

# The Theory of the New Economy Firm: A Dynamic Analysis of Human Capital Investment\*

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## Abstract

In the last decade a major change in the development of firms is that they have become more human capital oriented. Especially the new economy firm attracts talented employees in order to create comparative advantages. Physical assets have become less unique and are not commanding large rents anymore. Increased competition with many independent suppliers has increased demand for quality improvement, which can only be generated by talented employees. The implication is that new firms tend to be non-vertically integrated, human-capital-incentive organizations that operate in a highly competitive environment (Zingales (2000), *Journal of Finance*).

This paper analyzes this firm in the setting of "the dynamic theory of the firm". Human capital is the central state variable, and a framework is designed in which path dependency is accounted for. Optimal long run policies are determined, and it turns out that a quality trap exists, implying that, in order to be able to remain active in the long run, the firm's initial amount of human capital should be sufficiently large as we see in reality: if this amount is too small the firm will cease its operations in the long run and go bankrupt. Mathematically speaking:

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a multiple equilibria solution arises where the Skiba point separates success and failure.

*Keywords:* theory of the firm, new economy, human capital, Skiba point

## 1 Introduction

In the last decade the determinants of firm performance have changed considerably (Zingales (2000)). First, physical assets, which used to be the major source of rents, have become less unique, and thus do not automatically lead to large firm returns. Some reasons for this change are the easier access to financing due to the improvement of capital markets, and the drop in communication costs, which reduced the importance of expensive distribution channels, which in turn favors the access to the market of newly formed companies. Second, growing internationalization has increased interdependencies among firms. Former domestic market leaders now have to deal with competition. This has increased the demand for quality improvement which can only be generated by talented employees, implying an increased importance of human capital. For the firm human capital is a strategic asset, because it belongs to "the set of difficult to trade and imitate, scarce, appropriable, and specialized resources and capabilities that bestow the firm's competitive advantage" (Amit and Shoemaker (1993, p. 36)). During the last decade firms, and especially new economy firms, have thus become human capital-intensive organizations directed at innovation and quality improvement.

This paper belongs to the dynamic theory of the firm. Within the theory of the firm four streams can be identified which are (i) the first theoretical reflections (Gordon (1962), Marris (1964), Penrose (1959)) concerning the relation between the most important elements as dividends and profits, the value of the firm and the growth rate, (ii) the simulation models where the performance of the firm (profit, cash flow, investment level) is calculated using models with many equations (Burril and Quinto (1972)), (iii) the capital accumulation models, in which optimal control theory is applied to develop and analyze firm investment behavior, taking into account the relation between debt and equity, investment and dividend (Lesourne (1973), Van Hilten et al. (1993)), and (iv) partial models in the areas of finance (Jarrow et al. (1995)), marketing (Eliashberg and Lilien (1993)) and organization (Burton and Obel (1995)).

The implication of these developments for the existing theories of the firm is that they analyze the traditional economy, but they seem to be quite ineffective in helping us cope with the new type of firms that is emerging (Zingales (2000)). Therefore, this paper

focuses on investment in highly qualified human capital as a means to increase the firm's performance. As already recognized in the literature (e.g. Becker and Gerhart (1996)), two of the key factors in the development of human resources within organizations are causal ambiguity and path dependency. Especially the latter characteristic requires a dynamic approach in order to design a framework in which the development of human capital within a firm can be meaningfully analyzed. Therefore, we develop a dynamic model of the firm in which the central state variable represents the firm's human capital relative to some baseline level, thus taking into account that human capital is increasing also for the firm's competitors. Using an optimal control model we analyze optimal firm behavior regarding investments in human capital.

It turns out that a quality trap exists in the sense that if the initial level of (resources to attract) human capital is low it is not profitable for the firm to attract human capital. Therefore the firm will lose market share and this will lead to ceasing the firm's operations in the long run. If the initial level of human capital is sufficiently large the firm will invest in human capital. These investments are continuous over time and in the long run the firm enters a unique steady state where human capital (relative to its baseline level) is constant, and marginal revenue of human capital investments equals marginal investment costs.

The paper is organized as follows. Section 2 contains a formulation of the model, which is analyzed in Section 3. Section 4 concludes.

## 2 Model Formulation

For modern corporations human capital is emerging as the most crucial asset (Zingales (2000)). The aim of this paper is to provide a theoretical framework for human capital investments. As argued by Dulbecco and Garrouste (2000), to develop the new theory of the firm "it is necessary to take into account the processual dimension of both production and knowledge in order to analyze as a process the coordination of the firm's plans." To do so we formulate a dynamic model of the firm in which the state variable is human capital ( $H$ ) relative to some baseline level. The greater  $H$ , the more efficient the firm can produce, or, alternatively, the higher the quality of its products (relative to the product quality level of its competitors) will be, and the more attractive it is to customers.  $H$  can be increased by attracting promising human beings. This we call human capital investments which we denote by  $I$ . However, if the firm refrains from human capital investments, its relative standing, which is what  $H$  stands for, will tend to decline. It

is assumed that human capital of the firm's competitors grows with rate  $\delta$  ( $\delta > 0$  and constant). This brings us to the following dynamic equation governing  $H$ :

$$\dot{H}(t) = \frac{dH(t)}{dt} = I(t) - \delta H(t), H(0) = H_0 \geq 0. \quad (1)$$

Equation (1) indicates that the relative advantage of a firm increases with attracting human capital, but it reduces through human capital growth in other firms. The latter is taken exogenous in our model although it is assumed here that this growth is proportional to the own firm's human capital, which in a sense reflects the "the dialectics of progress". In general human capital growth will differ among competitors. Therefore, the constant  $\delta$  should be seen as a kind of weighted average that represents its competitive environment with regard to technological progress.

A straightforward inequality is

$$H \geq 0. \quad (2)$$

Define profit ( $\pi$ ) as the difference between revenue obtained from selling output on the market and variable production costs. An increase of human capital enables the firm to increase its profit, because if, relative to its competitors, increasing human capital leads to more efficient production its profit margin increases and/or it produces more goods. Alternatively, if increased human capital generates the production of higher quality goods, the firm can increase its output price. We conclude that profit is increasing in human capital, so that  $\pi = \pi(H)$  with  $\pi' > 0$ . To determine the second order derivative of  $\pi(H)$  it should be realized that it is easier to raise profit when human capital is still low, where the occurrence of synergetic effects could even lead to increasing returns to scale, but it becomes more difficult if human capital is already large. Making an already very efficient production process even more efficient, or increasing the quality of a high quality product, requires advanced R&D activities (see also Das and Van de Ven (2000)). Taking these arguments into account we impose that

$$\begin{aligned} \pi''(H) &\geq 0 \text{ for } H \leq \hat{H}, \\ \pi''(H) &< 0 \text{ for } H > \hat{H}. \end{aligned} \quad (3)$$

results in a convex-concave shape of  $\pi(H)$  where  $\pi'' \geq 0$  for  $H \leq \hat{H}$ , while  $\pi'' < 0$  for  $H > \hat{H}$ .<sup>1</sup>

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<sup>1</sup>Notice that in microeconomics the assumption of a profit function having this shape is quite common (see, e.g., Kreps (1990), p. 256).

In order to raise human capital with  $I$  the firm needs to incur adjustment costs given by  $C(I)$ . Concerning these adjustment costs we can think for instance about costs of moving, job advertising, tempting promising people to join the firm in the form of offering option arrangements, schooling new employees, and so on and so forth. In case human capital declines because people leave the organization, thus  $I < 0$ , no adjustment costs are generated. It holds that  $C' > 0$  and  $C'' > 0$  for  $I \geq 0$ , and  $C(I) = 0$  for  $I \leq 0$ . This implies that  $C(I)$  has a kink for  $I = 0$ , i.e. it holds that  $C'(0^-) = 0 < C'(0^+)$ . The firm's objective is to maximize the discounted cash flow stream. One interpretation is that earnings that are not used for investments are paid out as benefits to the specialists that are employed within the firm. The objective of the firm is to maximize this benefit stream over an infinite planning period, discounted with rate  $r$  ( $r > 0$  and constant). Hence, the specialists maximize their wealth: they receive the end-of-the-year-benefits. An alternative explanation is that the earnings are paid out as dividends to the shareholders. It is in the interest of the firm's specialists to increase dividends because of the positive influence of the dividends on the value of the stocks, which increases the value of the options owned by the specialists. In mathematical terms the objective is

$$\max_{I(t)} \int_0^{\infty} e^{-rt} [\pi(H(t)) - C(I(t))] dt. \quad (4)$$

The optimal control model is now given by (1), (2) and (4). The firm has to attract human capital in such a way, that the resulting values for  $I(t)$  and  $H(t)$  maximize (4), while taking into account the constraint (2).

To limit the possible scenarios we focus on the case where

$$\pi'(0) < (r + \delta) C'(0). \quad (5)$$

This means that for human capital equal to zero marginal profits are that low that they fall below marginal cost of human capital investment<sup>2</sup>. The implication of this assumption is that for sufficiently low initial values of human capital it does not pay to increase human capital by performing human capital investments.

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<sup>2</sup>The validity of this assumption is exemplified by the following story. According to the Tilburg University newspaper *Univers* (November 30, 2000) the Dutch government was only willing to subsidize the founding of the economic research institute CentER in 1987, in case Tilburg University itself would first attract at least three distinguished researchers.

### 3 Analysis of the Model

In order to shorten notation, we omit from now on, when no confusion can arise, the time argument  $t$ . To find the optimal solution we start out by deriving the necessary optimality conditions provided by Pontryagin's maximum principle (see, e.g., Feichtinger and Hartl (1986)). The current value Hamiltonian ( $\mathfrak{H}$ ) is

$$\mathfrak{H} = \pi(H) - C(I) + \lambda(I - \delta H),$$

where  $\lambda$  is the co-state variable representing a dynamic shadowprice of human capital. To take also into account the constraint (2) we state the Lagrangian:

$$L = \mathfrak{H} + \nu H,$$

where  $\nu$  is the Lagrange parameter corresponding to the constraint (2). Then the necessary optimality conditions are

$$\lambda = C'(I) \text{ for } I > 0, \tag{6}$$

$$\dot{\lambda} = (r + \delta)\lambda - \pi'(H) + \nu, \tag{7}$$

$$\nu H = 0, \nu \geq 0. \tag{8}$$

No adjustment costs are incurred when  $I \leq 0$ . Therefore, as long as an additional unit of human capital has enough value to the firm, i.e.  $\lambda > C'(0^+)$ , investments will be positive, otherwise the firm refrains from attracting human capital.

#### 3.1 Feasible steady states

To find feasible steady states we consider the interior of the feasible region, so that we put the Lagrange multiplier  $\nu$  equal to zero. Then it is obtained from (6) and (7) that for  $I \geq 0$  it holds that

$$\dot{I} = \frac{1}{C''(I)} [(r + \delta)C'(I) - \pi'(H)]. \tag{9}$$

The dynamic system governing the dynamics of the model is given by (1) and (9). The  $\dot{H} = 0$ -isocline thus equals the straight line  $I = \delta H$ . For the  $\dot{I} = 0$ -isocline it holds that it only exists for  $I \geq 0$ , in which case we have

$$\frac{dI}{dH} \Big|_{\dot{I}=0} = \frac{\pi''(H)}{(r + \delta)C''(I)} \begin{cases} > \\ < \end{cases} 0 \text{ for } H \begin{cases} < \\ > \end{cases} \hat{H}. \tag{10}$$

Since the  $\dot{H} = 0$ -isocline is monotonically increasing and the  $\dot{I} = 0$ -isocline first increases and then decreases, at most two feasible steady states exist. Straightforward calculations (see, e.g., Feichtinger and Hartl (1986)) show that the larger steady state, denoted by  $H_2$ , is a saddle point, which implies that it can be a long run equilibrium, while the lower steady state ( $H_1$ ) is unstable.

In case the larger steady state does not exist for a positive human capital level, it can be shown that the firm will always cease its operations in the long run (cf. Hartl and Kort (2000)). Since this is economically less interesting, we choose to disregard this case here. Hence, in what follows we only consider the case where (at least) the larger steady state exists for a positive human capital level. Due to (5) this implies that both large and small steady states are located in the first quadrant<sup>3</sup>. This happens when

$$\pi'(\hat{H}) > (r + \delta) C'(\delta\hat{H}). \quad (11)$$

Together with (5) this relation indicates that it is only worthwhile for the firm to invest in human capital if marginal profit is sufficiently large (see also the next section), that is, if marginal profits of the human capital increase exceeds the marginal cost of human capital investment.

If (11) does not hold, it follows that the larger steady state does not exist for a positive human capital level. What happens in this case is that the firm will reduce human capital investments, leading to a decline in human capital. The implication is that eventually competitors will drive this firm out of the market

## 3.2 The optimal trajectories

The information obtained in the previous subsection is used here to present the firm's optimal trajectories in a phase diagram, which is depicted in Figure 1. Due to assumption (5) it holds that for sufficiently low initial values of human capital it does not pay to perform human capital investments. Here marginal profits are even that low that they fall below marginal cost of human capital investment. The implication is the occurrence of a Skiba point (cf. Dechert (1983), Dechert and Nishimura (1983))  $H_S$ , to the left of which the firm converges to the point  $(0, 0)$  where it ceases its operations. Hence, the firm enters this trajectory when  $H(0) < H_S$ . This trajectory acknowledges the existence

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<sup>3</sup>When the inequality in (5) is reversed, it holds that only the larger steady state (and *not* the smaller one) lies in the first quadrant. This larger steady state is then a unique long run equilibrium to which the optimal trajectories of the firm converge.

of a 'quality trap', thus that a sufficiently high amount of initial quality is needed for the firm in order to survive in the long run.

[Place Figure 1 about here]

To the right of the Skiba point  $H_S$  it is again worthwhile for the firm to invest in human capital. We distinguish two starting points of the firm: one where the initial human capital level lies between the Skiba point  $H_S$  and the long run steady state level  $H_2$  (see Figure 1) and one where the initial human capital level exceeds  $H_2$ . For the trajectory with relatively low initial human capital level, given by  $H(0) = H_{0L} \in (H_S, H_2)$ , it holds that investments first increase and then decrease over time. Next, we provide the economic intuition for this optimal trajectory.

From (6)-(8) it can be obtained that human capital investments should satisfy

$$\int_t^\infty e^{-(r+\delta)(s-t)} \pi'(H(s)) ds - C'(I(t)) = 0. \quad (12)$$

Equation (12) represents the net present value of marginal investment (NPVMI, see Van Hilten et al. (1993)), which is thus zero. The NPVMI consists of the total cashflow caused by an additional unit of human capital investment at time  $t$  and consists of two terms. The first term contains the profit stream resulting from the marginal human capital investment at time  $t$ . This term is corrected for discounting and depreciation. The second term is the initial investment outlay including the adjustment costs.

The trajectory approaches the long run stationary equilibrium  $H_2$  at which the marginal profits equals marginal cost, while human capital is kept at the same level:

$$\pi'(H_2) = (r + \delta)C'(\delta H_2). \quad (13)$$

Next let us turn to the solution where the initial human capital is high, thus where  $H(0) = H_{0H} > H_2$ . Since we are on the concave part of the profit function, the profit increase due to an extra unit of human capital is low. This makes that initial investments are low. It can even be the case that it is not optimal to start out with human capital investments. This happens when the NPVMI is negative (cf. (6)-(8)):

$$\int_t^\infty e^{-(r+\delta)(s-t)} \pi'(H(s)) ds - C'(0) < 0. \quad (14)$$

Hence, the firm refrains from attracting new personnel since the immediate investment costs exceed the discounted extra profits implied by the human capital increase. The implication is that human capital starts to decrease. Then after a while human capital



is sufficiently low, and thus the marginal profit increase sufficiently high, to make it worthwhile to start up human capital investments. At this point the trajectory enters the saddle point path where the NPVMI equals zero. From this moment on the remaining part of the trajectory resembles the previous one with the difference that now investments increase on the saddle point path while human capital is decreasing all the way.

By now it is clear that the Skiba point separates the regions of the initial values of human capital that determine the long run viability of the firm. It is therefore worthwhile to investigate how the location of the Skiba point reacts to changes of the model parameters. From (9) it is obtained that the unstable state satisfies

$$\pi'(H_1) = (r + \delta)C'(\delta H_1). \quad (15)$$

As indicated by several contributions in the "Skiba point literature" (e.g. Ladron-De-Guevara et al. (1999)), the location of the unstable equilibrium does not coincide with the Skiba point although they are located "near" to each other. However, in what follows we implicitly assume that the Skiba point will react to changes in the model parameters in the same way as the unstable steady state does (moreover, unlike of what is known from the existing literature, recently Hartl et al. (2001) showed that situations exist where policy functions are continuous and the Skiba point is the unstable steady state itself). From (15) and remembering that the Skiba point occurs in the interval of  $H$ -values where the output price is a convexly increasing function of human capital, it is obtained that the Skiba point moves to the right in case the discount rate ( $r$ ), the growth in human capital of other firms ( $\delta$ ), and the marginal adjustment costs of human capital investments increase in value. The economic implication is that the viability of new economy firms reduces. This leads to the conclusion that on a stock exchange with many new economy firms an increase of the discount rate does not only result in the usual negative effect on the stockprices but this usual effect is strengthened by the upward shift of the Skiba point. That is, the domain for initial levels of human capital for which it is optimal to seize operations in the long run, and thus leading to bankruptcy, increases.

## 4 Conclusions

This paper deals with the incentive to invest in human capital. Since human capital has long run implications for the firm, a dynamic framework has to be considered. A

large amount of human capital implies that very talented employees are working for this firm, and they are able to generate product and process innovations leading to higher profits (Zingales (2000)). Here it is reasonable to assume that increased human capital especially raises profits when human capital is still low, since then there is much room for improvement. However, on the other hand extra human capital does not lead to significantly more profits if the human capital level is already high, since it is very difficult to improve an already efficiently operating organization. These features are incorporated in a dynamic model of the firm, where the objective is to maximize the discounted cash flow stream over the planning period. At each point in time the firm has to decide how much efforts it has to make in order to raise human capital. These efforts, which in the paper are denoted by human capital investments, generate adjustment costs in the sense of job advertising, schooling, option arrangements, etc.

It turns out that the firm fixes the human capital investments such that the net present value of marginal investment is zero, indicating that an extra unit of investment generates a cash inflow that exactly equals the immediate cash outflow. In the same fashion, the firm refrains from undertaking human capital investments at the moment that the net present value of marginal investment is negative. If the initial level of human capital is sufficiently large, it is always optimal for the firm to keep on producing during the whole planning period. Investments are continuous over time, and eventually a steady state is reached where marginal profits of human capital investments equals marginal costs.. If the initial human capital level is small, it turns out to be optimal to reduce human capital and stop production in finite time.

In conclusion, this paper applies optimal control theory and the insights gained from the "Skiba point literature" (Dechert (1983), Ladron-De-Guevara et al. (1999)) to the human capital investment decision. This is an important topic since recently increased competition at the worldwide level has raised the demand for process innovation and quality improvement, which can only be generated by talented employees. Thus, the quest for more innovation increases the importance of human capital. In this paper it is shown that our setting leads to clear economic conclusions. It is our belief that the methodological approach taken in this paper could be a starting point for the development of a solid theoretical foundation concerning the new economy firm and its human capital.

Another interesting topic for future research would be to endogenize the competitive environment with regard to technological progress. In the analysis of the present paper this is represented by a constant  $\delta$ . However, in reality this  $\delta$  will differ among firms, its

value will change over time, and strategic behavior will exist in the sense that firms will design their investment policy taking into account its influence on the competitors'  $\delta$ 's. However, it has to be remarked that incorporating elements like this into a fully dynamic framework like our model, will complicate the analysis considerably.

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**Figure Captions**

Figure 1. Optimal trajectories of the firm.

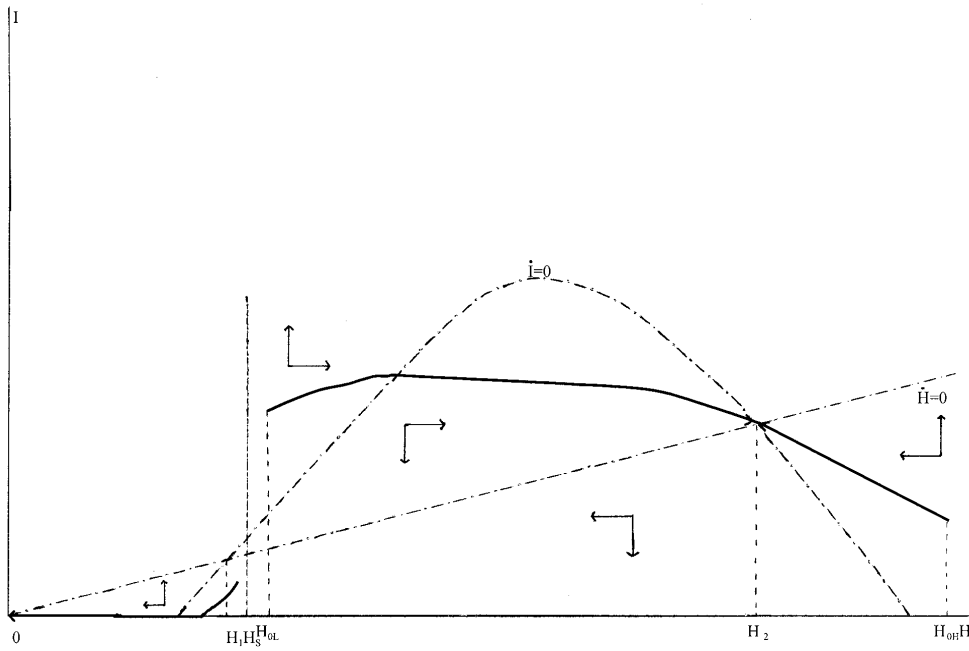


Figure 1