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Persistence of Business Cycles in Multisector RBC Models*

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Abstract

In this paper we explore whether the changing composition of output in response to technology shocks can play a significant role in the propagation of shocks over time. For this purpose we study two multisector RBC models, with two and a three sectors. We find that, whereas the two sectors model requires a high intertemporal elasticity of substitution of consumption to match the dynamic properties of the U.S. data, the three sector model has a strong propagation mechanism under conventional parameterizations, as long as the factor intensities in the three sectors are different enough.

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1 Introduction

Recently attention has been drawn to the fact that standard one sector, stochastic optimal growth models, the paradigm of RBC theory, do not have a strong enough endogenous mechanism to propagate shocks over time; as a consequence, these models are not capable of generating persistent business cycles.[1] The purpose of this paper is to show that multisector models have a potentially strong propagation mechanism which does not rely upon any extra features.

We identify the presence of a strong propagation mechanism with a hump–shaped impulse response of output growth, i.e. with the presence of a strong trend–reverting component, with an autocorrelation function of output growth which is significantly positive for at least three lags, as it seems to be in the U.S. data, and with a power spectrum of output growth which has a peak at business cycle frequencies, again as it is the case in the U.S. data.[2] Cogley and Nason, [6], have shown that the standard RBC model has a monotonically decreasing impulse response function of output to a non–permanent shock, an autocorrelation function of output growth which is always very close to zero and possibly negative, and a flat spectrum of output growth.

Several modeling strategies are known to mitigate the problem: quadratic adjustment costs to capital and labor, as in Cogley and Nason, [6]; variable factor utilization rates, as in Burnside and Eichenbaum, [5]; variable factor

[1]This fact was pointed out, for example, by Cogley and Nason, [6], and Rotemberg and Woodford, [10].

[2]This means that we follow the same definition of propagation mechanism given in Cogley and Nason, [6]; as consequence we feed strongly autocorrelated shocks to our models. Alternatively, we could have looked at the autocorrelation and spectra of levels, rather than growth rates, and have fed white noise shocks to the models. This is the approach taken by Beaudry and Devereaux, [3].

utilization rates within an efficiency wages framework, as in Beaudry and Devereaux, [3]; the embedding of a search–theoretic approach to the labor market within an otherwise standard RBC model, as in Andolfatto, [1]; habit formation in leisure coupled with increasing returns to scale, as in Wen, [11]; home production with enough productive externalities to generate multiple equilibria, as in Perli, [8]; sector-specific externalities, as in Benhabib and Farmer, [4]; the addition of a human capital sector with low elasticity of substitution between raw labor and human capital, as in Perli and Sakellaris, [9]; and human capital with variable factor utilization rates, as in DeJong *et al.*, [7].

All the previous papers essentially try to break the inverse relationship between consumption and labor after the impact of a non–permanent shock.[3] It is well known that, while in the period of impact consumption, labor and output all move in the same direction, after the impact the intratemporal marginal efficiency condition, which has to be satisfied in any one sector RBC model, forces consumption and labor to move in opposite directions. Since typically consumption continues to increase after a positive shock, labor decreases forcing output to decrease as well, unless capital responds extremely strongly.[4] The papers mentioned above that rely on adjustment costs or variable factor utilization rates or search, [6], [5], [3], [1], [7], introduce a delay in the response of labor to the technology shock, whereas the other papers, [11], [8], [9], [4], try to modify the working of the intratemporal marginal efficiency condition, to allow consumption and labor to move in the same direction for a few periods after impact.

[3]We assume in what follows that the shocks are always strongly autocorrelated, but do not contain a unit root, i.e., that they are not permanent.

[4]For a detailed discussion of this problem see Perli, [8], and Perli and Sakellaris, [9], who in turns build on the early paper by Barro and King, [2].

In this paper we pursue a different strategy: instead of adding extra features to a one sector model, we explore the propagation behavior of two multi–sector models. The idea is that looking at the composition of output may also help understanding the way shocks are propagated in the economy. While in one sector models output is defined simply as the output of a single production function, in multi-sector models output is the result of the composition of consumption and investment goods, i.e., $Y_t = C_t + \sum_{i=1}^{n} p_{i_t} X_{i_t}$. Thinking in terms of impulse response functions, it is clear that output can display a hump–shaped pattern in response to a shock even if neither the consumption nor the investment good have a hump–shaped response, or if only one of them has it. In the case of a two sector model, for example, output could have the appropriate impulse response if, after impact, consumption increases more than what investment decreases. In particular, we confine our attention to a one capital good, two sector model and a two capital goods, three sector model. We show that the two capital goods, three sector model has a strong endogenous propagation mechanism for a wide range of empirically plausible parameters, as long as the three sectors have sufficiently different factor intensities. The one capital good, two sector model, on the other hand, requires the strong additional assumption of a very low intertemporal elasticity of substitution of consumption to generate artificial data with the same dynamic properties as the real U.S. data. It is nonetheless useful to examine the behavior of this model, since it easier to grasp the intuition behind the persistence results within its simpler structure.

The paper is organized as follows: the next section discusses the one capital good, two sector model; section 3 discusses the two capital goods, three sector model; and section 4 concludes.

2 A Two–Sector Model

In this section we consider a standard model with a single capital good and two sectors, labelled consumption and investment. The representative agent chooses how to allocate capital and labor across the two sectors in order to maximize the discounted sum of each period utilities, subject to the production constraints in the two sectors and the law of motion of capital and of the technology parameters. Formally:

$$\max_{K_{C_t}, L_{C_t}, L_{I_t}} \mathrm{E}_0 \sum_t \rho^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \theta \frac{(1 - L_{C_t} - L_{I_t})^{1-\gamma}}{1-\gamma} \right]$$

subject to:

$$\begin{aligned}
C_t &= q_C z_{C_t} K_{C_t}^{\alpha} L_{C_t}^{1-\alpha} \\
X_t &= q_X z_{X_t} K_{X_t}^{\beta} L_{X_t}^{1-\beta} \\
K_{t+1} &= (1-\delta) K_t + X_t \\
z_{C_{t+1}} &= z_{C_t}^{\xi} e_t \\
z_{X_{t+1}} &= z_{X_t}^{\zeta} u_t \\
K_t &= K_{C_t} + K_{X_t}
\end{aligned}$$

To solve this problem we substitute C_t into the utility function; the first order conditions with respect to K_C, L_C, and L_I are respectively:

$$MUC_t \cdot MPK_{C_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} \cdot MPK_{X_t} \tag{1}$$

$$MUC_t \cdot MPL_{C_t} = MUL_{C_t} \tag{2}$$

$$MUL_{X_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} \cdot MPL_{X_t} \tag{3}$$

where MPw and MUw denote the marginal product and marginal utility respectively of variable w and v_{t+1} is the value function at time $t+1$. To find an expression for $\mathrm{E}(\partial v_{t+1}/\partial K_{t+1})$ consider that:

$$\frac{\partial v_t}{\partial K_t} = \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} \left(1 - \delta + MPK_{X_t}\right) \tag{4}$$

From (1):

$$\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} = \frac{MUC_t \cdot MPK_{C_t}}{MPK_{X_t}}$$

and therefore, substituting into (4) and updating one period :

$$\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{t+1}} = \frac{MUC_{t+1} \cdot MPK_{C_{t+1}}}{MPK_{X_{t+1}}} \cdot \left(1 - \delta + MPK_{X_{t+1}}\right) \tag{5}$$

The model has of course to be solved numerically; we use a log–linearization around the steady state. In this way we can write the system in the following way:

$$\begin{pmatrix} \widehat{K}_{t+1} \\ \widehat{L}_{C_{t+1}} \\ \widehat{z}_{C_{t+1}} \\ \widehat{z}_{X_{t+1}} \end{pmatrix} = J \cdot \begin{pmatrix} \widehat{K}_t \\ \widehat{L}_{C_t} \\ \widehat{z}_{C_t} \\ \widehat{z}_{X_t} \end{pmatrix} + Q \cdot \begin{pmatrix} e_t \\ u_t \end{pmatrix} \tag{6}$$

where J is the Jacobian matrix and $\widehat{w}$ indicates the percentage deviation of variable w from its steady state. All other variables can be expressed as (approximately) linear functions of total capital, labor in the consumption sector, and the two shocks. System (6) can be simulated numerically; of course an appropriate value of $\widehat{L}_C$ must be chosen at any point in time as a function of $\widehat{K}$, $\widehat{z}_C$ and $\widehat{z}_X$ since the model has a unique equilibrium.

Once we find artificial time series for all the variables we must compute output. One way to do it is:

$$Y_t = C_t + p_t X_t$$

where p_t is the price of the investment good in terms of the consumption good. We can obtain p_t from the static first order conditions of the firms with respect to capital:

$$MPK_{C_t} = r_t \qquad\qquad p_t MPK_{X_t} = r_t$$

Dividing one by the other we get:

$$p_t = \frac{MPK_{C_t}}{MPK_{X_t}}$$

i.e., the price of the investment good is equal to the ratio of the marginal product of capital in the two sectors. The same condition could have been obtained using the first order conditions with respect to labor. Output computed in this way would correspond to "nominal" output, since current prices are used. Alternatively, one could use $Y_t = C_t + pX_t$, where p is the steady state value of the price and also its steady state value in our simulation. In this way we would have the "real output", expressed in terms of the initial time prices.

2.1 Calibration and Results

We calibrate some of the parameters in a standard way, and the remaining ones we choose so that the model exhibits a degree of persistence of the shocks compatible with what we observe in the U.S. data. We then ask the question of whether the latter values are plausible or not in view of the available empirical evidence.

We set the depreciation of capital, δ, to 0.025 and the discount factor, ρ, to 0.9898, since we want to simulate quarterly data. Moreover, as it is standard in the RBC literature, we assume that the utility function is linear in leisure, i.e., we set $\gamma = 0$. We then set $\theta = 1.51$ so that, in the steady state, the number of hours worked is 1/3 of the total time available. Finally, we assume that the shocks to both sectors are highly persistent; in particular we set $\xi = \zeta = 0.95$.

We choose the remaining parameters, α, β, and σ, in order for the model to have a significant endogenous propagation mechanism. We provide an intuition for two cases under which endogenous persistence arises. As noted in the Introduction, persistence can arise in this model due to changes in the composition of output. Note that we say that we have persistence when the impulse response of output is "hump shaped", i..e., when output increases when a positive, non permanent shock first hits the economy, and also for a few periods after that. Since output is composed of consumption and investment, this can happen, of course, if both variables continue to increase after the impact of the shock; but it can also happen if one of the two variables increases more than the other decreases. Below we present a case in which the latter possibility occurs.

Suppose that both sectors are subject to the same technology shocks, i.e., that e_t and u_t are identical.[5] Assume that a positive, persistent shock hits both sectors. If the households like to smooth consumption, we will typically observe that both consumption and investment increase at impact, and then decrease monotonically; this clearly is not going to generate any persistence in output. If, however, households do not care a lot about consumption smoothing, i.e., if the utility of consumption is linear or close to linear, households are strongly willing to substitute future for present consumption since the interest rate is higher, and indeed they reduce consumption at date 0,

[5]The same results that we report below hold also for different innovations, as long as they are strongly positively correlated.

the period of impact. Investment, on the other hand, of course strongly increases, so output increases at date 0. After the shocks have hit the economy, however, households will start consuming more (the deferred consumption of date 0), and investment will decline. Given its low intertemporal elasticity of substitution, and given the high elasticity of the labor supply that we assume ($\gamma = 0$), consumption actually increases faster than investment decreases, and therefore output keeps increasing. Eventually both consumption and investment will go back to their steady states, and so output has to start decreasing and also go back to its steady state after a few periods; but, since at least initially C increases faster than pX decreases, output has a hump–shaped impulse response and therefore the model has a strong endogenous propagation mechanism.

The first restriction that we have to impose to our remaining parameters, therefore, is that σ has to be low; we choose $\sigma = 0.07$, the higher value that gives the desired persistence result. Note moreover that we can not have a low labor supply elasticity: if γ is high, total labor is practically constant, and the only movements in labor that we see are between the two sectors. These intersectoral movements are simply not sufficient to push up consumption enough to generate noticeable persistence; we need some extra labor going to the consumption sector from leisure after the impact of the shock. A linear utility of leisure is standard in RBC models. We view an almost linear utility of consumption however as a significant difficulty for the model above, and will address and correct it in the three-sector model in the next section.

For persistence alone these are the only restrictions that we need, in the sense that we could set the capital shares in the two sector at identical values around 1/3, as is customary. We want, however, to calibrate the model so that not only persistence, but also other statistics are in line with what we observe in the data. If the factor intensities are identical in the two sectors, we see that consumption remains below the steady state for several periods after impact; this has the unpleasant implication of making consumption weakly correlated with output, or even countercyclical, depending on parameter values.[6] The problem can be easily corrected assuming that the investment sector is more capital intensive than the consumption sector, i.e., that $\beta > \alpha$. In particular we choose $\alpha = 0.2$ and $\beta = 0.4$, although the same results below would be obtained with many other different combinations; what matters is the ratio of the two capital shares. In this way, after impact, labor will tend to move back faster to the consumption sector, which is labor intensive, and consumption output will respond more strongly and rapidly to the inflow of labor. With this calibration consumption is below its steady state only for the period of impact, which implies a much higher correlation with output.

	Y	C	X	L	K
Standard Deviation	1.00	0.94	4.52	0.93	0.41
	(1.00)	(0.49)	(2.82)	(0.86)	(0.34)
Correlation with Output	1.00	0.72	0.66	0.99	0.44
	(1.00)	(0.76)	(0.96)	(0.86)	(0.14)
AR(1) Coefficient	0.75	0.76	0.66	0.75	0.97
	(0.90)	(0.84)	(0.76)	(0.90)	(0.96)

Table 2.1 (U.S. Data in parenthesis)

The standard RBC set of statistics for several variables is in Table 2.1. We see that the model performs more or less like other RBC models, i.e., it captures several "static" aspects of the U.S. business cycle quite well. The biggest problem, however, is that consumption is almost twice as volatile as

[6] If the factor intensities are equal in the two sectors, and the shocks are the same, the model is equivalent to a one–sector model. The one–sector model, therefore, can exhibit persistence if the utility of consumption is very flat. The problem is that not only is this assumption unrealistic, but also standard RBC statistics are quite off the mark.

in the U.S. data; this was obviously to be expected, given the extremely high intertemporal elasticity of substitution of consumption.

The performance of the model in terms of persistence is shown in figures 1–3. The impulse response function of output is shown in figure 1; with respect to the impulse response of U.S. output, we see that Y takes too long to go back to its steady state, a sign that our model has too much, rather than too little persistence. Figure 2 shows the first ten lags of the autocorrelation function of output growth; they are all positive, which is another sign of too much persistence relative to the U.S. data. Finally, figure 3 shows the power spectrum of output growth; the presence of a peak is clear, although it occurs at frequencies slightly lower than the typical business cycle frequencies; this again says that the model has too much persistence.

While we are able to obtain a degree of persistence comparable to what is observed in the U.S. data with this parameterization, we have the problem that the intertemporal elasticity of substitution of consumption has to be too high. In the next section we show that a three–sector model can produce the same level of persistence with a more conventional logarithmic utility of consumption.

3 A Three–Sector Model

In the previous section we showed that a two sector model can generate persistence of technology shocks comparable to what we observe in the U.S. data if consumption increases faster than investment decreases in the periods subsequent to the impact of the shock. This required, however, a very high intertemporal elasticity of substitution of consumption. The idea of this section is to have a second investment good to absorb some of the role played by consumption in the two sector model. Here we assume a conventional intertemporal elasticity of substitution of consumption equal to one, and we introduce a second investment sector, which we denominate W. The model therefore has one consumption good and two capital goods. Even if here consumption does not react very strongly to technology shocks, as it appears to be the case in the U.S. data, the two investment goods are generating persistence in the same way that persistence was generated in section 2.

The working of the model is pretty standard: as in the previous section, the representative agent chooses how to allocate capital and labor across the three sectors in order to maximize the discounted sum of each period utilities, subject to the production constraints in the three sectors and the law of motion of capital and of the technology parameters. Formally:

$$\max_{K_{C_t}, L_{C_t}, L_{I_t}} \mathrm{E}_0 \sum_t \rho^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \theta \frac{(1 - L_{C_t} - L_{I_t})^{1-\gamma}}{1-\gamma} \right]$$

subject to:

$$\begin{aligned}
C_t &= q_C z_{C_t} L_{C_t}^{\alpha_0} K_{XC_t}^{\alpha_1} K_{WC_t}^{1-\alpha_0-\alpha_1} \\
X_t &= q_X z_{X_t} L_{X_t}^{\beta_0} K_{XX_t}^{\beta_1} K_{WX_t}^{1-\beta_0-\beta_1} \\
W_t &= q_W z_{W_t} L_{W_t}^{\gamma_0} K_{XW_t}^{\gamma_1} K_{WW_t}^{1-\gamma_0-\gamma_1} \\
K_{X_{t+1}} &= (1-\delta) K_{X_t} + X_t \\
K_{W_{t+1}} &= (1-\delta) K_{W_t} + W_t \\
z_{C_{t+1}} &= z_{C_t}^{\xi} e_t \\
z_{X_{t+1}} &= z_{X_t}^{\zeta} u_t \\
z_{W_{t+1}} &= z_{W_t}^{\eta} v_t \\
K_{X_t} &= K_{XC_t} + K_{XX_t} + K_{XY_t} \\
K_{W_t} &= K_{WC_t} + K_{WX_t} + K_{WW_t}
\end{aligned}$$

Total real output is again defined as $Y_t = C_t + p_{X_t} X_t + p_{W_t} W_t$, where p_X and p_W are the prices of the two capital goods in terms of the consumption good and are computed in the same way as with the two–sector model; total investment is $I_t = p_{X_t} X_t + p_{W_t} W_t$. The agent chooses how much to work and how much capital to use in the three sectors; total capital is just the sum of the capital stocks used in the individual sectors. The first order conditions of this problem can be written as:

$$
\begin{aligned}
MUC_t \cdot MPL_{C_t} &= MUL_{C_t} \\
MUL_{X_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPL_{x_t} \\
MUL_{W_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPL_{W_t} \\
MUC_t \cdot MPK_{XC_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{XW_t} \\
\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPK_{XX_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{XW_t} \\
MUC_t \cdot MPK_{WC_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{WW_t} \\
\rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} \cdot MPK_{WX_t} &= \rho \mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} \cdot MPK_{WW_t}
\end{aligned}
$$

where the two derivatives of the value function next period are given by:

$$
\begin{aligned}
\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{X_{t+1}}} &= \frac{MUL_{X_{t+1}}}{MPL_{X_{t+1}}} \cdot \left(1 - \delta + MPK_{XX_{t+1}}\right) \\
\mathrm{E} \frac{\partial v_{t+1}}{\partial K_{W_{t+1}}} &= \frac{MUC_{t+1} \cdot MPK_{WC_{t+1}}}{MPK_{WW_{t+1}}} \cdot \left(1 - \delta + MPK_{WW_{t+1}}\right)
\end{aligned}
$$

This model is again of course too complicated to be solved analytically. We solve it numerically using the same technique as with the two–sector model, i.e., we linearize the Euler equations around the steady state. Since we have now two state variables (the two capital stocks) plus three shocks, the resulting linearized dynamical system consists of seven difference equations in seven variables (the two states, two controls, and the three shocks):

$$\begin{pmatrix} \widehat{K}_{X_{t+1}} \\ \widehat{K}_{W_{t+1}} \\ \widehat{L}_{X_{t+1}} \\ \widehat{L}_{W_{t+1}} \\ \widehat{z}_{C_{t+1}} \\ \widehat{z}_{X_{t+1}} \\ \widehat{z}_{W_{t+1}} \end{pmatrix} = J \cdot \begin{pmatrix} \widehat{K}_{X_t} \\ \widehat{K}_{W_t} \\ \widehat{L}_{X_t} \\ \widehat{L}_{W_t} \\ \widehat{z}_{C_t} \\ \widehat{z}_{X_t} \\ \widehat{z}_{W_t} \end{pmatrix} + Q \cdot \begin{pmatrix} e_t \\ u_t \\ v_t \end{pmatrix}$$

All the other choice variables can be expressed in terms of the above variables using the first order conditions. Appropriate values of $\widehat{L}_{X_t}$ and $\widehat{L}_{W_t}$ have to be chosen in every period to make sure that the transversality conditions are satisfied, i.e., to make sure that the dynamics takes always place on the stable branch of the saddle point.

3.1 Calibration and Results

The model is calibrated using the same strategy as in the previous section, In particular, we set the depreciation of capital, δ, to 0.025, the discount factor, ρ, to 0.9898, and the inverse of the labor supply elasticity, γ, to zero. We set $\theta = 1.73$, so that, again, total hours worked in the steady state are $1/3$ of the total time available. Unlike the previous case we also set $\sigma = 1$ here, which implies that the utility is logarithmic in consumption, a standard feature of RBC models. We also assume that the shocks to all three sector are highly persistent, $\xi = \zeta = \eta = 0.95$, and identical.

The factor shares are set in order to get the desired persistence and acceptable results in terms of other types of statistics. As it turns out there are several combinations of parameter values that yield relatively good results. One such case involves assuming that the consumption sector is relatively labor intensive, and that the investment sector W is relatively more capital W intensive than the investment sector X. For example figures 4, 5 and 6 and Table 3.1 below were obtained with the following parameters: $\alpha_0 = 0.58$, $\alpha_1 = 0.32$; $\beta_0 = 0.56$, $\beta_1 = 0.24$; $\gamma_0 = 0.39$, $\gamma_1 = 0.12$.

	Y	C	I	L	K
Standard Deviation	1.00	0.70	3.23	1.22	0.65
	(1.00)	(0.49)	(2.82)	(0.86)	(0.34)
Correlation with Output	1.00	0.34	0.88	0.74	0.29
	(1.00)	(0.76)	(0.96)	(0.86)	(0.14)
AR(1) Coefficient	0.93	0.93	0.85	0.82	0.75
	(0.90)	(0.84)	(0.76)	(0.90)	(0.96)

Table 3.1 (U.S. Data in parenthesis)

As one can see, the impulse response function in figure 4 is clearly hump–shaped and the autocorrelation function of output growth in figure 5 is always positive for the first 10 lags, so also in this case we have too much, rather than too little persistence. The results of Table 3.1 are also similar to those in Table 2.1 for the two–sector model, with some differences as far as consumption is concerned. As was to be expected C is now less volatile than before, since σ is smaller; it is, however, only mildly procyclical. Another problem is that total labor is too volatile.

The reason why we have persistence for those parameters has again to do with the changing composition of output. When a positive shock hits all three sectors in the same way at date 0, we see again that consumption still slightly decreases, but very little in percentage terms. The two investment goods, however, respond much more strongly; in particular X increases and W decreases. Since X increases more than what C and W decrease, the response of output to the shock is positive at date 0. Note that here consumption decreases at impact even if the intertemporal elasticity of substitution of consumption is not high as in the two sectors model; this happens because the response of the investment goods to the shock is very pronounced, generating a strong incentive to postpone consumption through higher interest rates. After the impact of the shock, we see that consumption and the second investment good, W, start increasing, while X starts decreasing. Again the key here is that W and C increase more than what X decreases, and so total output continues to increase. This lasts for several periods, and therefore output has the pronounced hump that is the characteristic of persistence. Note that here we are exploiting the same effects that yielded persistence in the two–sector model when the investment sector was relatively capital intensive. Only, here we do not need a high intertemporal elasticity of substitution of consumption, thanks to the extra reallocation of resources across the two investment sectors. In other words, here one of the two investment sectors, W in particular, plays the role that was played by consumption in the two–sector model.

For this mechanism to work, investment good W must be countercyclical (a lagging rather than a leading sector), and very volatile. Empirically it seems more plausible that the output of an investment sector is more volatile than output and countercyclical rather than the output of the consumption sector.[7] The fact that investment good W is countercyclical (its correlation with output is -0.41) does not of course mean that total investment is also countercyclical. Since the other investment good, X, is strongly procyclical, total investment is indeed procyclical, as shown in table 3.1.

The particular parameter values that we chose are not the only ones that give persistence. Simulations show that there is a wide region in the parameter space that yields results equivalent to those shown above. This region is characterized by the fact that the factor intensities in the three sectors have

[7] One could interpret the output of the W sector as human capital, making its countercyclicality more natural.

to be different. Further restrictions are necessary, however, for the model to capture other properties of the business cycle, like the statistics reported in Table 3.1. There is a wide set of parameter values that yields countercyclical consumption, for example; this happens if the factor intensities are different but too close to each other. Unlike the two sector model, it is also possible that some parametrizations. lead to a negative response of output to the shocks; this happens when the decline in the production of good W is too strong.

Another interesting feature of this model is its ability to magnify extremely small shocks. To generate artificial time series for output that have the same variance of the real U.S. output we need shocks in each sector with a standard deviation of only 0.00023. Altogether, the three shocks have a standard deviation which is about 10 times smaller than what is required by other standard one–sector RBC models. Although we do not pursue this argument here any further, multisector models seem therefore to be interesting not only for their intrinsic propagation mechanism, but also for their amplification mechanism.

4 Conclusion

In this paper we presented two multisector real business cycle models and explored their implications for the propagation of shocks over time. We found that both models, and the three–sector, two capital goods in particular, are able to generate artificial data with the same degree of persistence observed in the U.S. data. The models are also able to match several other key business cycle statistics to the same degree that other standard one sector models do. The message that we think is coming out clearly from our analysis is that sectorial reallocations of productive factors in response to productivity shocks may be one of the fundamental reasons why we see persistent effects of the shocks themselves in the U.S. output.

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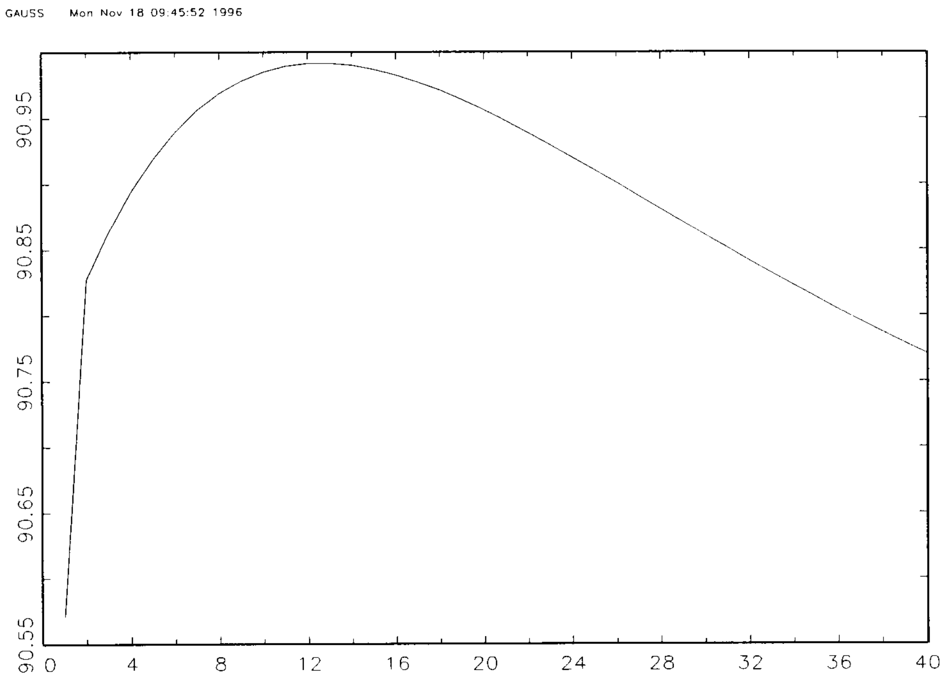

Figure 1: Impulse Response Function of Output – Two Sector Model

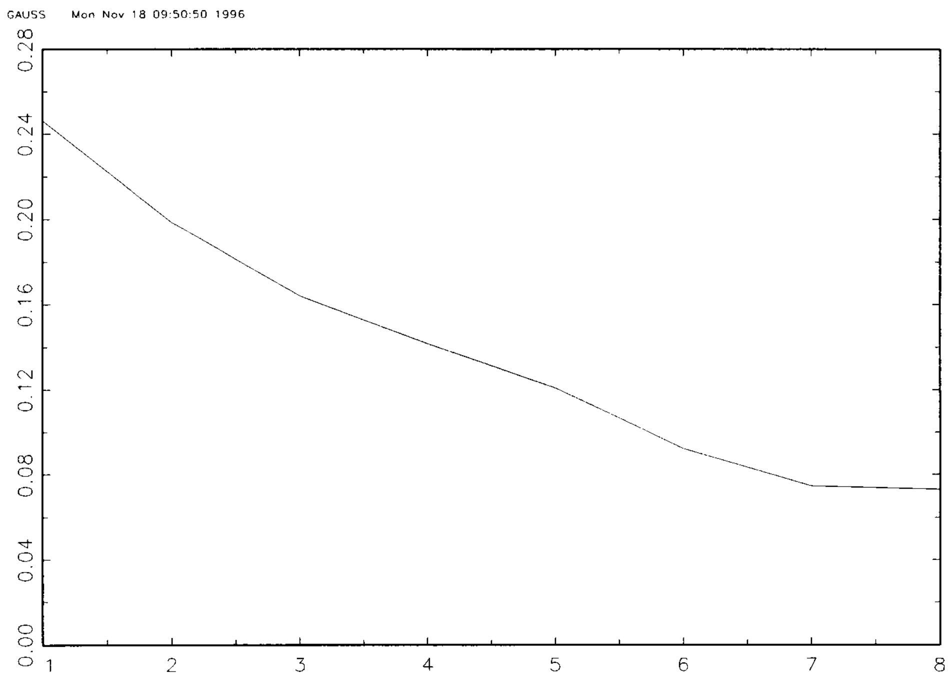

Figure 2: Autocorrelation Function of Output Growth – Two Sector Model

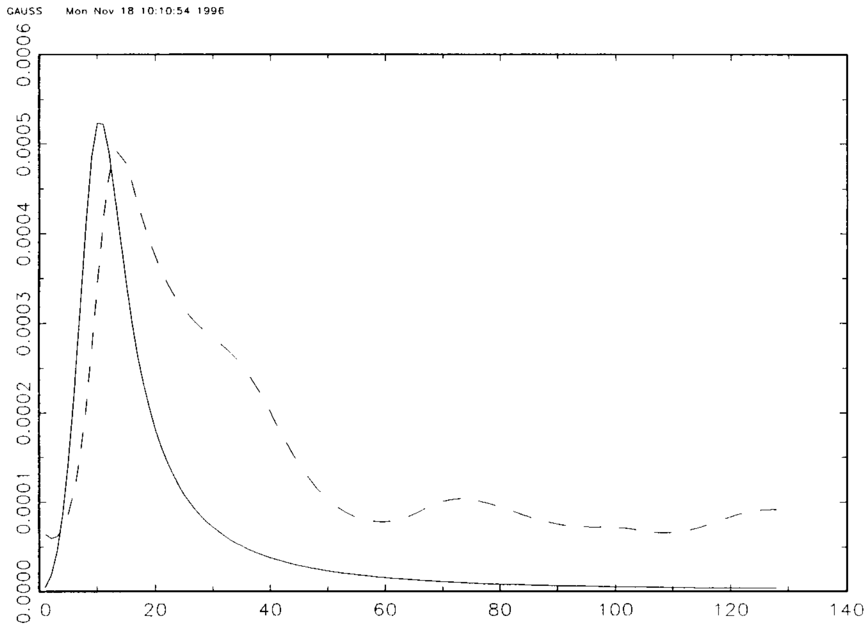

Figure 3: Power Spectrum of Output Growth – Two Sector Model

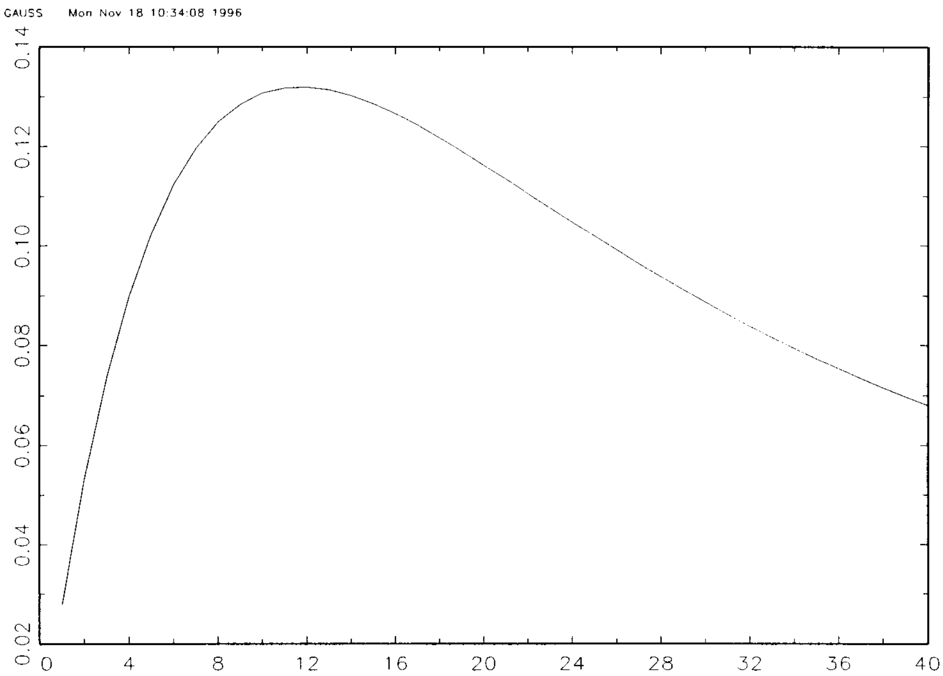

Figure 4: Impulse Response Function of Output – Three Sector Model

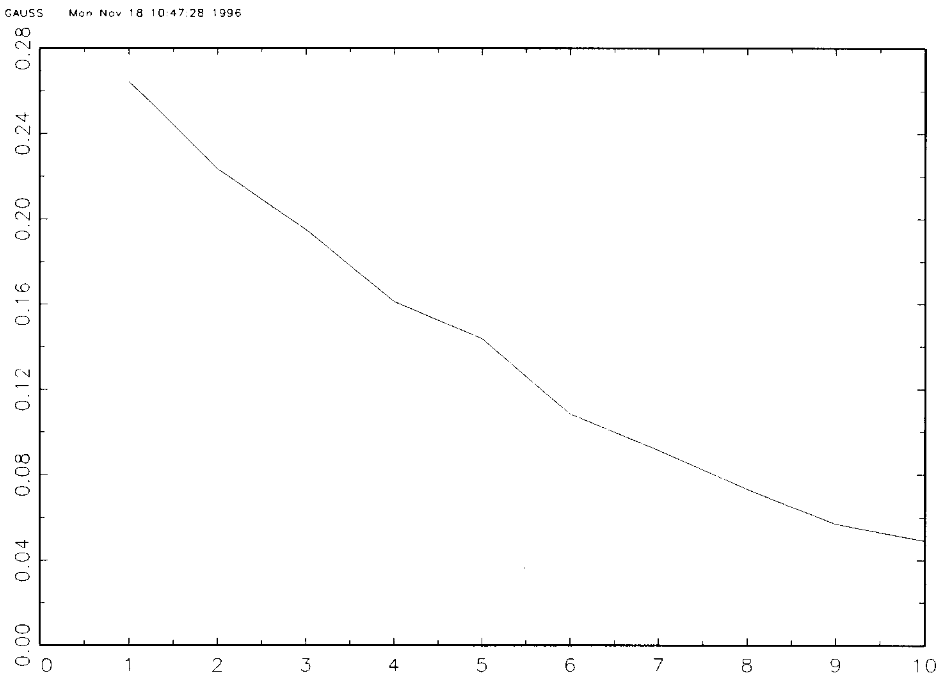

Figure 5: Autocorrelation Function of Output Growth – Three Sector Model

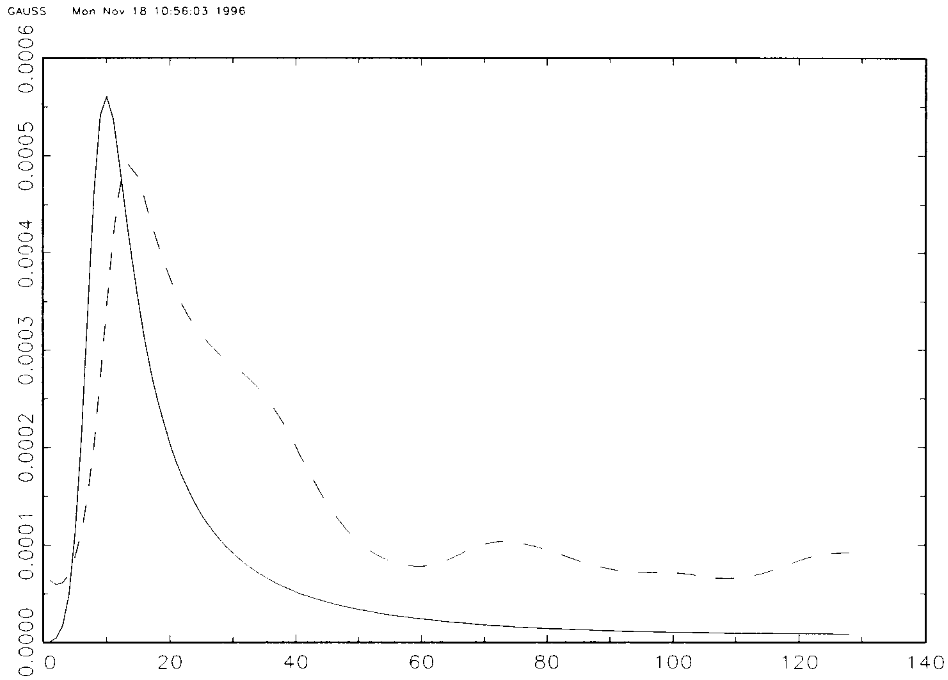

Figure 6: Power Spectrum of Output Growth – Three Sector Model