REVEALING CONSUMERS’ WILLINGNESS-TO-PAY: A COMPARISON OF THE BDM MECHANISM AND THE VICKREY AUCTION

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Abstract
We compare the capability of the Becker-DeGroot-Marschak mechanism and the Vickrey auction to reveal willingness-to-pay information for a sample of French consumers. We measure the bias and dispersion of bids relative to valuations. We find that the Vickrey auction, for the particular training procedure we employ, is more effective as a willingness-to-pay elicitation device than the BDM process. We conjecture that differences in the shape of the payoff function account for some of the differences.

1. INTRODUCTION
Demand revealing auctions are potentially powerful vehicles for eliciting willingness-to-pay information. The dominant strategy of truthful bidding and the commitment of real money create an incentive to truthfully reveal limit prices, regardless of the risk attitude of the bidder and the strategies other participants use. A demand-revealing auction has the advantage over the study of purchase decisions with field data that it allows an

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individual’s limit price to be measured directly. Observing only whether or not an individual purchases a product merely establishes whether or not his limit price exceeds the current market price. Accurate willingness-to-pay information is particularly useful for new products because other sources of demand estimates on which to base profit or cost-benefit calculations are not readily available.¹ Experimental economists have employed demand revealing auctions to study limit prices for goods as varied as consumer products (see for example Hoffman et al., 1993; Bohm et al., 1997; List and Shogren, 1998; and List and Lucking Reilly, 2001), food safety (Hayes et al., 1995; Fox et al., 1998; Busby et al., 1998; Huffman et al., 2000; and Lusk et al., 2001), and lotteries (Grether and Plott, 1979; Cox and Grether, 1996).

The two most widely used demand-revealing mechanisms in experimental economics are the second price sealed bid auction, also called the Vickrey auction (Vickrey, 1961), and the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al., 1964). In a second price sealed bid auction, each subject simultaneously submits a bid to purchase a good. The agent who submits the highest bid wins the auction and receives the item, but pays an amount equal to the second highest bid among the bidders in the auction. In a BDM each subject simultaneously submits an offer price to purchase a good. Afterwards, a sale price is randomly drawn from a distribution of prices with support on an interval from zero to a price greater than the anticipated maximum possible willingness-to-pay among bidders. Any bidder who submits a bid greater than the sale price receives a unit of the good and pays an amount equal to the sale price.

¹ See for example Hoffman et al. (1993), who use demand revealing auctions to elicit willingness to pay information for new packaging for meat products or Noussair et al (2002b), who study consumer valuation for genetically modified foods.
There is a substantial literature studying the behavior of the two mechanisms in the laboratory when university student subjects are bidding for goods. Some of this research has used the technique of induced values (Smith, 1982) to create limit prices for fictitious goods. The experimenter offers a guarantee that bidders can resell goods at prices that are specified in advance, should they purchase the items in the auction. Several authors, including Coppinger et al. (1980), Cox et al. (1982), Kagel et al. (1987) and Kagel and Levin (1993), have studied the behavior of the Vickrey auction, and Irwin et al. (1998) and Kellar et al. (1993) have studied the BDM process using goods with induced values. These studies reach a variety of conclusions about bids relative to valuations, and some suggest that average bids are biased away from valuations. For example Kagel et al. (1987) and Kagel and Levin (1993) find that most winning bids in the Vickrey auction are higher than valuations. Irwin et al. (1993) find that the BDM process is more successful at eliciting true valuations for certain distributions of sale prices than others. Furthermore, all of the studies show that there is heterogeneity in bidding behavior that leads to a dispersion of bids relative to valuations. In the case of auctions for goods with homegrown (and therefore unobservable) valuations, such as consumer products, the evidence that bids tend to differ from valuations is indirect. Bohm et al. (1997) find that bids in the BDM are sensitive to the choice of endpoints of the distribution of possible transaction prices. List and Shogren (1999) find that bids in the Vickrey auction tend to increase as the auction is repeated. This suggests a bias in bidding either in the early or the late periods. Rutstrom (1998) finds that the two mechanisms generate different mean bids for the same objects, indicating that at least one of the two must be biased.
In this paper we explore the demand revelation capabilities of the two mechanisms with a diverse sample of consumers. In theory, of course, the mechanisms lead to the same result irrespective of the population that is participating. However, the behavior of our subjects is of interest because they constitute a more appropriate sample than university students for evaluating preferences over many products that are not targeted to student populations. Our subjects have an age and gender profile close to that of the general population and consist exclusively of people who make purchase decisions for their household.

Although both the BDM mechanism and the second price sealed bid auction are incentive compatible in the sense that under each technique there is a dominant strategy to truthfully reveal preferences, the two processes differ in substantive ways that might affect behavior.\(^2\) The most obvious is that the Vickrey auction is a game in which a participant interacts with other players, whereas the BDM places subjects in a situation of individual choice. However, as we describe in section four, the cost of suboptimal behavior also varies considerably between the two mechanisms, unless the distribution of sale prices for the BDM is appropriately adjusted, and unless all subjects in the Vickrey auction are following the dominant strategy. Though strategic uncertainty about other players’ actions in the Vickery auction does not affect the optimal strategy, it does affect the expected cost of deviating from it. In addition to the above considerations, there is

\(^2\) The BDM mechanism is formally equivalent to a second price sealed bid auction played against one other bidder, who bids her valuation, and whose value is drawn from the same distribution of valuations as that of the BDM prices.
also a difference between the framing of the two auctions that might lead to differences in their behavioral properties.\footnote{Multi-unit generalizations of the Vickrey auction have also been studied. When there are \( k \) units to be sold, and each demander wishes to purchase at most one unit, a sealed bid auction in which the \( k \) highest bidders each receive a unit and pay a price equal to the \( k+1 \)st highest bid induces a dominant strategy of bidding an amount equal to one’s valuation. When agents wish to purchase multiple units there is no single price demand revealing mechanism. A demand revealing mechanism exists in which the \( k \) highest bidders receive units and each bidder, for his \( j \)th accepted unit, pays an amount equal to \( j \)th highest rejected bid submitted by a bidder other than himself (Vickrey, 1961). Hoffman et al. (1993) employ a random \( k \)th price auction for the single unit demand case. The number of units sold, \( k \), is drawn randomly after the bids have been submitted. The \( k \) highest bidders each receive a unit and pay a price equal to the \( k \)th highest bid. The use of a random \( k \) is intended to give bidders who are unlikely to be marginal for a fixed \( k \), either because they have valuations much higher or lower than the marginal valuation, a stronger incentive to truthfully bid their valuation. Shogren et al. (2001) compare bidder behavior in the random \( k \)th price auction and the second price sealed bid auction. They find that the second price sealed bid auction generates bids closer to valuations for players whose valuations are close to the margin between winning and failing to win a unit. However, bids are closer to valuations under the random \( k \)th price auctions for off-margin bidders.}

In our analysis of the data, the questions we focus on are the following: (1) Does either or both of the systems contain a bias toward under or over-revelation of the willingness-to-pay? (2) Under which system are individuals more likely to bid near their true value? (3) Under which system is convergence by repetition toward demand revelation, if it occurs, more rapid? We pose these questions under specific conditions, when the population considered is a diverse sample of the population, when the goods considered have induced valuations, and when specific training procedures are in effect that our experience and intuition suggest would enhance the performance of the mechanisms. We argue that the Vickrey auction, accompanied by an appropriate training procedure, functions quite reliably in eliciting valuation information. As we report in the pages that follow, the Vickrey auction generates initial average bids closer to valuations and then more rapid convergence of bids toward valuations than the BDM. Both auctions have a bias toward underbidding in early periods as agents seem to “test” the market (Plott, 1996) with low bids, but this effect is more severe in the BDM than in the Vickrey auction. Under our training procedures, 90% of subjects in the Vickrey auction bid within
10% of their valuations and 77% bid within 2%, after forty-five minutes of training. We turn to these procedures in section 2.

2. PROCEDURES

The participants in our experiment were residents of the Grenoble, France area. 199 subjects took part in the experiment, each taking part in one of the 26 sessions. The ages of the 199 subjects that participated in our study ranged from 18 to 79 years, and averaged 34 with a standard deviation of 14 years. 43.8% were male. The subjects were varied in their educational attainment and their occupations. 26% of our subjects were college graduates, and 38% had at least some education beyond high school. 74% had completed their Baccalaureate, the equivalent of a high school diploma.

In 16 of the sessions, subjects participated in Vickrey auctions and the remaining ten sessions, the BDM process was used. The sessions were conducted between June 1999 and July 2000, at the Ecole Nationale Supérieure de Génie Industriel (ENSGI) at the Institut National Polytechnique de Grenoble (INPG) in Grenoble, France. In each session there were between 3 and 14 participants. Details of the experimental procedures are available from the authors.

Candidates were chosen by drawing names randomly from the Grenoble, France telephone directory for recruitment by telephone. At the time of recruiting, subjects were asked if they were consumers of orange juice or cookies and if they made purchase decisions for their household. If they responded affirmatively to both questions, they were invited to come to the laboratory to sample food products for a publicly funded research project conducted by university professors. They were given no other details
about what would take place during the session. 11% of telephone calls yielded a participant. No subject participated in more than one session. Subjects did sample food products in each experiment as advertised, but after they were trained in the use of the bidding mechanisms, that is, after they participated in the auctions reported here. Subjects were told that they were learning a special buying procedure that would be used for the entire experiment. This created an additional incentive to find an optimal strategy, since it was likely to yield future gains.  

The procedures of the sessions were designed to facilitate learning of the dominant strategy, rather than to provide an unbiased test of theoretical propositions, which is the case with most experimental research. The instructions are available from the authors. They reflect special considerations that come into play when studying participants with varied educational backgrounds. Experiments conducted with university students, who share a common age and educational background, permit the use of a particular vocabulary. This was a resource not available to us in this study.

Upon arrival at the laboratory, each subject received 150 francs (roughly US$21) in cash. They were then told that they would be bidding for consumer products later in the session, but beforehand there would be a phase in which they could learn about and practice the bidding process that would be used later on. Subjects then bid for several periods, in either a Vickrey auction or a BDM depending on the session, for an item called X, which had an induced value. In each period, each subject drew a valuation for X independently from a common uniform distribution. The distribution changed from period to period, and differed between sessions. The highest possible valuation range was

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4 The focus of this paper is on comparing the behavior of the two auctions in the training phase in which goods with induced values were auctioned. For the results of the study of food products, see Noussair et al. (2002a, 2002b).
221-389 FF (US$ 29-52), and the lowest was 13-56 FF (US$ 2-7). Ranges were typically wider for the Vickrey auction, to compensate for the greater expected earnings due to the higher probability and greater conditional gains from receiving a unit in the BDM process. The payoff functions of the two mechanisms are discussed in section 4 of this paper in more detail.\(^5\) At the time of bidding under both mechanisms, subjects were not aware of the valuations of other bidders nor the distribution from which their own and the others’ valuations were drawn.

During sixteen of the sessions we used Vickrey auctions.\(^6\) In each auction, subjects simultaneously submitted a bid to purchase a good. The agent who submitted the highest bid won the auction, but paid an amount equal to the second highest bid among the bidders in the auction. The other bidders did not receive items and paid zero. No communication between subjects was allowed during the bidding process.

In the remaining ten sessions, the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al, 1964) was studied. The rules of the BDM mechanism were the following. Each subject simultaneously submitted an offer price to the experimenter in a closed envelope. The experimenter then randomly drew a sale price from a range of prices spanning the endpoints of the distribution, from which bidders’ willingness to pay was drawn. This was done by drawing a piece of paper from a box containing the possible prices. The fact that the box was chosen before bids were submitted made it common knowledge that the sale prices were determined independently of the bids. The range was

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\(^5\) It is, in principle, possible that the use of a different scale of valuations for the two mechanisms could influence the comparison between them. However, we felt that it was preferable to compare the mechanisms under conditions in which the expected payoff from the process was similar rather under conditions where the valuations are similar and the expected payoff to bidders differed greatly.

\(^6\) There were between 5 and 9 participants in the sessions in which the Vickrey auction was used. This meant that a bidder always had between 4 and 8 competing bidders.
pre-specified but unknown to the subjects, though subjects knew that the sale prices would be drawn from the range of possible valuations. Any subject who submitted a bid greater than the sale price received an item and paid an amount equal to the sale price.

Under both processes, after the subjects submitted their bids, the experimenter wrote all of the valuations and bids on the blackboard, without identifying to whom they belonged. He then asked the following four questions to the group of subjects, who were free to volunteer responses and to engage in open discussion on the topics. The experimenter did not participate in the discussion beyond posing the initial question. A) Which bid was yours? B) Which bidder(s) won items? C) How much did the winner(s) pay? D) Do you regret the bids you submitted, now that you know how much the others bid?

The purpose of the first three questions was to verify that subjects were paying attention to their behavior, that they understood the rules of the mechanism, and that they understood how to earn money. The fourth question was intended to stimulate discussion about how subjects’ decisions might be improved.

After the discussion, the experimenter announced publicly which subjects purchased units. Those subjects then received an amount of money equal to their induced valuation minus the price, as determined in the auction. The cash was physically placed on the desk in front of the subject after the auction, to emphasize that there were real monetary earnings at stake. A series of identical auctions was conducted using the same procedure, but with new randomly chosen induced values each time. The bidding continued until the time available was exhausted or until at least 80% of bids in a period were within 5% of valuations.
3. RESULTS

We use two measures to compare the mechanisms. The first measure is the overall average bias of the mechanisms in period $t$, normalized by the valuation. It indicates the extent to which average bids are higher or lower than valuations. The bias for period $t$ is calculated as $\sum [b_{jt} - v_{jt}] / v_{jt} n_t$, where $b_{jt}$ denotes player $j$’s bid in period $t$, $v_{jt}$ is her valuation in period $t$, and $n_t$ is the total number of bidders in period $t$. The second measure is the average dispersion, defined for period $t$ as $\sum |b_{jt} - v_{jt}| / v_{jt} n_t$. The dispersion is equal to the average absolute value of the difference between bids and valuations, normalized by the valuation. For an individual bid, the dispersion is the absolute value of the bias. Table 1 illustrates the average value of each measure over the course of the sessions under both processes. The standard deviations are indicated in parentheses.

Figures 1a-j indicate the number and percentage of bids under each process and in each period that were less than, equal to, and greater than the valuation of the participant submitting the bid and the distribution of the size of the deviations of bids from valuations.

[Table 1 and figures 1a-j: About Here]

The figures reveal the following patterns.\(^7\) Both auctions are highly biased in period 1, with bids tending to be below valuations. This bias is larger and the dispersion is greater under the BDM mechanism. Overall, 90% of subjects bid less than their

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\(^7\) Figures 1g and 1h, and the column entitled “Period 4” in table 2 also contain data from period 3 for the sessions that only lasted three periods. Figures 1i and 1j and the column entitled “Last Period” in table 2 contain only the data from the last period of all sessions.
valuations and only a very small percentage bid more than their valuations. 2.44% of participants bid more than their valuations in the BDM process and 5.98% did so in the Vickrey auction. The percentage bidding an amount equal to their valuations is also small in both auctions, between 6 and 7 percent of subjects under both systems, though 17% bid within 2% of their valuations in the Vickrey auction.

On average, under the BDM mechanism, bids are 39.87% lower than valuations with standard deviation (of the percentage difference between bid and valuation) of 28.89%. In the Vickrey auction, the period 1 average bid is 30.16% less than the corresponding valuation with a standard deviation is 32.53%. Pooled variance t-tests indicate that the bias is significant at the \( p < .01 \) level for both mechanisms. The proportion bidding less than their values is greater in the BDM than in the Vickrey auction. The magnitude of the average underbid is less severe in the Vickrey auction. The average underbid was 44.56% of the valuation for BDM and 35.98% for the Vickrey auction. The average absolute difference between bids and valuations, our measure of dispersion, is 41.65% in the BDM compared to 32.57% in the Vickrey auction. Both the average bias and the average dispersion are significantly greater than in the BDM than the Vickrey auction at the \( p < .05 \) level. Thus, in the practice period, the Vickrey auction is less biased, exhibits less dispersion, and has a greater percentage of agents bidding within 2% of values.

In period 2, the first auction that counted toward subjects’ earnings, both auctions remain biased, but less so than in period 1. The introduction of monetary payments as well as repetition appears to improve decisions. 87.8% of bids in the BDM and 76.07% of those in the Vickrey auction are less than valuations. The bias is –28.06% of valuation for
the BDM and −11.50% for the Vickrey auction. The decline in the bias is steeper in the Vickrey auction than in the BDM. The bias in the BDM decreases by 29.6%, whereas in the BDM the decline is of 63.3%. The decline is mainly due to a reduction in the amount that agents underbid, and not to a decrease in the percentage of agents underbidding. The percentage bidding equal to valuations increases to over 10% overall and is slightly higher in the Vickrey auction than in the BDM. The overall dispersion shrinks in both systems but the decrease is steeper in the Vickrey auction (51.5% versus 31.4%). Thus, the overall data from periods 1 and 2 suggest that the Vickrey auction is less biased, exhibits lower dispersion, induces a greater percentage to reveal their exact valuations, and improves its performance more quickly over time.

These trends continue in subsequent periods. The overall bias decreases in each subsequent period for both processes, reaching zero in the Vickrey auction and 6% in the BDM mechanism. In each of the five periods, the bias in the BDM is significantly greater in magnitude than in the Vickrey auction at \( p < .05 \) (according to a pooled variance t test). Beginning in period 4, the bias is no longer different from zero at conventional significance levels in the Vickrey auction. However in all 5 periods, the bias is significant at the 5% level in the BDM. The percentage of agents bidding an amount equal to their valuations increases from period to period under both processes, reaching 41.46% for the BDM and 68.38% for the Vickrey auction in the last period. If latitude of 2% of the valuation is allowed in considering a bid to equal the corresponding valuations, the data in figure 1i show with the Vickrey auction, 77% of bids are within 2% of valuations and 90% are within ten percent of valuations. The dispersion between bids and valuations decreases in each period of the BDM. Though the same measure increases between
periods three and four in the Vickrey auction, the overall trend is clearly downward. The average absolute difference in the last period is 3.89% in the last period of Vickrey compared to 11.75% in the BDM. In the Vickrey auction, the dispersion is significantly less than in the BDM at \( p < .01 \) in all periods except for period 4. Therefore, in the time horizon, for which it was feasible to study the behavior of our subject pool, the Vickrey auction generated data much closer to truthful bidding than did the BDM.

Tables 2a-d illustrate the dynamics from period to period. Though subjects were not paid for their period one activity, they calculated their profits as if they were being paid and could use the hypothetical profit information to change their behavior in period 2. Tables 2a and 2b illustrate dynamics in the BDM. Between periods 1 and 2, the tendency for underbidding remains strong. 67 of the 75 who underbid in period one continued to do so in period 2. Of the seven subjects who did not underbid their valuations by at least two percent in period 1, four underbid by more than two percent in period 2. Table 2b shows that all nine subjects who bid within 2 percent of their values in period 2, continued to do so in period 3. 15 of 71 of the period 2 underbidders bid within 2% of their valuations in period 3. Overall 15% change their category in period 2 and 26% do so in period 3. A similar dynamic continues into the later periods, as is suggested from figures 1 and 2.

Tables 2c and 2d illustrate the corresponding data for the Vickrey auction. It shows a clear dynamic of subjects moving from bidding less than their values toward bidding equal to their values. 23.6% of those who underbid in period 1 bid an amount close to their valuations in period 2. Overall the percentage bidding within 2% of their values increases from 17% to 30% in period 2. 44% of those bidding less than their
valuations in period 2 proceed to bid near valuations in period 3, whereas 92% of those who were within 2% of valuations in period 2 remained so in period 3. 32% of bidders in period 2 and 36% in period 3 changed category, higher percentages than under the BDM. Once our subjects have begun to bid truthfully in the Vickrey auction, as in the BDM, they continue to do so. However, the convergence dynamic toward truthful revelation of values is faster in the Vickrey auction.

[Tables 2a-d: About Here]

4. DISCUSSION

The expected loss of deviating from truthful revelation for a typical bidder in the experiment is shown in figure 2, which indicates the payoff function for a bidder in the 50th percentile of valuations under a uniform distribution on the interval [0,1000]. The figure shows the expected payoff, divided by the maximum possible expected payoff, for all possible bids from an agent in a Vickrey auction facing five other demand revealing bidders, whose valuations are drawn from a uniform distribution. The figure also includes the expected payoff as a percentage of the payoff at the optimum of a player facing a uniform distribution of sale prices under the BDM. The payoff functions of bidders with different valuations involve fixed shifts along the range of valuations and thus have the same properties as those described below.

The figure suggests two relationships of note. The first is a marked difference between the consequences of deviating from the dominant strategy under the two mechanisms. In the Vickrey auction, bidding either under or over one’s valuation by a
given amount is always costlier than under the BDM in terms of foregone expected earnings as a percentage of expected earnings at the optimum. The second relationship is that in the Vickrey auction, if there are three or more bidders, a deviation of a given magnitude from the dominant strategy is costlier when it involves bidding higher than when it involves bidding lower than the optimum. This asymmetry does not exist for the BDM mechanism. The BDM is equivalent to bidding against one demand-revealing opponent, while the Vickrey auction can involve bidding against multiple players. Underbidding reduces the probability of winning in the Vickrey auction more rapidly than in the BDM as the bid decreases. Bidding more than one’s value raises the probability of winning more rapidly than the BDM as the bid increases. However, the likelihood that this overbidding translates into losses, because the second highest bidder bids an amount above one’s value but also below one’s bid, also increases rapidly.

The two relationships are general if the distribution of valuations is uniform. To see this, consider the BDM mechanism. The expected price conditional on bidder $j$, who has valuation $v_j$, purchasing a unit, if the distribution of sale prices is uniform on the interval $[0, v_{\text{max}}]$, equals $b_j/2$, where $b_j$ is the bid that $j$ submits. The expected payoff conditional on winning the auction then equals $v_j - b_j/2$. Since the overall probability that bidder $j$ wins the auction with a bid $b_j$ is given by $b_j/v_{\text{max}}$, the expected payoff from submitting bid $b_j$ is $\Pi_{\text{bdm}}(v_j,b_j) = [v_j - b_j/2] \cdot [b_j/v_{\text{max}}]$. It can be readily verified that the expression is maximized at $b_j = v_j$, where the expected payoff is

$$\Pi_{\text{bdm}}(v_j,v_j) = \frac{v_j^2}{2v_{\text{max}}}.$$  \hspace{1cm} (1)

On the other hand, in a Vickrey auction with $n > 2$ bidders, the expected purchase price for bidder $j$ given that she purchases a unit and is facing $n-1$ demand revealing
competitors is equal to $(n-1)b_j/n$ and the probability that bid $b_j$ is a winning bid equals $[b_j/v_{\text{max}}]^{n-1}$. Thus the expected payoff for bidder $j$ in competition with $n-1$ other bidders equals $\Pi_{vn}(v_j,b_j) = [b_j/v_{\text{max}}]^{n-1}*[v_j - (n-1)b_j/n]$. It also has a maximum at $b_j = v_j$ but with an expected payoff at the optimum of

$$\Pi_{vn}(v_j,v_j) = [v_j/v_{\text{max}}]^{n-1}*[v_j - (n-1)v_j/n]. \quad (2)$$

To show that any possible individual deviation from optimal behavior is more costly in percentage terms in the Vickrey auction than in the BDM mechanism, it is sufficient to show that for any $b_j \neq v_j$, $\Pi_{bdm}(v_j,b_j)/\Pi_{bdm}(v_j,v_j) > \Pi_{vn}(v_j,b_j)/\Pi_{vn}(v_j,v_j)$. It is clear from the above derivation of the objective functions that the payoff function for the BDM mechanism is identical to that of a Vickrey auction with two bidders. We now show that in any Vickrey auction, the expected percentage loss from deviation to any $b_j \neq v_j$ is increasing in $n$, which implies that the loss is greater in the Vickrey auction with three or more bidders than in the BDM. We show that, for any $b_j \neq v_j$,

$$\Pi_{vn}(v_j,b_j)/\Pi_{vn}(v_j,v_j) > \Pi_{vn+1}(v_j,b_j)/\Pi_{vn+1}(v_j,v_j). \quad (3)$$

Substitution of the expressions obtained earlier for $\Pi_{vn}(v_j,b_j)$, $\Pi_{vn}(v_j,v_j)$, $\Pi_{vn+1}(v_j,b_j)$, and $\Pi_{vn+1}(v_j,v_j)$, and cancellation of terms reduces the inequality to $v_j^2 + b_j^2 > 2v_jb_j/n$. The inequality is always satisfied since $n > 2$, and $v_jb_j < \max\{v_j^2, b_j^2\}$ for all $b_j \neq v_j$. It is always more costly to deviate by a given amount from the optimal strategy under the Vickrey auction than under the BDM mechanism.
While in the BDM the expected payoff function is symmetric about the optimum, under the Vickrey auction, the function is not symmetric. Bidding greater than $v_j$ by any amount $\varepsilon < \max\{v_j, v_{\text{max}} - v_j\}$ yields a lower expected payoff than bidding $v_j - \varepsilon$. In other words $\Pi_{vn}(v_j, v_j - \varepsilon) > \Pi_{vn}(v_j, v_j + \varepsilon)$. To see this property, consider the function $\Phi(v_j, \varepsilon, n) = \Pi_{vn}(v_j, v_j - \varepsilon) - \Pi_{vn}(v_j, v_j + \varepsilon)$. $\Phi(v_j, \varepsilon, n)$ equals

$$[(v_j - \varepsilon)/v_{\text{max}}]^{n-1}[v_j - (n-1)(v_j - \varepsilon)/n] - [(v_j + \varepsilon)/v_{\text{max}}]^{n-1}[v_j - (n-1)(v_j + \varepsilon)/n].$$  \hspace{1cm} (4)

The function $\Phi$ is continuous and equals zero when $\varepsilon = 0$. Therefore, to show that the expected payoff from bidding $v_j - \varepsilon$ is greater than from bidding $v_j + \varepsilon$ for all $\varepsilon > 0$, it is sufficient to show that the first derivative is greater than zero for all positive $\varepsilon$. Taking the first derivative and simplifying, we have:

$$\frac{d\Phi}{d\varepsilon} = n\varepsilon \left[ n-1 \left( \frac{v_j + \varepsilon}{v_{\text{max}}} \right)^{n-2} - \left( \frac{v_j - \varepsilon}{v_{\text{max}}} \right)^{n-2} \right] \hspace{1cm} (5)$$

Since $\varepsilon > 0$, $n > 2$, $v_{\text{max}} > 0$, the overall expression is positive. The difference in payoff between bidding $\varepsilon$ lower than one’s valuation and $\varepsilon$ greater is increasing in $\varepsilon$, and therefore always positive. The difference is larger the greater the value of $\varepsilon$.

Given that there is an initial bias under both mechanisms, the Vickrey auction contains more powerful feedback leading agents to adjust their bids in the direction of the optimum than the BDM does. It is more costly to deviate from bidding one’s valuation, and there is more to be gained from improvement of a given suboptimal strategy, in the Vickrey than in the BDM. We believe that the shape of the payoff function is a major factor accounting for the lower observed absolute deviation of bids from values in the
Vickrey auction after the first period, and the greater speed of convergence of the Vickrey auction toward the dominant strategy. The bias is smaller in the Vickrey auction because a bias is more costly: an agent’s earnings fall more quickly as his bid decreases in the Vickrey than in the BDM. However, the existence of more effective feedback under the Vickrey auction does not explain the initial downward bias in both auctions.

Our results invite comparisons to earlier work. Irwin et al. (1998), who concluded that the BDM was effective in revealing willingness-to-pay, also observed that the shape of the payoff function had no effect when the BDM decision task was transparent to subjects, but promoted learning when the optimal strategy was not transparent. We believe that for the subject pool used in our study, and for many other non-student subject pools, the optimal strategy is not transparent for either the BDM or the Vickrey auction. The conclusion of Irwin et al., that the shape of the payoff function plays a major role in the changes in subject behavior over time when the optimum is not deduced from logic, is consistent with what we have observed in this study.

We are also in agreement with earlier studies that have found that the Vickrey auction is biased, and that subjects do not easily discover the dominant strategy. However, we believe that the direction of the bias may be sensitive to certain aspects of the experimental design: the distribution of valuations, the information given to subjects about the distribution, the maximum and minimum possible bids, the subject pool
employed, the wording of instructions, and other factors.\(^8\) Our procedures in this study differ drastically from previous studies (e.g. Kagel and Levin, 1993) where bidding greater than valuations was widely observed. However, unlike those studies, we have not attempted to design our experiment to provide an unbiased test of a theoretical model. Our protocol choices were guided by our desire to find a simple willingness-to-pay revelation system with low bias and dispersion. We believe that we have successfully developed a training procedure that leads to a small bias and reduces dispersion after relatively few periods, and we believe it to be effective regardless of the direction of the initial bias.

5. CONCLUSION

Our research indicates that, given the procedures we have used in our study, the Vickrey auction is preferable to the BDM mechanism as an instrument for the elicitation of the willingness-to-pay for private goods. We observe that the BDM is subject to more severe biases, greater dispersion of bids, and slower convergence to truthful revelation than the Vickrey auction. With our techniques, neither auction could be made into a perfect tool to reveal valuations with our subjects, at least not during the time horizons

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\(^8\) One possible explanation for bids above valuations is that placing subjects in a role of buyers suggests that one of the objectives of the experiment was to make purchases even when it might be not in their economic self-interest to do so. Submitting bids higher than valuations increases the probability of making a purchase even though it lowers the expected payoff of the bidder.

One of our referees has suggested an interesting possible explanation for the low incidence of bidding greater than valuations in our Vickrey auction data relative to other studies using student subjects. Bidding above valuations in previous studies of the Vickrey auction may be due to concern about relative payoffs on the part of bidders. Players may bid more than their values to prevent others from winning the auction and earning a positive profit. However, because the level of anonymity between the subjects in our study, who are recruited from a diverse city of 150,000 people rather than from courses at a university, is much greater than in the typical experiment conducted with student subjects, relative payoffs are less of a concern. The small number of bids greater than valuations that we observed here may be due to such a phenomenon.
that were available to us. However, the Vickrey auction performs better than the BDM by the three criteria we have set for it. Our experimental protocol was effective in debiasing the Vickrey auction over several periods, but less effective on the BDM. Of course, it remains unknown whether unbiased bidding for goods with induced values carries over to subsequent bidding for goods with homegrown values.  

Our research affirms that the Vickrey auction can be an effective tool for demand revelation with non-student subject pools, but also cautions that sufficient practice and appropriate training in the rules of the auction is important. Based on observations we have made during the course of our research, we propose some training techniques to use with the Vickrey auction to enhance its demand revelation capability. Revealing publicly subjects’ bids, corresponding valuations, and earnings seems to help participants emulate successful strategies. Open discussion between subjects, where they themselves volunteer analyses of the properties of the mechanism, appears to facilitate learning. The use of our particular discussion questions focuses subjects on those aspects of the auction, which are essential for the comprehension of their incentives. The sale of goods with induced values that differ by subject for training purposes creates a clear distinction between learning the rules of the process and updating preferences. This difference can become blurred if subjects are trained through the sale of products with homegrown values, and convergence to common values may occur. It also appears to us that explaining and suggesting the optimal strategy to subjects directly is less effective than a technique that allows them to learn on their own. Subjects do not learn the process by reading the rules,
but rather by active participation. Giving subjects their cash immediately after every transaction is valuable in establishing the credibility of both gains and losses in the experiment. The fact that we do not use computers seems to make the decision situation less abstract.

We close with a conjecture on how learning to use the dominant strategy in the Vickrey auction might be accelerated by the use of Japanese auctions (such as the English clock auction studied by McCabe et al., 1990; and Kagel and Levin, 2001). In a Japanese auction, the experimenter increases the price of the product in small increments, and subjects indicate whether or not they wish to purchase a unit at that price. Exit is irrevocable in the sense that once an individual indicates a quantity demanded of zero, he may not re-enter the auction. When quantity demanded equals quantity supplied, the auction stops and the remaining demanders each receive a unit and pay the current price.

Under our proposed system, training would proceed in three periods. In the first period, subjects would participate in a Japanese auction and afterward engage in a discussion along the lines of our protocol. In this first period, the auction would be “open” in that agents would observe the dropout prices of other bidders. In the second period, they would participate in another Japanese auction, in which they do not observe the dropout prices of other bidders until after the end of the auction. They would then engage in a discussion as in our experiment. In the third period, they would be asked to simply state their dropout prices without playing out the auction. The experimenter would determine the winner on the basis of the dropout prices. This third auction is formally equivalent to and as rapid to implement as a Vickrey auction.
REFERENCES


### Table 1 – Deviations of Bids from Valuation (All bids)

<table>
<thead>
<tr>
<th></th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Last Period</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BDM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average bias</td>
<td>-39.87%</td>
<td>-28.06%</td>
<td>-12.76%</td>
<td>-8.19%</td>
<td>-6.33%</td>
</tr>
<tr>
<td>( B_{BDM} = \sum_j [b_{jt} - v_{jt}] / v_{jt} n_t )</td>
<td>(28.89%)</td>
<td>(22.32%)</td>
<td>(23.38%)</td>
<td>(23.60%)</td>
<td>(20.95%)</td>
</tr>
<tr>
<td>Average dispersion</td>
<td>41.65%</td>
<td>28.59%</td>
<td>16.86%</td>
<td>13.94%</td>
<td>11.75%</td>
</tr>
<tr>
<td>( D_{BDM} = \sum_j</td>
<td>[b_{jt} - v_{jt}]</td>
<td>/ v_{jt} n_t )</td>
<td>(26.24%)</td>
<td>(21.62%)</td>
<td>(20.58%)</td>
</tr>
<tr>
<td><strong>Vickrey</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average bias</td>
<td>-30.16%</td>
<td>-11.50%</td>
<td>-5.57%</td>
<td>+1.33%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>( B_V = \sum_j [b_{jt} - v_{jt}] / v_{jt} n_t )</td>
<td>(32.53%)</td>
<td>(27.76%)</td>
<td>(11.94%)</td>
<td>(26.49%)</td>
<td>(11.13%)</td>
</tr>
<tr>
<td>Average dispersion</td>
<td>32.57%</td>
<td>16.79%</td>
<td>6.25%</td>
<td>9.27%</td>
<td>3.89%</td>
</tr>
<tr>
<td>( D_V = \sum_j</td>
<td>[b_{jt} - v_{jt}]</td>
<td>/ v_{jt} n_t )</td>
<td>(30.10%)</td>
<td>(24.89%)</td>
<td>(11.59%)</td>
</tr>
<tr>
<td><strong>Significance Tests</strong></td>
<td>(t-stats from pooled variance t-tests)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ho: ( B_{BDM} = B_V )</td>
<td>2.17</td>
<td>4.48</td>
<td>2.84</td>
<td>2.71</td>
<td>2.78</td>
</tr>
<tr>
<td>Ho: ( B_{BDM} = 0 )</td>
<td>-12.50</td>
<td>-11.38</td>
<td>-4.94</td>
<td>-3.14</td>
<td>-2.74</td>
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<tr>
<td>Ho: ( B_V = 0 )</td>
<td>-10.03</td>
<td>-4.48</td>
<td>-5.05</td>
<td>.57</td>
<td>-.06</td>
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<tr>
<td>Ho: ( D_{BDM} = D_V )</td>
<td>2.20</td>
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<td>4.63</td>
<td>1.39</td>
<td>3.82</td>
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<td>Ho: ( D_{BDM} = 0 )</td>
<td>14.37</td>
<td>11.97</td>
<td>7.42</td>
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<td>5.77</td>
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<tr>
<td>Ho: ( D_V = 0 )</td>
<td>11.70</td>
<td>7.30</td>
<td>5.83</td>
<td>4.01</td>
<td>4.04</td>
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</table>
### Table 2a: Bid Strategies in Period 2 as a Function of Bid in Period 1: BDM Mechanism

<table>
<thead>
<tr>
<th>Period 1</th>
<th>Period 2</th>
<th>B &lt; .98v</th>
<th>.98v ≤ b &lt; 1.02v</th>
<th>b ≥ 1.02v</th>
<th>Total period 1 bids</th>
</tr>
</thead>
<tbody>
<tr>
<td>b &lt; .98v</td>
<td></td>
<td>67</td>
<td>6</td>
<td>2</td>
<td>75</td>
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<td>.98v ≤ b &lt; 1.02v</td>
<td></td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>b ≥ 1.02v</td>
<td></td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Total period 2 bids</td>
<td></td>
<td>71</td>
<td>9</td>
<td>2</td>
<td>82</td>
</tr>
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</table>

### Table 2b: Bid Strategies in Period 3 as a Function of Bid in Period 2: BDM Mechanism

<table>
<thead>
<tr>
<th>Period 2</th>
<th>Period 3</th>
<th>b &lt; .98v</th>
<th>.98v ≤ b &lt; 1.02v</th>
<th>b ≥ 1.02v</th>
<th>Total period 2 bids</th>
</tr>
</thead>
<tbody>
<tr>
<td>b &lt; .98v</td>
<td></td>
<td>50</td>
<td>15</td>
<td>6</td>
<td>71</td>
</tr>
<tr>
<td>.98v ≤ b &lt; 1.02v</td>
<td></td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>b ≥ 1.02v</td>
<td></td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total period 3 bids</td>
<td></td>
<td>50</td>
<td>24</td>
<td>8</td>
<td>82</td>
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### Table 2c: Bid Strategies in Period 2 as a Function of Bid in Period 1: Vickrey Mechanism

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<th>Period 1</th>
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<th>.98v ≤ b &lt; 1.02v</th>
<th>b ≥ 1.02v</th>
<th>Total period 1 bids</th>
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<td>b &lt; .98v</td>
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<td>64</td>
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<td>93</td>
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<tr>
<td>.98v ≤ b &lt; 1.02v</td>
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<td>6</td>
<td>14</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>b ≥ 1.02v</td>
<td></td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Total period 2 bids</td>
<td></td>
<td>72</td>
<td>36</td>
<td>9</td>
<td>117</td>
</tr>
</tbody>
</table>

### Table 2d: Bid Strategies in Period 3 as a Function of Bid in Period 2: Vickrey Mechanism

<table>
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<tr>
<th>Period 2</th>
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<th>b &lt; .98v</th>
<th>.98v ≤ b &lt; 1.02v</th>
<th>b ≥ 1.02v</th>
<th>Total period 2 bids</th>
</tr>
</thead>
<tbody>
<tr>
<td>b &lt; .98v</td>
<td></td>
<td>39</td>
<td>32</td>
<td>1</td>
<td>72</td>
</tr>
<tr>
<td>.98v ≤ b &lt; 1.02v</td>
<td></td>
<td>2</td>
<td>33</td>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>b ≥ 1.02v</td>
<td></td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Total period 3 bids</td>
<td></td>
<td>44</td>
<td>68</td>
<td>5</td>
<td>117</td>
</tr>
</tbody>
</table>
Figure 1a: Vickrey Period 1 (without payment)

Figure 1b: BDM Period 1 (without payment)

Figure 1c: Vickrey Period 2 (with payment)

Figure 1d: BDM Period 2 (with payment)
Figure 1e: Vickrey Period 3

Figure 1f: BDM Period 3

Figure 1g: Vickrey Period 4

Figure 1h: BDM Period 4
Figure 1i: Vickrey Last Period

Figure 1j: BDM Last Period
Figure 2: Payoff Function in BDM and Vickrey as Percentage of Maximum Profit: Valuation at 50th Percentile