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by Tapan Mitra
and
Efe A. Ok

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C. V. STARR CENTER FOR APPLIED ECONOMICS

NEW YORK UNIVERSITY
FACULTY OF ARTS AND SCIENCE
DEPARTMENT OF ECONOMICS
WASHINGTON SQUARE
NEW YORK, N.Y. 10003

On the Equitability of Progressive Taxation*

TAPAN MITRA†

AND

EFE A. OK ‡

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Abstract

We propose the principle of equal sacrifice to determine the class of "vertically inequitable" progressive taxes. A necessary condition for an income tax function to be equal sacrifice is formulated, and hence, a subclass of progressive taxes which cannot inflict the same sacrifice upon all individuals relative to *any* strictly increasing and concave utility function is determined. Conversely, it is shown in a very general framework that any convex (thus progressive) tax function satisfies the principle of equal sacrifice. Our findings point to the fact that equal sacrifice under progressive income taxation depends heavily upon the degree of marginal rate (as opposed to average rate) progressivity.

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Key words: Income taxation, tax progressivity, equal sacrifice.

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†Department of Economics, Cornell University, 416 Uris Hall, Ithaca NY 14853. Telephone: 607 255 4062. E-mail: tm19@cornell.edu

‡[*Corresponding author*] Department of Economics, New York University, 269 Mercer St., New York NY 10003. Telephone: 212 998 8920. E-mail: okefe@fasecon.econ.nyu.edu

1 Introduction

An important criterion of redistributive justice formulated in the realm of income taxation is the following:

> *An income tax function should decrease income inequality (in the sense of relative Lorenz dominance) for any given pre-tax income distribution.*

This criterion is typically referred to as the "*principle of progressivity*", for it is now well-known that an income tax function satisfies this principle if, and only if, it is a progressive tax (that is, its average tax rate is increasing).[1] This observation may very well account for the striking fact that all countries (and certainly all OECD countries) use (statutory) progressive income tax schemes. But a fundamental question remains: *are all progressive taxes equitable*?

A reformulation of John Stuart Mill's famous maxim of income "taxation so as to inflict equal sacrifice" leads us to another compelling redistributive justice principle:

> *An income tax function must yield equal sacrifice to all individuals relative to at least one acceptable social norm (or, a utility function for the representative agent of the society).*

Following Young (1988), we call this maxim "*the principle of equal sacrifice*" and contend that it is a useful fairness criterion. True, it is by no means sufficient for determining "*equitable*" taxes perforce, for such a determination can only be done relative to the 'actual' social norm that summarizes the preferences of the society, or even better, relative to the 'true' preferences of the individuals. But the principle is certainly very effective in elucidating "*inequitable*" taxes, for, by implication, an income tax function not satisfying the principle of equal sacrifice guarantees *unequal* sacrifice relative to *any* possible social norm (utility function for income), and thus in particular, relative to the true preferences of the constituents of the society.[2]

In this note, we attempt to understand the equity properties of progressive income taxes in the light of the principle of equal sacrifice (or put differently, from the perspective of the doctrine of "*ability to pay*" (cf. Musgrave (1985))). An immediate question is then the following: *do all progressive tax functions satisfy the principle of equal sacrifice*? We show that the answer is negative by determining a subclass of progressive taxes which fail to satisfy the principle. Roughly speaking, progressive

[1] See, for instance, Jakobsson (1976), Fellman (1976), Eichhorn *et al.* (1984), and Lambert (1993).

[2] There is now a small literature on the various aspects of equal sacrifice income taxation: see Richter (1983), Buchholz *et al.* (1987), Young (1987, 1988, 1990), Yaari (1988), Berliant and Gouveia (1993), Mitra and Ok (1994) and Ok (1995).

taxes which are "sufficiently non-convex on a neighborhood" *cannot* yield equal sacrifice for *any* concave and strictly increasing utility function.[3] This result illustrates that there is merit in combining the principle of progressivity with the principle of equal sacrifice to pave way towards a theory of equitable income taxation.

The next question is, of course, whether or not the principles of progressivity and equal sacrifice are compatible. We find that they are, and establish that all *convex* progressive taxes do satisfy the principle of equal sacrifice. These results show that equal sacrifice under progressive personal income taxation depends heavily upon the degree of *marginal rate progressivity* (as opposed to the more conventional average rate progressivity), and is an issue far from trivial.[4]

Our results remain silent with respect to a particular subclass of tax functions, roughly speaking that of "mildly non-convex" tax functions. We show that this class contains both kinds of the tax functions; those that satisfy the principle of equal sacrifice and those that do not. Finding out exactly which members of this set are actually equal sacrifice taxes is of interest, for only then a full characterization of non-equitable progressive taxes will be achieved. This problem remains open for the moment.

We proceed by Section 2 where the precise formulations of the key concepts of the present note are presented. Section 3 states and discusses our main results. It is in this section we determine some useful subclasses of the sets of progressive equal sacrifice and progressive unequal sacrifice post-tax functions. The above mentioned open question is also formally put forth in this section. In Section 4, we discuss the robustness of our results and find that they are not tight with respect to the relaxation of technical hypotheses. Potential extensions of our findings are also pointed out in this section by means of several examples. The final section supplies the proofs of our main results.

2. Preliminaries

By a *post-tax function*, we mean a continuous, right differentiable and surjective function $f : \mathbf{R}_+ \to \mathbf{R}_+$ (that associates to pre-tax income x a post-tax income $f(x)$) such that the following conditions hold:

(A1) $f(0) = 0$ and $0 < f(x) < x$ for all $x > 0$,

[3]This observation is by no means inconsequential from a practical point of view. For example, one can check that Turkish (statutory) personal income tax was "sufficiently non-convex" around TL 25,000,000 between 1981 and 1985 to guarantee (by Theorem 1) that it was a progressive but not an equal sacrifice income tax (cf. OECD (1986), p. 286.) See also Example 1.

[4]1991 U.S. Federal (Statutory) Income Tax is, therefore, found to be respecting the principle of equal sacrifice by virtue of its marginal rate progressivity (see footnote 5).

(A2) $0 < f'_+(x) < 1$ for all $x \geq 0$,

(A3) $x \mapsto f(x)/x$ is a Lipschitz continuous mapping near origin; that is, there exists $(y, K) \in \mathbf{R}^2_{++}$ such that

$$\left|\frac{f(x)}{x} - f'_+(0)\right| \leq Kx \quad \text{for all } x \in (0, y].$$

The set of all post-tax functions are denoted by $\mathcal{F}$. (Notice that, given a post-tax function $f \in \mathcal{F}$, the tax liability levied on income level $x > 0$ is $t(x) := x - f(x)$.)

(A1) is a fairly standard assumption positing that zero income earners do not pay any taxes and that if one earns a positive income, he/she has to pay a positive amount of taxes which must be less than his/her taxable income. (A2) is also quite standard and assures that a higher income earner pays a higher level of taxes than a lower income earner and that the ranking of taxpayers by pre-tax income and post-tax income is the same. (In other words, by virtue of (A2), we focus only on *non-confiscatory* taxation schemes. Such tax functions are sometimes referred to as *incentive preserving* in the literature (cf. Fei (1981), Eichhorn *et al.* (1984) and Ok (1995)).)

In the literature on income taxation, analyses are typically conducted in terms of differentiable tax functions. Although there is nothing wrong with the differentiability assumption, it clearly makes it difficult to relate the study to the actual taxation practice since the statutory income taxes are typically designed as continuous piecewise linear functions.[5] On the other hand, if one concentrates only on continuous piecewise linear tax functions, then relating the analysis to the existing literature on income taxation becomes a problem. By assuming only right differentiability of $f(.)$ and (A3), our framework covers both smooth tax functions and continuous piecewise linear tax functions as special cases. Therefore, although they are a bit tedious to state, these assumptions should be viewed as weak regularity conditions which allow for a definitive generality of analysis.[6]

A post-tax function $f \in \mathcal{F}$ is said to be *progressive* if the average post-tax function $x \mapsto f(x)/x$ is decreasing. One can easily show that a concave (*marginal*

[5] For example, 1991 U.S. Federal Income Tax for single persons was of the following form:

$$t(x) = \begin{cases} 0.15x, & 0 \leq x < 20250 \\ 0.28x - 2632.5, & 20250 \leq x < 49300 \\ 0.31x - 4111.5, & 49300 \leq x \end{cases}.$$

The associated post-tax function is of course given by $f(x) = x - t(x)$ for all $x \geq 0$.

[6] We shall, in fact, later demonstrate that (A3) is not a necessary condition for our results to hold.

rate progressive) post-tax function is progressive but the converse statement does not hold.[7] We shall denote the class of all progressive post–tax functions by $\mathcal{F}^{prog}$.

By an *equal sacrifice* post-tax function, we mean a post-tax function $f \in \mathcal{F}$ such that

$$\exists c > 0 : [\forall x > 0 : [u(x) - u(f(x)) = c]] \tag{1}$$

holds for at least one concave and strictly increasing utility function $u : \mathbf{R}_{++} \to \mathbf{R}$.[8] This definition is identical to that of Young (1988) except that Young requires (1) to be satisfied by a continuous and strictly increasing utility function. But Ok (1995) shows that *any* $f \in \mathcal{F}$ is, in fact, an equal sacrifice post-tax function with Young's definition. Therefore, demanding the concavity of the utility function of the individuals is essential to the theory (cf. Mitra and Ok (1994)). Moreover, the assumption of decreasing marginal utility is almost exclusively made in the related public finance literature. By virtue of the usual arguments favoring risk averse behavior, we feel that it is a well-justified assumption.

The above definition of equal sacrifice taxation is best interpreted by considering $u(.)$ as standing for the preferences of a *representative agent* of the society, and thus acting as a *social norm* (cf. Musgrave (1959) and Young (1990)). We stress that this interpretation saves the principle of equal sacrifice from necessitating interpersonal utility comparisons.

Finally, let us emphasize that if a post-tax function is an equal sacrifice post-tax, all we know is the existence of a well-behaved utility function relative to which everyone sacrifices equally. Since this utility function may not be a good approximation of the agents' true preferences for income, one cannot conclude that an equal sacrifice post-tax is, in fact, *vertically equitable.* However, if a post-tax function is not equal sacrifice, then we can infer that it cannot inflict the same sacrifice upon everyone relative to *any* sensible utility function. It follows that there is a clear sense in which such taxes are *vertically inequitable.* Therefore, the principle of equal sacrifice is not an inclusion principle identifying the equitable taxes, but is an *exclusion* principle determining the inequitable taxes from the perspective of ability to pay doctrine.

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[7]Define $f \in \mathcal{F}$ as $f(x) := \begin{cases} 3x/4, & 0 \leq x < 1 \\ (x/4) + 1/2, & 1 \leq x < 2 \\ x/2, & 2 \leq x \end{cases}$. One can easily check that $x \mapsto f(x)/x$ defines an everywhere decreasing mapping while $f(.)$ is not concave around 2.

[8]One can easily show that (1) holds for some concave and strictly increasing $u : \mathbf{R}_{++} \to \mathbf{R}$ if and only if

$$\exists c > 0 : [\forall x > 0 : [v(x) = cv(f(x))]]$$

holds for some concave and strictly increasing $v : \mathbf{R}_{++} \to \mathbf{R}$. Therefore, an equal sacrifice post-tax function can be thought of as both an equal *absolute* sacrifice and an equal *proportional* sacrifice post-tax function.

3. Results

There seems to be a consensus that the concept of progressive taxation carries a considerable degree of egalitarianism with it. Almost all countries use progressive (statutory) taxation schemes and this widespread usage is usually justified on the basis of income inequality aversion. (See Lambert (1993) for an extensive survey.) Indeed, it is well known that a progressive post-tax function maps a pre-tax income distribution to a more equal post-tax distribution (in the sense of relative Lorenz dominance). Therefore, all progressive post-tax functions are inequality reducing, and hence, they all pass a specific test of distributive justice. We propose another test based on the principle of equal sacrifice; the question is if all progressive post-tax functions are equal sacrifice. If the answer to this question was yes, then one would conclude that the principle of equal sacrifice is a very weak principle in that it is not useful in further refining the broad class of progressive taxes on the basis of redistributive justice. On the other hand, if the answer was no, then this would mean that the principle of equal sacrifice can be effectively used in assessing the normative properties of progressive taxation.

This appears to be a natural way of making use of the principle of equal sacrifice. It seems to us that the reason why this question is not at all addressed in the literature is because the analysis of Samuelson (147, p. 227) is taken to imply that the principle of equal sacrifice has no selective power.[9] Many authors appear to indicate that any progressive post-tax function can be equal sacrifice with respect to a strictly increasing and concave utility function $u(.)$ with a relative risk aversion coefficient greater than 1. Our first result identifies a subclass of progressive post-tax functions which are not equal sacrifice, and hence, shows that this contention is unwarranted.

Theorem 1. *Let $f \in \mathcal{F}^{\text{prog}}$. If there exists $x_0 > y > 0$ such that $y \geq f(x_0)$ and*

$$f'_+(x_0)f'_+(f(x_0)) > f'_+(y),$$

then there does not exist a strictly increasing and concave utility function $u : \mathbf{R}_{++} \to \mathbf{R}$ such that

$$\exists c > 0 : [\forall x > 0 : [u(x) - u(f(x)) = c]].^{10}$$

[9]Put precisely, what Samuelson (1947) proves is the following: Given a post-tax function $f \in \mathcal{F}$ and a concave and strictly increasing utility function $u : \mathbf{R}_{++} \to \mathbf{R}$ satisfying (1), $f(.)$ is progressive if and only if

$$\left| \frac{u'_+(x - f(x))(x - f(x))}{u'_+(f(x))f(x)} \right| \geq 1 \quad \text{for all } x > 0.$$

But his observation is far from clarifying under what conditions (1) can be satisfied for a given $f \in \mathcal{F}$ (cf. Mitra and Ok (1994)).

[10]The theorem remains intact if we drop the assumptions of surjectivity of $f(.)$ and (A3); these properties are not used in the proof of Theorem 1 given in Section 5.

To deal with the converse of this theorem we need to study the progressive post-tax functions $f \in \mathcal{F}^{prog}$ such that

$$\forall x > 0 : [\forall y \in [f(x), x) : [f'_+(x) f'_+(f(x)) \leq f'_+(y)]]. \tag{2}$$

Unfortunately, even condition (2) is not strong enough to guarantee the existence of a strictly increasing and concave utility function $u(.)$ such that (1) holds. However, if we assume a slightly stronger condition than (2), namely that

$$\forall x > 0 : [\forall y \in [f(x), x) : [f'_+(x) \leq f'_+(y)]] \tag{3}$$

we obtain a definitive answer:

Theorem 2. *Let $f \in \mathcal{F}$. If (3) holds, then $f(.)$ is an equal sacrifice post-tax function.*

Theorems 1 and 2 remain silent with respect to the progressive post-tax functions which satisfy (2) but does not satisfy (3). In the next section, we shall show that such post-tax functions may or may not satisfy (1). The characterization of such post-tax functions which satisfy the principle of equal sacrifice (preferably by a set of easily checkable conditions) stands as an open problem at the moment.

4. Examples

Theorems 1 and 2 together give a very practical way of checking if a given post-tax function is equal sacrifice or not.[11] Our first example illustrates the applicabilility of these results to the actual taxation practice.

Example 1. Define

$$f(x;\alpha) := \begin{cases} 0.9x, & 0 \leq x < 80 \\ \alpha x + (72 - 80\alpha), & 80 \leq x < 90 \\ 0.7x + (10\alpha + 9), & 90 \leq x \end{cases},$$

and notice that for any $\alpha \in (0,1)$, $f(.;\alpha) \in \mathcal{F}$. Moreover, $f(.;\alpha)$ is a progressive post-tax function if and only if $\alpha \in (0, 0.9)$. Now, one can easily check that if $f(90;\alpha) \geq 80$, then (2) holds. So let $f(90;\alpha) = 10\alpha + 72 < 80$; that is, $\alpha < 0.8$. Choose $x_0 = 90$ and

[11] Although our primary focus in this paper is on progressive post-tax functions, we note that Theorems 1 and 2 remain valid if we replace $\mathcal{F}^{prog}$ by $\mathcal{F}$ in their statements.

notice that, for any $y \in [f(90;\alpha), 90) \subset (80, 90)$, we have $f'_+(90;\alpha)f'_+(f(90;\alpha);\alpha) > f'_+(y;\alpha)$ if and only if $0.9(0.7) = 0.63 > \alpha$. Thus, one concludes that

$$\exists x > 0 : [\exists y \in (f(x;\alpha), x) : [f'_+(x;\alpha)f'_+(f(x;\alpha);\alpha) \leq f'_+(y;\alpha)]]$$

if and only if $\alpha \in (0, 0.63)$. Therefore, in view of Theorem 1, $f(.;\alpha)$ is a progressive post-tax function which is *not* equal sacrifice as long as $0 < \alpha < 0.63$. Conversely, if $0.7 \leq \alpha < 0.9$, then by Theorem 2, $f(.;\alpha)$ is a progressive tax which inflicts the same sacrifice upon all income levels relative to a strictly increasing and concave utility function. The indeterminacy region for α corresponding to the case where (2) holds but (3) does not, is [0.63,0.7). As noted above, whether $f(x;\alpha)$ with $0.63 \leq \alpha < 0.7$ is equal sacrifice or not is an open question.[12] □

In the next to examples, we shall demonstrate that a progressive post-tax function which satisfies (2) but does not satisfy (3) may or may not be an equal sacrifice post-tax function. Therefore, it is proved that the subclass of $\mathcal{F}^{prog}$ where Theorems 1 and 2 are silent contain both equal sacrifice and unequal sacrifice post-tax functions. In other words, the converse of neither Theorem 1 nor Theorem 2 holds true: Example 2 illustrates that a progressive post-tax function that does not satisfy the antecedent of Theorem 1 can be unequal sacrifice; and Example 3 shows that a progressive post-tax function can be equal sacrifice without being concave (that is, without satisfying (3)).

Example 2. (*Lindsey II*)[13] Let

$$h(x) := \begin{cases} 0.1, & \text{if } 0 \leq x < 1 \\ 0.04, & \text{if } x \in \bigcup_{k\in\{0,2,\ldots,98\}} [1 + \frac{k}{100}, 1 + \frac{1+k}{100}) \\ 0.05, & \text{if } x \in \bigcup_{k\in\{1,3,\ldots,99\}} [1 + \frac{k}{100}, 1 + \frac{1+k}{100}) \\ 0.04, & \text{if } 2 \leq x, \end{cases}$$

and define

$$f(x) := \int_0^x h(u)du \quad \text{for all } x \geq 0.$$

It easily follows that $f \in \mathcal{F}$. One can also directly verify that, for any $x > 0$, $\int_0^x h(u)du \geq xh(x)$ so that $f \in \mathcal{F}^{prog}$. Here we find that $\max_{x,z>0} h(x)h(z) < \min_{y>0} h(y)$ so that (2) is trivially satisfied (while (3), of course, fails). We claim that $f(.)$ is not

[12]More generally, let $f \in \mathcal{F}$ be defined by $f(x) := \begin{cases} \alpha_1 x, & x \in [0, b_1) \\ \alpha_2 x + \theta_1, & x \in [b_1, b_2) \\ \alpha_3 x + \theta_2, & x \in [b_2, \infty) \end{cases}$. If and only if $\alpha_3 b_2 + \theta_2 < b_1$ and $\alpha_2 < \alpha_1\alpha_3$, the hypothesis of Theorem 1 holds, and thus, under these conditions one can conclude that $f(.)$ is *not* an equal sacrifice post-tax function. On the other hand, if $\alpha_1 > \alpha_2 > \alpha_3$, then Theorem 2 entails that $f(.)$ is equal sacrifice.

[13]This interesting example is communicated to us by Professor John Lindsey II; we gratefully acknowledge our debt to him.

an equal sacrifice post-tax function. Assume, by way of contradiction, that (1) holds for some strictly increasing and concave $u : \mathbf{R}_{++} \to \mathbf{R}$. Then, upon iteration, we must have $u(x) = u(f^n(x)) + nc$, and hence,

$$u'_+(x) = u'_+(f^n(x))d_n(x) \quad \text{for all } x > 0 \text{ and } n \in \{..., -1, 0, 1, ...\} \tag{4}$$

where $d_n(x) := (f^n)'_+(x)$ for all $x > 0$ and $n \in \mathbf{Z}$, and where the right differentiability of $u(.)$ follows from its hypothesized concavity.[14] Now define

$$s_k := f\left(1 + \frac{k-1}{100}\right) \quad \text{for all } k \in \{1, ..., 99\}, \quad \text{and } n_k = \begin{cases} 1, & \text{if } k \text{ is odd} \\ 0, & \text{if } k \text{ is even} \end{cases}.$$

Choosing $x \in \{f^{-n_k}(s_k), f^{-n_k}(s_{k+1})\}$ and $n = n_k$ in (4), we obtain, for all $k \in \{1, ..., 98\}$,

$$u'_+(f^{-n_k}(s_k)) = u'_+(s_k)d_{n_k}(f^{-n_k}(s_k)) \quad \text{and } u'_+(f^{-n_k}(s_{k+1})) = u'_+(s_{k+1})d_{n_k}(f^{-n_k}(s_{k+1})).$$

By concavity of $u(.)$ and the fact that $f^{-n_k}(s_{k+1}) > f^{-n_k}(s_k)$, we thus have

$$\frac{d_{n_k}(f^{-n_k}(s_k))}{d_{n_k}(f^{-n_k}(s_{k+1}))} \geq \frac{u'_+(s_{k+1})}{u'_+(s_k)} \quad \text{for all } k \in \{1, ..., 98\}. \tag{5}$$

On the other hand, choosing first $n = n_{99} - 1$ and $x = f^{-n_{99}+1}(s_{99})$, and then $n = n_{99}$ and $x = f^{-n_{99}}(s_1)$ in (4), we have $u'_+(f^{-n_{99}+1}(s_{99})) = u'_+(s_{99})d_{n_{99}-1}(f^{-n_{99}+1}(s_{99}))$, and $u'_+(f^{-n_{99}}(s_1)) = u'_+(s_1)d_{n_{99}}(f^{-n_{99}}(s_1))$, respectively. Thus, by concavity of $u(.)$ and the fact that $f^{-n_{99}+1}(s_{99}) < f^{-n_{99}+1}(1) = f^{-n_{99}}(f(1)) = f^{-n_{99}}(s_1)$, we have

$$\frac{d_{n_{99}-1}(f^{-n_{99}+1}(s_{99}))}{d_{n_{99}}(f^{-n_{99}}(s_1))} \geq \frac{u'_+(s_1)}{u'_+(s_{99})}.$$

Combining this with (5) yields that

$$A := \left(\prod_{k=1}^{98} \frac{d_{n_k}(f^{-n_k}(s_k))}{d_{n_k}(f^{-n_k}(s_{k+1}))}\right) \frac{d_{n_{99}-1}(f^{-n_{99}+1}(s_{99}))}{d_{n_{99}}(f^{-n_{99}}(s_1))} \geq \left(\prod_{k=1}^{98} \frac{u'_+(s_{k+1})}{u'_+(s_k)}\right) \frac{u'_+(s_1)}{u'_+(s_{99})} = 1.$$

But by direct computation, $\frac{d_{n_k}(f^{-n_k}(s_k))}{d_{n_k}(f^{-n_k}(s_{k+1}))}$ is found to be equal to $\frac{0.04}{0.05} = 0.8$ if $k \in \{1, 3, ..., 97\}$ and 1 if $k \in \{2, 4, ..., 98\}$ so that $A = (0.8)^{49}(1/0.04) \simeq 0.00044 < 1$, a contradiction. We conclude that $f(.)$ is not an equal sacrifice post-tax function. □

[14] For any function $\varphi : \mathbf{R}_+ \to \mathbf{R}$ and any $n \in \{0, 1, ...\}$, we define the nth iterate of φ as the function $\varphi^n(x) := (\varphi \circ \cdots \circ \varphi)(x)$ for all $x > 0$, where the composition operator is applied n times.

Example 3. Define

$$g(x) := \begin{cases} 3x/4, & \text{if } 0 \leq x \leq 1 \\ (x/4) + (1/2), & \text{if } 1 < x \leq 2 \\ x/2, & \text{if } 2 < x \end{cases}.$$

While proving Theorem 2 in Section 5, we shall show that, for any $f \in \mathcal{F}$, the iteration sequence $f^n(x)/\left(f'_+(0)\right)^n$ converges in $\mathbf{R}_{++}$ and, for any $c > 0$, $u : \mathbf{R}_{++} \to \mathbf{R}$ defined as

$$u(x) := \frac{-c}{\log f'_+(0)} \log \lim_{n\to\infty} \frac{f^n(x)}{(f'_+(0))^n} \quad \text{for all } x > 0,$$

satisfies $u(x) - u(f(x)) = c$ for all $x > 0$. Therefore, since that $g \in \mathcal{F}^{prog}$ can easily be checked, showing that $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ is a concave and strictly increasing function is enough to conclude that it is, in fact, an equal sacrifice post-tax function. Now, for $n \in \{3, 4, ...\}$ we have

$$\left(\frac{4}{3}\right)^n g^n(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 1 \\ \left(\frac{4}{3}\right)\left(\frac{1}{4}x + \frac{1}{2}\right), & \text{if } 1 < x \leq 2 \\ \left(\frac{4}{3}\right)^2\left(\frac{1}{4}\left(\frac{x}{2}\right) + \frac{1}{2}\right), & \text{if } 2 < x \leq g^{-1}(2) \\ \dots\dots\dots, & \dots\dots\dots \\ \left(\frac{4}{3}\right)^{n-1}\left(\frac{1}{4}\left(\frac{x}{2^{n-2}}\right) + \frac{1}{2}\right), & \text{if } g^{2-n}(2) < x \leq g^{1-n}(2) \\ \left(\frac{4}{3}\right)^n\left(\frac{1}{4}\left(\frac{x}{2^{n-1}}\right)\right) + \frac{1}{2}, & \text{if } g^{1-n}(2) < x \leq g^{-n}(2) \\ \left(\frac{4}{3}\right)^n \frac{x}{2^n}, & \text{otherwise.} \end{cases}$$

Observe that, for each n, $\left(\frac{4}{3}\right)^n g^n(.)$ is concave on $[0, g^{-n}(2)]$ and that $\{g^{-n}(2)\}_{n=1}^{\infty}$ is a strictly increasing sequence such that $\lim_{n\to\infty} g^{-n}(2) = \lim_{n\to\infty} 2^{n+1} = \infty$. Therefore, $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ is concave on $[0,1] \cup [1,2] \cup \bigcup_{n=1}^{\infty}[2, g^{-n}(2)] = [0, \infty)$. On the other hand, notice that $\mathbf{D}_+[\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(x)] > 0$ for all $x \in [0,2]$ so that $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ is strictly increasing on $[0,2]$. Let $x_1, x_2 \in g^{-1}([0,2])$ and $x_2 > x_1$. Then $(x_1, x_2) = (g^{-1}(a), g^{-1}(b))$ for some $a, b \in [0,2]$ such that $b > a$. Thus by strict monotonicity of $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ on $[0,2]$,

$$\begin{aligned} \tfrac{3}{4}\lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(x_2) = \tfrac{3}{4}\lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(g^{-1}(b)) &= \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(b) \\ &> \lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(a) \\ &= \tfrac{3}{4}\lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(g^{-1}(a)) \\ &= \tfrac{3}{4}\lim_{n\to\infty} \left(\tfrac{4}{3}\right)^n g^n(x_1) \end{aligned}$$

so we learn that $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ is strictly increasing on $g^{-1}([0,2])$. But then, by an easy induction argument, it follows that $\lim_{n\to\infty} \left(\frac{4}{3}\right)^n g^n(.)$ must be strictly increasing on $\lim_{n\to\infty} g^{-n}([0,2]) = [0,\infty)$. We conclude that $g(.)$ is an equal sacrifice post-tax function.[15] □

Our final example is about the robustness of our findings with respect to the boundary condition (A3). One can easily check that (A3) is not necessary for Theorem 1. Indeed, the proof of Theorem 1 given in the next section makes no reference to this assumption. On the other hand, since this assumption is used crucially in the proof of Theorem 2, it is not at all clear if it is necessary for this result. The following example shows that it is not. Therefore, Theorem 2 is also not tight with respect to the relaxation of (A3).

Example 4. Define

$$f(x) := \begin{cases} \beta x - (\beta x^{3/2}/2), & 0 \leq x < 1 \\ \beta x/(x+1), & 1 \leq x \end{cases}.$$

$f(.)$ is a differentiable, surjective and concave function which satisfies (A1) and (A2). Since, for any $K > 0$, there exists $x_0 > 0$ such that $|\beta - f(x)/x| = \beta x/2\sqrt{x} > Kx$ for all $x \in (0, x_0)$, it is clear that $f(.)$ does not satisfy (A3). We claim that $f(.)$ satisfies (1), however. Let us first note that if $\lim_{n\to\infty} f^n(x)/\beta^n \in (0,\infty)$ then, for any $c > 0$,

$$u(x) := \left(\frac{-c}{\log \beta}\right) \log \lim_{n\to\infty} \frac{f^n(x)}{\beta^n} \quad \text{for all } x > 0,$$

defines a concave and strictly increasing utility function such that $u(x) - u(f(x)) = c$ for all $x > 0$. (The detailed proof of this assertion is given in the next Section.) Therefore, all we have to show to conclude that $f(.)$ is equal sacrifice is that $\lim_{n\to\infty} f^n(x)/\beta^n \in (0,\infty)$ for all $x > 0$. Now, for any $x > 0$, $\lim_{n\to\infty} f^n(x)/\beta^n = \prod_{n=0}^{\infty} f^{n+1}(x)/\beta f^n(x) \in (0,\infty)$ if and only if $\sum_{n=0}^{\infty}(1 - f^{n+1}(x)/\beta f^n(x))$ converges (cf. Theorem 4 of Knopp (1990), p. 220).[16] But

$$\sum_{n=0}^{\infty}\left(1 - \frac{f^{n+1}(x)}{\beta f^n(x)}\right) = \frac{1}{2}\sum_{n=0}^{\infty}\sqrt{f^n(x)} \quad \text{for all } x \in [0,1],$$

[15] This example establishes that a progressive but not concave post-tax function may be equal sacrifice. In fact, even a convex post-tax function can be equal sacrifice; in a private communication, Professor John Lindsey II showed that the post-tax function $f(x) := \frac{1}{2}(x - a\log(1+x))$ with $a \in (1, 1/10)$ is equal sacrifice yielding with respect to a concave and strictly increasing utility function. But, if one insists that an 'admissible' utility function for income is differentiable in an arbitrary neighborhood of origin, then we have a very strong result: *Almost all non-concave post-tax functions are not equal sacrifice!* (See Corollary 3.8 of Mitra and Ok (1994).)

[16] Notice that, by strict concavity of $f(.)$, $x \mapsto f(x)/x$ is strictly decreasing, and this implies that $\beta > ... > f^{n+1}(x)/f^n(x) > ... > f(x)/x$ for any $x > 0$ and positive integer n. Therefore, $\lim_{n\to\infty} f^n(x)/\beta^n = \prod_{n=0}^{\infty} f^{n+1}(x)/\beta f^n(x) \in [0,\infty)$ is guaranteed for any $x > 0$; all we have to check is that $\prod_{n=0}^{\infty} f^{n+1}(x)/\beta f^n(x) > 0$ for all $x > 0$.

so that choosing $x = 1$ and noting that

$$\sqrt{f^n(1)} \leq \frac{(\sqrt{\beta})^n}{\sqrt{2}} \quad \text{for all } n \in \{0, 1, ...\},$$

we learn that $\sum_{n=0}^{\infty} \sqrt{f^n(1)} < \infty$, and therefore $\lim_{n\to\infty} f^n(1)/\beta^n > 0$. By the monotonicity of $f(.)$, $\lim_{n\to\infty} f^n(.)/\beta^n$ is a non-decreasing function, and hence by the previous observation, we must have $\lim_{n\to\infty} f^n(x)/\beta^n > 0$ for all $x \geq 1$. Finally, let

$$\hat{x} := \max\{x \in [0, 1) : \lim_{n\to\infty} f^n(x)/\beta^n = 0\}$$

which is well-defined since $\lim_{n\to\infty} f^n(.)/\beta^n$ is a concave function (being the limit of a convergent sequence of concave functions), and thus, is continuous on $(0, 1)$. If $\hat{x} > 0$, using again the monotonicity of the limit function, we see that $\lim_{n\to\infty} f^n(.)/\beta^n$ vanishes on $[0, \hat{x}]$ and is strictly positive on $(\hat{x}, 1]$, and this implies that $\lim_{n\to\infty} f^n(.)/\beta^n$ is not concave around $\hat{x}$, a contradiction. Therefore, $\hat{x} = 0$ and $\lim_{n\to\infty} f^n(x)/\beta^n > 0$ for all $x \in (0, 1]$ as well, and the claim is proved.[17] □

5. Proofs

Proof of Theorem 1

Assume the hypothesis of the theorem, and let

$$u(x) - u(f(x)) = c \quad \text{for all } x > 0,$$

for some $c > 0$ and $u : \mathbf{R}_{++} \to \mathbf{R}$ which is strictly increasing and concave. Then, for any $x > 0$ and $\epsilon > 0$,

$$u(x + \epsilon) - u(f(x + \epsilon)) = u(x) - u(f(x))$$

so that by the Taylor's expansion from the right

$$\begin{aligned} u(x + \epsilon) - u(x) &= u(f(x + \epsilon)) - u(f(x)) \\ &= u(f(x) + f'_+(x)\epsilon + o(\epsilon)) - u(f(x)) \end{aligned}$$

[17] In effect, this example establishes the fact that (A3) is not necessary for $\lim_{n\to\infty} f^n(x)/\beta^n \in (0, \infty)$ for all $x > 0$. The following result due to Seneta (1969) gives necessary and sufficient conditions for this convergence to hold: $\lim_{n\to\infty} f^n(x)/\beta^n \in (0, \infty)$ for all $x > 0$ if, and only if,

$$\int_1^{\infty} \left(\frac{\beta}{a} - f\left(\frac{1}{a}\right)\right) da < \infty.$$

(A3), of course, implies this integral condition, but not conversely. (In the case of the above example, $\int_1^{\infty} \left(\frac{\beta}{a} - f\left(\frac{1}{a}\right)\right) da = \beta$.)

where $\lim_{\epsilon \downarrow 0} \frac{o(\epsilon)}{\epsilon} = 0$. Therefore, for any $x > 0$ and $\epsilon > 0$,

$$\frac{u(x+\epsilon)-u(x)}{\epsilon} = \left(\frac{f'_+(x)\epsilon + o(\epsilon)}{\epsilon}\right)\left(\frac{u(f(x)+f'_+(x)\epsilon+o(\epsilon))-u(f(x))}{f'_+(x)\epsilon+o(\epsilon)}\right).$$

By letting $\epsilon \downarrow 0$ (and recalling that $u(.)$ is right differentiable since it is concave) we obtain

$$u'_+(x) = f'_+(x)u'_+(f(x)) \quad \text{for all } x > 0. \tag{6}$$

Since $x_0 > y$ and $u(.)$ is concave, $u'_+(y) \geq u'_+(x_0)$ and (6) yields

$$u'_+(f(y)) \geq \left(\frac{f'_+(x_0)}{f'_+(y)}\right) u'_+(f(x_0)). \tag{7}$$

Letting $x = f(x_0)$ in (6), $u'_+(f(x_0)) = f'_+(f(x_0))u'_+(f^2(x_0))$ and so (7) gives

$$u'_+(f(y)) \geq \left(\frac{f'_+(x_0)f'_+(f(x_0))}{f'_+(y)}\right) u'_+(f^2(x_0)). \tag{8}$$

But since $f(x_0) < y$ and $f(.)$ is strictly increasing, $f^2(x_0) < f(y)$ and by concavity of $u(.)$, $u'_+(f^2(x_0)) \geq u'_+(f(y))$. Therefore, (8) gives

$$u'_+(f^2(x_0)) \geq \left(\frac{f'_+(x_0)f'_+(f(x_0))}{f'_+(y)}\right) u'_+(f^2(x_0))$$

and this contradicts the hypothesis that $f'_+(y) < f'_+(x_0)f'_+(f(x_0))$.

Proof of Theorem 2

Let $f \in \mathcal{F}$ and assume that (3) holds. Since $f(.)$ is continuous and right differentiable on $\mathbf{R}_+$, by an obvious modification of the mean value theorem, for any $b > a > 0$,

$$|f(b) - f(a)| \leq f'_+(\xi)(b-a) \quad \text{for some } \xi \in (a, b).$$

But by hypothesis (A2), $f'_+(\xi) < 1$ so that we obtain $|f(b) - f(a)| \leq (b-a)$ for all $b > a > 0$. It immediately follows that f is absolutely continuous. Therefore, we may apply the second fundamental theorem of calculus for Lebesque integral to write, for any $a > 0$,

$$f(x) = f(a) + \int_a^x f'_+(t)dt \quad \text{for all } x > a. \tag{9}$$

Suppose that there exist $x_0 > 0$ and $y > 0$ such that $x_0 > y$ and $f'_+(x_0) > f'_+(y)$. Since $0 < f(x_0) < x_0$, we have $\lim_{n\to\infty} f^n(x_0) = 0$, and therefore, there must exist a positive integer n_0 such that $y \in [f^{n_0+1}(x_0), f^{n_0}(x_0))$. Applying (3) at $x = f^{n_0}(x_0)$, we then have

$$f'_+(f^{n_0}(x_0)) \leq f'_+(y) < f'_+(x_0).$$

But this is impossible, for by applying (3) successively,

$$f'_+(x_0) \leq f'_+(f(x_0)) \leq f'_+(f^2(x_0)) \leq \cdots \leq f'_+(f^{n_0}(x_0)).$$

We therefore conclude that $f'_+(x) \leq f'_+(y)$ whenever $0 < y < x$; that is, $f'_+(.)$ is decreasing on $(0,\infty)$. But then, in view of (9) and a well-known characterization of concavity (cf. Theorem A of Roberts and Varberg (1973), p. 9-10), we may conclude that $f(.)$ is concave on (a,∞) for any $a > 0$. It follows that $f(.)$ is concave on $(0,\infty)$.

Now let $f'_+(0) = \beta$ and define, for any $x > 0$,

$$G_n(x) := \frac{f^n(x)}{\beta^n} \quad \text{for all } n \in \{0,1,...\} \quad \text{and} \quad G(x) := \lim_{n\to\infty} G_n(x),$$

where $f^0(x) = x$, and for any $n \geq 1$, $f^n(.)$ is the nth iterate of $f(.)$ (see footnote 14). Assume for the moment that $G(x) \in (0,\infty)$ for all $x > 0$, and, for any $c > 0$, define the function $u : \mathbf{R}_{++} \to \mathbf{R}$ as

$$u(x) := \frac{-c}{\log\beta}\log G(x) \quad \text{for all } x > 0.$$

First thing to note is that $G(.)$ is the limit function of a convergent sequence of concave functions, and hence, it is concave on $(0,\infty)$. This implies that $u(.)$ is a concave function. Moreover, for all $x > 0$,

$$\frac{-c}{\log\beta}\log\lim_{n\to\infty}\frac{f^{n+1}(x)}{\beta^n} = \frac{-c}{\log\beta}\log\left(\beta\lim_{n\to\infty}\frac{f^{n+1}(x)}{\beta^n}\right) = \frac{-c}{\log\beta}\log\lim_{n\to\infty}\frac{f^n(x)}{\beta^n} - c,$$

that is, $u(f(x)) = u(x) - c$.

We next claim that $G(.)$ is, in fact, strictly increasing. Since $f^n(.)$ is strictly increasing for each n, $G(.)$ must be non-decreasing, and hence $G'_+(x) \geq 0$ for all $x > 0$. But by (A1), $G(0) = 0$, and hence, that $G(x) > 0$ for all $x > 0$ implies that there exists $y > 0$ such that $G'_+(y) > 0$. Then, by concavity of $G(.)$, $G'_+(x) > 0$ for all $x \in (0,y)$; that is $G(.)$ is strictly increasing on $(0,y)$. Now, let $x_1, x_2 \in f^{-1}((0,y))$ and $x_2 > x_1$. Then, $x_1 = f^{-1}(a)$ and $x_2 = f^{-1}(b)$ for some $a, b \in (0,y)$ such that $b > a$. Since G satisfies $G(f(x)) = \beta G(x)$ for all $x \geq 0$, we have $\beta G(f^{-1}(b)) = G(b) > G(a) = \beta G(f^{-1}(a))$ since $G(.)$ is known to be strictly increasing on $(0,y)$. Thus, $G(f^{-1}(b)) = G(x_2) > G(x_1) = G(f^{-1}(a))$ proving that $G(.)$ is strictly increasing on $f^{-1}((0,y))$. By induction, it follows that $G(.)$ is strictly increasing on $f^{-n}((0,y))$ for all $n \in \{0,1,...\}$ which, in turn, implies that $G(.)$ is strictly increasing on $\bigcup_{n=0}^{\infty} f^{-n}((0,y)) = (0, \lim_{n\to\infty} f^{-n}(y))$. But by (A2), $\cdots > f^{-2}(y) > f^{-1}(y) > y$ so that $\{f^{-n}(y)\}_{n=0}^{\infty}$ is a strictly increasing sequence. If $\{f^{-n}(y)\}_{n=0}^{\infty}$ was bounded, then we would have $\lim_{n\to\infty} f^{-n}(y) = M$ for some $M > 0$, and by continuity of $f^{-1}(.)$ and (A2), we would obtain the following contradiction:

$$\lim_{n\to\infty} f^{-(n+1)}(y) = \lim_{n\to\infty} f^{-1}(f^{-n}(y)) = f^{-1}(\lim_{n\to\infty} f^{-n}(y)) = f^{-1}(M) > M.$$

Therefore, $\{f^{-n}(y)\}_{n=0}^{\infty}$ cannot be bounded, and we conclude that $\lim_{n\to\infty} f^{-n}(y) = \infty$. Consequently, $G(.)$, and thus $u(.)$, is strictly increasing on $(0, \infty)$.

The above analysis shows that the proof of Theorem 2 will be complete if we can show that $G(x) \in (0, \infty)$. Let $x > 0$ be arbitrary. We have, for all $n \in \{0, 1, ...\}$,

$$\frac{f^{n+1}(x)}{\beta^{n+1}} = \frac{f(f^n(x))}{\beta f^n(x)} \frac{f(f^{n-1}(x))}{\beta f^{n-1}(x)} \cdots \frac{f(x)}{\beta x} x$$

so that

$$G(x) = \lim_{n\to\infty} \frac{f^n(x)}{\beta^n} = \left(\prod_{n=0}^{\infty} \frac{f^{n+1}(x)}{\beta f^n(x)} \right) x.$$

Therefore, $G(x) \in (0, \infty)$ if and only if

$$\prod_{n=0}^{\infty} \frac{f^{n+1}(x)}{\beta f^n(x)} \in (0, \infty). \tag{10}$$

Since $f(.)$ is concave, $t \mapsto f(t)/t$ is decreasing. Moreover, $\lim_{t\to 0} f(t)/t = \beta$, and hence $f^n(x)/\beta f^{n-1}(x) \leq 1$ for each n. Consequently, by Theorem 4 of Knopp (1990), p. 220, (10) holds if and only if

$$\sum_{n=0}^{\infty} \left(1 - \frac{f^{n+1}(x)}{\beta f^n(x)} \right) \tag{11}$$

is convergent. By (A3) and the fact that $\lim_{n\to\infty} f^n(x) = 0$, there must exist an integer N and $K > 0$ such that

$$\left| \frac{f(f^n(x))}{f^n(x)} - \beta \right| \leq K f^n(x) \quad \text{whenever } n \geq N$$

and thus,

$$\sum_{n=N}^{\infty} \left(1 - \frac{f^{n+1}(x)}{\beta f^n(x)} \right) \leq \frac{K}{\beta} \sum_{n=N}^{\infty} f^n(x). \tag{12}$$

We shall next show that $\sum_{n=N}^{\infty} f^n(x)$ is convergent. Let $\gamma \in (\beta, 1)$. Notice that since $\{f^{n+1}(x)/f^n(x)\}_{n=0}^{\infty}$ is a decreasing sequence converging to β, there exists an integer L such that

$$n \geq L \quad \text{implies } f^{n+1}(x) < \gamma f^n(x). \tag{13}$$

Of course, there must exist an integer n_L such that $f^L(x) \leq \gamma^{n_L} x$ so that by (13), $f^{L+1}(x) < \gamma^{n_L+1} x$. This can immediately be generalized to

$$f^{L+\ell}(x) < \gamma^{n_L+\ell} x \quad \text{for all } \ell = 1, 2, ...,$$

and therefore, we must have

$$\sum_{n=L}^{\infty} f^n(x) < x \sum_{\ell=1}^{\infty} \gamma^{n_L+\ell} < \infty.$$

Combining this observation with (12), we learn that the series in (11) is convergent. As noted earlier, this proves (10) and hence completes the proof.

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