ECONOMIC RESEARCH REPORTS

OPTIMAL CONTRACT INDEXATION: THE CHANGE IN THE TERMS OF BANK LENDING 1977-1984

by

Clive Bull

and

Antonis L. Karydakis

R.R. #84-14 September 1984

C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

OPTIMAL CONTRACT INDEXATION: THE CHANGE IN THE TERMS OF BANK LENDING 1977-1984

Clive Bull and Antonis L. Karydakis*

Department of Economics
New York University
269 Mercer Street
New York, NY 10003

(212) 598-7891

*Clive Bull's research was supported by the N.S.F. Antonis Karydakis' research was supported by the German Marshall Fund and the C. V. Starr Center for Applied Economics, New York University.

ABSTRACT

This paper describes the dramatic changes that have occurred over the period 1977-1984 in the terms of the short-term, commercial and industrial loans made by U.S. banks. In brief, the average maturity of loans over $1 million made by the 48 largest banks dropped nearly 80% from over three months to under three weeks, while the proportion of such loans that used floating rate loans dropped from 68% to 28%. In contrast, for loans below $1 m. maturity rose as did the proportion using floating rate loans.

A general model of optimal contract indexation, which substantially generalizes Gray's model, is developed and used to explain the changes in the terms of these loans and why these changes differ qualitatively and quantitatively by loan size and by size of bank. We conclude that the change in the real size of loans in each category was the principal cause of these changes with the increased volatility of the Treasury Bill rate and its reduced correlation with market loan rates playing a minor and ambiguous role in the process.

0. Introduction

Over the last five years there has been a substantial increase in the amplitude of high frequency fluctuations in the growth rate of the money supply and the volatility of U.S. interests both of which have usually been attributed to the change in the Federal Reserve's operating procedures in October 1979. While these increases in variability have attracted considerable attention from economists and bankers, the behavior of several other financial variables has changed perhaps even more dramatically during the same period. One area in which very dramatic but unpublicized changes have occurred is in the terms of the short-term (less than one year) commercial and industrial loans of the commercial banks. Such loans make up almost 95% of all of the commercial and industrial loans made by large banks (the 48 largest banks by assets) and so any change in the terms of lending in this sector has a major impact on industry. To get a flavor of these changes the weighted average maturity of the large (greater than $1 m.) short-term commercial and industrial loans made by the 48 largest banks was 3.2 months in 1977. In 1983 the average maturity of the same category of loans was only 0.7 months. Perhaps even more surprisingly, the proportion of these loans which carried floating interest rates dropped over the same period from 68% to 28%. What is more, the changes of maturity of these loans and the proportion of them that carried floating rates differs both qualitatively and quantitatively across the size of loans (greater or less than $1 m.) and across the size of banks (the 48 largest versus the rest).

This paper has two major objectives. The first, carried out in Section 1, is to describe the changes in bank lending practices between the late seventies and 1984 and the qualitative variation in these changes across banks and sizes of loan. These have to our knowledge gone largely unnoticed in the economics and banking literatures despite the fact that they possibly indicate an unforeseen impact of the Federal Reserve's change in monetary policy. Moreover, to the extent that corporations' real activities are affected by the structure of their liabilities this change in bank lending practices must have had a real impact.

The second objective of the paper is to explain with a single model the different changes in maturity and the use of floating rates across bank and loan sizes. We do this by applying a model of optimal indexation of contracts which is an adaptation and improvement on the classic model in this area due to Gray (1978). In our model the parties to the loan contract wish to keep the contractual loan interest rate equal to the market loan interest rate. Informational and strategic reasons prevent direct and perfect indexation of the contractual rate to the market rate and force instead indexation to the 90-day Treasury Bill rate. The optimal degree of indexation to the T bill rate can (cf. Gray) lie outside the unit interval and indeed was greater than unity prior to October 1979. As very significant differences in the maturity of loans and the use of indexation are observed across loan size, we generalize Gray's model to allow for variations in the size of the trades governed by the contract. While, in general, the impact of changing the size of the loan on the use of indexation is ambiguous, under the reasonable assumption that the indexing costs over the life of the loans are lower than the costs of negotiating the loan (or rolling it over) we obtain an inverse relationship between loan size and indexation of the loan rate. At a rather more technical level we improve on Gray's model by trying to minimize the discounted costs associated with contracting rather than minimizing the average per period costs. Although this complicates the analysis somewhat, present value maximizations or minimizations are usually preferred by economists. More importantly they allow one to study the impacts of changing discount rates on the optimal contract. Finally, we study in a more general setting than Gray the impact of increasing the noise in the variable to which the price of concern is indexed on the indexing/fixed price choice.[1]

While we believe these extensions to the theory of indexation to be of value in their own right, the importance of this analysis is that it provides the first attempt (to our knowledge) to use the theory of optimal contract indexation to explain actual, and important, empirical changes in contractual terms. This explanation is given in section 3. Section 4 contains some concluding remarks.

---

[1] Gray treats a special case in which the optimal degree of indexation is fixed independently of the level of noise. As noted, she also does not allow for discounting.

1. The Changes in the Terms of Commercial and Industrial Bank Loans[2]

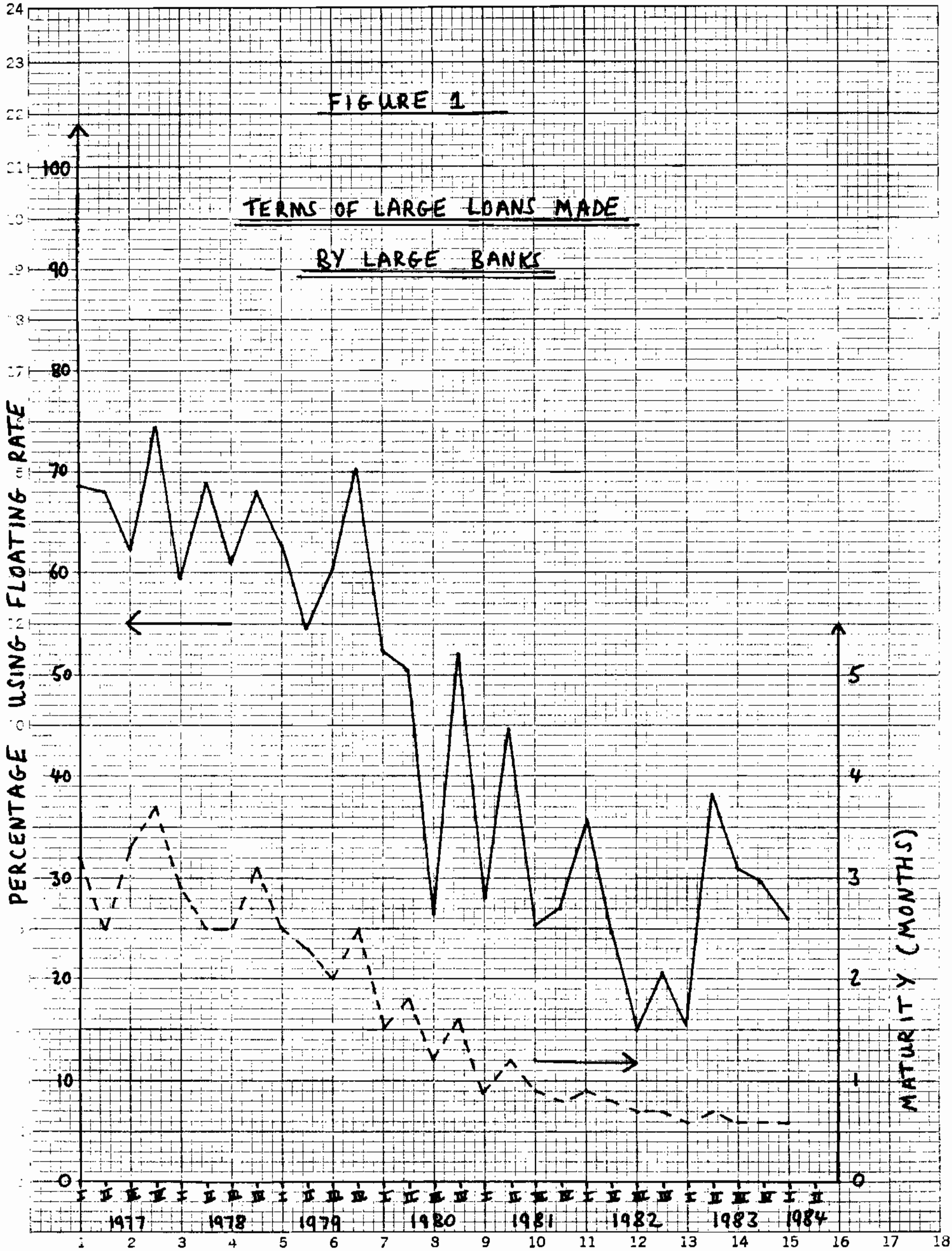
Throughout this paper the term "loans," unless explicitly stated otherwise, refers to the short-term (initial maturity of less than one year) commercial and industrial loans made by banks. This is the single largest category of loans made by banks. In the week February 6-10, 1984, $38.3 billion of such loans were made representing 84% of the total loans made by banks in that week. Notice that as these loans are commercial and industrial rather than financial they are directly related to real economic activity.

This class of loans is subdivided along two dimensions: by the size of the loan and by the size, in terms of assets, of the banks making the loans. By size of loan we refer to loans of $1 million and over as "large" and to loans of less than $1 million as "small." While there are differences in behavior within the small loans category, the most dramatic contrast is between the changes in the terms of lending of the large and small loans. Along the bank size dimension we will refer to the 48 largest banks as "large" banks and the other banks as "small." Of course, the small banks can in fact be large institutions. For instance, in early 1983 the largest of the small banks had assets of almost $2.7 billion. However, this division of banks is dictated by the source of the data for this paper which is the Statistical Release E2 (Survey of the Terms of Bank Lending) produced by the Board of Governors of the Federal Reserve System.[3] We have then four different groups of

---

[2]For a much more detailed description of the changes outlined in this section, see Karydakis (1984) upon which this section is based.

[3]Hereinafter referred to as Statistical Release E2. This data has been collected since 1977. Prior to 1979 it was called Statistical Release G14.



5 Squares to the Centimeter

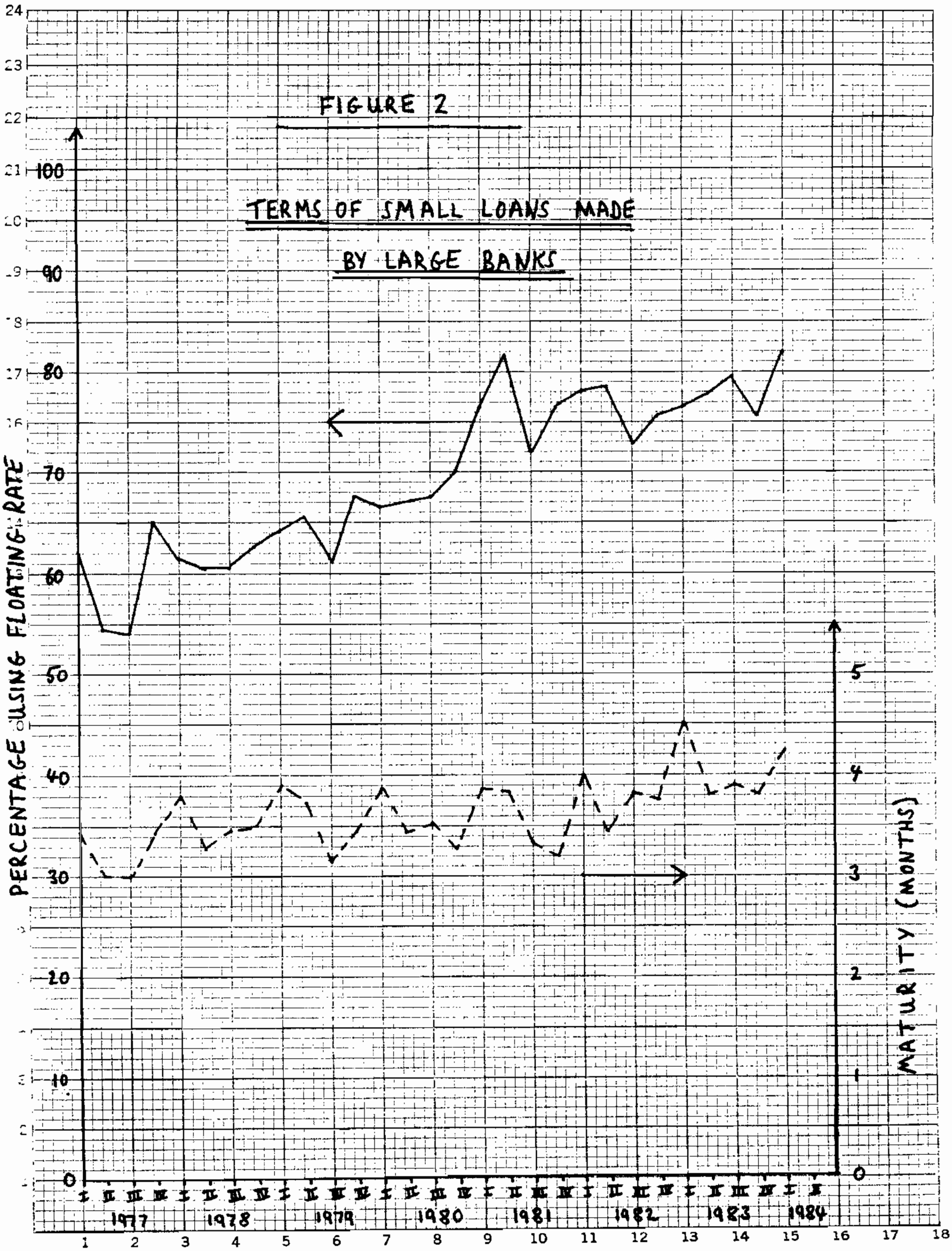
loans according to whether the bank and the loan are small or large. First we will describe the changes in the terms of lending of the large banks.

The large banks make relatively few large loans, but because of their size such loans dominate their loan portfolio. Moreover, this domination has grown rapidly in recent years. In 1977, 68.6% of the large banks' short-term, commercial and industrial loans by value were large. By 1983, this number had grown to 95.8%. Thus when we talk of the changing terms of lending of the large banks we are effectively talking about the changes in their terms for large loans. These changes are depicted in Figure 1. Clearly there has been a very major reduction in both the average maturity of these loans and the use of floating rate loans. In 1977, large loans had an average maturity of 3.225 months and 68.2% of these loans had floating interest rates. By 1983, the average maturity had dropped by over 75% to 0.725 months and the proportion that used floating interest rates had more than halved to 28.5%. Not only do the series for average maturity and the proportion of loans with floating rates show the same trend behavior but they also appear to be highly correlated at higher frequencies. This is borne out by the following regression result. Using quarterly observations (this is the frequency of the surveys contained in Statistical Release E2) for the period 1977.I - 1984.I, a regression of the proportion of large loans at large banks using floating rates (FLL_t) on time (T) and their average maturity (MLL_t) yielded (t statistics in parentheses),

$$FLL_t = \underset{(1.34)}{19.32} + \underset{(3.93)}{16.65}\, MLL_t - \underset{(0.27)}{0.13}\, T$$

$$F = 147.27 \qquad R^2 = 0.85 \qquad DW = 1.85$$

12-189



5 Squares to the Centimeter

The behavior of the terms of small loans at the large banks as depicted in Figure 2 contrasts sharply with that of the large loans. In 1977 the average maturity of these small loans was 3.223 months, while 60% of these small loans used floating rates. By 1983 the average maturity of the small loans had risen to 4.02 months and the proportion using floating rates had risen to 77.3%. These are exactly the opposite of the changes in the terms for the large loans. Moreover, the changes in the terms of lending were far greater for the smaller loans rather than the larger loans within the small loan category. Between 1977 and 1983 the average maturity of loans in the $1,000 - $24,000 group rose from 3.63 months to 4.5 months, while in the $500,000 - $999,999 group maturity grew little, rising from 3.0 to 3.5 months over the same period. Thus for very small loans maturity rose over the period, for very large loans it fell, and for those of one half to one million dollars it remained roughly constant. The same type of behavior occurred with respect to the use of floating rates. The proportion of loans in the $1,000 - $24,000 group that used floating rates rose from 42.1% in 1977 to 71.2% in 1983. In the $500,000 - $999,000 group, however, the rise in the use of floating rates was very modest, from 69.2% in 1978 to 71.7% in 1983. So again we see that the change in the terms of lending vary directly with the size of loan. In this case, very small loans adopted the use of floating interest rates while very large loans abandoned it. Loans of around $1 million appear to be the switch point between these positive and negative changes.

A comparison of Figures 1 and 2 show that there is a considerably weaker link between the changes in maturity and floating rates for the

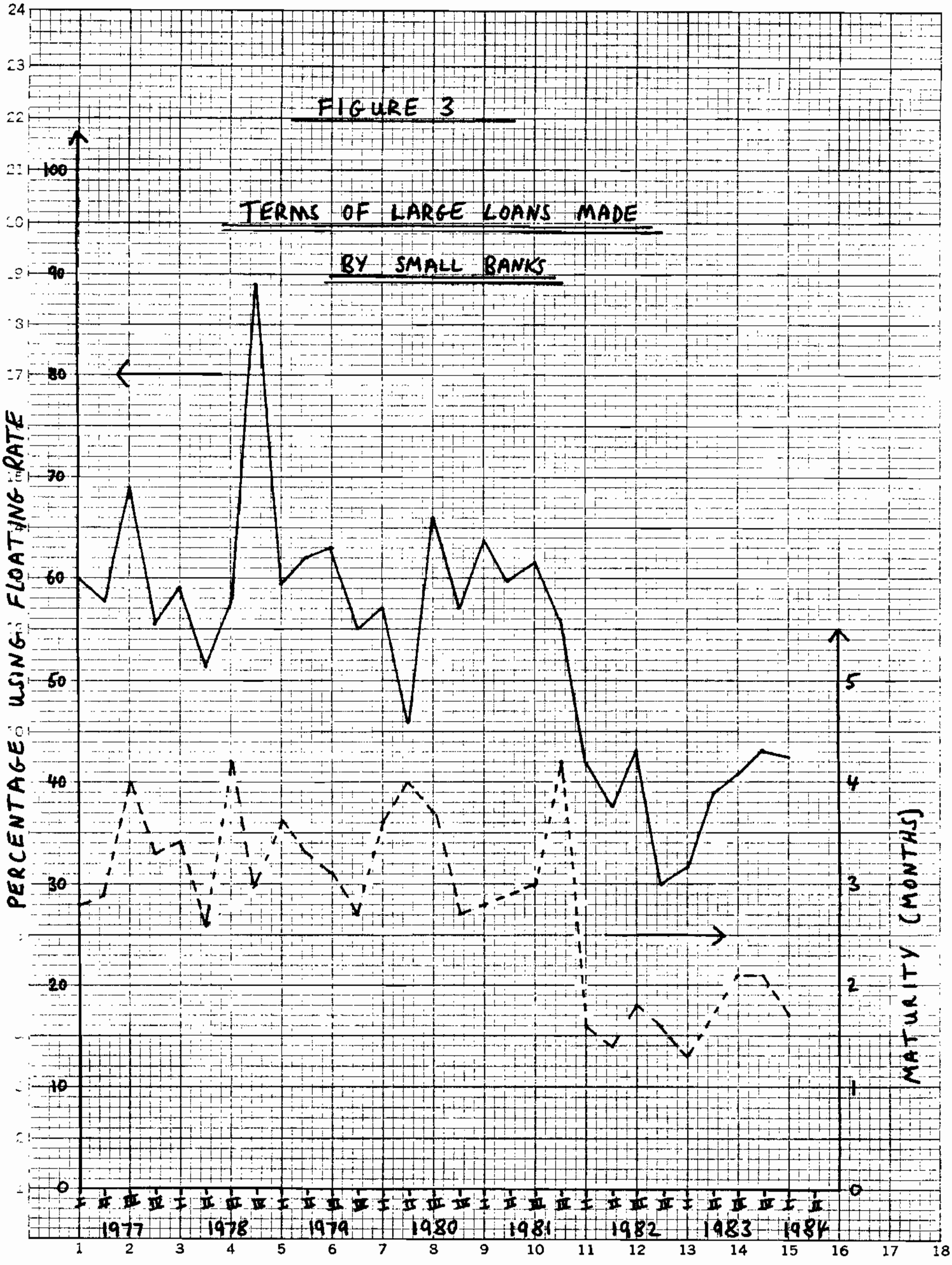


FIGURE 3

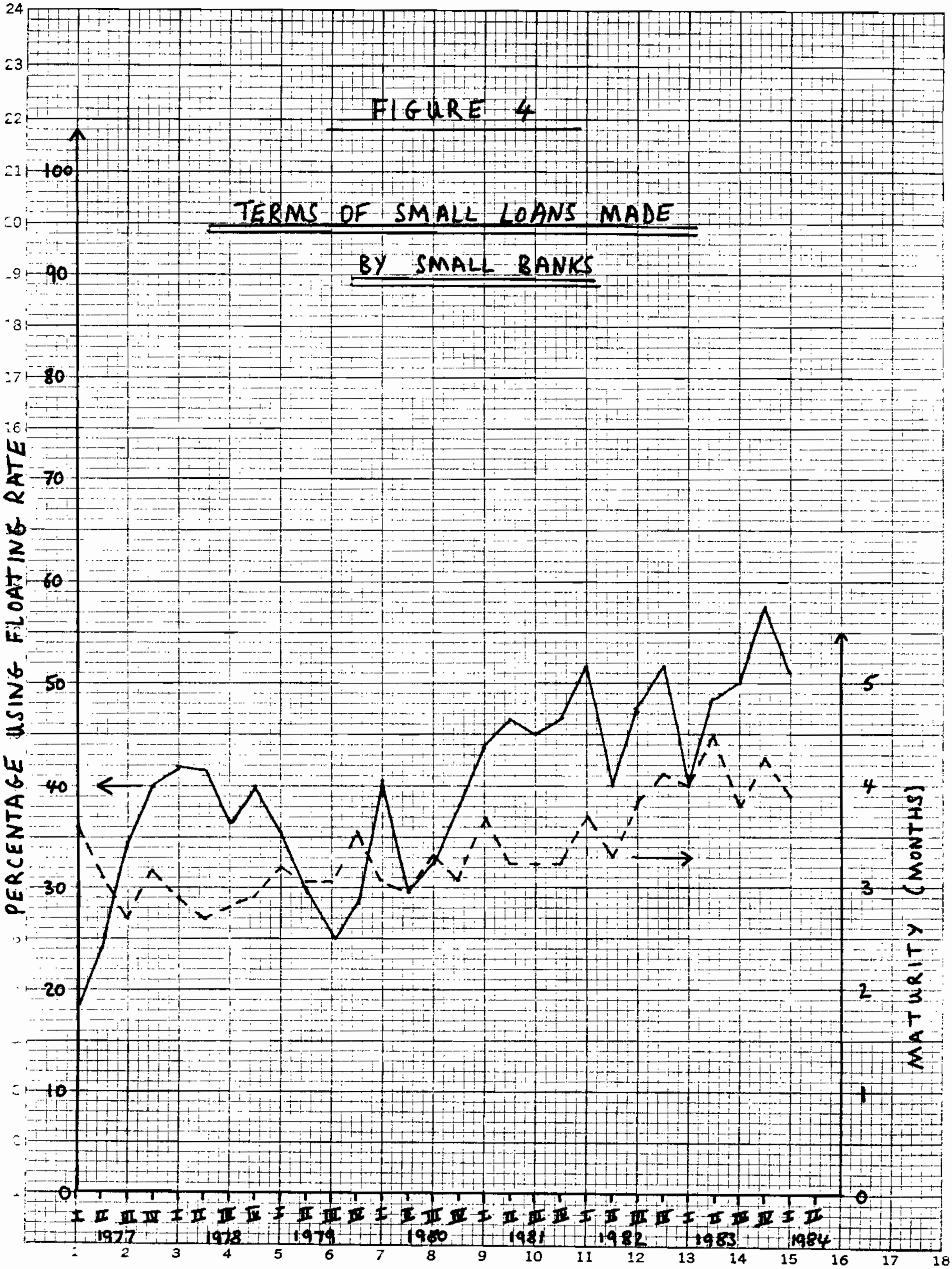
TERMS OF LARGE LOANS MADE BY SMALL BANKS small loans compared with the large loans. This arises from mixing the qualitatively different behaviors of the larger and smaller loans with the group of all small loans. Quantitatively this is shown in the regression below in which FLS_t and MLS_t represent the terms of lending of the small loans of the large banks (t statistics in parentheses).

$$FLS_t = \underset{(6.48)}{49.97} + \underset{(0.88)}{2.04}\, MLS_t + \underset{(8.73)}{0.81}\, T$$

$$F = 62.21 \qquad R^2 = 0.83 \qquad DW = 1.55$$

The small banks, unsurprisingly, make smaller loans than the large banks. Large loans accounted for only 23% (in value terms) of the small banks' loans in 1977, although this had risen to 49.1% by 1983. This tendency for small banks to make small loans also shows up within the small loans class,with the smaller subcategories of loans being much more important for the small banks than the large banks. Considering large loans, the terms of the small banks were comparable to those of the large banks. In 1977, the average maturity of a large loan made by a small bank was 3.25 months, and 60.7% of such loans used floating rates. By 1983 average maturity had shortened to 1.8 months and only 38.6% of large loans bore floating interest rates. The only major difference, then, between the large loans of the two classes of banks were that the large loans of small banks were consistently of longer maturity than those of the large banks and both the declines in maturity and the use of floating interest rates were somewhat less dramatic for the small banks.

Figure 3 shows the behavior of the terms of lending over the period for the large loans of the small banks. A comparison of Figures 1 and 3 shows clearly that although the trend changes in the terms were similar



5 Squares to the Centimeter

for large and small banks, the variations within the period were very different. This is most notable for loan maturity, which declines quite smoothly for the large banks but looks much more like a step function, with the step in the last quarter of 1981, for the small banks. Moreover, the very close relationship between changes in maturity and the use of floating rates that was found for large banks is much less close for small banks, as the regression below indicates. Here FSL_t and MSL_t are, respectively, the proportion of floating rate and the average maturity of the large loans of small banks (t statistics in parentheses).

$$FSL_t = \underset{(4.50)}{44.93} + \underset{(2.50)}{6.20}\, MSL_t - \underset{(2.24)}{0.57}\, T$$

$$F = 16.33 \qquad R^2 = 0.56 \qquad DW = 2.02$$

The behavior of the terms of lending for small loans is shown in Figure 4. Again the changes over 1977-1983 qualitatively mimic those for the large banks and are also comparable in magnitude. A regression of the proportion of small loans using floating rates (FSS_t) on average maturity (MSS_t) yields results very close to those of the small loans at large banks,

$$FSS_t = \underset{(3.26)}{35.57} - \underset{(0.74)}{2.88}\, MSS_t + \underset{(4.35)}{0.94}\, T$$

$$F = 17.25 \qquad R^2 = 0.57 \qquad DW = 1.12 \qquad \rho = 0.37$$

The only significant difference between the two types of banks is that the maturity and the use of floating rates was lower for the small banks than for the large. This is peculiar because if one looks at maturity across loan size for the large banks, smaller loans correspond

Table 1

Changes in the Terms of Bank Lending, 1977-1983

	Weighted (by value) Average Maturity of loans (months)		Proportion of Loans Using Floating Interest Rates	
	Large Loans (≥ $1m.)	Small Loans (< $1m.)	Large Loans (≥ $1m.)	Small Loans (< $1m.)
Largest 48 Banks ("large banks")	3.2 → 0.7	3.2 → 4.0	68% → 28%	60% → 77%
Other Banks ("small banks")	3.3 → 1.8	3.1 → 4.1	61% → 38%	29% → 49%

to longer maturities. If this pattern were followed in the small banks, given that their large loans are of longer maturity than those of large banks, we would expect small banks' loans to be of longer average maturity than those of the large banks. In fact there seems to be little or no correlation between size of loan and maturity within the small loan category for the small banks.

The major movements of the terms of lending over the period are summarized below in Table 1. Although this summary ignores many of the interesting details of the time paths of the terms of lending and of the variations across finer subdivisions of loans, it is valuable because it brings home the startling movements in these terms over a seven-year period. The purpose of the rest of this paper is to explain the qualitative changes presented in Table 1. More precisely, we will try to answer the following questions:

(i) Why did the maturity and the use of floating interest rates fall at both types of banks for large loans?

(ii) Why was the above fall smaller at the small banks?

(iii) Why did the maturity and the use of floating interest rates rise at both types of banks for small loans?

2. A Model of Optimal Indexation and Contract Length

On being presented with the data contained in Table 1 most people express surprise that such major changes have occurred in the terms of lending, especially the massive changes in the terms of the large loans of large banks. What is most puzzling about these changes is the fall in the use of floating rate loans in the face of very large increases in the volatility of short-term interest rates after October 1979. This increased volatility raises the interest rate risk on a fixed rate loan of given length and so presumably would raise the demand for some form of loan interest rate indexation.[4] On reflection this apparently perverse behavior with regard to indexation is "explained" by pointing to the 80% reduction in the initial maturity of loans over the period. Obviously this greatly reduces the interest rate risk of a loan and could well more than offset the effect of the increased interest rate volatility, thereby reducing the use of floating rate loans. The problem with this explanation is that it treats one aspect of the outcome of the bargaining between a bank and a borrower, the maturity of the loan, as exogenous and uses it to explain another aspect of the outcome, namely, whether the loan carries a fixed or floating rate. Moreover, even if we accepted the exogeneity of loan maturity we would then have to live with the fact that this variable moved in different directions for different loans.

Similar objections can be raised to the other commonly offered explanation which appeals to the rise of the commercial paper market. This market is an intercorporate loan market where the typical instrument

---

[4] As we will see shortly, this intuition is very imprecise.

is fixed rate paper of about one month maturity. The volume (at 1977 prices) of paper outstanding on this market rose from slightly under $15 billion in 1977 to a peak of approximately $40 billion in mid 1982. Because only large corporations have credit ratings that are public information, only they have access to the corporate paper market, which means that the market is most closely competitive with the large loans made by large banks. The proposed explanation for the behavior of the terms of such loans then goes as follows. The commercial paper market grew rapidly (at least until mid 1982) because it offered paper that corporations increasingly wished to issue and buy. Given this change in tastes, the competition from the commercial paper market forced banks to offer large loans with similar terms, i.e., fixed rate loans of about one month maturity. The weakness of this explanation is that it appeals to an exogenous change of tastes for the terms of loans and yet these "tastes" should simply be the optimal programs resulting from an underlying profit maximization problem. Moreover, it entirely fails to explain the change in terms of small loans.

Both of the above explanations fail to treat the maturity of the loan and whether it is indexed or not as endogenous results of an optimization problem. Recently models have been developed, notably that of Jo Anna Gray (1978), to do precisely this and so it makes sense to look first at what such models would predict would happen to the terms of bank loans over this period. The rest of this section develops a model which extends and generalizes Gray's to do exactly this.

Consider a firm j and a bank. At the current market interest rate for loans i_t the firm and bank agree on a loan size L_j. The cost of signing this agreement[5] is $K_j > 0$, which is independent of the size and length of the loan and so the cost, per dollar borrowed, of arranging the loan is $k_j \equiv K_j/L_j$. In accordance with the efficient markets hypothesis let the market loan rate evolve according to a Wiener process with a variational term $\sigma_i dt$. This means that at t the covariance between i_t and i_{t+s}, $s > 0$, is simply $\sigma_i^2 s$. Let us assume that the firm expects to borrow loan L_j forever. This can be done at one extreme by taking out an infinite length loan (e.g., U.K. government "consols") or at the other extreme by signing a new loan every instant. The latter strategy is clearly suboptimal given that the signing costs are independent of the length of the loan. Whether the infinite length loan is optimal or whether it is better to roll over a finite length loan depends on the loss function associated with deviations of the market loan rate from the loan rate in the loan contract. If these two rates diverge there will exist a misallocation of resources and consequently the usual triangle of welfare loss which represents foregone gains from trade which could be captured by the firm and the bank. Unfortunately, in order to get closed form solutions for the optimal length of the contract and degree of indexation we are forced to use a quadratic loss function as in Gray (1978). In the absence of indexing we can write the expected losses per dollar associated with writing a fixed interest rate loan L_j of length T, v(t;T), as

$$v_j(t;T) = k_j + \int_t^{t+T} E_t(i_\tau - i_t)^2 e^{-\rho(\tau-t)} d\tau + e^{-\rho T} E_t v_j(t + T;T') \qquad (1)$$

[1]Note that both the bank and the firm take the market loan rate as given. The signing costs involve credit rating, management time, legal fees, etc.

where E_t is the expectation operator conditional on information available at t. Define the value of $v_j(t;T)$ when T is chosen to minimize $v_j(t;T)$ as $V_j(t;\hat{T})$.[6] Given the stationarity of the problem we know that the minimized value of $v_j(t + \hat{T};T')$ is $V_j(t + \hat{T};\hat{T}) = V_j(t;\hat{T})$. Thus

$$V_j(t) = \underset{T}{\text{Min}}\ (1 - e^{-\rho T})^{-1}[k_j + \int_t^{t+T} E_t(i_\tau - i_t)^2 e^{-\rho(t-\tau)} d\tau$$

or

$$V_j(t) = \underset{T}{\text{Min}}\ (1 - e^{-\rho T})^{-1}[k_j + \sigma_i^2 \int_0^T \tau e^{-\rho\tau} d\tau] \tag{2}$$

Carrying out the minimization[7] in (2) yields the following expression for the optimal contract length, T_j^*:

$$T_j^* = \frac{\rho V_j(t)}{\sigma_i^2} \tag{3}$$

The intuition behind (3) is as follows. Consider arriving at T_j^* and asking whether to extend the contract by dt. From the point of view of t we would expect such an extension to raise the losses incurred by the difference between the contract and market loan rates by $T^*\sigma_i^2 dt$. However, by not terminating the loan and beginning a new one we are foregoing $\rho V_j(t)dt$ costs. (3) simply equates these marginal costs and benefits. Note that by using the convexity of $V_j(t)$ it is easy to show that $dT_j^*/dk_j < 0$ and $dT_j^*/d\sigma_i^2 < 0$ as one would expect.

---

[6] Uniqueness is ensured by the strict convexity of $v_j(t,T)$ in T.

[7] At several points in this section the differentiability of the value function is assumed. Differentiability of the appropriate order can be demonstrated in all cases using theorem 3.1 of Blume, Easly and O'Hara, 1982.

Given the loss function there is clearly an incentive to index the loan rate in the loan contract. The natural rate to which to index the contractual loan rate would be the market loan rate. This is never observed. Instead almost all the loans under consideration are tied to the 90-day T bill rate. The T bill rate has the advantage that it is determined on a large auction market for a homogeneous, risk-free asset. Moreover, the T bill rate is instantaneously available at very low cost. In contrast the loan market is not an auction market because the loans vary in size and in quality, i.e., default risk. It is also costly to obtain quotations on loans and this information is usually private. In short, it would be both expensive and strategically unwise to index to the spot loan rate.

Let r_t denote the 90-day T bill rate at t and let the rate evolve according to a Weiner process with variational term $\sigma_r dt$. We will restrict our analysis to unconditional,[8] linear indexing rules of the form $i^c_{t+s} = i_t + \gamma(r_{t+s} - r_s)$, $s > 0$, where i^c_{t+s} denotes the interest rate in the loan contract signed at t that must be paid at t + s. Indexing is, however, costly as the adjustment of the loan rate involves management time and accounting costs. Let the per unit time cost of updating the loan rate be $C_j > 0$, which is independent of the loan size. Therefore, the cost per dollar of the loan, per unit time, of updating the loan rate is $c_j \equiv C_j/L_j$. Notice that this means that the total cost of indexing a loan, $c_j\int_0^T \tau e^{-\rho\tau}d\tau$, unlike the signing cost, depends

[8]The indexing rule is conditional on neither i_{t+s}, r_{t+s} or s. This does violence to the empirical loan contracts only by imposing continuous updating of i^c. In fact the loan rate is updated at predetermined intervals.

linearly on the length of the contract. This is unrealistic. The indexing costs occur in "lumps" at the time of updating the interest rate on the loan. However, treating indexing costs symmetrically with signing costs greatly complicates the analysis.

We can rewrite (2) to allow for using a floating rate as

$$V_j^F(t) = \underset{T,\gamma}{\text{Min}} \{k_j + \int_t^{t+T} E_t[i_\tau - i_t - \gamma(r_\tau - r_t)]^2 e^{-\rho(\tau-t)} d\tau + $$

$$+ c_j \int_t^{t+T} e^{-\rho(\tau-t)} d\tau\}(1 - e^{-\rho T})^{-1}$$

or

$$V_j^F(t) = \underset{T,\gamma}{\text{Min}} \{k_j + [\sigma_i^2 + \gamma^2\sigma_r^2 - 2\gamma\sigma_{i,r}]\int_0^T \tau e^{-\rho\tau} d\tau\}(1 - e^{-\rho T})^{-1} + \frac{c_j}{\rho} \qquad (4)$$

where $\sigma_{i,j}$ is the "instantaneous" covariance between i and r.

Minimizing (4) with respect to γ yields the optimal value of $\gamma, \hat{\gamma}$, which is independent of T, k_j and c_j

$$\hat{\gamma} = \sigma_{i,r}/\sigma_r^2 \qquad (5)$$

Note that $\hat{\gamma}$ can be greater than one in absolute value. Substituting (5) into (4) yields

$$V_j^F(t) = \underset{T}{\text{Min}} \{k_j + \sigma_i^2(1 - \beta^2)\int_0^T \tau e^{-\rho\tau} d\tau\}(1 - e^{-\rho T})^{-1} + \frac{c_j}{\rho} \qquad (6)$$

where β is the (contemporaneous) correlation coefficient between i and r. This minimization gives the optimal contract length, $\hat{T}$,

$$\hat{T}_j(1 - e^{-\rho\hat{T}_j}) = \frac{\rho k_j}{\sigma_i^2(1-\beta^2)} + \rho\int_0^{\hat{T}_j} \tau e^{-\rho\tau}d\tau \qquad (7)$$

If we rewrite (7) as

$$\hat{T}_j\sigma_i^2(1-\beta^2) + c_j = \rho V_j^F(t)$$

we see that the intuitive interpretation of $\hat{T}_j$ is exactly the same as that for T_j^* once we allow for the impact of indexing in reducing the per period interest rate loss but raising the per period transaction costs. From (7) we see that, as we would expect, $d\hat{T}_j/dk_j > 0$, $d\hat{T}_j/d\sigma_i^2 < 0$, $d\hat{T}_j/d|\beta| > 0$. Note also that c_j has no impact on $\hat{T}_j$.[9] As a loose check on the reasonableness of (7) we might note that if the cost of signing a loan contract, k_j, is zero, (7) implies that $\hat{T}_j = 0$. Conversely, if $\beta = +1$ or -1, $\hat{T}_j = \infty$. If on the other hand $\beta = 0$, i.e., indexing buys you nothing, (7) can be shown to imply that $\hat{T}_j = T_j^*$. Finally, notice that there is a clear ranking of $\hat{T}_j$ and T_j^*. Provided σ_i^2, σ_r^2, β and $k_j > 0$, $\hat{T}_j > T_j^*$. To prove this it is enough to note that at $c_j = 0$, $V_j(t) = V_j^F(t)$ and so from (3) and (7) $T_j^* < \hat{T}_j$. As T_j^* and $\hat{T}_j$ are independent of c_j this implies that $T_j^* < \hat{T}_j$ for all values of c_j.

The final task in this section is to establish when a floating rate will be adopted and how this decision is affected by certain parametric changes. The decision itself is quite straightforward. Given the parameters, the bank and firm simply compare $V_j(t)$ and $V_j^F(t)$ and adopt a floating rate if, and only if, $V_j^F(t) < V_j(t)$. While the decision itself is simple the comparative statics of the decision are far less obvious.

---

[9]This is a peculiarity due to the asymmetric cost structures of signing and indexing.

Specifically, we will analyze how the indexation decision changes with changes in the volatility of the market loan rate, σ_i^2, the correlation between the loan rate and the T bill rate, β, and the size of the loan, L_j.

Intuitively one would expect an increase in the volatility of the market loan rate to raise the demand for floating rate loans because such an increase raises, for any given length of loan, the expected loss from a fixed rate loan by a proportion β^2 more than that for a floating rate loan. This intuition is, however, seriously incomplete as it ignores the fact that the optimal length of a floating rate loan is greater than that of a fixed rate loan. Thus the increased loss incurred by the rise in σ_i^2 is incurred for longer under a floating rather than a fixed rate. These two opposing forces show up clearly in the algebra. Taking the partial derivatives of $V_j^F(t)$ and $V_j(t)$ (equations 6 and 2) with respect to σ_i^2 yields

$$\frac{\partial V_j^F(t)}{\partial \sigma_i^2} = (1 - e^{-\rho\hat{T}})(1 - \beta^2)\int_0^{\hat{T}} \tau e^{-\rho\tau} d\tau \tag{8}$$

$$\frac{\partial V_j(t)}{\partial \sigma_i^2} = (1 - e^{-\rho T^*})\int_0^{T^*} \tau e^{-\rho\tau} d\tau \tag{9}$$

Recalling that $\hat{T} > T^*$ for all strictly positive values of the parameter vector it is not obvious from (8) and (9) whether a rise in σ_i^2 will have a bigger impact on the costs of a fixed or a floating rate loan. Indeed, it is straightforward to pick parameter values such that a rise in σ_i^2 raises $V_j^F(t)$ by <u>more</u> than $V_j(t)$, i.e., a rise in the variance of the market loan rate reduces the relative attractiveness of a floating rate loan.

Fortunately we are interested in the impact of a change in σ_i^2 on the decision of a firm to take a floating or a fixed rate and so we can confine our attention to firms who are currently indifferent between using either type of loan. This is a great help because for such a firm $V_j^F(t) = V_j(t)$ and so from (3) and (7) we know that the optimally chosen lengths of loans for such a firm will be related by

$$\hat{T}(1 - \beta^2) + \frac{c_j}{\sigma_i^2} = T^* \tag{10}$$

Thus for a firm that is indifferent between the two types of loans the optimal lengths of the loans cannot diverge too much and in particular $T^* > (1 - \beta^2)\hat{T}$. This restricts the adverse impact of σ_i^2 on $V_j^F(t)$ and enables us to get definite comparative static results. The proof of the results is tedious and so is relegated to the appendix. Here we just state the result that

$$\frac{\partial V_j^F(t)}{\partial \sigma_i^2} - \frac{\partial V_j(t)}{\partial \sigma_i^2} < 0 \quad \text{at} \quad V_j^F(t) = V_j(t). \tag{11}$$

In other words a rise in the variance of the market loan rate will cause a previously indifferent firm to choose a floating rate loan.

The impact of a change in $|\beta|$ is considerably more direct. As $|\beta|$ increases the gains from indexing rise, i.e., $\partial V_j^F(t)/\partial \beta^2 < 0$, while $V_j(t)$ is unaffected. Thus an increase in the absolute value of the contemporaneous correlation coefficient between the market loan rate and the T bill rate reduces the expected loss on a floating rate loan thereby causing a previously indifferent firm to change to a floating rate loan.

Finally, we wish to assess the impact of a change in loan size. For a firm with a fixed rate loan the impact of an increase in L_j is to lower per dollar signing costs k_j. Thus, from (2)

$$\frac{\partial V_j(t)}{\partial L_j} = \frac{-k_j}{L_j}(1 - e^{-\rho T^*})^{-1} < 0 \tag{14}$$

For a firm using a floating rate loan the rise in loan size also lowers the per period per dollar indexing cost c_j. Thus from (4)

$$\frac{\partial V_j^F(t)}{\partial L_j} = \frac{-k_j}{L_j}(1 - e^{-\rho\hat{T}})^{-1} - \frac{c_j}{L_j}\frac{1}{\rho} < 0 \tag{15}$$

Unfortunately, nothing can be said analytically about the ranking of (14) and (15) for a firm initially indifferent between floating and fixed rates. Certainly if $c_j = 0$, then $\frac{\partial V_j(t)}{\partial L_j} < \frac{\partial V_j^F(t)}{\partial L_j}$ and the firm would shift to fixed rates. However, for a given β one can find a c_j sufficiently large that this result can be reversed and so in order to obtain definite results we must put some restrictions on the values the parameters may take.

Given that the indexing cost is a purely accounting cost and given the use of computers it seems reasonable that the capital value of indexing costs, $c_j/\rho \leq k_j$. If we set $c_j/\rho = k_j$ then $\partial V_j(t)/\partial L_j < \partial V_j^F(t)/\partial L_j$ if and only if $(1 - e^{-\rho T^*})^{-1} - (1 - e^{-\rho\hat{T}})^{-1} > 1$. Using the argument made previously, for a firm that is indifferent between floating and fixed rates $T^* > \hat{T}(1 - \beta^2)$. Taking a value for ρ of 0.1 per annum, a value of β of 0.9[10] and setting $T^* = \hat{T}(1 - \beta^2)$ we find that $\hat{T}$ must be greater than 30 years for the above inequality to be violated. Thus we will assume that in the market and time period under consideration a rise in the size of the loan relative to K_j and C_j will lower the incentive to choose a floating rate loan.

We turn now to the explanation of the changes portrayed in Table 1 and the three questions they gave rise to.

---

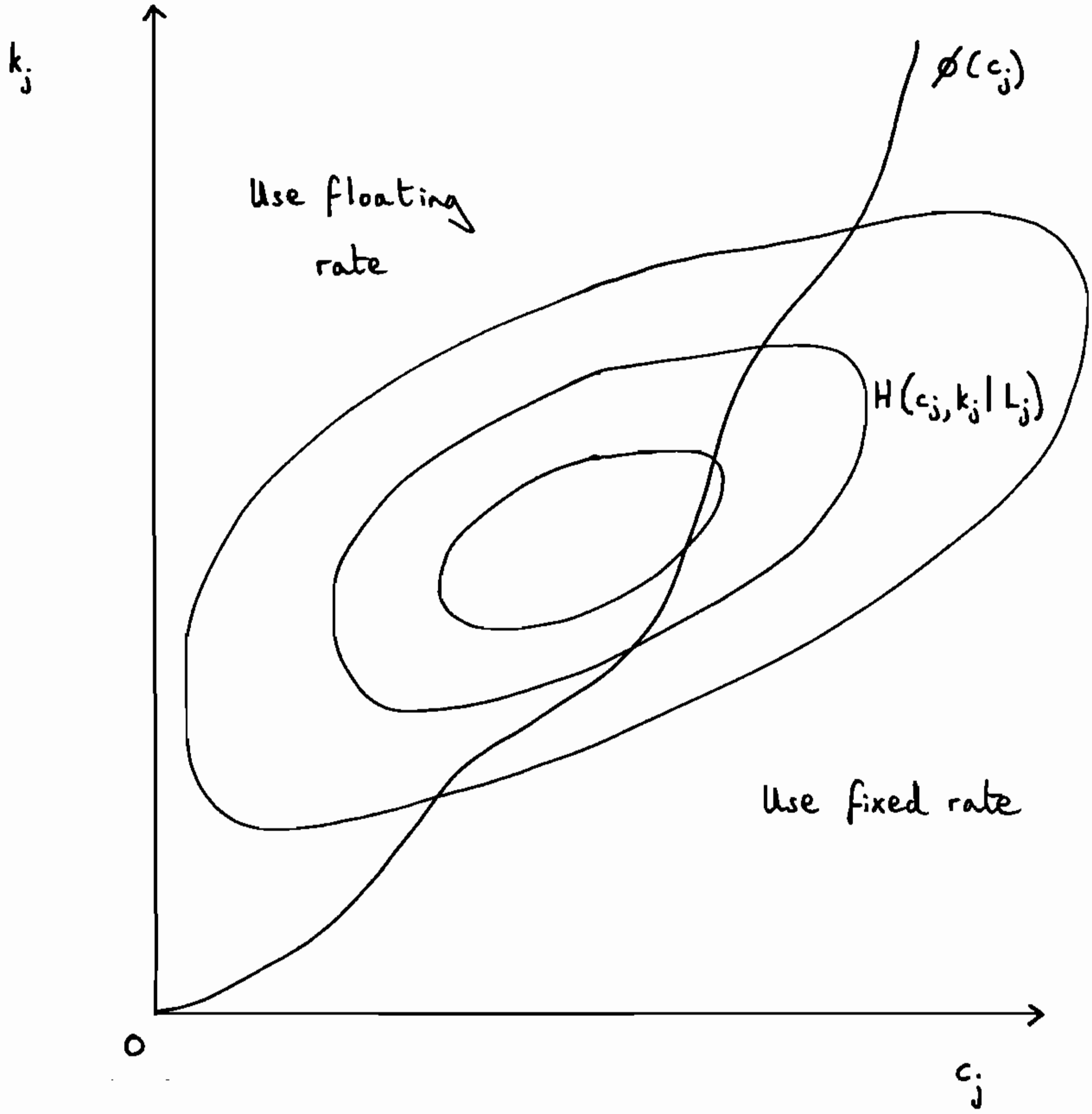
[10]As will be shown in the next section, this is a realistic value for β.

3. Explaining the Data

Notice from Table 1 that in each class of loans there is diversity of behavior with respect to indexing. In each class some loans use floating rates and some use fixed rates. If all firms face the same market parameters this diversity of behavior, given the model of the previous section, must stem from diversity in the costs of rolling over a loan, K_j, and of indexing the loan rate, C_j, within each class of loans. Let us assume then that, independently of the size of loans or the size of banks making them, there is a joint distribution of such costs $G(C_j,K_j)$ within each class of loans. Given the size of loan in that class, L_j, there will be a conditional (on L_j) distribution of per dollar loan costs $H(c_j,k_j|L_j)$. Now consider the locus of L_j sized loans in c_j,k_j space for which firms are indifferent between using a fixed or a floating rate, i.e., for which $V_j^F(t) = V_j(t)$. From (2) and (6) and the fact that $\hat{T} > T^*$ we see that this locus $k_j = \phi(c_j)$ is a smooth, monotonically increasing function. The intuition behind this is that for any given value of k_j because $\hat{T} > T^*$ the discounted per dollar rollover costs involved in the floating rate loan will be less than for the fixed rate loan. The larger is k_j the larger is this cost advantage and so the higher c_j must be in order to make the firm indifferent between the two types of loan. This function together with $H(c_j,k_j|L_j)$ is depicted in Figure 5. Those loans for which (c_j,k_j) lies to the northeast of ϕ will use floating rates while those to the southwest will use fixed rates.

In any given class of loans, for instance the large loans of the large banks, any change in the proportion of loans that use floating rates

FIGURE 5



must come about either from a shift in the ϕ_j function, which in turn can be caused by a change in σ_i^2 or β, or from a shift in the distribution $H(c_j,k_j|L_j)$ which can be caused by a change in the loan size relative to C_j and K_j. Let us first consider changes in the behavior of interest rates.

There are two marked changes in interest rate behavior that occur over the 1977-1983 period. The first, which is quite well known, is that first differences of interest rates became much more variable after October 1979. Taking the lending rates of the 48 largest banks on their large loans and assuming these rates to be a random walk, the variance of the changes in these rates 1977.I - 1979.III was 0.697 while in the period 1979.IV - 1984.I it was 7.720. This change is significant at the 1% level. Using the average daily 90-day T bill rate for the weeks in which the loan survey was carried, we find that the variance of the changes in the T bill rate (σ_r^2) rose from 0.514 to 5.075 across the same time periods.[11] This increase is also significant at the 1% level.

The second marked change in interest rate behavior is that the correlation across interest rates dropped, i.e., spreads became more volatile. In the case of the loan rate and T bill rate described above, the contemporaneous correlation coefficient (β) between the first differences of the rates, assuming both rates to be random walks, was 0.91 in the period 1977.I - 1979.III. In the post October 1979 period (1979.IV - 1984.I) this had dropped to 0.73. Thus even allowing for the

---

[11]In both cases if we drop the assumption of the interest rates being random walks and take instead the variance of the first differences in the rate around the sample mean of first differences, similar significant increases in variance are obtained.

increased variance of the changes in the loan and T bill rates the spread between the two became considerably more variable. This would have a major impact on the use of floating rate loans and the maturity of loans in this class of loans. First, note that changes in β affect only $V_j^F(t)$ and $\hat{T}$ but as almost 70% of the large loans of the large banks used floating rates in 1977 this does not diminish the impact of the drop in β. From the comparative static results of the previous section we know that $\partial V_j^F(t)/\partial|\beta| < 0$ and so the fall in β will raise the cost of using a floating rate loan for any pair (c_j, k_j). Thus in terms of Figure 5, the function ϕ rotates counterclockwise[12] thereby reducing the proportion of loans use floating rates.[13] The reduction in β also has a direct and indirect effect on loan maturity. From (7) we see that for the 70% of loans that used a floating rate, the fall in β would reduce the optimal loan length $\hat{T}$. The indirect effect on maturity is that those loans that switch from floating to fixed rates will switch from length $\hat{T}$ to T^* where $T^* < \hat{T}$. So as well as a marginal reduction in loan length for all floating rate loans there will be a discrete drop in loan length for those loans which switch to a fixed rate. Moreover, we know that for firms that are indifferent between using floating and fixed rate loans $T^* \simeq (1 - \beta^2)\hat{T}$. If $\beta \simeq .8$, then $T^* \simeq 0.36\hat{T}$ and so the drop in length of loan as these loans switch from floating to fixed rates could be of the order of 50%.

---

[12]More accurately, the fall in β causes ϕ_j to shift left at all points other than the origin.

[13]From (5) we might note that taking the sample variances and covariances as point estimates of the true instantaneous variational terms, the optimal indexation coefficient dropped from 1.06 to 0.90 across the two periods.

The impact of the rise of the variance of the changes in the loan rate (σ_i^2) to some extent offsets that of the fall in β. From the previous section we know that although a rise in σ_i^2 raises both $V_j^F(t)$ and $V_j(t)$ for loans lying along ϕ the rise in $V_j(t)$ exceeds that for $V_j^F(t)$ and so ϕ will rotate clockwise. Thus the proportion of floating rate loans will rise. While this effect is clear cut, the impact of a rise in σ_i^2 on maturity is indeterminate. Note that a rise in σ_i^2 lowers both $\hat{T}$ and T^*. However, those loans that switch to a floating rate will at the same time lengthen in a discrete fashion and so the net effect on average maturity is indeterminate. Nevertheless, the fact that all inframarginal loans will have their optimal maturity reduced suggests that the net impact of a rise in σ_i^2 will be to lower average maturity.

The combined impact of the changes in interest rate behavior, the fall in β and rise in σ_i^2, is ambiguous as far as the use of floating rates is concerned and, though technically ambiguous, suggests a fall in average maturity. Clearly, as a prediction about the behavior of any one of the four classes of loans in Table 1, this is at best weak and does nothing to explain the variation in behavior across classes of loans. Fortunately, there was a third change that occurred over the period, namely a change in the real size of loans in each class. We have data on the average size of large loans. For the large banks this average size rose from \$3.247 m. in 1977 to \$8.491 m. in 1983 - 1984.I. At 1977 prices, the average size of a large loan was \$5.488 m. in 1983 so that in real terms the average size of large loans made by large banks rose by almost 70%. In contrast, the average size of large loans of the small banks rose over the period from \$2.115 m. to

\$3.619 m. or by a little over 10% in real terms. While no data is available on average loan size within the small loan categories given that these categories are fixed in nominal terms inflation by itself would have lowered the real size of these loans by over 50%. Thus there was a modest rise in the real size of large loans made by small banks and very large increases and decreases respectively in the real size of large loans made by large banks and small loans made by small banks.

How would these changes affect the terms of the different classes of loans? A rise in the real value of the loan lowers for any given real K_j and C_j, the real per dollar costs of rolling over the loan k_j and indexing c_j. In terms of Figure 5, the location of the distribution $H(c_j,k_j|L_j)$ is shifted down and to the left. Notice though that we assume that $c_j\int_0^{\hat{T}} e^{-\rho\tau}d\tau$ is of the order of k_j and so c_j is very small relative to k_j. Thus $H(c_j,k_j|L_j)$ will shift considerably more downwards than leftwards. As ϕ is fixed this will mean that the proportion of loans using floating rates will drop. The converse is true for a reduction in the real value of the loan. In view of this we would expect the very large increase in size of the large loans of large banks to result in a very dramatic drop in the proportion of such loans using floating rates. As fixed rate loans are optimally shorter than floating rate loans this would result in an even more dramatic fall in the average maturity of such loans, especially given that the change in interest rate behavior also suggests a drop in maturity. Given that the increase in the real size of large loans made by large banks was more modest, the switch to fixed rate loans and the shortening of maturity should also be more modest, which it was. In the case of the small loans at both banks, a sharp reduction in their real size should result in an increased use of floating rates and consequently a lengthening of average maturity. However, in this case the change in interest rate behavior which works to shorten maturity and so would tend to offset to some extent the impact of the fall in real size of these loans. In consequence, although the fall in the real size of these loans may have been close to the size of the increase in the real size of the large loans of the large banks, the rise in the maturity of the small loans will be less than the fall in maturity of the large loans.

4. Conclusions

The analysis above shows that a quite standard model of optimal contract indexation can account well for the major and diverse changes in the terms of bank loans over the period 1977 - 1984.I. However, contrary to what one might expect, the changes in the behavior of interest rates over this period, notably the increased volatility of the 90-day T bill rate and the drop in the correlation between this rate and the market loan rate, plays a minor and ambiguous role in this explanation. The principal explanatory variable and the only variable which explains the differences in changes across loan classes is the change in real size of the loan.

While the explanation given above works quite well for the changes in terms of lending, it works far less well in explaining the initial level of the terms. Why were the average maturities of all four classes of loans almost identical in 1977? Why were the proportions that used floating rates very similar for all classes of loans except for the small loans of small banks in 1977? These questions are still puzzles at this stage.

Finally it should be noted that there are many other aspects of the terms of a loan other than its length and whether it is fixed or floating rate. These other aspects, such as the size of compensating balances and free bank or accounting services are not treated here because of a lack of data. In particular one marked change that has occurred is the growth of fixed rate loans made under commitment, that is, where a firm buys a call option on a fixed rate loan. More than 60% of loans are now of this form. Why an interest rate option is tied in to the loan and what impact this has on the choice between a fixed and floating rate is an open issue.[14] Certainly, the model provided in this paper has nothing to say about changes in these terms of the loan contracts.

---

[14]See Ho and Saunders (1983) for a description of such loans and their pricing. One possible explanation for the tie-in is that within some corporations rules restrict the finance director from using interest rate futures and options. The banks are providing such constrained directors with disguised access to such markets. Of course, this leaves unexplained why such rules exist within these corporations.

APPENDIX

Integration of (8) and (9) yields

$$\frac{\partial V_j^F(t)}{\partial \sigma_i^2} - \frac{\partial V_j(t)}{\partial \sigma_i^2} = \frac{(1-\beta^2)}{1-e^{-\rho\hat{T}}}[1-(1+\rho\hat{T})e^{-\rho\hat{T}}] -$$

$$- \frac{1}{1-e^{-\rho T^*}}[1-(1+\rho T^*)e^{-\rho T^*}] \equiv Y \qquad \text{(A1)}$$

Note that $Z(x) = \frac{1-(1+x)e^{-x}}{1-e^{-x}}$ is increasing in x, $x \geq 0$.

$$\frac{dZ}{dx} = \frac{(1-e^{-x})xe^{-x} - [1-(1+x)e^{-x}]e^{-x}}{(1-e^{-x})^2}$$

$\therefore$ sign $\frac{dZ}{dx}$ is same as sign of the numerator $= xe^{-x} - e^{-x} + e^{-2x}$

$$= e^{-x} + x - 1 > 0$$

We know that for a firm that is indifferent between using a floating rate and a fixed rate, from (3) and (7), $T^* = \hat{T}(1-\beta^2) + c_j/\sigma_i^2 > \hat{T}(1-\beta^2)$. Thus substituting $T^* = \hat{T}(1-\beta^2)$ into (A1) raises the value of the expression in (A1).

$$Y < \frac{(1-\beta^2)}{1-e^{-\rho\hat{T}}}[1-(1+\rho\hat{T})e^{-\rho\hat{T}}] -$$

$$- \frac{1}{1-e^{-\rho\hat{T}(1-\beta^2)}}[1-(1+\rho\hat{T}(1-\beta^2))e^{-\rho\hat{T}(1-\beta^2)}] \qquad \text{(A2)}$$

Notice that the expression on the RHS of (A2) is of the form

$(1 - \beta^2)Z(x) - Z((1 - \beta^2)z)$. This expression will be negative, and hence Y will be negative, if Z(x) is concave in x. We now prove this. First rewrite dZ/dx as

$$\frac{dZ}{dx} = \frac{xe^{-x}}{1 - e^{-x}} - \frac{Z(x)e^{-x}}{1 - e^{-x}} \equiv A(x) - B(x)$$

$$\frac{dA(x)}{dx}(1 - e^{-x})^{-2} = (1 - e^{-x})(e^{-x} - xe^{-x}) - xe^{-2x} = e^{-x} - xe^{-x} - e^{-2x}$$

$$\frac{dB(x)}{dx}(1 - e^{-x})^{-2} = (1 - e^{-x})[-Z(x)e^{-x} + dZ(x)/dx \cdot e^{-x}] - e^{-x}Z(x)e^{-x}$$

$$= (1 - e^{-x})\{-Z(x)e^{-x} + \frac{xe^{-2x}}{1 - e^{-x}} - \frac{Z(x)e^{-2x}}{1 - e^{-x}}\} - Z(x)e^{-2x}$$

$$= xe^{-2x} - Z(x)e^{-x} - Z(x)e^{-2x}$$

$$= e^{-x}[xe^{-x} - Z(x) - Z(x)e^{-x}]$$

Thus the sign of d^2Z/dx^2 = sign (dA/dx - dB/dx), which is the same as

$$1 - x - e^{-x} - xe^{-x} + Z(x) + Z(x)e^{-x}$$

or

$$(1 - e^{-x})Z(x) - x + Z(x) + Z(x)e^{-x}$$

or

$$2Z(x) - x < 0 \quad \text{for } x \geq 0$$

Thus we have proved that Z(x) is concave and so that Y < 0.

References

Blume, Laurence, David Easley and Maureen O'Hara, "Characterization of Optimal Plans for Stochastic Dynamic Programming," Journal of Economic Theory, 28 (2), December 1982, 221-234.

Board of Governors of the Federal Reserve System, Survey of the Terms of Bank Lending, Statistical Release G14 (1977-1978), Statistical Release E2 (1979-present).

Gray, Jo Anna, "On Indexation and Contract Length," Journal of Political Economy, 86 (1), February 1978, 1-18.

Ho, S.Y. and Anthony Saunders, "Fixed Rate Loan Commitments, Take-Down Risk and the Dynamics of Hedging with Futures," Journal of Financial Quantitative Analysis, December 1983.

Karydakis, Antonis, "Changes in Bank Lending Terms and the Effects on Competition Between Large and Small Banks," mimeo, New York University, August 1984.