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# Dynamic Adjustment and Debt Accumulation in a Small Open Economy

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# 1. Introduction

This paper is the result of an attempt to build a simple model of an open economy exhibiting some interesting open economy dynamics. According to the latest fashion the model had to be based on intertemporal optimization and anticipation of agents.

As a point of reference, we start with a model of a closed economy. We choose the smallest number of agents necessary to construct an interesting economy: two, being a representative firm and a representative consumer. We model anticipation by rational expectations. As we use a simple non-stochastic setting, this implies perfect foresight. The model describes four markets: a market for goods, a market for labour, a market for money and a share market. Prices and interest rates are perfectly flexible. In order to get more realistic dynamics we impose an imperfection on the labour market. This is done by including a Phillips-curve: wages react sluggish to excess demand for (supply of) labour. The resulting model is a microfounded intertemporal analogon of the standard IS-LM model.

Section 2 describes the closed economy model. It is shown that the intertemporal behaviour of the model depends critically on the form of the utility function used. A numerical simulation experiment with the closed economy model is shown.

Then we start opening up the economy. The simplest way to model openness is by the concept of the small open economy, which we use. In the literature there are basically two roots for optimizing small open economies with perfect foresight. One is a paper by Turnovsky (1985), the other consists of two papers, one by Obstfeld (1981) and one by Hodrick (1982).

The Turnovsky model is based on clearing of all markets, purchasing power parity and imperfect capital mobility. Apart from money, there are two assets in his model: domestic bonds denominated in domestic currency and internationally traded bonds denominated in foreign currency. Although agents have perfect foresight. Turnovsky includes some degree of aversion against the exchange risk (in a certainty equivalence framework). As a consequence, there is no (uncovered) interest rate parity and the demand for the internationally traded bonds is proportional to the uncovered interest rate differential on traded and domestic bonds. This way of modeling implies that some ad hoc relation between the domestic and the foreign rate of time preference has to be postulated in order to get a well defined stationary state. Turnovsky presents transitional dynamics based on the saddlepoint stability of this stationary state. However, there are no true dynamics in his model. Van de Klundert and Van der Ploeg (1987) present numerical simulation results for the Turnovsky model with the inclusion of (nominal and real) wage rigidity. They also present a version of the model with imperfect substitution between domestic and foreign goods. In a recent paper Van de Klundert (1988) extends this model to allow for capital accumulation. This model approaches the model we had in mind when we started the research for this paper. As a consequence of the Turnovsky approach to the capital account based on exchange risk aversion, it lacks, however, one of the interesting features of a dynamic open economy. In the long run there is no accumulation of foreign claims (or debt). Therefore, we based the open economy presented in this paper on the other, older, root.

The papers by Obstfeld and Hodrick present a model for a small open economy in a one good/one non-monetary asset world where all bonds are denominated in foreign currency and internationally traded. Hence, in these papers there is perfect capital mobility and interest rate parity as well as purchasing power parity.

Obstfeld uses a very simple model with an exogenous fixed output. He endogenizes the rate of time preference in a way suggested by Uzawa (1968). This enables analytical calculation of the stationary state and saddlepoint analysis. A considerable change in the dynamics of the model results. This becomes clear when we compare his results to Hodrick's.

Hodrick uses basically the same model, extended by a simple production sector. Firms use labour as the only input and absorb all labour supplied at the constant exogenous nominal wage rate. His analysis is based on a constant domestic rate of time preference, which has to be equal to the foreign rate of time preference for a stationary state to exist. The stationary state of the model cannot, however, be determined from the model equations. It depends on the net bond holdings. Given the net bond holdings that are determined by history, the stationary state can be computed. Because there is no sluggishness in the model, in case of an unexpected shock, the economy jumps to a new stationary state. This implies that there is no time for bond accumulation (decumulation) and the new stationary state can be computed given the history-determined bond holdings. Things are different in case of an anticipated shock, however. In that case there is time for the agents to save and accumulate bonds. The final level of bond holdings varies with the amount of time that elapses between the moment of arrival of the news about the shock and the actual shock. The stationary state depends on the stock of bond holdings, hence it cannot be computed analytically.

Because his model is intuitively more appealing, we choose Hodrick's model as a starting point for our open economy. The model is extended to a full macroeconomic model comparable to the closed economy model presented in section 2. The poor dynamics of the model are enriched by allowing for capital accumulation and wage stickiness. Furthermore, the asset portfolio is enlarged. Apart from bonds denominated in foreign currency, there are domestic shares denominated in domestic currency paying a variable dividend and thus having a variable price. As there is perfect foresight and perfect capital mobility, both assets are treated as perfect substitutes. The exchange rate is assumed to be perfectly flexible and hence a current account deficit leads to debt accumulation i.e. assets (bonds or shares) leaving the country. As in the Hodrick model, the stationary state depends on asset (debt) accumulation in history and cannot be computed analytically. This history dependency is sometimes referred to as hysteresis. Though not analytically tractable, we show that it is possible to trace the stationary state and the adjustment process by numerical simulation.

The open economy model is presented in section 3.1. Section 3.2 presents some numerical simulation experiments with this model. As we use the same parameter values and the same initial state, a direct comparison with the closed economy results can be made. This illustrates the (dis-)advantages of openness in a one good world. Section 4 is a concluding section.

### 2. Modeling the Closed Economy

# 2.1 Household Behaviour

The welfare of households depends on consumption of goods c and of leisure  $(l_m - l)$ , being the maximum available time  $l_m$  minus the supply of labour l, and the amount of real cash balances  $(M/P_v)$  held.<sup>1</sup> The instantaneous utility function can be written as:<sup>2</sup>

$$u = u \left[ c, \left( \frac{M}{P_y} \right), \left( l_m - l \right) \right].$$
 (2.1.1)

Because households are assumed to be infinitely lived, they choose a time path of consumption, real cash balances and leisure that, given a constant (utility) rate of time preference v, maximizes the present value of utility:

$$U = \int_{l}^{\infty} \left\{ u \left[ c, \left( \frac{M}{P_{y}} \right), \left( l_{m} - l \right) \right] \cdot e^{-v \cdot (z-l)} \right\} dz. \quad (2.1.2)$$

The household's nominal wealth W consists of human capital H, shares issued by firms E and the nominal stock of money M:

$$W = H + E + M.$$
 (2.1.3)

Human capital is defined as follows:

$$H = \int_{l}^{\infty} \left\{ l_m \cdot P_l \cdot e^{-\int_{l}^{z} R(s) ds} \right\} dz, \qquad (2.1.4)$$

where  $P_i$  stands for the nominal wage rate. The return on shares consists of dividend D and the increase of the value of shares  $\dot{E}$ . The total return on shares in absence of uncertainty equals the

<sup>&</sup>lt;sup>1</sup> An alternative approach to "money-in-the-utility-function" is the "cash-in-advance" approach (see for instance Fischer, 1988). An optimizing continuous time closed model with a cash-in-advance constraint can be found in Meijdam and Van Stratum (1988). A number of characteristics of the model used here is taken from the latter paper.

<sup>&</sup>lt;sup>2</sup> Time subscripts are generally dropped for convenience.

risk-free nominal interest rate R:

$$\frac{\dot{E}+D}{E}=R.$$
(2.1.5)

Holding cash balances is not giving a return (in terms of money). The dynamic budget constraint of the household can, given the dividend-policy of the firm as shown in the next section, be written as follows:

$$\dot{W} = \mathcal{R} \cdot (W - M) - (l_m - l) \cdot P_l - c \cdot P_y, \qquad (2.1.6)$$

where  $P_v$  is the price of the one good in this economy.

Though the good- and money-market are assumed to clear all the time, the labour market exhibits slow nominal wage adjustment. Therefore, the households face a constraint with respect to their supply of labour:

$$l \le \bar{l}_d, \tag{2.1.7}$$

where  $\bar{l}_d$  stands for the amount of labour demanded by firms. This amount is given for the households and indicates the maximum of labour that can be sold to firms. Furthermore, a life-time budget constraint is imposed on households:

$$\int_{1}^{\infty} \left\{ (c \cdot P_{y} + R \cdot M + (l_{m} - l) \cdot P_{l}) \cdot e^{-\int_{1}^{l} R(s) ds} \right\} dz \leq W. \quad (2.1.8)$$

The latter constraint may be seen as a transversality condition that guarantees that the present value of all net expenses does not exceed the initial wealth of households. This condition is sometimes known as a no-Ponzi-game condition.

The households have perfect foresight with respect to the time paths of wages, prices, interest rates, dividends and the value of shares. It is also assumed that they anticipate the constraints on the labour market correctly. The households maximize the intertemporal utility function (2.1.2) with respect to c,  $M/P_y$  and  $(l_m - l)$ , subject to contraints (2.1.6), (2.1.7) and (2.1.8). Using Pontryagin's maximum principle, the following necessary and sufficient first order conditions for an optimal solution can be found:

$$u_c = x \cdot P_{\gamma}, \qquad (2.1.9)$$

$$u_{(M/P_y)} = x \cdot P_y \cdot R, \qquad (2.1.10)$$

$$u_{(l_m-l)} = x \cdot P_l - \lambda_h,$$
 (2.1.11)

$$\dot{x} = (v - R) \cdot x,$$
 (2.1.12)

together with the Kuhn-Tucker condition:

$$\lambda_h \cdot (l_d - l) = 0, \quad \lambda_h \ge 0. \tag{2.1.13}$$

The symbol x stands for the costate variable associated with the dynamic budget constraint (2.1.6), denoting the marginal utility of a unit wealth. The symbol  $\lambda_h$  is a Lagrange multiplier associated with the inequality constraint (2.1.7).

The following equation, which can be derived from (2.1.2), is added to enable tracing the time path of life-time utility:

$$\dot{U} = v \cdot U - u. \tag{2.1.14}$$

## 2.2 Firm Behaviour

Firms are assumed to exist infinitely long and produce nonstorable output y using two factors of production, capital k and labour l, according to a neo-classical production function with constant returns to scale and appropriate derivatives:

$$y \le f(l, k). \tag{2.2.1}$$

Capital stock depreciates exponential at a rate  $\delta$ . The accumulation equation is:

$$\dot{k} = i - \delta \cdot k, \tag{2.2.2}$$

where, as usual, i is investment. The opportunity costs of investment j, consisting of purchase costs i and installation costs h(i, k), are given by a linearly homogeneous function that is well behaved for the problem under consideration:

$$j = i + h(i, k).$$
 (2.2.3)

Once in history firms issued a number of shares, say for simplicity one big folio-"share", that can be split up and rearranged in any way, to start their business. Investment is financed out of retained earnings and no new shares are ever issued. Holding a share of the firm gives right to a stream of dividends, which can by consequence be either positive or negative. The dividend of the firm is given by:

$$D = y \cdot P_{\nu} - l \cdot P_l - j \cdot P_{\nu}. \qquad (2.2.4)$$

Firms have perfect foresight with respect to the time paths of wages, prices and interest rates. It is assumed that they anticipate the constraints on the labour market correctly. Firms maximize the value of outstanding shares or, equivalently, the value of the firm:

$$E = \int_{t}^{\infty} \left\{ D \cdot e^{-\int_{t}^{s} R(s) ds} \right\} dz. \qquad (2.2.5)$$

Because of sluggish nominal wage adjustment firms may not be able to hire all the labour they ask for:

$$l \le \bar{l}_s, \tag{2.2.6}$$

where  $\bar{l}_s$  is the amount of labour that households are willing to supply. It is treated as an exogenous parameter for the firm.

The following equations can be derived from the necessary (and under the "usual" transversality conditions: sufficient) conditions for an optimal solution of the firm's optimization problem, i. e. maximizing (2.2.5) with respect to l and i, subject to constraints (2.2.1), (2.2.2), (2.2.3) and (2.2.6), using Pontryagin's maximum principle:

$$f_l = \frac{P_l + \lambda_l}{P_y}, \qquad (2.2.7)$$

$$q = 1 + h_i, \tag{2.2.8}$$

$$\dot{q} = \left(R + \delta - \frac{\dot{P}_y}{P_y}\right) \cdot q - f_k + h_k, \qquad (2.2.9)$$

together with the Kuhn-Tucker condition:

$$\lambda_l \cdot (\bar{l}_s - l) = 0, \quad \lambda_l \ge 0. \tag{2.2.10}$$

The symbol q stands for the costate variable associated with the accumulation equation (2.2.2) and can be interpreted as the real marginal profit of capital. When the market-constraints are never binding this "marginal q" coincides with Tobin's "average q". The symbol  $\lambda_i$  is a Lagrange parameter associated with the inequality constraint (2.2.6).

# 2.3 The Markets

Nominal wages are assumed to respond slowly to discrepancies between demand and supply on the labour market:

$$\dot{P}_l = \theta \cdot (l_d - l_s) \cdot P_l. \tag{2.3.1}$$

Actual employment is determined by the minimum of demand and supply:

$$l = \min(l_d, l_s).$$
 (2.3.2)

Given the stock of capital, actual employment determines the supply of goods. Due to perfect price adjustment, a price that ensures equality of demand for and supply of goods results. Price flexibility as well as the fact that the supply and demand of goods are derived from value-maximizing behaviour surmounts the standard IS-LM framework. However, parametric on the market-clearing price one can imagine the equivalent of the traditional IS-curve. Together with the exogenous supply of money, the equilibrium price determines the supply of real cash-balances. In combination with the demand for money from intertemporal optimizing consumer behaviour this delivers the equivalent of the traditional LM-curve. The intersection of IS- and LM-curve reads as the equilibrium rate of interest:

$$R = \frac{u_{(M/P_y)}}{x \cdot P_y}.$$
 (2.3.3)

Evidently, the corresponding level of production equals demand and supply of goods:

$$y = c + j.$$
 (2.3.4)

# 2.4 Stationary State and Simulation

The following specifications for the utility function, the production function and the function that describes the installation costs are used throughout the remainder of this text:

$$u = \ln \left\{ \gamma_c \cdot c^{-\rho} + \gamma_m \cdot \left(\frac{M}{P_y}\right)^{-\rho} + \gamma_l \cdot (l_m - l)^{-\rho} \right\}^{-\frac{1}{\rho}}, (\gamma_c + \gamma_m + \gamma_l = 1),$$
(2.4.1)

$$y = \varepsilon \cdot \left\{ \alpha \cdot k^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot l^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \qquad (2.4.2)$$

$$h(i,k) = \frac{(i-\delta \cdot k)^2}{2 \cdot \psi \cdot k}.$$
(2.4.3)

The stationary state of the model is a Walras-equilibrium  $(l_d = l_s)$  and, consequently, the market-constraints are not binding. This implies:

$$\lambda_h = \lambda_l = 0, \qquad (2.4.4)$$

and therefore from (2.2.7):

$$f_l = \frac{P_l}{P_y}.$$
 (2.4.5)

Combining (2.2.2) and k=0 gives:

$$i = \delta \cdot k. \tag{2.4.6}$$

Inserting this result in (2.4.3), together with (2.2.3), gives:

$$j = i, \tag{2.4.7}$$

and, using the specification of the installation costs:

$$h_i = h_k = 0. (2.4.8)$$

In combination with (2.2.8) and (2.2.9), together with  $\dot{q} = 0$  and  $\dot{P}_{\nu} = 0$ , this leads to:

$$q = 1,$$
 (2.4.9)

$$f_k = \mathbf{R} + \delta. \tag{2.4.10}$$

From (2.1.12) and  $\dot{x} = 0$  it follows that:

$$R = v. \tag{2.4.11}$$

Combining (2.4.2), (2.4.5), (2.4.10) and (2.4.11) gives expressions for the real wage and the net-output/labour ratio, which are crucial for solving the rest of the model:

$$\frac{P_l}{P_y} = (1 - \alpha) \cdot \varepsilon \cdot \left\{ (1 - \alpha) + \frac{(1 - \alpha) \cdot \alpha}{\left\{\frac{\nu + \delta}{\alpha \cdot \varepsilon}\right\}^{\sigma - 1} - \alpha} \right\}^{\frac{1}{\sigma - 1}},$$

$$(2.4.12)$$

$$A^{\#} = \frac{y - \delta \cdot k}{l} = \frac{y/k - \delta}{l/k} = \frac{\varepsilon \cdot \left\{\frac{\nu + \delta}{\alpha \cdot \varepsilon}\right\}^{\sigma} - \delta}{\left\{\frac{1 - \alpha}{\left\{\frac{\nu + \delta}{\alpha \cdot \varepsilon}\right\}^{\sigma - 1} - \alpha}\right\}^{\frac{\sigma}{1 - \sigma}}}.$$

$$(2.4.13)$$

For the closed economy we have y = c + i and  $i = \delta \cdot k$ . Therefore it follows that:

$$\frac{y-\delta\cdot k}{l} = \frac{c}{l}.$$
 (2.4.14)

(2.4.13)

Using the specification of the utility function and first order conditions (2.1.9) to (2.1.11) we find:

$$\frac{c}{l_m - l} = \left\{ \frac{\gamma_c}{\gamma_l} \cdot \frac{P_l}{P_y} \right\}^{\frac{1}{p+1}}, \qquad (2.4.15)$$

$$\frac{c}{(M/P_y)} = \left\{\frac{\gamma_c}{\gamma_m} \cdot R\right\}^{\frac{1}{\rho+1}}.$$
 (2.4.16)

Combining (2.4.13) to (2.4.15) gives an expression for the amount of leisure in the stationary state:

$$l_{m} - l = \frac{A^{\#} \cdot l_{m}}{A^{\#} + \left\{\frac{\gamma_{c}}{\gamma_{l}} \cdot \frac{P_{l}}{P_{y}}\right\}^{\frac{1}{1+\rho}}}.$$
 (2.4.17)

Now the amount of consumption can be determined from (2.4.15) and (2.4.17):

$$c = \frac{A^{\#} \cdot I_{m}}{1 + \frac{A^{\#}}{\left\{\frac{\gamma_{c}}{\gamma_{l}} \cdot \frac{P_{l}}{P_{y}}\right\}^{\frac{1}{1 + \rho}}}}.$$
 (2.4.18)

From the last expression, (2.4.11) and (2.4.16) the price level can be determined and hence, from the expression for the real wage-rate, the nominal wage-rate. From the dynamic budget constraint and  $\dot{W}=0$ , W can be found and by now all remaining variables.

Before turning to a numerical simulation of the adjustment process of this economy after it is hit by a shock, we state the following proposition:

#### **Proposition 1:**

If, in the model described above, the instantaneous utility function is of the Cobb-Douglas type ( $\rho=0$ ) and the (exogenous) money supply is fixed then:

$$x=\frac{\gamma_m}{M\cdot\nu},\quad R=\nu,$$

at any moment.

Proof: See appendix.

This proposition implies that the forward looking variable xdoes not jump as long as the time preference rate, the money supply and the parameter for the real cash balances in the utility function is not changed. This is due to the special feature of the Cobb-Douglas utility function that there are no static spill-overs between markets and that the demand for money does not depend on the price level. The result does not hold for a CES utility function with  $\rho > 0.3$  Furthermore, the result of Proposition 1 breaks down when the households do not anticipate the future constraints correctly. Perfect anticipation of constraints enables the households to allocate wealth over time according to their time preference. When the actual constraint in a period differs from the anticipated constraint this implies an unexpected accumulation (decumulation) of wealth and thus an interest rate different from the expected level v. Confronted with this new information the households will revise their plan for the following periods. This implies a change of x. An example of a model that displays perfect foresight with respect to all variables but without correct anticipation of future constraints can be found in Meiidam (1988).

In order to have a point of reference with respect to the open economy, we discuss the numerical results of Table 1. A technological shock hits upon the closed economy in its stationary state. The evolution from the old to the new stationary state is shown by

| Period →<br>Variable ↓ | 0      | 1      | 2     | 5      | 10    | Stationary state |
|------------------------|--------|--------|-------|--------|-------|------------------|
| v                      | 3.95   | 4.17   | 4.32  | 4.72   | 5.17  | 5.87             |
| c                      | 4.14   | 4.41   | 4.63  | 5.16   | 5.76  | 6.69             |
| j                      | 3.38   | 3.43   | 3.42  | 3.41   | 3.41  | 3.40             |
| k                      | 0      | 0.32   | 0.61  | 1.33   | 2.15  | 3.40             |
| 1                      | -0.10  | 0.00   | 0.01  | 0.06   | 0.13  | 0.23             |
| $P_{v}$                | -3.97  | -4.23  | -4.42 | -4.91  | -5.45 | -6.27            |
| $\vec{P_l}$            | 0      | -0.03  | 0.03  | 0.32   | 0.71  | 1.32             |
| R                      | 0.00   | 0.00   | 0.00  | 0.00   | 0.00  | 0.00             |
| D                      | 0.21   | 0.06   | -0.07 | -0.75  | -1.66 | -3.08            |
| Ε                      | - 1.23 | - 1.37 | -1.52 | - 1.91 | -2.37 | - 3.08           |
| W                      | 0.00   | 0.00   | 0.00  | 0.00   | 0.00  | 0.00             |
| U                      | 58.52  | 59.71  | 60.79 | 63.43  | 66.37 | 70.86            |

Table 1. A Technological Shock in a Closed Economy ( $\rho = 0$ )

<sup>3</sup> This is illustrated for an open economy by the simulation results of section 3.2.

the time paths of the percentual deviation of a number of variables from their old stationary state values.

The numerical results can be obtained by solving a two-point boundary value problem, which is done by using the method of multiple-shooting (Ascher et al., 1988). A technological shock is symbolized by a more efficient production process ( $\varepsilon = 0.26$ ). This leads to an increase of production, which can only be sold when prices go down sufficiently. Real wealth increases and hence consumption rises as well as leisure. Investment demand goes up because it is profitable to build up capital for future production. The purchasing power of a share of the firm rises, though the nominal share-price falls. Nominal wages initially go down because less labour is needed in a more productive economy. Later on, expansion of production surpasses the initial fall in labour demand and nominal wages start to rise. As can be seen from life-time utility, this shock is welcome to this economy.

Note that the rate of interest is constant over time, an illustration of Proposition 1. Nominal wealth is constant over time in this case as households are not rationed in their supply of labour during adjustment.

### 3. Modeling the Open Economy

### 3.1 Turning the Closed Economy into an Open One

As we strive for analogy with the closed economy model, we assume there is one good in the world and the small country can export or import any amount of the good at the existing world price. Exporting or importing is done to ensure equilibrium on the goods market of the small country: when home demand for goods is too small to absorb all home production, the excess supply is simply exported to the rest of the world and vice versa. Producers are in the first place willing to sell on the home market and only sell abroad to get rid of their excess supply. The story behind this behaviour is that exporting (implicitly) takes more trouble than selling at home. So, net exports are determined according to:

$$b = y - c - j.$$
 (3.1.1)

Because there are no trading barriers, it seems plausible to assume the law of one price in this one good world:

$$P_{y} = P^* \cdot e, \qquad (3.1.2)$$

where  $P^*$  stands for the price of the good denominated in foreign currency and *e* represents the exchange rate. For simplicity we take the foreign price constant and equal to one:

$$P^* = 1.$$
 (3.1.3)

This, together with the purchasing power parity of (3.1.2), implies that the exchange rate always equals the price of the good denominated in home currency ( $e = P_y$ ). Because capital is assumed to be perfectly mobile the return on capital must be the same in all parts of the world at all times ((uncovered) interest rate parity):

$$R = R^* + \frac{\dot{e}}{e}.\tag{3.1.4}$$

 $R^*$  denotes the nominal interest rate in the rest of the world and is assumed to be given and equal to the (utility) rate of time preference:

$$R^* = v.$$
 (3.1.5)

Money is held only by the inhabitants of the country where it was issued. Shares of domestic firms can be held by everyone and form an international store of value. The value of shares issued by firms is, as before, denoted by E. The total value of assets that are in possession of inhabitants of the small country is denoted by A. When the total value of assets in possession is greater than the value of domestic shares, the inhabitants of the small country have a net claim on the rest of the world. The difference (A - E) can be called the net value of foreign claims. A foreign claim can be thought of as a (foreign) bond or equivalently as a (foreign) share with a constant dividend payout, giving a rate of return equal to  $R^*$ . In our numerical simulations we have used the counterpart of the net value of foreign claims, i. e. debt, as a fraction of total sales:

$$s = \frac{(E-A)}{y \cdot P_y}.$$
 (3.1.6)

The change of this variable over time represents a measure of debt accumulation of the small country.

Total wealth of households by consequence changes slightly from (2.1.3) into:

$$W = H + A + M.$$
 (3.1.7)

A perfectly flexible exchange rate ensures that the balance of

payments  $(S_b)$  is always in equilibrium:

$$S_b = b \cdot P_v + R \cdot (A - E) - (\dot{A} - \dot{E}) = 0.$$
 (3.1.8)

The two constituent parts of the balance of payments are the current account  $(b \cdot P_y + R \cdot (A - E))$  plus the capital account  $(\dot{E} - \dot{A})$ . As now can easily be checked, the formulation of the dynamic budget constraint of (2.1.6) remains intact for the open economy.

Household behaviour and firm behaviour as described in the relevant sections before continue to be relevant for the open economy. The characteristics of the markets stay as they were. The labour market shows sluggish adjustment in response to disequilibrium due to the, ex hypothesis, immobility of labour. The other markets keep ex ante demand and ex ante supply in pace by infinitely fast adjustment.

By now we have formulated a dynamic and micro-founded equivalent of the "IS-LM-BP-analysis" with a flexible exchange rate (see for instance Parkin and Bade, 1988).

### 3.2 Stationary State and Simulation

Determining the stationary state of the open economy gives rise to some troubles not encountered in case of the closed economy. In the closed model, equations (2.4.13) to (2.4.15) determined the amount of consumption, leisure and so on. Equations (2.4.13) and (2.4.14) must now be replaced by:

$$A^{*} = \frac{y - \delta \cdot k}{l} = \frac{c+b}{l}.$$
(3.2.1)

The latter equation together with (2.4.15) gives rise to:

$$l_{m} - l = \frac{A^{*} \cdot l_{m} - b}{A^{*} + \left\{\frac{\gamma_{c}}{\gamma_{l}} \cdot \frac{P_{l}}{P_{y}}\right\}^{\frac{1}{1 + \rho}}},$$
(3.2.2)  
$$c = \frac{A^{*} \cdot l_{m} - b}{1 + \frac{A^{*}}{\left\{\frac{\gamma_{c}}{\gamma_{l}} \cdot \frac{P_{l}}{P_{y}}\right\}^{\frac{1}{1 + \rho}}}.$$
(3.2.3)

Now, from (2.4.16), the amount of real cash-balances can be determined. From  $\dot{A} = \dot{E} = 0$  together with (3.1.8) and (2.4.11) it follows that:

$$b = \frac{v \cdot (E - A)}{P_v}.$$
 (3.2.4)

Given the value of net foreign claims (A-E), the model can be solved by inserting (3.2.4) in (3.2.2) and (3.2.3). The problem now is that this value cannot be determined uniquely from the stationary state conditions. The value of net foreign claims at which the open economy stabilizes after it is hit by an exogenous shock depends on the initial state of the economy and the speed of the adjustment processes. This phenomenon is called hysteresis and is characterized by a zero root of the system. The economic reason behind this is that compared with the closed economy there is one extra degree of freedom. An open economy can lend or borrow abroad. In principle this is comparable with lending or borrowing by a consumer. However, in a general equilibrium model assets and liabilities cancel out at the aggregate level. This is clearly not the case when the concept of a small open economy is used. The essence of this concept is that there is no feedback from the rest of the world to the small country. Hence assets and liabilities do not cancel out at the aggregate level, i. e. on world scale. In this sense one can say that the model of the small open economy is not a "closed" one.

The total number of assets held by households consists of home shares and foreign shares, or, equivalently, foreign bonds. The total value of home shares equals E, while the value of a foreign share (in home currency) is  $E^* \cdot e$ , where  $E^*$  is assumed to be constant over time, denoting the value of a foreign share in foreign currency. When the numbers of home- and foreign shares that are in possession of the home country are denoted by n and mrespectively, the following can be written:

$$A = n \cdot E + m \cdot E^* \cdot e, \quad 0 \le n \le 1, \ m \ge 0.$$
 (3.2.5)

The numbers n and m are determined by history. Given these numbers, the initial stationary state can be computed straightforward as is shown in the appendix.

Note that the real value of net foreign claims is not predetermined and can jump. But, given a certain number for  $n_0$  and  $m_0$ , the jump of the real value of net foreign claims can be related to the jump of *E*, using equation (3.2.5). This assures that there is a unique stable arm converging to the new stationary state and enables numerical simulation. The value of net foreign claims is predetermined in the special case that all home-shares are held by the households of the small country  $(n_0 = 1)$ . This is e.g. the case when there has been no exchange of shares between the small economy and the rest of the world  $(n_0 = 1, m_0 = 0)$ . In fact, the latter configuration is used in the numerical simulations that are to follow. We start with an economy that is effectively closed from the outset but grows into an open one after it is hit by some unexpected event. When the economy is recovering from the disturbance, shares go from the small economy to the rest of the world and vice versa, i. e. the numbers n and m will eventually change over time.

Again, before turning to the numerical simulations, we discuss a proposition that refers to the open economy:

#### **Proposition 2:**

If, in the model described above, the instantaneous utility function is of the Cobb-Douglas type ( $\rho = 0$ ) and the (exogenous) money supply is fixed then no exchange rate overshooting occurs despite non-clearing on the labour market and households will smoothen out their consumption over time.

Given the result of Proposition 1, which is valid for this open economy as well, it follows from interest rate parity (equation (3.1.4)) that the exchange rate is constant over time. From purchasing power parity (equation (3.1.2)) and a given foreign price level it follows that the home-price level comoves with the exchange rate, so the home-price level is constant over time. Therefore, real cash-balances are constant over time and together with (2.4.11) and (2.4.16) it follows that consumption is constant over time.

We now turn to the simulation of a technological shock that is of the same calibre as the one presented in Table 1. A comparison of Tables 1 and 2 shows the effects of opening up a closed economy. The economy that is hit in Table 1 is a closed economy and stays that way. The economy of Table 2 is initially the same as the closed economy of Table 1. The difference is that the economy of Table 2 is opposed to the rest of the world when restoring equilibrium. The open economy has extra opportunities to cope with difficulties vis-à-vis the closed economy, but here are no gains without losses. The open economy for example is bound to the dictate of purchasing power parity and to interest rate parity. Whether the inhabitants of the open economy are better off than their counterparts of the closed economy is impossible to say in a general sense. Table 2 shows that in this case the open economy is slightly better off than the closed one. The outcome of Table 2 will be discussed briefly now.

| Period →<br>Variable ↓ | 0      | 1      | 2      | 5      | 10     | Stationary<br>state |
|------------------------|--------|--------|--------|--------|--------|---------------------|
| v                      | 3.44   | 4.04   | 4.34   | 4.84   | 5.39   | 6.09                |
| c                      | 5.42   | 5.42   | 5.42   | 5.42   | 5.42   | 5.42                |
| <i>i</i> .             | 4.14   | 4.20   | 4.14   | 3.96   | 3.81   | 3.62                |
| Ъ                      | -2.41  | - 1.56 | -1.09  | -0.31  | 0.55   | 1.63                |
| k                      | 0      | 0.39   | 0.75   | 1.59   | 2.48   | 3.62                |
| 1                      | -1.08  | -0.32  | -0.09  | 0.03   | 0.21   | 0.44                |
| $P_{v}$                | - 5.14 | - 5.14 | - 5.14 | - 5.14 | - 5.14 | - 5.14              |
| $\vec{P}_{I}$          | 0      | -0.46  | -0.48  | 0.17   | 1.20   | 2.55                |
| Ŕ                      | 0.00   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00                |
| D                      | - 4.13 | -2.51  | -1.72  | -1.21  | - 1.37 | -1.71               |
| Ε                      | - 1.65 | - 1.48 | -1.42  | - 1.44 | - 1.55 | -1.71               |
| W                      | 0.04   | 0.00   | 0.00   | 0.00   | 0.00   | 0.00                |
| S                      | 0      | 1.36   | 2.41   | 4,82   | 7.38   | 10.57               |
| U                      | 58.54  | 58.01  | 57.77  | 57.27  | 56.67  | 55.91               |

Table 2. A Technological Shock in an Open Economy ( $\rho = 0$ )

As before, all variables are shown in percentual deviations from their old stationary state values, except for the variables band s, where the actual values ( $\times$  100) are shown.

The first striking result is that, though the economy starts producing more, the economy also starts importing goods. Apparently, the economy is taking a short cut to future returns that are coming along with the greater productivity. The possibility to sell goods to or buy goods from the rest of the world enables households to realize a constant consumption over time by lending or borrowing abroad at the constant world interest rate. Because the households' time preference is just offset by the economy's rate of return, it is that they want to smoothen their consumption over their life-span. The result is that a rich land is going into debt, which is perfectly intelligible in this case. During transition the current account is in deficit all of the time. In the end this economy is exporting the amount of goods to pay off the return on their outstanding debt. Alternatively, it can be said that the small country is paying dividends to foreign shareholders directly in the form of exported goods. Because of the greater wealth, households want to hold more real balances and therefore the home price-level will fall. A constant foreign price-level means that the homecurrency has undergone an appreciation in this world of purchasing power parity. The exchange rate directly jumps to its equilibrium value and no overshooting occurs even though the labour market displays sticky wages in the short run. This result runs contrary to the received view that, though "the explanations of the overshooting phenomenon vary, ... they all rely on the short run fixity of some nominal quantity" (Frenkel and Rodriguez, 1982, p. 1). Apparently, the short run fixity of some variable is not a sufficient condition to account for overshooting. Furthermore, it can be seen from the table that the initial unemployment is making place for "overemployment" after some periods of time.

| Period →<br>Variable ↓ | 0              | 1              | 2               | 5             | 10             | Stationary state |
|------------------------|----------------|----------------|-----------------|---------------|----------------|------------------|
| у                      | 3.46           | 4.03           | 4.30            | 4.78          | 5.31           | 5.96             |
| с<br>j                 | 5.42<br>4.07   | 5.38<br>4.10   | 5.37<br>4.04    | 5.37<br>3.86  | 5.36<br>3.69   | 5.34<br>3.50     |
| b<br>k                 | -2.37          | -1.51<br>0.38  | -1.07<br>0.73   | -0.30<br>1.55 | 0.54<br>2.42   | 1.58<br>3.50     |
| l<br>P.                | -1.04<br>-5.09 | -0.33<br>-5.09 | -0.15<br>-5.09  | -0.04 - 5.08  | 0.12 - 5.08    | 0.32 - 5.07      |
| $P_l$                  | 0              | -0.40          | -0.39           | 0.28          | 1.30           | 2.62             |
| D<br>D                 | -3.91          | -2.40          | - 1.66          | -1.21         | -1.40          | -1.75            |
| E<br>W                 | -1.65<br>0.08  | -1.49<br>0.06  | $-1.44 \\ 0.05$ | -1.47<br>0.06 | - 1.59<br>0.06 | $-1.75 \\ 0.07$  |
| $\overset{s}{U}$       | 0<br>56.78     | 1.33<br>56.29  | 2.34<br>56.07   | 4.69<br>55.59 | 7.17<br>55.03  | 10.21<br>54.33   |

Table 3. A Technological Shock in an Open Economy ( $\rho = 0.1$ )

Table 3 shows the effects of the same technological shock in an open economy, be it that this time no Cobb-Douglas utility specification is used. There are no qualitatively different results in this table. Showing it serves the purpose to demonstrate that the constant interest rate and consumption crucially hinge on the Cobb-Douglas assumption. The overshooting of the exchange rate can be observed from Table 3. Finally, Table 4 serves to demonstrate the non-neutrality of money in the open economy. Non-neutrality in the short run does not come as a surprise due to the adjustment processes in the economy. The non-neutrality that is shown to exist in the long run however must be attributed to the aforementioned hysteresis phenomenon of our open economy. The stock of money is (unexpectedly) increased by 2.5%. Numerical simulations have shown that money is neutral in this model

| Period →<br>Variable ↓ | 0     | 1      | 2      | 5      | 10    | Stationary state |
|------------------------|-------|--------|--------|--------|-------|------------------|
| v                      | -0.22 | -0.11  | - 0.05 | - 0.01 | 0.00  | 0.01             |
| c                      | -0.03 | -0.03  | -0.03  | -0.03  | -0.03 | -0.03            |
| i                      | -0.14 | - 0.06 | -0.02  | 0.00   | 0.01  | 0.01             |
| b                      | -0.24 | -0.10  | -0.03  | 0.02   | 0.03  | 0.04             |
| k                      | 0     | -0.01  | -0.01  | -0.01  | 0.00  | 0.01             |
| 1                      | -0.44 | -0.21  | -0.10  | -0.01  | 0.00  | 0.01             |
| $P_{v}$                | 2.53  | 2.53   | 2.53   | 2.53   | 2.53  | 2.53             |
| $\vec{P_I}$            | 0     | 1.32   | 1.95   | 2.45   | 2.52  | 2.53             |
| Ŕ                      | 0.00  | 0.00   | 0.00   | 0.00   | 0.00  | 0.00             |
| D                      | 7.67  | 4.98   | 3.70   | 2.66   | 2.54  | 2.54             |
| Ε                      | 3.15  | 2.83   | 2.68   | 2.56   | 2.54  | 2.54             |
| W                      | 2.50  | 2.50   | 2.50   | 2.50   | 2.50  | 2.50             |
| S                      | 0     | 0.12   | 0.17   | 0.23   | 0.26  | 0.29             |
| U                      | -0.02 | - 0.22 | -0.31  | -0.38  | -0.40 | -0.40            |

Table 4. A Monetary Shock in an Open Economy ( $\rho = 0$ )

when there would be no sluggish wage adjustment. In the latter case immediate adjustment of the economy takes place and there is no accommodation of the economy's position of debt. When (for instance) wage adjustment takes time it is optimal to adjust capital stock and bond holdings. The fact that adjustment takes time itself changes the stationary state of the economy. Given that the level of consumption must be constant, it must be such that the present discounted value of current and future trade imbalances are zero. This condition can be seen as the no-Ponzi-game condition for a small open economy. In anticipation of all future events it appears that the constant level of consumption must fall. The greater stock of money will lead to a higher price level in the small country. Along with this there is a depreciation of the home currency, dictated by the purchasing power parity assumption. As prices are higher and nominal wages sticky in the short run firms find it optimal to ask for more labour and expand production. The lower real wage rate will, however, induce consumers to withdraw labour supply so that firms will be quantity rationed when demanding labour. As a consequence it is inevitable for firms to shrink production. Perfect anticipation of future events will dictate that it is necessary to decumulate the stock of capital for some time. Most of future production will be down for labour rationing also. This implies a loss of real wealth to the economy. Therefore the constant level of consumption has to fall. In the absence of binding consumer constraints nominal wealth is proportional to the money stock.<sup>4</sup> Consequently the price level has to rise by more than 2.5% in order to decrease real wealth. Nominal wage adjustments over time relax the binding labour constraint slowly. Due to this relaxation output will rise over time. The fall in the constant level of consumption is smaller than the initial fall in net output. The difference is catered for by borrowing abroad and importing goods. At some date in the future there must be a reversal from importing to exporting goods. Nevertheless there is a continuous capital inflow because a rent must be paid over existing debts. Because debt is accumulated during adjustment to the new steady state, the current account is in deficit all time.

Note that these results in general do not coincide with standard textbook analysis.

### 4. Concluding Remarks

A perfect foresight model for a small open economy with micro foundations for households and firms has been analyzed in an intertemporal setting. The model has been kept as simple as possible. True dynamics are introduced by incorporation of capital accumulation and slow wage adjustment. Stock prices have been incorporated in the same way as was done before in Meijdam and van Stratum (1988). A closed economy model has been used as a starting point for a small open economy model. Purchasing power parity and interest rate parity were assumed in order to get the above-mentioned desideratum of simplicity. Shares of firms are used as the only international store of value. A perfectly flexible exchange rate keeps the balance of payments in equilibrium and hence a current account deficit is mirrored by debt accumulation, i. e. shares go out of the country. The open model exhibits the phenomenon of hysteresis, in other words: the steady state of the economy is history dependent. This history-dependency troubles the determination of the steady state.

Both Obstfeld (1981) and Turnovsky (1985) have circumvented this particular difficulty, but it seems that neither has solved the problem in a satisfactory way. We have shown (in fact by using numerical simulations) that another solution is viable in order to obtain optimal debt accumulation. In doing so, we discovered the

<sup>&</sup>lt;sup>4</sup> This relation can be derived from (2.9) to (2.12) in combination with the budget constraint (2.1.6) and the transversality condition (2.1.8).

Hodrick (1982)-paper as our main source of inspiration. In retrospect, we think that the research-activities on the optimizing small open economy have been undertaken too strongly in line with the Turnovsky-paper, while the earlier contribution of Hodrick was dismissed unjustly.

The numerical simulations are carried out in such a way that a comparison directly can be made between a closed economy and an open economy. It appears that nothing in general can be said about the desirability to be closed or open. A number of characteristics of our model depends crucially on the Cobb-Douglas utility specification. In the latter case, the open economy is able to generate a constant consumption pattern over time by importing or exporting goods and accumulation or decumulation of national debt. Intertemporal consumption smoothing goes together with a constant rate of interest equal to the (exogenous) world rate of interest, which in turn implies that no exchange rate overshooting occurs. The intertemporal optimizing character of the model generates results that are difficult to reconcile with standard textbook analysis. The open economy shows non-neutrality of money in the long run, due to the hysteresis-phenomenon. Non-neutrality per se does not imply, however, that increasing the (exogenous) supply of money has positive welfare effects.

In future research, it is interesting to relax the purchasing power parity assumption in the first place and allow for imperfect substitution between home- and foreign-goods, while keeping the interest rate parity rule intact. This opens up the possibility to investigate exchange rate overshooting in an intertemporal optimizing equivalent of the original Dornbusch (1976)-setting with sticky (good-) prices and a flexible exchange rate.

#### Appendix

### The Model for the Closed Economy

$$c_{d} \mid \gamma_{c} \cdot c_{d}^{-(\rho+1)} = x \cdot P_{y} \cdot \left\{ \gamma_{c} \cdot c_{d}^{-\rho} + \gamma_{m} \cdot \left(\frac{M}{P_{y}}\right)^{-\rho} + \gamma_{l} \cdot (l_{m} - l_{s})^{-\rho} \right\}$$
(A.1)

$$l_{s} \mid \gamma_{l} \cdot (l_{m} - l_{s})^{-(\rho+1)} = x \cdot P_{l} \cdot \left\{ \gamma_{c} \cdot c_{d}^{-\rho} + \gamma_{m} \cdot \left(\frac{M}{P_{y}}\right)^{-\rho} + \gamma_{l} \cdot (l_{m} - l_{s})^{-\rho} \right\}$$
(A.2)

$$l_{d} = k \cdot \alpha^{\frac{\sigma}{\sigma-1}} \left\{ \left( (1-\alpha) \cdot \varepsilon \cdot \frac{P_{y}}{P_{l}} \right)^{1-\sigma} - (1-\alpha) \right\}^{\frac{\sigma}{\sigma-1}}$$
(A.3)

$$l = \min\left(l_d, l_s\right) \tag{A.4}$$

$$c \mid \gamma_c \cdot c^{-(\rho+1)} = x \cdot P_y \cdot \left\{ \gamma_c \cdot c^{-\rho} + \gamma_m \cdot \left(\frac{M}{P_y}\right)^{-\rho} + \gamma_l \cdot (l_m - l_s)^{-\rho} \right\}$$
(A.5)

$$i = \delta \cdot k + \psi \cdot k \cdot (q - 1) \tag{A.6}$$

$$j = i + \frac{(i - \delta \cdot k)^2}{2 \cdot \psi \cdot k}$$
(A.7)

$$y = \varepsilon \cdot \left\{ \alpha \cdot k^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot l^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}$$
(A.8)

$$P_{y}|y = c + j \tag{A.9}$$

$$R = \frac{\gamma_m \cdot \left(\frac{M}{P_y}\right)^{-(\rho+1)}}{x \cdot P_y \cdot \left\{\gamma_c \cdot c^{-\rho} + \gamma_m \cdot \left(\frac{M}{P_y}\right)^{-\rho} + \gamma_l \cdot (l_m - l)^{-\rho}\right\}}$$
(A.10)

$$D = y \cdot P_y - l \cdot P_l - j \cdot P_y \tag{A.11}$$

$$H = W - E - M \tag{A.12}$$

$$\dot{W} = R \cdot W - R \cdot M - (l_m - l) \cdot P_l - c \cdot P_y \qquad (A.13)$$

$$\dot{E} = R \cdot E - D \tag{A.14}$$

$$\dot{P}_l = \theta \cdot (l_d - l_s) \cdot P_l \tag{A.15}$$

$$\dot{x} = (v - R) \cdot x \tag{A.16}$$

$$\dot{q} = \left(R + \delta - \frac{\dot{P}_{y}}{P_{y}}\right) \cdot q - \left\{\varepsilon^{\frac{\sigma-1}{\sigma}} \cdot \alpha \cdot k^{\frac{1}{\sigma}} \cdot y^{\frac{1}{\sigma}}\right\} - \left\{\frac{\left(\frac{\dot{i}}{k}\right)^{2} - \delta^{2}}{2 \cdot \psi}\right\}$$
(A.17)

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Dynamic Adjustment and Debt Accumulation

$$\dot{k} = i - \delta \cdot k \tag{A.18}$$

$$\dot{U} = v U - \ln \left\{ \gamma_c \cdot c^{-\rho} + \gamma_m \cdot \left(\frac{M}{P_y}\right)^{-\rho} + \gamma_l \cdot (l_m - l)^{-\rho} \right\}^{-\frac{1}{\rho}}$$
(A.19)

# The Model for the Open Economy

See the model for the closed economy. Replace equations (A.9) and (A.12) by:

$$b = y - c - j, \tag{A.9'}$$

$$H = W - A - M. \tag{A.12'}$$

Add the following three equations to complete the model:

$$\dot{A} = b \cdot P_{y} + R \cdot A - D, \qquad (A.20)$$

$$\dot{P}_{y} = P_{y} \cdot (R - R^{*}), \qquad (A.21)$$

$$s = \frac{(E-A)}{y \cdot P_y}.$$
 (A.22)

# **Proof of Proposition 1**

It can be seen from the consumer's optimization problem that with a Cobb-Douglas utility specification ( $\rho = 0$  in equation (2.4.1)), money-market clearance and a fixed supply of money, the following two relationships hold:

$$\frac{M}{P_{y}} = \frac{\gamma_{m}}{P_{y} \cdot R \cdot x},$$
 (A.23)

$$\dot{x} = (v - R) \cdot x. \tag{A.24}$$

From (A.23) it follows that:

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$$R = \frac{\gamma_m}{x \cdot M}.$$
 (A.25)

Substitution of (A.25) in (A.24) gives:

$$\dot{x} = v \cdot x - \frac{\gamma_m}{M}.$$
 (A.26)

The only non-exploding solution for (A.26), bearing in mind a

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constant stock of money, is:

$$x = \frac{\gamma_m}{M \cdot \nu}.$$
 (A.27)

Exploding solutions in general must be excluded because the transversality conditions will not hold anymore. Because the jump-variable x turns out to be a constant over time, it follows from (A.24) that the nominal interest rate always equals the time preference rate: R = v.

## Determination of the Stationary State for an Open Economy

From the dynamic budget constraint together with  $\dot{W}=0$  and the equations (3.2.2), (3.2.3), (2.4.11) and (2.4.16) we find that consumption and leisure can be written as functions of real wealth:

$$l_{m} - l = B^{*} \cdot \frac{W}{P_{y}} = \begin{cases} \frac{\left\{\frac{\gamma_{l} \cdot \nu}{\gamma_{m}}, \frac{P_{l}}{P_{y}}\right\}^{\frac{1}{1+\rho}} \cdot \nu}{\left\{\frac{\gamma_{l} \cdot \nu}{\gamma_{m}}\right\}^{\frac{1}{1+\rho}} \cdot \left(\frac{P_{l}}{P_{y}}\right)^{\frac{\rho}{1+\rho}} + \left\{\frac{\gamma_{c} \cdot \nu}{\gamma_{m}}\right\}^{\frac{1}{1+\rho}} \end{cases} \begin{cases} \frac{W}{P_{y}}, \\ (A.28) \end{cases}$$

$$c = C^{*} \cdot \frac{W}{P_{y}} = \begin{cases} \frac{\left\{\frac{\gamma_{c} \cdot \nu}{\gamma_{m}}\right\}^{\frac{1}{1+\rho}} \cdot \nu}{\left\{\frac{\gamma_{c} \cdot \nu}{\gamma_{m}}\right\}^{\frac{1}{1+\rho}} + \left\{\frac{\gamma_{c} \cdot \nu}{\gamma_{m}}\right\}^{\frac{1}{1+\rho}} \end{cases} \end{cases} \end{cases} \cdot \frac{W}{P_{y}}.$$

$$(A.29)$$

In these expressions real wealth can be written as follows (using 3.2.4):

$$\frac{W}{P_{y}} = \frac{A^{\#} \cdot l_{m} - b}{A^{\#} \cdot B^{\#} + C^{\#}} = \frac{A^{\#} \cdot l_{m} + v \cdot \frac{(A - E)}{P_{y}}}{A^{\#} \cdot B^{\#} + C^{\#}}.$$
 (A.30)

From (3.2.5) it follows that (keeping in mind that  $e = P_y$ ):

$$\frac{(A-E)}{P_{y}} = (n-1) \cdot \frac{E}{P_{y}} + m \cdot E^{*}.$$
 (A.31)

From (3.2.1) and (A.28) together with  $\dot{E} = 0$ :

$$\frac{E}{P_{y}} = \left\{ A^{*} - \frac{P_{l}}{P_{y}} \right\} \cdot \frac{l_{m} - B^{*} \cdot \frac{W}{P_{y}}}{v}.$$
(A.32)

Inserting (A.31) and (A.32) in (A.30):

$$\frac{W}{P_{y}} = \frac{A^{\#} \cdot l_{m} + (n-1) \cdot l_{m} \cdot \left\{A^{\#} - \frac{P_{l}}{P_{y}}\right\} + v \cdot m \cdot E^{*}}{A^{\#} \cdot B^{\#} + C^{\#} + (n-1) \cdot B^{\#} \cdot \left\{A^{\#} - \frac{P_{l}}{P_{y}}\right\}}.$$
 (A.33)

This expression determines the stationary state of the open economy.

# **Parameter Values**

| α        | = 0.25  | $\gamma_c$ | = 0.85            | $l_m$ | =9.0    |
|----------|---------|------------|-------------------|-------|---------|
| Е        | = 0.25  | YI         | = 0.1             | M     | = 1.0   |
| $\sigma$ | = 0.40  | Υm         | = 0.05            | $n_0$ | = 1.0   |
| δ        | = 0.1   | v          | $= 0.1 \ (= R^*)$ | $m_0$ | = 0     |
| Ψ        | = 0.125 | $\theta$   | = 0.1             | ρ     | = 0/0.1 |
| $E^*$    | = 3.0   |            |                   | -     |         |

# **Stationary State**

| if $\rho$        | = | 0:    | if | ρ                | _  | 0.1:   |
|------------------|---|-------|----|------------------|----|--------|
| у                | = | 1.454 |    | y                | _  | 1.458  |
| с                | = | 1.089 |    | с                | =  | 1.092  |
| b                | = | 0     |    | b                | =  | 0      |
| j                | = | 0.365 |    | j                | == | 0.366  |
| k                | = | 3.653 |    | k                | =  | 3.664  |
| l                |   | 7.646 |    | l                | =  | 7.669  |
| $P_{y}$          | = | 1.561 |    | $P_{v}$          | =  | 1.483  |
| $\vec{P_l}$      | = | 0.148 |    | $\vec{P_l}$      | =  | 0.140  |
| D                | _ | 0.570 |    | D                | =  | 0.543  |
| $\boldsymbol{E}$ | = | 5.703 |    | Ε                | =  | 5.434  |
| W                | = | 20.0  |    | W                | -  | 19.068 |
| S                | _ | 0     |    | S                | =  | 0      |
| $\boldsymbol{U}$ | _ | 0.804 |    | $\boldsymbol{U}$ | =  | 0.830  |

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