NONMYOPTIC EQUILIBRIA

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ABSTRACT

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A new concept of equilibrium in normal-form games, based on the idea that players can look ahead and anticipate where a process may end up if they are allowed to make an indefinite number of sequential moves and countermoves from any outcome in a game, is defined and illustrated. Unlike the more myopic equilibrium concepts of Nash and Stackelberg in noncooperative game theory, and bargaining and solution concepts in cooperative game theory, a nonmyopic equilibrium is a look-ahead concept that places no arbitrary limit on the extent of bargaining or kinds of threats that might be made. Among its advantages are that it (1) coincides with the generally accepted minimax solution to two-person, zero-sum games with saddlepoints, (2) shows up long-term stability that more myopic concepts fail to find in other two-person games, including Prisoners' Dilemma and Chicken, (3) is applicable to *n*-person games--with and without coalitions--(4) is calculable by means of an alogrithm, and (5) is invulnerable to "theory absorption."

NONMYOPIC EQUILIBRIA[1]

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1. Introduction

In this paper we shall define a new concept of equilibrium in normal-form, or matrix, games, based on the idea that players can look ahead and anticipate where a process might end up if they are allowed to make an indefinite number of sequential moves and countermoves from any outcome in a game. We call this concept a nonmyopic equilibrium and shall show how it differs from the more myopic equilibrium concepts of Nash and Stackelberg, which consider only unilateral deviations of players, or single best responses by players to another player's strategy choice. We shall also point out that solution concepts in cooperative game theory are encumbered by a similar myopia.

Because the concept of a nonmyopic equilibrium is a rather subtle one, we shall, after reviewing previous research in Section 2, begin our analysis by defining it for 2 x 2 ordinal games, in which each player can rank the four possible outcomes from best to worst. We shall then show that it exists in 37 of the 78 distinct 2 x 2 games (47 percent), generally coinciding with pure-strategy Nash equilibria in these games.

But in two of the 37 games, the nonmyopic equilibria do not coincide with Nash equilibria, or the less myopic Stackelberg and dual Stackelberg equilibria (to be defined in Section 3) in these games. Significantly, these nonconforming cases are game theory's most famous--and problematic--

games, Prisoners' Dilemma and Chicken. In both these games, the "cooperative" outcome is singled out as stable in the nonmyopic but not the Nash or Stackelberg sense, providing a new rationale for its choice by players who are farsighted.

We compare this rationale with that of metagame theory, in which the cooperative outcomes of Prisoners' Dilemma and Chicken are stable in the metagames of these games, but show in Section 4 that metagame theory's equilibrium predictions are not the same. We also compare nonmyopic equilibria with other equilibria in games and supergames.

In Section 5 we turn to general two-person (finite) games and prove several results, including the coincidence of saddlepoints and nonmyopic equilibria in strictly competitive (zero-sum) games. In nonstrictly competitive games, we demonstrate the existence of nonmyopic equilibria that are (1) Pareto-inferior and unique or (2) Pareto-inferior and not coincidental with Nash.

We also offer in Section 5 a general definition of nonmyopic equilibria in two-person games, and in Section 6 indicate how coalitions, and nonmyopic coalition equilibria, can be defined in _n_-person games. In addition, we consider problems associated with finding such equilibria and argue that the computational difficulties should not detract from the theoretical importance of the concept in understanding bargaining processes, and its philosophical significance as an equilibrium concept sustained by, not rendered vulnerable to, "theory absorption" in dynamic games.

Our approach to dealing with the problem of anticipating moves and countermoves in a game might be interpreted as laying the basis for a a theory of threats and counterthreats.[2] Threats, after all, are designed to deter opponents from making moves whose ultimate consequences may not be fully appreciated. We assume in the subsequent analysis that the players can work out the consequences that are set off by deviations from a particular outcome, so communication of these, in a strict sense, is superfluous. In the proverbial real world, however, since we cannot assume players possess all the information we posit, threats and counterthreats may reinforce consequences that are not immediately evident to the players.

More generally, we seek to develop a dynamic theory of strategy that taps the complex and subtle aspects of bargaining implicit in an outcome matrix. It may seem a contradiction in terms to make an equilibrium concept the cornerstone of a dynamic theory, but we believe the foundation for such a theory must be built on the assumption that a process ends somewhere (i.e., there is equilibrium), or, if it does not, the path the process follows in a nonequilibrium state must be specified. In Section 2 we review some other approaches that have been taken to the development of a dynamic theory of strategy.

2. Previous Research

It is well known, and was readily admitted by its creators (von Neumann and Morgenstern [43, pp. 44-45]), that game theory is essentially a static theory. Many efforts have been made to introduce dynamic elements into the theory. In cooperative game theory, for example, a variety of bargaining-type solutions concepts has been proposed, following from the pioneering work of Aumann and Maschler [2].

Generally speaking, however, the solution concepts proposed allow for only limited dynamic interplay, such as "objections" and "counter-objections" in Aumann and Maschler's bargaining-set solution. Their concept does not permit subsequent objections to the counterobjections, and so on, which less myopic bargaining might encompass.

Metagame theory, as developed by Howard [20], is perhaps the most significant attempt to introduce bargaining considerations into non-cooperative games. Essentially, metagame theory provides a rationale for new equilibria in normal-form games, based on the successive expansion of each player's strategies, conditional on other players' strategies.

The theory is innovative but tends to be cumbersome because the conditional calculations of players quickly produce a veritable explosion of strategies, requiring an analysis of very large matrix games indeed. (Just how large can be seen from Howard's [23] analysis of the "general metagames" of Prisoners' Dilemma.) Moreover, the theory is largely developed only for two-person games. Also, the meaningfulness of metaequilibria in the play of a game has been questioned (Harsanyi [17]; Howard [21]). Howard [22] has sought to extend his original theory by introducing various "stability-reinforcing" properties which, in our opinion, point up the difficulties of developing a dynamic theory within his framework. The dynamic theory is further elaborated and illustrated in Howard [24].

Research on differential games, stimulated by Isaacs [25], has been an area of recent ferment. Unfortunately, there are no compelling solution concepts that have meaningful interpretations in a variety of differential games. Indeed, the notion of a "solution" in a differential game is itself controversial (Isaacs [26]). Related to work on differential games are dynamic-process models of the kind developed by Kramer [28, 29], in which paths or trajectories are found in electoral-competition situations that demonstrate party/candidate convergence under specified conditions. But here, too, the parties/candidates are very myopic and do not consider future moves.

Our work is perhaps closer in spirit to Cyert and DeGroot [6,7,8] and Marschak and Selten [32], who consider sequential responses to moves. However, Cyert and DeGroot artificially limit the number of responses, and Marschak and Selten do not consider the nature of the equilibria. We will return to their work later in our paper.

Finally, supergames have been used to analyze the extended play of different games in order to determine optimal strategies in a series of trials. Taylor [42], for example, has analyzed supergames of Prisoners' Dilemma, but the results tend to be quite sensitive to such parameters as the discount rate of players. Additionally, repeated trials of a single game hardly capture the intricacies of many real-life bargaining situations. We shall, however, briefly compare in Section 3 the proposed nonmyopic equilibrium with equilibrium concepts that have been suggested for supergames.

In a capsule review, of course, we cannot do justice to all the interesting and important work on dynamic approaches to strategic analysis. Moreover, our criticisms of the work cited are not meant to suggest that we have solutions to all the specific problems to which we alluded. Our point rather is that there remain problems in the development of an adequate dynamic theory of strategy.

We close this section by specifying three criteria that we think an adequate dynamic theory should satisfy:

1. It should be nonmyopic--there should be no arbitrary limitation on the extent of conjecturing, or where it may lead.
2. There should be readily calculable solutions--those for which an algorithm can be specified--that indicate where the process will end up. If such equilibria do not always exist, conditions for their existence should be given.
3. The concept of a solution should be general--applicable to both two-person and *n*-person games, with and without coalitions.

We would also add a fourth and less tangible criterion: the concept of a solution should be interpretable and meaningful in a variety of contexts. It is hard to operationalize this criterion, but perhaps two measures of a solution's usefulness are that it (1) coincide with solution concepts that are generally accepted (e.g., minimax in two-person, zero-sum games with saddlepoints), and (2) offer new insights into games with no generally accepted solutions (e.g., two-person, nonzero-sum games).

3. Nonmyopic Equilibria and Related Concepts

We begin by offering an intuitive idea of the equilibrium concept we shall shortly define formally. The idea is that players look ahead and ascertain where, from any outcome in an outcome matrix, they will end up if they depart from this starting outcome. Comparing the final outcome with the starting outcome, if they are better off at the starting outcome--taking account of their departures, possible responses to their departures, and so on--they will not depart in the first place. In this case, the starting outcome will be an equilibrium in an extended, or nonmyopic, sense. This sense implies that, in a game of complete information, players act on an extended notion of rationality: they not only prefer better to worse outcomes but, in addition, base their comparisons on the assumption that all players will make optimal departures, looking ahead, in a strictly alternating sequence.

It is not necessary to assume that the sequential departures (first row, then column, etc.) are physical moves. Rather, they may go on in the minds of the players, tracing out the consequences of their possible departures, and stopping them--or not starting them in the first place--when they anticipate they will end up worse off. Thus, the nonmyopic equilibrium we shall define can be interpreted as an outcome stable in the face of either physical moves or (putative) threats and counterthreats.

Although it is not necessary to assume that the sequential departures are physical, we would argue that sequential physical moves are a more accurate description of real-world behavior than simultaneous choices (possibly repeated in a supergame). As a case in point, the Iranian hostage crisis of 1979-80 did not involve simultaneous decisions by Iran and the United States; rather, each side responded to the other, trying to anticipate where each move might lead.

In effect, our sequential-move analysis can be seen as a discrete analog of a continuous game, wherein behavior in past plays of the game continuously determines possible moves available in the present. To make these ideas more concrete in the discrete case, consider a 2 x 2 game, in which there are two players, each of whom has just two strategies. For convenience, assume that each player's best outcome is indicated by "4," next best by "3," next worst by "2," and worst by "1." Thus, the higher the number, the better the outcome, but the numbers indicate only an ordinal ranking, not cardinal utilities, that the players associate with the outcomes.

How can the look-ahead idea underlying a nonmyopic equilibrium be formalized for 2 x 2 games (see Figure 1)? For each outcome, one con-

FIGURE 1

2 X 2 GAME

	C	
R	(a_1,a_2)	(d_1,d_2)
	(b_1,b_2)	(c_1,c_2)

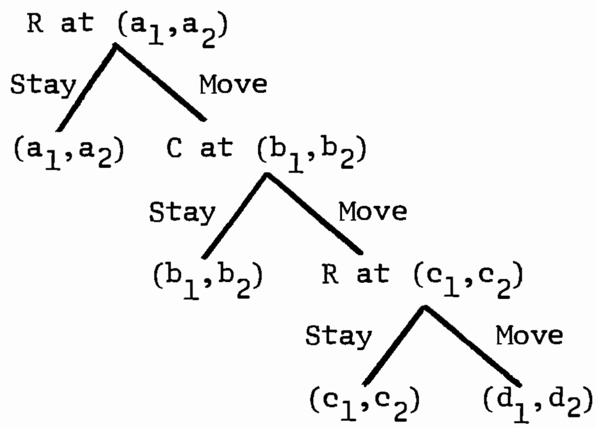
ducts a test to ascertain where departures by row (R) and column (C), from any starting outcome, will terminate (if anywhere). In the outcome matrix of Figure 1, outcomes subscripted by 1 are R's, outcomes subscripted by 2 are C's, and preferences are assumed to be strict.

Without loss of generality, let (a_1,a_2) be the starting outcome. Moves first by R, then by C, and so forth, from this outcome can be represented by a game in extensive form, or game tree, as shown in Figure 2. A "final outcome" on the game tree is reached when the player with the next move has no incentive to move and so stays at that outcome.

To determine whether an outcome is final, it is necessary to work backward up the game tree. Consider R's move at (c_1,c_2). Whether R prefers (d_1,d_2) (i.e., $d_1 > c_1$) or (c_1,c_2) (i.e., $c_1 > d_1$), call his

FIGURE 2

GAME TREE FOR MOVES, STARTING WITH R, FROM (a_1,a_2)



preferred outcome (x_1,x_2). Substitute (x_1,x_2) for the endpoint of the second-to-bottom "Move" branch emanating from "C at (b_1,b_2)."

Working backward again, one would then compare (b_1,b_2) with (x_1,x_2) Call the outcome which C prefers in this comparison (y_1,y_2). This outcome would then move up to a final comparison with (a_1,a_2) by R. Whichever outcome "survived" this comparison would be the surviving outcome, which we label (z_1,z_2), at the top, or root, of the tree.

To determine whether (z_1,z_2) is a final outcome, or the process is intransitive, one has to ascertain whether the surviving outcome would indeed terminate the process of moves and countermoves. A simple test suffices: (z_1,z_2) is a final outcome if, moving from the top of the game tree of Figure 2 to the bottom, a player at any branch (node) will stay because, by doing so, he obtains his best outcome. Clearly, if $a_1 = 4$ or $c_1 = 4$, R will have no incentive to move from (a_1,a_2) or (c_1,c_2), respectively; and if $b_2 = 4$, C will have no incentive to move from (b_1,b_2). Similarly, although we do not show a move by C in the game tree of Figure 2 from (d_1,d_2) back to (a_1,a_2), which would complete the cycle, such a move would not occur if $d_2 = 4$, giving C his best outcome at (d_1,d_2).

In sum, for an outcome to be final, two conditions must be met:

1. <u>That outcome survives</u>: the backward comparisons of the game-tree analysis eliminate all other outcomes.
2. <u>There is termination</u>: there exists a node in the game tree such that the player with the next move can ensure his best outcome by staying at it.

Basically, Condition 2 is needed to guarantee that the process is not intransitive--that is, would not cycle back to (a_1,a_2). Given this is the case, Condition 1 gives an algorithm for finding the rational outcome the players will choose when they look ahead.

It is important to note that the final outcome will not necessarily be where the process would terminate simply because it is best for the player with the next move. For example, if $(d_1,d_2) = (1,4)$, the second condition would be satisfied, but the process would never reach this outcome. The reason is that (d_1,d_2) would be eliminated by (c_1,c_2) as the surviving outcome since, necessarily, $c_1 > d_1 = 1$ in the comparison at the bottom of the game tree in Figure 2. Whether (c_1,c_2), (b_1,b_2), or (a_1,a_2) would then emerge as the (final) surviving outcome would depend on the players' preferences for the other outcomes.

If Condition 2 is not met, an extension of the game tree of Figure 2 to include the repetition of moves in a cycle could lead to surviving outcomes different from those given by ending the tree at (d_1,d_2). For example, an extension of the tree from, say, (d_1,d_2) back to (a_1,a_2)

could switch the surviving outcome from (d_1,d_2) to (a_1,a_2), rendering the claim for either outcome as "final" a tenuous one. To avoid making the final outcome dependent on where the tree analysis stops, and therefore essentially arbitrary, we insist that there be termination: only if the process assuredly reaches an outcome best for a player with the next move--before cycling--will the surviving outcome be indisputably final.

If there is no cycling, and hence no intransitivity, because Condition 2 is satisfied, the backward comparisons we have specified will allow the players to determine at what node the process will stay. If the process stays immediately at the starting outcome, (a_1,a_2), R would have no incentive to depart initially; otherwise he would, and (a_1,a_2) would not be a nonmyopic equilibrium.

More formally, an outcome (a_1,a_2) is a nonmyopic equilibrium for R in a 2 x 2 ordinal game iff (if and only if) the final outcome, as determined by backward comparisons starting from (d_1,d_2) in the game tree of Figure 2, is (a_1,a_2). Analogous comparisons, starting with a first move by C at (a_1,a_2), define a nonmyopic equilibrium for C. We say that (a_1,a_2) is a nonmyopic equilibrium iff it is a nonmyopic equilibrium for both R and C. Clearly, a departure from such an outcome by either player would lead him to an unequivocally worse outcome.

Note that if (a_1,a_2) is not a nonmyopic equilibrium, the final outcome that could be triggered by an initial move by either R or C might be (b_1,b_2), (c_1,c_2), or (d_1,d_2), given the process is not intransitive. Whichever outcome it is, it will be preferred by the departing player to (a_1,a_2), for otherwise he would not have departed from (a_1,a_2) in the first place.

The process, however, will not reach a final outcome if Condition 2 is not satisfied and it is, therefore, always rational to move at each node, including (d_1,d_2) if R departs initially. Although one cannot say in the case of such an intransitivity that R would do unequivocally worse if he departed from (a_1,a_2), he would certainly not be any better off if the process cycled completely. More important, however, is the fact that there is no stable outcome if cycling occurs, for it may continue indefinitely. Indeed, if (a_1,a_2) is not a nonmyopic equilibrium because it is in a cycle, neither is any other outcome in the cycle.

Our principal interest, however, is in equilibrium outcomes; we assume that disequilibrium outcomes contribute negligible value to the players as they pass through them. For example, if payoffs are a function of time, and moves through the disequilibrium outcomes are per day while moves in equilibrium are per year, it is the equilibrium solution, if it exists, that is consequential for the players.

Our notion of a nonmyopic equilibrium differs racically from more myopic concepts. Consider that of Nash [36] and Stackelberg (as described in Henderson and Quandt [18, pp. 229-231]). An outcome is a Nash equilibrium if neither player (in a two-person game, which is what we shall focus on here), by departing unilaterally, can improve the outcome for himself.[3] An outcome is a Stackelberg equilibrium if neither player, after anticipating the best response of the other (follower) to his (the leader's) choice of a strategy, can obtain a better outcome for himself--each player alternately being considered the leader, who departs initially, in the test. If the equilibrium is the same whichever player is considered the leader (follower), the outcome is a dual Stackelberg equilibrium.

Stackelberg's concept is clearly less myopic than Nash's, but it does not allow for best responses to best responses, and so on. Its restriction of best responses--to just one by the follower--is arbitrary in games in which subsequent responses are rational.

Moreover, Stackelberg's concept assumes responses to an initial strategy choice of a leader, whereas the nonmyopic concept assumes possible departures from an outcome.[4] Put another way, in ascertaining nonmyopic equilibria, players are assumed to be at a particular outcome (based on previous strategy choices), or to consider the possibility of being at that outcome.

To assess this outcome's long-run stability, players evaluate the consequences of departures from it, or that of other outcomes to which they might move sequentially in a series of steps. The process starts at an outcome, perhaps a status quo point, and stability depends on where sequential departures may move the process. It should be stressed that departures are assumed not to be simultaneous but rather alternating, which we think mirrors the reality of most bargaining processes, wherein players react successively--not instantaneously--to each other's choices.

The concept of a nonmyopic equilibrium differs in another significant way from Stackelberg's concept. Leaving aside the fact that an outcome, rather than a strategy, is the starting point, a nonmyopic equilibrium does not simply extend Stackelberg to an endless series of (myopic) best responses. Departures are triggered by a full anticipation of how each player at each stage will respond, vis-à-vis his present outcome and his anticipation of the entire chain of consequences set off by each (rational) departure.

This calculation, unlike that for a parlor game like chess, has ramifications for the extended play of all games in which sequential physical or mental moves are possible. Of course, constructing a normal-form representation of chess, not to mention finding nonmyopic equilibria in it, is well-nigh impossible, but in principle the concept of a nonmyopic equilibirum is defined for this game.

Recently, a similar equilibrium concept was proposed by Marschak and Selten [32], based on an "extended response function" that specifies the outcome of a sequence of deviations by some player from any outcome. Marschak and Selten show that there may not exist response functions in games with certain desirable properties--for example, that they are "restabilizing," or rationality-preserving.

In our view, the significant problem is not to find "reasonable" response functions but rather to define rationality in terms of survival and termination conditions and ask what outcomes in games (if any) will be stable in the look-ahead sense. Marschak and Selten offer no explicit definition of rational behavior but instead analyze response functions in supergames, which seem to us an unnecessary complication.

Supergame equilibria are also studied in Aumann [1], Friedman [12, 13,14,15], and Roth and Murnigham [39], among other places. Essentially, these are Nash equilibria in a supergame--that is, a game comprising repeated plays of the same (constituent) game--in which constituent game strategies are assumed to be chosen simultaneously by the players in each play of the constituent games. (The Nash equilibria in the supergame are not necessarily Nash equilibria in the constituent games strung together.) This, of course, is a very different concept of stability from that based on sequential moves of players, looking a ahead, in a single game.

Cyert and DeGroot [6] make this point emphatically in justifying their own sequential model, but they offer a solution only in a duopoly (two-person) game in which the number of periods (plays) is known in advance. These restrictions remain unaltered in their more sophisticated learning models (Cyert and DeGroot [7,8]). By comparison, we assume an indefinite number of sequential moves (dependent on the game being played) in the determination of equilibria. Moreover, the concept we have formally defined for 2 x 2 games is readily generalizable to general two-person as well as *n*-person games in normal form, about which we shall say more later.

A recent and still different approach to modeling game dynamics, utilizing summary measures of past outcomes, is developed in Smale [41], but it is based on very different assumptions from ours.

4. Classification of Nonmyopic Equilibria in 2 x 2 Games

A complete enumeration of the 78 distinct 2 x 2 ordinal games in which preferences are strict is given in Rapoport and Guyer [37] and Brams [5]. These games are distinct in the sense that no interchange of strategies by R or C, or interchange of players, or both, will transform one of the 78 games into any other.

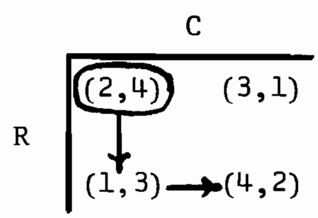
Of the 78 games, 37 (47 percent) have nonmyopic equilibria, in all of which--except one--the equilibrium is unique. These games can be divided into three mutually exclusive classes:

I. 21 no-conflict games (57 percent). These games all have one outcome mutually best for both players [(4,4)], so neither player has an incentive to depart from it. All Class I equilibria are also pure-strategy Nash equilibria.[5]

II. 9 games (24 percent) in which one player obtains his best outcome but the other does not. An example is game 36 in Rapoport and Guyer [37] and Brams [5], which is shown in Figure 3 with its nonmyopic

FIGURE 3

CLASS II GAME



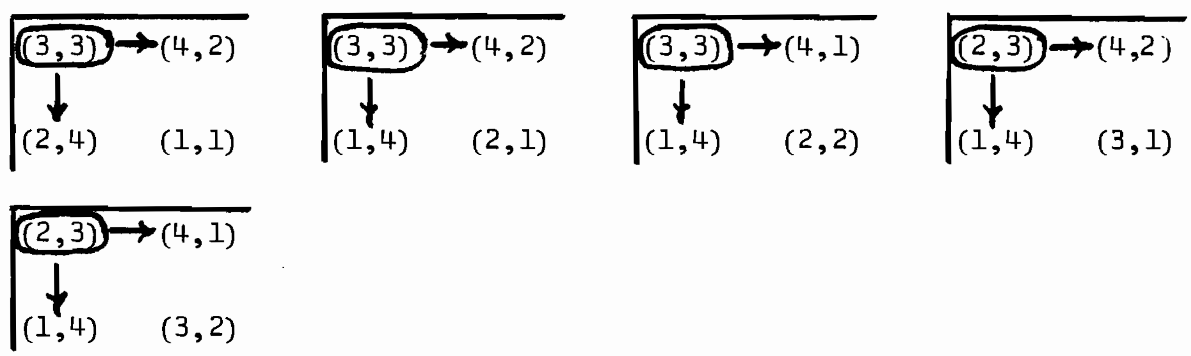
equilibrium circled. Obviously, since (2,4) is C's best outcome, R is the only player with a possible incentive to depart from (2,4). Should R move the process to (1,3), C would have no incentive to move it to (4,2), because this is R's best outcome and he therefore would not be motivated to move from it. Since (4,2) is only C's next-worst outcome, he would have no incentive to move from (1,3). But now R would have no incentive to move to (1,3) in the first place, so (2,4) is a nonmyopic equilibrium.

In this manner, the backward reasoning described in Section 3 establishes $(a_1,a_2) = (2,4)$ as the surviving outcome, which is also final because there would be termination at (4,2) if this outcome were reached. However, since departures from (2,4) would be irrational for for both players, this outcome is a nonmyopic equilibrium. All Class II nonmyopic equilibria are also pure-strategy Nash equilibria.

III. 7 games (19 percent) in which neither player obtains his best outcome. Five of these games are shown in Figure 2 (games 7-11 in Rapoport and Guyer [37]; games 16-20 in Brams [5]). If *either*

FIGURE 4

FIVE CLASS III GAMES



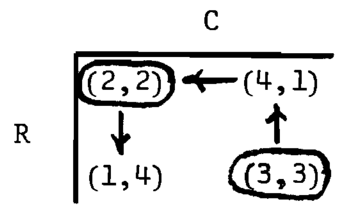
player departs from the nonmyopic equilibria in any of these games, the outcome reached would not only terminate the process but also be best for the *other* player. Because this terminal outcome is inferior to the starting outcome for the initially departing player, he would not be motivated to depart from it. The nonmyopic equilibrium in each of these five games is also a pure-strategy Nash equilibrium.

The remaining two games in Class III are the most interesting for two reasons: (i) a nonmyopic equilibrium in each of these games is *not* a pure-strategy Nash equilibrium; (ii) despite the extensive discussion of these games in the literature, there is no generally agreed-upon "solution" to either.

Consider first the game shown in Figure 5, which is called Prisoners' Dilemma and has a rich and colorful history (Brams [4, chs. 4 and 8, and citations therein]). The reason (2,2) is a nonmyopic equilibrium in

FIGURE 5

PRISONERS' DILEMMA



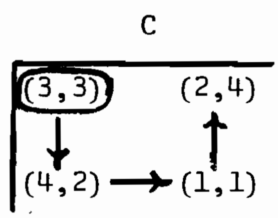
this game is the same as that given for the five games in Figure 4. By comparison, outcome (3,3) is a nonmyopic equilibrium because if, say, R departed from it, the terminal outcome reached would be (1,4), as shown by the arrows in Figure 5, which is inferior for R.

In fact, however, the penultimate outcome reached, (2,2), would have its own "finality," because it itself is a nonmyopic equilibrium. In either event, comparing (3,3) with either (2,2) or (1,4) for R, R would have no incentive to depart from (3,3), and neither of course would C because of the symmetry of the game. Hence, (3,3), as well as (2,2), is a nonmyopic equilibrium in Prisoners' Dilemma; however, only (2,2) is a pure-strategy Nash equilibrium, in addition to being a (dual) Stackelberg equilibrium. Moreover, note that from (1,4) or (4,1) the process would move to (2,2), not (3,3), making (2,2) the <u>absorbing</u> equilibrium in this game: from any outcome except the other nonmyopic equilibrium [(3,3)], (2,2) would be the outcome to which the process would gravitate in a series of moves and countermoves by the players.

The last game in Class III, shown in Figure 6, is known in the literature as Chicken. It has a unique nonmyopic equilibrium, (3,3), which is not also a pure-strategy Nash equilibrium. Neither is it a dual Stackelberg equilibrium: if R is the leader, the equilibrium is

FIGURE 6

CHICKEN



(4,2); if C is the leader, the equilibrium is (2,4). Each of these Stackelberg equilibria is also a Nash equilibrium because, from each neither player would have an incentive to depart unilaterally.

As shown by the arrows in Figure 6, the terminal outcome that would be reached if R departed initially from (3,3) is (2,4), which is inferior for him. Similarly, by the symmetry of the game, C has no incentive to depart from (3,3), so this is a nonmyopic equilibrium but, like (3,3) in Prisoners' Dilemma, it does not absorb other outcomes.

It is worth pointing out that if we relax our assumption of strictly alternating departures from an outcome, and instead allow players to backtrack or not respond, then the nonmyopic equilibrium in games like Chicken might not be sustained. Specifically, assume a player moves the process in Chicken to, say, (1,1), but it is not incumbent on the second player to move next. To the contrary, the second player can respond by not moving, hoping to force the first player to backtrack to a previous (and more favorable) outcome than one he (the second player) can achieve by making the next move himself. Backtracking, of course, implies that moves are not irrevocable--one can renege on a commitment; permitting it would leave indeterminate whether a player can force a new response by another.

We prefer to assume that, perhaps for reasons external to the game, each player can always force a new move, different from the preceding one, unless the game terminates at that move. To be sure, one player may be more powerful than another, and therefore able to hold out longer at (1,1), forcing the other player back. But then the other player should be able to recognize this fact before he makes his initial move. To avoid complications like this, based on information not contained in the outcome matrix, we assume that preferences alone dictate moves, and moves always occur in an alternating sequence.

Chicken contains two competitive pure-strategy Nash equilibria, (4,2) and (2,4), one best for R and the other best for C. This is why the cooperative (3,3) outcome in Chicken, as well as Prisoners' Dilemma, has been so difficult to justify as a solution: it is unstable (in a myopic sense), and there exist pure-strategy Nash equilibria which are not. In Prisoners' Dilemma, the claim of (2,2) as a solution also rests on the fact that it is the product of dominant strategy choices by both players, even if it is inferior for both to (3,3).

Taking a more farsighted view, however, the (3,3) outcome in both Prisoners' Dilemma and Chicken is stable--though not uniquely so in Prisoners' Dilemma--but it does not absorb other outcomes, making it, in a sense, only locally stable. Strikingly, these two games, long considered pathological because the obvious compromise solution, (3,3), is not a Nash equilibrium, are the only ones in which nonmyopic equilibria do <u>not</u> coincide with pure-strategy Nash equilibria. The abiding interest shown by theorists in these two games seems to stem from an implicit recognition that cooperation can somehow be justified. But how? Nonmyopic equilibria that allow for the possibility of sequential moves and countermoves offer one justification.

It should be noted that (3,3) is also in equilibrium in the metagames of Prisoners' Dilemma and Chicken, but so too are (2,2) in Prisoners' Dilemma, and (4,2) and (2,4) in Chicken. Also, in the metagame of Chicken, (3,3) is associated with a dominated strategy of one player, rendering it controversial as a solution (Harsanyi [17]; Howard [24]; see Brams [4, ch. 5] for a review of the controversy). By contrast, (3,3) is the unique nonmyopic equilibrium in the game of Chicken.

To summarize, the 37 2 x 2 games containing at least one nonmyopic equilibrium can be assigned to three disjoint classes, depending on the ranking the players associate with these equilibria. The large majority of these games (81 percent) are no-conflict games, or games in which one player but not the other obtains his best outcome at the nonmyopic equilibrium. The seven games in which neither player obtains his best outcome include Prisoners' Dilemma and Chicken, which are the only 2 x 2 games to have nonmyopic equilibria which are not also Nash (though Prisoners' Dilemma has a second nonmyopic equilibrium which is Nash).

5. General Two-Person Games

We shall not attempt to classify nonmyopic equilibria in m x n games, where $m > 2$ or $n > 2$ or both, and there are thus more than four outcomes that can be ranked by the players. Instead, we shall prove four theorems (and one lemma) about general two-person finite games, two of which are restricted to strictly competitive (or zero-sum) games.

A *strictly competitive game* is one in which the best outcome for one player is worst for the other, the next-best for one is next-worst for the other, and so on. If the players associate cardinal utilities with the outcomes, a game is <u>zero-sum</u> if the sum of the utilities, or payoffs to each player, is zero at each outcome.

An example of a strictly competitive game is the last one shown in Figure 4, which has a unique nonmyopic equilibrium. This outcome is also a saddlepoint, because for R the minimum of the row (2) is also the maximum of the column in which it falls. Since R cannot do better by switching to his second strategy, nor can C do better by switching to his second strategy, this outcome is also a Nash equilibrium.

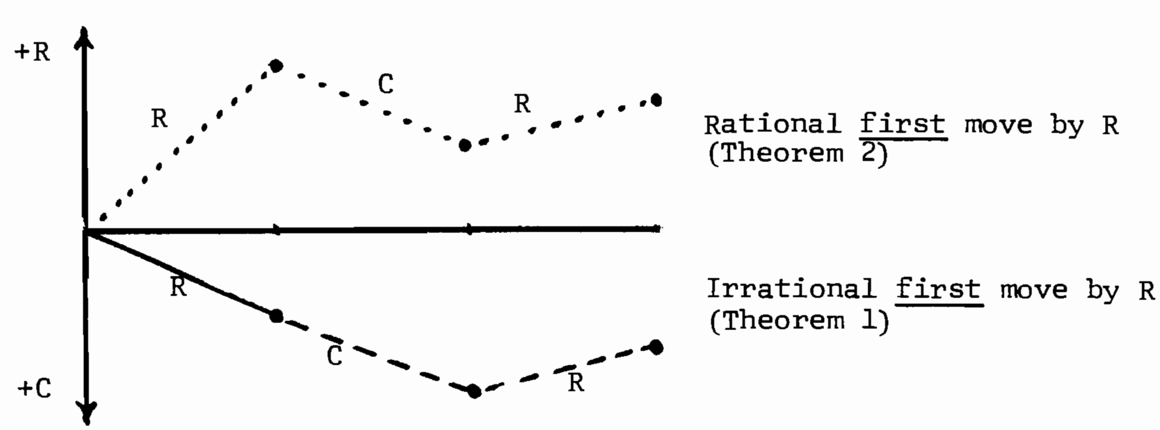
To prove our first results about strictly competitive games, we first prove

LEMMA: <u>In a finite, two-person, strictly competitive game, a player will depart from an outcome only if it is immediately better for him</u>.

PROOF: Without loss of generality, assume it is R who departs initially, and the outcome to which he departs is worse for himself (solid line in Figure 7). Then either (i) C will not depart subsequently

FIGURE 7

DEPARTURES BY PLAYERS IN A STRICTLY COMPETITIVE GAME



or (ii) C will depart subsequently if he can anticipate the process will terminate at an outcome better for himself (dashed lines in Figure 7). In either case, R would end up worse by departing initially to an inferior outcome. Hence, he should not consider departing unless his initial departure leads to an immediately better outcome for himself. Q.E.D.

THEOREM 1: _If a finite, two-person, strictly competitive game contains a saddlepoint, it is a nonmyopic equilibrium._

PROOF: By the definition of a saddlepoint, any departure by R or C will immediately lead to an inferior outcome. The outcome at which the process terminates, whether reached after the first departure or subsequently (i.e., at endpoint of solid line, or one of dashed lines, in Figure 7), will be inferior to the saddlepoint from the proof of Lemma, so the saddlepoint is a nonmyopic equilibrium. Q.E.D.

THEOREM 2: _In a finite, two-person, strictly competitive game, an initial departure (say, by R), and a subsequent departure (by C), bracket the ranks of the outcome at which the process terminates (if it does)._

PROOF: By Lemma, R will depart from a starting outcome, and C from the subsequent outcome, only if both do immediately better after each move (dotted line in Figure 7). Clearly, a terminal outcome will not exceed the first-departure outcome (for R), because C, looking ahead, would otherwise have terminated the process at this outcome; and it will not fall below the second-departure outcome (for C), because R, looking ahead, would otherwise not have made his initial departure that would then allow C to make a subsequent departure, worse for R than the starting outcome. Therefore, the first two moves from the starting outcome by each player establish the upper and lower limits within which the final outcome will fall. Q.E.D.

We have shown that if a finite, two-person, strictly competitive game contains a saddlepoint, it is a nonmyopic equilibrium (Theorem 1); if an outcome in such a game is not a saddlepoint, departures by R and C will "zig and zag," but they will never terminate at an outcome that exceeds the top of the first zig or that falls below the bottom of the subsequent zag (Theorem 2). Because R can always ensure the maximum of his row minima (maximin)--by moving to the row containing it--and C an ensure his corresponding minimax, the zigs and the zags will necessarily stay within these bounds in a strictly competitive game.

We are unable to characterize, in any simple way, nonmyopic equilibria in general two-person, finite games, or even the kind of path that departures from a nonequilibrium outcome might follow in such a game. However, it is possible to establish certain existence results, which do not obtain in 2 x 2 games, that show nonmyopic equilibria are not immune from the social pathologies that can afflict Nash equilibria.

THEOREM 3: In general two-person, finite games, a unique nonmyopic equilibrium may be Pareto-inferior.

PROOF: The game in Figure 8 contains a unique nonmyopic equilibrium, (2,2): should R or C depart from (2,2) to a (1,4) or (4,1) outcome, respectively--as indicated by the arrows--this outcome would be terminal and inferior for the departing player. Outcome (2,2) is Pareto-inferior because two outcomes, (4,3) and (3,4), are preferred by both players. It can readily be established that neither (4,3) nor (3,4)--or any other outcomes--are nonmyopic equilibria (see Definition below). Q.E.D.

FIGURE 8

GAME WITH UNIQUE NONMYOPIC EQUILIBRIUM THAT IS PARETO-INFERIOR

	C		
R	(2,2) →	(4,1) →	(4,1)
	↓ (1,4)	(2,2)	(4,3)
	↓ (1,4)	(3,4)	(1,1)

It is useful to recall that of the 78 2 x 2 games, Prisoners' Dilemma is the only game that contains a nonmyopic equilibrium, (2,2), that is Pareto-inferior. However, because (3,3) is also a nonmyopic equilibrium in Prisoners' Dilemma, (2,2) is not unique--as it is the unique nonmyopic equilibrium in the game in Figure 8.

The 3 x 3 game in Figure 8 raises the question of how we determine what, if any, outcomes are nonmyopic equilibria in games larger than 2 x 2. In general, the following algorithm can be applied to two-person, finite games:

DEFINITION: *A starting outcome* (a_1,a_2) *is a nonmyopic equilibrium if it is final*: (1) *the surviving outcome, as determined by backward comparisons for each of the game trees defined by initial departures of R and C, is* (a_1,a_2); *and* (2) *departures from* (a_1,a_2) *always lead to a terminal outcome best for the player with the next move*.

A few things implicit in this definition should be clarified. First, we assume that each game tree, starting with an initial departure by either R or C, specifies *all* possible ways of reaching each outcome in a sequence of moves by the two players. Second, we assume there is no repetition of outcomes in the game tree; the repetition of an outcome defines a cycle, and should it be rational to return to an outcome because Condition 2 above is not satisfied, then we assume that outcome cannot be final. Third, we assume, as before, that the tree is analyzed from the bottom up, and the final outcome is the one that survives all comparisons, given the process is not intransitive.

Naturally, even for games only somewhat larger than 2 x 2, the number of branches, and hence paths to be investigated, on expanded versions of the game tree shown in Figure 2 enormously increases the difficulty of finding nonmyopic equilibria. The reader can test his checking skills by verifying the existence of the two nonmyopic equilibria we claim exist in the example that constitutes the proof of

THEOREM 4: <u>In general two-person, finite games, nonmyopic equilibria may be Pareto-inferior and not pure-strategy Nash</u>.

PROOF: It can be demonstrated that the two circled outcomes in the game in Figure 9 are the only nonmyopic equilibria in this game.

FIGURE 9

GAME WITH PARETO-INFERIOR, NONMYOPIC EQUILIBRIUM THAT IS NOT NASH

	C			
R	((3,2))	(4,1)	(4,1)	(1,4)
	(1,4)	(2,2)	(4,3)	(1,4)
	(1,4)	(3,4)	(1,1)	(1,4)
	(4,1)	(4,1)	(4,1)	((2,2))

Unlike the Pareto-inferior nonmyopic equilibrium in the game in Figure 8, (3,2) is not a pure-strategy Nash equilibrium [though (2,2) is]. Q.E.D.

We have now established that not only may nonmyopic equilibria be unique and Pareto-inferior (Theorem 3) but also that Pareto-inferior equilibria may be distinct from Nash (Theorem 4).

6. Is a New Equilibrium Concept Needed?

At first blush, it may seem terribly demanding to require of players that they plot out the consequences of each and every move they can make if one departs from an outcome in the matrix. But is it? We know that such activities as bicycle riding and billiards require awesome mathematical-physical calculations, if made explicit, to sustain one's balance or make a good shot, but human beings make these kinds of calculations, implicitly, in acts they perform every day.

So at a cognitive level in games, we believe, especially when the consequences, as in Chicken, may be disastrous for both players. As Theodore Sorenson described American deliberations in the Cuban missile crisis, which has been modeled as a game of Chicken (Brams [3]),

> We discussed what the Soviet reaction would be to any possible move by the United States, what our reaction with them would have to be to that Soviet reaction, and so on, trying to follow each of those roads to their ultimate conclusion (quoted in Holsti, Brody, and North [19]).

Nonmyopic equilibria, in our opinion, furnish a new basis both for studying dynamic behavior in games and for determining stable outcomes in the face of an anticipated series of sequential moves by the players, looking ahead. In terms of the criteria we suggested earlier, they are (1) nonmyopic, in the sense of imposing no arbitrary limitation on the extent of conjecturing or where it may lead, (2) calculable by means of an algorithm, and (3) general, being applicable to all normal-form games, including those with more than two players, as described below.

We would also contend that nonmyopic equilibria meet the fourth, less tangible criterion of being readily interpretable, in a variety of contexts, wherein the thinking of players extends over time. This concept, it will be recalled, coincides with a saddlepoint in two-person, strictly determined games, which is generally agreed to be the most compelling solution concept in game theory.

Moreover, these equilibria depend only on preferences, making them less demanding than many cooperative solution concepts in game theory that require that cardinal utilities be associated with outcomes. Indeed, even the noncooperative notion of mixed strategies in two-person, zero-sum games without a saddlepoint assumes cardinal-utility calculations.

In our previous analysis, we assumed two-person games of complete information in which players not only have full knowledge of the strategies and outcomes but also can make anticipatory strategic calculations of the consequences of all possible moves and countermoves. In principle, these calculations can be extended to *n*-person games, though we acknowledge the practical difficulty of checking the consequences of all possible moves and countermoves by more than two players.

Coalitions, whose members adopt joint strategies, have a natural interpretation in the nonmyopic framework. Define a coalition to be a subset of $k \geq 2$ players in an *n*-person game $(k < n)$ whose strategy set is given by the Cartesian product of individual coalition-member strategies. In the sequential-move analysis, its members would make simultaneous moves in the k dimensions, or axes, they define. A question yet to be explored is under what circumstances coalitions are more effective than individual players in inducing favorable nonmyopic equilibria for their members, or in upsetting equilibria that are stable when coalitions cannot form.

In n-person games, with or without coalitions, the problems of finding nonmyopic equilibria are magnified. It seems unlikely that there exist efficient algorithms, akin to linear-programming algorithms for finding minimax solutions, that can rapidly locate nonmyopic equilibria. In fact, we conjecture that the problem of finding nonmyopic equilibria is NP-complete (Garey and Johnson [16]), and only an exhaustive search of virtually all paths through an outcome matrix, from each outcome to every other, can settle the question of where (if anywhere) the process will terminate when players make optimal look-ahead choices. In fact, it appears that the computer will be indispensable in finding nonmyopic equilibria in specific games much larger than 2 x 2.

Despite these practical difficulties, we believe that the theoretical concept of a nonmyopic equilibrium is an important one. Philosophically, it offers an approach to understanding implications of "theory absorption," as originally propounded by Morgenstern [33] and analyzed and reviewed most recently in Dacey [9]. The underlying idea is that once players have accepted a theory, and act on the basis of it, their behavior may change from what it was before there was feedback from the theory to its object of study. In particular, an understanding of the theory may destroy the predictions it makes, which is a fact that has also been noted by Lefebvre [31] and whose recognition in the economic sphere has given rise to the "rational expectations" school of thought (Kantor [27]). This school, in our view, has been encumbered by its undue reliance on myopic equilibrium concepts, which are vulnerable to theory absorption.

Nonmyopic equilibria, we contend, are stable in the face of theory absorption. Rather than being vulnerable to moves and countermoves, as are pure-strategy Nash and Stackelberg equilibria, nonmyopic equilibria are sustained by the very information that no sequence of departures can lead to a better stable outcome for the departing player(s), and, if preferences are strict, will always be worse. Nonmyopic equilibria are thus "absorption-proof"--given the alternating, nonretractable moves we have specified are possible--and hence provide a stronger conceptual foundation on which to build a theory of rational expectations than do other equilibria.

We indicated earlier that sequential choices in games might take the form of either physical moves or verbal moves, such as threats and promises. Whatever form they take, we think nonmyopic equilibria offer unique insight into the stability, or lack thereof, of outcomes in ongoing games, wherein play does not abruptly terminate after strategy choices have been made. Most real-life games, we submit, are ongoing, and static equilibrium concepts say little about their stability.

As important as the stability of an outcome, however, is the dynamics of a game, including the possible paths bargaining and negotiation processes might follow as moves and countermoves are contemplated or chosen. The nonmyopic viewpoint provides both a concept of stability and contingent calculations for understanding exactly how (if at all) stability is achieved in a dynamic game. In our opinion, it is a denial of rationality to freeze the outcome of a game when it is not in the interest of players to think outcomes are static, and the strategy choices that lead to them are irrevocable.

FOOTNOTES

1. We are grateful to Morton D. Davis, Marek Hessel, Laura Scalia, and Philip D. Straffin Jr. for valuable comments on an earlier draft of this paper.
2. For parallel work on threats, see Schelling [40], Moulin [34], and Laffond and Moulin [30].
3. See Rosenthal [38] for an interesting variation on Nash.
4. The concept of a sophisticated equilibrium, proposed by Farquharson [10] and recently discussed by Moulin [35] and Ferejohn, Grether, and McKelvey [11], is found by the successive elimination of dominated strategies in a normal-form game. It has certain desirable properties, especially in the implementation of social choice functions. However, like Nash and Stackelberg, this equilibrium concept is not relevant to assessing the consequences set off by sequential moves and countermoves of players from outcomes, as opposed to their choosing different strategies.
5. It should be noted that 9 of the 78 games (12 percent) do not have pure-strategy Nash equilibria; since their mixed-strategy equilibria (assuming their payoffs are cardinal) do not have an obvious non-myopic equilibrium counterpart, we do not consider them here in comparing different equilibria in the 2 x 2 games.

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