set of all possible orderings of the candidates grouped by their respective posts.

Definition: An assembly-based procedure is a function $f: \Gamma(Æ_m)^v \text{---}> Æ_m$.

Definition: A candidate-based procedure is a function $g: \Gamma(N)^v \text{----}> Æ_m$.

We assume that each voter i has a well-defined preference ordering $R_i(Æ_m) \epsilon \Gamma(Æ_m)$ over assemblies, with "A > B" meaning assembly A is strictly preferred to assembly B and "A ≥ B" indicating weak preference.

We note that while our definitions do not immediately apply to party systems such as party-list proportional representation, our ensuing discussion (in particular, theorem 1) can be applied to these systems.

2.2 Separability. In some situations, complementarities among the candidates may determine the desirability of an assembly. In such cases, it may be that some voters' favorite assembly consists, for instance, of candidates a_1 and a_{10}, but that these voters only like a_1 because of how well she works with a_{10}. If a_{10} is not present, these voters do not want a_1 on the assembly. Other voters may like the way that a_6 and a_8 interact and so forth.

When preferences over assemblies have this interactive nature, choosing assemblies by voting for individual candidates does not seem sensible; the "wrong" combination of candidates might easily be elected. Indeed, with this type of preferences, there is no immediate way for voters to rank individual candidates.

On the other hand, it may be that the desirability of a candidate can be determined independently of the composition of the rest of the assembly. That is, preferences may be <u>separable</u> so that if a voter prefers the assembly $\{a_1, a_{10}\}$ to $\{a_2, a_{10}\}$ then she prefers $\{a_1, a_7\}$ to $\{a_2, a_7\}$ and in fact she prefers a_1 to a_2 on any assembly. In this case there is a natural ranking of the candidates and one might think that candidate-based procedures are then appropriate. However, as we shall show, even in this "good" case, common candidate-based procedures fail to have minimally desirable properties.

Formally, we define separability as follows. For any two candidates a_i and a_j, let X_{ij} be a set of (m-1) candidates not including a_i or a_j.[4]

<u>Definition</u>: An individual has <u>separable assembly preferences</u> if and only if, for all a_i, a_j, A_{ij}, and B_{ij}, $\{a_i\}UA_{ij} > \{a_j\}UA_{ij}$ implies $\{a_i\}UB_{ij} \geq \{a_j\}UB_{ij}$.

A separable assembly ranking <u>generates</u> a unique candidate ranking in the obvious way; $a_i > a_j$ if and only if there exists an A_{ij} such that $\{a_i\}UA_{ij} > \{a_j\}UA_{ij}$ and a_i is indifferent to a_j if and only if $\{a_i\}UA_{ij}$ is indifferent to $\{a_j\}UA_{ij}$ for all A_{ij}.

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[4]For designated-post assemblies, a_i and a_j must contest the same post and X_{ij} draws its (m-1) candidates from each of the remaining posts.

Many assembly rankings, however, are consistent with any single candidate ranking; that is, a single candidate ranking could be generated from many different separable assembly preferences. The following proposition from Benoit and Kornhauser [1991] elaborates this idea:

Proposition 1: Let $a_1 > a_2 > \ldots > a_n$ be a candidate ranking and A and A′ be two assemblies. If and only if there exist i and j such that the ith most highly ranked candidate in A is more highly ranked than the ith most highly ranked candidate in A′ but the jth most highly ranked candidate in A′ is more highly ranked than the jth most highly ranked candidate in A, then A is ranked above A′ by some consistent assembly rankings, and below A′ by others.

2.3 Non-Strategic and Simple Voting. A strategic voter tries to anticipate the votes of others when deciding how she will vote. A non-strategic voter, on the other hand, casts her votes solely as a function of her own preferences. Of course, voting non-strategically may not be optimal. Nonetheless, in what follows we often assume that voters act non-strategically.

We make this assumption for three reasons. First, in many situations non-strategic voting may reasonably describe voter behavior. Second, an understanding of non-strategic voting is an important first step to the understanding of the candidate-based procedures which predominate in our political institutions.

Finally and most importantly, our results do not hinge upon this assumption.[5]

When preferences are separable, a salient non-strategic way of voting is for a voter with k votes to cast these votes for the top k candidates in the candidate ranking generated from her assembly ranking. Such voting is akin to sincere voting and maximizes a voter's expected utility with respect to a uniform distribution on the behavior of other voters. Elsewhere we have termed this simple voting.[6]

3. Why Assembly Preferences Matter.

Candidate-based procedures ought to be evaluated in terms of preferences over assemblies. Put differently, preferences over assemblies, not individual candidates, are fundamental. Adopting this perspective has significant consequences for an understanding of representative institutions. In this section, we offer examples concerning efficiency and proportional representation that illustrate this claim. In subsequent sections we offer some general theorems concerning the efficiency properties of candidate-based procedures.

3.1 An Example. Our first example shows that candidate-

[5] For instance, see the discussion following example 1. Also see example 2, where voters use dominant strategies.

[6] See Benoit and Kornhauser [1992], where there is also a fuller discussion of simple voting. Separability is a sufficient but not necessary condition for simple voting. A necessary and sufficient condition is "top-separability".

based procedures may elect Pareto inferior assemblies.

There are five candidates contesting two seats on an at-large assembly. The voters divide into four groups. The first group ranks the ten assemblies as follows:

$\{a_4,a_2\} > \{a_4,a_3\} > \{a_4,a_1\} > \{a_4,a_5\} > \{a_2,a_3\} > \{a_2,a_1\} > \{a_3,a_1\} > \{a_2,a_5\} > \{a_3,a_5\} > \{a_1,a_5\}$.

Notice that preferences are separable. This assembly ranking generates the following ranking of the candidates:

$$a_4 > a_2 > a_3 > a_1 > a_5.$$

Other separable assembly rankings are consistent with this same candidate ranking. For instance, reversing the preference between $\{a_3,a_1\}$ and $\{a_2,a_5\}$ in the above assembly ordering produces separable preferences that generate the same candidate ranking.

The remaining three groups also have separable assembly preferences, which generate candidate rankings. The figure below presents the four candidate rankings. The number at the top of each column gives the number of voters in that group.

Example 1

Rank	8	10	5	12
1st	a_4	a_1	a_1	a_3
2nd	a_2	a_2	a_5	a_5
3rd	a_3	a_3	a_3	a_1
4th	a_1	a_4	a_4	a_4
5th	a_5	a_5	a_2	a_2

Suppose, as is often the case with two-person assemblies, that everybody casts two votes for two different candidates and that the top two vote-getters form the assembly. We assume that everyone simply votes for their top two candidates. The assembly $\{a_2,a_5\}$ is elected with 18 votes for a_2 and 17 votes for a_5. But <u>every</u> voter prefers $\{a_1,a_3\}$ to $\{a_2,a_5\}$. In fact, the last three groups of voters rank $\{a_1,a_3\}$ second while they rank $\{a_2,a_5\}$ no higher than seventh. (More precisely, in every assembly ranking consistent with the above candidate rankings $\{a_1,a_3\}$ is ranked second by the last three groups while $\{a_2,a_5\}$ is ranked seventh or eighth.)

The election of the inefficient assembly $\{a_2,a_5\}$ is not due to an assumption of non-strategic voting or a failure of the voters to recognize the superiority of $\{a_1,a_3\}$. If the voters agreed to vote for a_1 and a_3, the first two groups, for instance, would have a strong incentive to deviate from the agreement and vote for a_2 instead of a_3, since each member of these groups prefers $\{a_1,a_2\}$ to $\{a_1,a_3\}$.[7]

Although the prevailing candidates in example 1 do not win decisively, it is easy to write down examples in which the winning candidates receive an overwhelming majority of the votes or form a Condorcet set of candidates capable of beating all other candidates pairwise, and still the winning assembly is

[7]This reasoning is in the spirit of Nash equilibrium analysis. Any assembly, however, could be elected in a Nash equilibrium.

inefficient.[8] Furthermore, an elected assembly could rank, say, in the bottom 20% of all possible assemblies for all voters, while there is another assembly that all voters agree ranks among the top 10%. Hence, the election results themselves provide no indication of the existence or severity of a problem.

3.2 Strategic Voting and Endogenous Preferences.

In example 1, the list of candidates is given exogenously. We now present a simple game-theoretic example in which the positions of candidates is determined endogenously, voters use dominant strategies, and the final outcome is inefficient. Thus, the cause of the inefficiency is the use of the candidate-based procedure and not that the "wrong" candidates have been assumed at the outset or that voters are not acting strategically.

The assembly is a numbered-post one in which the three posts consist of different offices (for instance, sheriff, school superintendent and sanitation chief). Two candidates are running for each office. For each office a candidate can adopt a position which is described by a real number from 0 to 1.

The voter's divide into three equal-sized groups. Group 1's

[8]If there exists a Condorcet assembly, however, then those candidates also form a Condorcet set; in this case, a Condorcet set is necessarily efficient. As the text implies, the existence of a Condorcet set of candidates does not imply the existence of a Condorcet assembly (Benoit and Kornhauser [1991] Kadane [1972]). Note also that even when there is a Condorcet assembly common voting procedures do not guarantee the election of the Condorcet candidates, and these procedures may still be inefficient.

favorite positions on the three offices is given by {1,0,0}. Members in Group 1 care most about the first office so that they prefer the assembly {1,1,1} to the assembly {0,0,0}. Group 2's and Group 3's favorite outcomes are given by {0,1,0} and {0,0,1}, respectively. Individuals in these groups care most about the second and third offices respectively so that they too prefer the assembly {1,1,1} to {0,0,0}. Preferences are separable.[9]

The game is in two stages. In the first stage candidates choose a position. In the second stage individuals cast their votes, one for each office. Each office is decided by majority rule. We look for a subgame perfect equilibrium in undominated strategies.

Since preferences are separable and there are only two candidates per office, it is always dominant for an individual to vote for the candidate that is closest to her ideal position for each office. Since for each office 2/3 of the voters have 0 as ideal point, in equilibrium both candidates will enter at 0 (the median). Thus, we have the following:

Proposition 2: The unique subgame perfect equilibrium

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[9]The preferences described here model reasonably well some actual situations. For example, from 1964 through 1986 the four resident commissioners of Etowah County (Alabama) were elected by a county-wide vote although each resident commissioner had to reside in her district. The Commission as a whole allocated money among districts, but each commissioner controlled funds allocated to her district. Residents of a district presumably had preferences like those described in the text. The Commission is described in Presley v. Etowah County Commission (1992).

outcome in undominated strategies is the election of the assembly {0,0,0}.

The assembly {0,0,0} is elected although all voters prefer {1,1,1}.

3.3 Proportional Representation.

At its most basic level, the concept of proportional representation holds that if x% of the electorate has identical preferences, then that group is entitled to choose x% of the members of an assembly. In this section we argue that if preferences are only assumed to be separable, this candidate-based concept is suspect. The concept of proportional representation, however, can be defended if assembly preferences satisfy a stronger restriction on preferences.[10]

Consider 10 candidates running for five seats and suppose that each voter has separable preferences over assemblies. Suppose 40% of the voters have identical preferences and that the voting procedure perfectly realizes the group's entitlement under proportional representation to elect 40% of the assembly. That is, the group can assure the election of its top two candidates. As 56 of the 252 possible assemblies include these two candidates, the voting procedure eliminates 196 possible

[10]Our discussion abstracts from many of the issues that have prompted most discussions of proportional representation. For example, we ignore problems of implementation as well as problems that arise when voters have heterogeneous preferences.

assemblies.

But these are not just any 196 assemblies. If the coalition were instead given the right to eliminate any 196 assemblies, it might well have chosen to exclude assemblies which contained undesirable candidates rather than those that did not include desirable ones. Put differently, although the outcomes have been restricted to 56 assemblies, the resulting assembly does not necessarily rank among the coalition's top 56. In fact the elected assembly may rank as low as 232nd. Even a coalition that finds its top four candidates (80% of the assembly) elected may rank the resulting assembly no higher than 125th!

Thus, allowing a group to include its top candidate(s) in an assembly may accomplish very little. To make sense of proportional representation, an additional assumption is required. Separable preferences are lexicographic when assembly A is preferred to assembly B if and only if the most highly ranked candidate in $A/(A \cap B)$ is preferred to the most highly ranked candidate in $B/(A \cap B)$. An individual with lexicographic preferences always chooses to eliminate first those assemblies that do not include her top candidates. Under proportional representation, candidates are also eliminated in this way and the resultant assembly is efficient. In the above example, with lexicographic preferences the coalition of 40% of voters elects an assembly that is ranked no lower than 56th (or in the top 23%

of all assemblies). While it may be too strong to say that the assumption of lexicographic preferences "justifies" proportional representation, at least there is a certain sense to it in this case.

4. The Efficiency of Constant Scoring Systems.

The inefficiency in examples 1 and 2 is due to the candidate-based nature of the voting method used. This inefficiency arises with a wide range of candidate-based voting methods and with a wide range of assumptions about voter behavior. Rather than give the most general possible theorem, however, we offer a fairly broad theorem which is easily stated (and proved).

Definition: A constant scoring system for an at-large assembly of size m is one in which each voter is given some number, $k \leq m$, of votes to be distributed among k different candidates, and the assembly elected consists of those m candidates receiving the m greatest total number of votes.

Assumption 1. The size of the assembly is $m \geq 2$, the number of candidates is $n \geq m + 3$, and the number of voters is $v \geq \max(m,6)$. When preferences are separable, individuals vote simply for their top k candidates.

Theorem 1: Under assumption 1, even if voters have separable preferences, a constant scoring system may result in the election of an at-large assembly A although there is an at-large assembly B that everyone prefers.

Pareto optimality is generally considered to be a rather weak requirement and is satisfied when individuals vote directly for the outcomes using, for instance, plurality rule, plurality with a runoff, and a Borda count. Theorem 1 shows, however, that, when assemblies are being chosen but candidates are being voted upon, this requirement is, in general, too strong.

We now offer a condition on preferences that is sufficient to guarantee that a constant scoring system yields a Pareto optimal assembly. Let $|S|$ denote the number of elements in S.

Definition: An individual has k-block preferences over assemblies if and only if there exists a set B(k) of candidates such that, for any assemblies A and A′, if $|A \cap B(k)| > |A' \cap B(k)|$, then A is strictly preferred to A′.[11]

We will say that an individual has block preferences if her preferences are k-block for some k. Preferences which are block may or may not be separable. In either case, voting for the block set of candidates is simple.

An individual with block preferences has favorite candidates and is most concerned with getting the maximum number of these

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[11]Fishburn [1981] assumes that A is strictly preferred to A′ if and only if $|A \cap B(k)| > |A' \cap B(k)|$. This implies blockness and is in fact a much stronger assumption. For instance, suppose $B(k) = \{a_1, a_2\}$. Let $A = \{a_1, a_3\}$ and $A' = \{a_2, a_4\}$. Fishburn's definition requires indifference between A and A′ but our definition permits $A > A'$, $A < A'$ or $A = A'$. Fishburn restricts the assembly ordering to one with k+1 indifference classes.

candidates elected. Thus, someone with 1-block preferences has a most preferred candidate and prefers any assembly with this candidate to any assembly without her. A person with 2-block preferences has two preferred candidates and prefers any assembly with one of these candidates to any assembly without either, and any assembly with both these candidates to one with fewer than both. No further restrictions are imposed. An individual with strong dislikes who is concerned with keeping certain candidates out of an assembly is not likely to have k-block preferences.

As the next theorem indicates, k-blockness guarantees Pareto optimality in constant scoring systems. In the proof of the theorem, we ignore the possibility of two candidates receiving the same number of votes. Ties may be broken by using an arbitrary voter as tie-breaker. Breaking ties by randomly selecting candidates, however, may be inefficient.

Assumption 2: A voter with k-block preferences and k votes, casts these votes simply for her block set. The electorate is sufficiently heterogenous that at least m candidates receive votes.

Theorem 2: Under assumption 2, if every voter has k-block preferences and k votes, then a Pareto optimal assembly is elected.

While this is a positive theorem for voting over candidates, several comments are in order. First, the theorem requires not only that voters have block preferences, but also that all these preferences be k-block for the same k, and that this k be the number of votes which each person is to cast. This is a lot to ask. However, if individuals are given either too few or too many votes, efficiency cannot be guaranteed.

Approval voting offers some hope in this regard. Under approval voting, each person can cast as many votes as she likes. One reasonable way for a person with k-block preferences to vote in such a system would be to cast k votes for her block set.[12] This assumption gives the following:

Theorem 3: If every voter has block preferences then approval voting (with each individual voting for her block set) elects a Pareto optimal assembly. If preferences are only separable, a Pareto inferior assembly may be elected.

Blockness is quite a strong assumption. Nevertheless, it is not easily relaxed. If even a few people have non-block preferences a Pareto inferior assembly may be elected. Furthermore, as the next theorem indicates, even if all preferences are block or satisfy stronger restrictions, a "nearly inefficient" assembly may be elected. To state the result we need some preliminaries.

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[12] Voting in this manner is simple, but it is not the only simple way to vote. Fishburn [1981] shows that under a stronger assumption than blockness (see footnote 11), voting in this manner is a dominant strategy.

A restriction is on individual preferences, as opposed to across preferences, if it can be verified by checking preferences in isolation. Thus, separability is a restriction on individual preferences, since any single preference ranking can be checked for separability; single-peakedness is a restriction across preferences since it can only be violated by a set of preferences.

A restriction is neutral if it is independent of the labeling of candidates. That is, if a preference ordering P satisfies a neutral restriction, then P′ will satisfy the same restriction, where P′ is defined as follows: for all assemblies A_i, let A_i' equal A_i with a_r replaced by a_m and a_m replaced by a_r (if a_r or a_m occurs in A_i. Otherwise A_i' equals A_i). Then in the ordering P′, A_i' is ranked ahead of A_j' if and only if A_i is ranked ahead of A_j in the ordering P.

All the restrictions we have considered so far are neutral restrictions on individual preferences. An example of a non-neutral restriction on individual preferences would be that the assembly A be ranked first.

A neutral non-strategic voting rule for a voter with k votes is a function h: $\Gamma(Æ_m) \dashrightarrow \mathbf{C}_k$ such that f(P) = C_k implies f(P′) = C_k' where $\mathbf{C}_k$ is the set of all subsets of size k of the set N of candidates, C_k is an element of $\mathbf{C}_k$ and C_k' is the permuted element.

The next theorem indicates that no matter what neutral restriction on individual preferences is imposed, an assembly which is nearly inefficient may be chosen by a constant scoring system.

Assumption 3: The size of the assembly is $m \geq 2$, the number of candidates is $n \geq 4$ and the number of voters is $v \geq \min(3,m)$. All voters use the same neutral non-strategic voting rule.

Theorem 4: Consider any neutral restriction on strict individual preferences. Under assumption 3, there exist preference profiles in which all individual orderings satisfy this restriction, an assembly A is elected, and all but one or two voters prefer an assembly B.

5. Designated Posts

Theorem 1 was stated for at-large assemblies. With suitable modifications the theorem extends to the case of districted and numbered-post elections. For these assemblies too, candidate-based elections may produce inefficient assemblies or governments.[13]

Designated-post assemblies are of special interest because they are so common. These assemblies may have two very distinct characters. The designated-posts may be different seats in a legislature or they may be different offices in a government.

[13] See Benoit and Kornhauser [1991]. The Ostrogorski paradox (Anscombe [1976]) can be understood as a special instance of our inefficiency results for numbered-post assemblies.

One might thus characterize the President of the United States and the members of both houses of Congress as a designated-post assembly with the President elected at-large and senators and representatives elected by different districted procedures.

We turn now to assumptions which guarantee the election of efficient designated-post assemblies. As the ensuing discussion indicates, these assumptions may be more plausible then the assumption of blockness used in the previous section.

In a numbered-post election, suppose a voter assigns an order of importance to the posts (e.g., the mayoralty is most important, followed by district attorney and so forth).

Definition: Let A and A′ be two assemblies with the posts listed in declining order of importance and suppose that the two assemblies first differ in the jth position. Then preferences are said to be top-lexicographic if an individual always prefers A to A′ when the jth candidate in A is the individual's most preferred candidate for post j.

Assumption 4: Preferences are separable. Each voter gets one vote per post and votes simply for her top candidate for each post.

Theorem 5: Suppose each individual agrees on the order of importance of the posts in a numbered-post assembly, and that preferences are top-lexicographic. Then, under assumption 4, a Pareto optimal assembly is always elected.

For simplicity, our analysis of districted procedures considers only those elections in which each individual votes in a single district. In this context, an individual's preferences are 1-block if there is a district and a candidate in that district, such that the individual prefers any assembly with that candidate to any assembly without that candidate.

Theorem 6: If each individual votes in a single district and has 1-block preferences with respect to that district, then a Pareto optimal assembly will be elected.

If voters in a numbered-post election have top-lexicographic preferences, but disagree on the order of importance of the posts, then an inefficient assembly may be elected. Similarly, if voters in a districted election have 1-block preferences but not with respect to the district in which they vote, then an inefficient assembly may be elected.

The assumptions on preferences in theorems 5 and 6 are both non-neutral since they are tied to specific posts. In a numbered-post assembly where the different posts have different titles and responsibilities, a non-neutral assumption may be reasonable. If the differently numbered posts are identical in duties and numbered merely for electoral purposes, a non-neutral assumption is unwarranted. In such a case there are neutral assumptions which will guarantee the election of an efficient assembly.[14] However, this efficiency can only be assured with respect to those assemblies which can be formed respecting the posts for which each candidate has declared. It cannot be guaranteed with respect to the at-large assemblies which could be formed. In the neutral case, this broader notion of efficiency would seem appropriate.

The non-neutrality in the assumption of theorem 6 may be warranted when voters elect local representatives whose responsibilities are primarily to their district. Again, if all representatives have identical duties, a non-neutral assumption is not justified. In districted elections, no neutral assumption can guarantee the election of a Pareto optimal assembly, even among the restricted set of assemblies.

When there exists a Condorcet winning assembly in a numbered-post election with only two candidates per post (for instance, a Democrat and a Republican), each candidate in this assembly is a majority rule winner in her individual post election (Kadane 1972). Thus, majority rule will produce this Condorcet winner. In a districted election, however, majority rule may fail to produce a Condorcet winner (Benoit and Kornhauser 1992).

7. Conclusion.

[14]For instance if assembly A has more first choices than assembly B, then A is preferred to B.

Jurisdictions generally adopt candidate-based procedures, in particular constant scoring rules, to elect assemblies. This common practice poses a problem since such procedures do not possess minimal efficiency properties; the elected assembly may be nearly inefficient.

These results suggest two types of inquiry. First, one might design and study fully assembly-based procedures for the election of assemblies. This task faces both practical and theoretical problems. Consider, for example, three different assembly-based procedures for an at-large election with ten candidates running for five seats. In procedure (a), each of the 252 possible assemblies are listed on the ballot and the assembly that receives the most votes is elected. Procedure (a) presents practical difficulties. In procedure (b), each voter marks five candidates on her ballot, ballots are counted as groups of five, and the group that receives the most votes is elected. Under this procedure (as under procedure (a)), the winning assembly might garner a very small percentage of the ballots cast and thus appear undesirable. Voting for candidates, of course, only gives the illusion of performing well in this respect. Although the winning candidates may receive many votes, the resultant assembly may well have received no votes in a direct assembly election. In procedure (c), the ten candidates divide into two slates of five, between which the voters choose by majority vote. Here the electorate might all prefer some other group of five candidates as the assembly. Finally, note that while any procedure in which individuals vote for candidates will be dominated by some assembly-based procedure, there is no presumption that a particular assembly-based procedure is superior to a particular candidate-based procedure.

A second inquiry pursues further the investigation of candidate-based procedures. While no ordinal restriction on individual preferences can justify the use of candidate-based procedures, one might identify restrictions across preferences that assure the good performance of specific procedures. Alternately, one might conjecture that individuals do not perceive much difference between many of the assemblies so that candidate-based procedures do not perform too badly.

We underscore our initial observation: to evaluate an election method, one must start with the recognition that preferences over assemblies, not candidates, are fundamental. Any discussion of preference restrictions must be made on assembly preferences and the selected assembly must be judged with respect to these assembly preferences.

Appendix
Proofs of Propositions and Theorems

Proof of Proposition 2: Suppose the two candidates enter at positions $x,y \neq 0$, and suppose y is elected. This is not an equilibrium. If x enters at position 0 instead, 2/3 of the voters will switch and vote for x, since this will be a dominant strategy for them. The unique subgame perfect equilibrium in undominated strategies is for both candidates to enter at 0. Voters then vote at random and each candidate has a 50% chance of being elected.

Proof of Theorem 1: The proof consists of constructing preference profiles such that the assembly $\{a_1,a_2,\ldots,a_{m-1},a_m\}$ gets elected even though everyone prefers $\{a_1,a_2,\ldots,a_{m-2}, a_{m+1}, a_{m+2}\}$. Assume that all preferences are separable. The proof considers two cases:

Case 1: $k < m$. Let the population be divided into m equal sized groups (plus or minus one, since all groups must be integral numbers) with the following generated candidate rankings:

	Group				
Rank	1	2		m-1	m
1	a_1	a_2	...	a_{m-1}	a_m
2					
.	{In positions 2 through m-2, the first m-2 groups rank candidates from a_1 through a_{m-2} arbitrarily (making sure no one lists the same candidate twice, of course). Groups m-1 and m arbitrarily rank candidates a_2 through a_{m-2}}				
m-2					
m-1	a_m	a_m	...	a_1	a_1
m	a_{m+1}	a_{m+1}	...	a_{m+1}	a_{m+1}
m+1	a_{m+2}	a_{m+2}	...	a_{m+2}	a_{m+2}
m+2	a_n	a_n	...	a_n	a_n
	{the remaining candidates}				
	.	.		.	.
n	a_{m-1}	a_{m-1}	...	a_m	a_{m-1}

Note that for $k<m$, candidates a_1 through a_m each receive at least one vote, and no other candidate receives any votes. Therefore the assembly $\{a_1,a_2,...,a_m\}$ is chosen. However, we can assume that everyone would like to replace $\{a_{m-1}, a_m\}$ by $\{a_{m+1}, a_{m+2}\}$; i.e., these candidate rankings are consistent with assembly preferences which rank $\{a_1,...,a_{m-2},a_{m+1},a_{m+1}\}$ over $\{a_1,...,a_{m-1},a_m\}$.

Case 2: $k=m$. Let the population be divided into four groups of two different sizes (plus or minus 1), p and q, $p < q$, as follows:

	Group Size			
Rank	p	p	q	q
1	a_1	a_1	a_1	a_1
2	a_2	a_2	a_2	a_2
.	.	.	.	.
.	.	.	.	.
m-2	a_{m-2}	a_{m-2}	a_{m-2}	a_{m-2}
m-1	a_{m-1}	a_m	a_m	a_{m-1}
m	a_{m+1}	a_{m+1}	a_{m+2}	a_{m+3}
m+1	a_{m+3}	a_{m+3}	a_{m+3}	a_{m+1}
m+2	a_{m+2}	a_{m+2}	a_{m+1}	a_{m+2}
m+3	a_n	a_n	a_n	a_n
.	[the remaining candidates]			
.	.	.	.	.
n	a_m	a_{m-1}	a_{m-1}	a_m

Again, $\{a_1,a_2,...,a_m\}$ is elected although we can assume that everyone would prefer to replace candidates a_{m-1} and a_m with a_{m+1} and a_{m+2}. ▲

Proof of Theorem 2: (reductio) Suppose that A is elected but B is Pareto superior. Let $A^* = A/(A \cap B)$ and $B^* = B/(A \cap B)$. Consider any individual. Since she likes B at least as much as A, B* must have at least as many elements which are in her block set as A* does. Since she has k votes, she casts at least as many votes among the elements of B* as among A*. This is true for all individuals, so the total number of votes received by the members of B* is at least as great as the total received by the members of A*. In particular, the maximum number of votes received by an element of B* must be greater than the minimum number received by an element of A* (assuming no ties) so that an element of B* must have been elected. A contradiction. ▲

Proof of Theorem 3: The proof is virtually identical to the proofs of theorems 1 and 2.

Proof of Theorem 4: First, it is easy to see that if the voting rule does not involve voting for a subset of a voter's top assembly, an inefficient assembly can be elected. For instance, if k=m and the preferences of all votes are identical, an assembly different from the voters' unanimous first choice will be elected. Therefore, assume that the voting rule involves voting for a subset of a voter's top assembly.

Suppose that k=m. The population divides into three groups, and each individual preference ordering satisfies some neutral restriction. Without loss of generality, assume that each individual in the first group has as favorite assembly $\{a_1,a_2\}\cup B(m-2)$, where $B(m-2)$ is a set of m-2 candidates disjoint from $\{a_1, a_2, a_3, a_4\}$. Furthermore, we can assume that $\{a_1,a_3\}\cup B(m-2)$ is ranked above $\{a_2,a_4\}\cup B(m-2)$. (Suppose instead that $\{a_2,a_4\}\cup B(m-2) > \{a_1,a_3\}\cup B(m-2)$. By neutrality (interchanging a_1 and a_2) a group with favorite assembly $\{a_1,a_2\}\cup B(m-2)$ and $\{a_1,a_4\}\cup B(m-2) > \{a_2,a_3\}\cup B(m-2)$ also satisfies the restriction. Again by neutrality (interchanging a_3 and a_4), a group with favorite assembly $\{a_1,a_2\}\cup B(m-2)$ and $\{a_1,a_3\}\cup B(m-2) > \{a_2,a_4\}\cup B(m-2)$ also satisfies the restriction). Similarly, we assume that the second group's favorite assembly is $\{a_3,a_4\}\cup B(m-2)$ and that they all also rank $\{a_1,a_3\}\cup B(m-2)$ above $\{a_2,a_4\}\cup B(m-2)$. Finally, the third group's favorite assembly is $\{a_2,a_4\}\cup B(m-$

2).

We assume that the three groups are, respectively, of size (V-1)/2, (V-1)/2, and 1 if V is odd, and (V-2)/2, (V-2)/2, 2 if V is even. The assembly $\{a_2,a_4\}UB(m-2)$ is elected, but all but two people prefer $\{a_1,a_3\}UB(m-2)$.

Now suppose that $k<m$. The population divides into m groups. The first m-1 groups are of (approximately) equal size. All members of these groups have $\{a_1,a_2,...,a_{m-1},b_m\}$ as their favorite assembly. Afterwards their preferences differ in such a way that each element of $\{a_1,a_2,...,a_{m-1}\}$ receives at least one vote and only these candidates receive any votes. (This can be done because each voter uses a neutral non-strategic voting rule.) The last group has one member who votes for a_m and candidates drawn from $\{a_1,a_2,...,a_{m-1}\}$. The assembly $\{a_1,...,a_m\}$ is chosen, but all save one voter (possibly) prefers $\{a_1,...,a_{m-1},b_m\}$. ▲

<u>Proof of Theorem 5</u>: Let A be the elected assembly and let A′ be a different assembly. Suppose that the two assemblies first differ in position j. The candidate chosen to fill that post must be someone's most preferred candidate for that post, and that individual prefers A to A′ since preferences are top lexicographic. ▲

<u>Proof of Theorem 6</u>: Obvious. ▲

References

Anscombe, G.E.M., On frustration of the majority by fulfillment of the majority's will, 36 Analysis 161-8 (1976).

Austen-Smith, D. and Banks, Elections, Coalitions, and Legislative Outcomes, 82 Am. Pol. Sci. Rev. 405-422 (1988).

Benoit, J.-P. and Kornhauser, L.A., Voting Simply in the Election of Assemblies mimeo May 1991.

Bezembinder, T and Van Acker, P., The Ostrogorski Paradox and its Relation to Nontransitive Choice, 11 J. Math. Sociology 131-58 (1985).

Daudt, H. and Rae, D., The Ostrogorski Paradox: A peculiarity of compound majority decision, 4 European J. of Political Research 391-98 (1976).

Deb, R. and Kelsey, D., On constructing a generalized Ostrogorski Paradox: Necessary and Sufficient Conditions, 14 Mathematical Social Sciences 161-74 (1987).

Downs, A., **An Economic Theory of Democracy** New York: Harper & Row (1957).

Fishburn, P., An Analysis of Simple Voting Systems for Electing Committees, 41 SIAM J. Applied Math. 499-502 (1981).

Kadane, J.B., On Division of the Question, 13 Public Choice 47-54 (1972).

Presley v. Etowah County Commission, 502 U.S. --, 117 L.Ed 2d 51 (1992).