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Individual discounting and the purchase of durables with random lifetimes

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Abstract

A number of papers have suggested that the returns consumers require on energy efficiency investments are much higher than the capital market rate of return. The earlier literature has typically assumed that the lifetime of the energy-using durables is fixed. I show that if risk-neutral consumers anticipate that the lifetime is random, ignoring the randomness results in an upward bias of estimated discount rates. The bias may be as large as 35%.

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1. Introduction

A number of papers have suggested that the returns consumers require on energy efficiency investments are much higher than the capital market rate of return. A widely cited study is the article by Hausman (1979) on the purchase and utilization of air-conditioners. Hausman estimated an average annual discount rate of 26.4%, considerably higher than the capital market rate of return. In a recent paper, Loewenstein and Thaler (1989) discussed two explanations that might be offered: information barriers, in particular with respect to the operating costs of durables, and liquidity constraints.

The present paper supplements these explanations. When comparing the discounted purchase and utilization costs of low-efficiency and high-efficiency versions of durables, the existing literature has typically assumed that the lifetime of the durables is fixed; examples are Hausman (1979), Gately (1980), Dubin and McFadden (1984), and Ruderman et al. (1986). In this paper I show that if risk-neutral consumers anticipate that the lifetime is random, the assumption of a deterministic lifetime results in an upward bias of estimated discount rates. The bias may be as large as 35%.

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2. Framework and main proposition

Consider a low-efficiency version ($i = \mathrm{L}$) and a high-efficiency version ($i = \mathrm{H}$) of a consumer durable good. Let τ_i be the operating costs per time period and let R_i be the purchase price of type i, $i = \mathrm{L,H}$. I assume $\tau_\mathrm{L} > \tau_\mathrm{H}$ and $R_\mathrm{L} < R_\mathrm{H}$. L and H generate identical services and are identical in all other respects. Consumers are risk-neutral, i.e. the choice between L and H is made on the basis of expected cost minimization, at a given service demand. The per-period subjective discount rate is denoted by r.

The lifetime t of both types is a random variable with distribution function $f(\cdot)$. As in the literature referred to above, time is discrete. In case of failure the durable is replaced by an identical one. Operating costs are paid at the end of each period. Let $0 < s_1 < s_2 < s_3 < \dots$ be the random points in time at which the durable fails and is replaced. Since s_1, $(s_2 - s_1)$, $(s_3 - s_2), \dots$ are independent drawings from $f(\cdot)$, the expected discounted purchase costs of type i over an infinite horizon can be written as

$$\begin{aligned}
&\mathrm{E}[R_i(1 + (1+r)^{-s_1} + (1+r)^{-s_2} + \cdots)] \\
&\quad = \mathrm{E}[R_i(1 + (1+r)^{-s_1} + (1+r)^{-s_1}\cdot(1+r)^{-(s_2-s_1)} + \cdots)] \\
&\quad = \mathrm{E}\Big[R_i \sum_{k=0}^{\infty} (1+r)^{-tk}\Big] = R_i \sum_{k=0}^{\infty} [\mathrm{E}(1+r)^{-t}]^k = \frac{R_i}{1-\mathrm{E}(1+r)^{-t}} . \qquad (1)
\end{aligned}$$

Consider the sum of expected discounted purchase and operating costs:

$$C_i(r) = \frac{R_i}{1-\mathrm{E}(1+r)^{-t}} + \frac{\tau_i}{r} . \qquad (2)$$

The *reservation value* of the discount rate is defined as the value of r that solves the equation $C_\mathrm{L}(r) = C_\mathrm{H}(r)$. A consumer will purchase a high-efficiency version if and only if his subjective discount rate does not exceed the reservation discount rate.

Using (2), $C_\mathrm{L}(r) = C_\mathrm{H}(r)$ can be rewritten as

$$h(r) \equiv \frac{1-\mathrm{E}(1+r)^{-t}}{r} = \frac{R_\mathrm{H} - R_\mathrm{L}}{\tau_\mathrm{L} - \tau_\mathrm{H}} \equiv B . \qquad (3)$$

B is the payback period, i.e. the period of time before the additional investment for type H is recovered from its lower operating costs.

Lemma. Let $0 < B < \mathrm{E}t$. *Then for any lifetime distribution* $f(\cdot)$ *there exists a unique positive value of r that solves Eq.* (3).

Proof. Consider the function

$$g(r) = \mathrm{E}(1+r)^{-t} = \sum_{k=1}^{\infty} P(t=k)\cdot(1+r)^{-k} , \qquad (4)$$

with

$$g'(r) = -\sum_{k=1}^{\infty} P(t = k) \cdot k(1 + r)^{-k-1} . \tag{5}$$

Since $\lim_{r \to 0} g'(r) = -\mathrm{E}t$, it follows from l'Hôpital's rule that $\lim_{r \to 0} h(r) = \mathrm{E}t$. Thus a consumer with $r = 0$ will purchase type H if and only if the expected lifetime exceeds the payback period. We therefore assume $0 < B < \mathrm{E}t$.

The existence and uniqueness of a positive solution to Eq. (3) follows from $\lim_{r \to 0} h(r) = \mathrm{E}t > B$, $\lim_{r \to \infty} h(r) = 0 < B$, and $h'(r) < 0$ for all $r > 0$. To prove that $h'(r) < 0$, note that $h'(r) = \{-rg'(r) - 1 + g(r)\}/r^2$. Thus, it suffices to show that $\phi(r) \equiv -rg'(r) - 1 + g(r) < 0$. Now

$$\phi(r) = \sum_{k=1}^{\infty} P(t = k) \cdot \psi(r,k) , \tag{6}$$

with

$$\begin{aligned} \psi(r,k) &= rk(1 + r)^{-k-1} - 1 + (1 + r)^{-k} \\ &= (1 + r)^{-k-1}[rk - (1 + r)^{k+1} + (1 + r)] \\ &= (1 + r)^{-k-1}[1 + (k + 1)r - (1 + r)^{k+1}] \\ &= (1 + r)^{-k-1}\left[1 + (k + 1)r - \sum_{j=0}^{k+1} \binom{k+1}{j} r^j\right] \\ &= (1 + r)^{-k-1}\left[-\sum_{j=2}^{k+1} \binom{k+1}{j} r^j\right] . \end{aligned} \tag{7}$$

Thus $\psi(r,k) < 0$ for all $k = 1,2,\ldots$ and $r > 0$. As a consequence, $\phi(r) < 0$ and $h'(r) < 0$. □

The main result can now be formulated as follows.

Proposition. Let r_{F} denote the reservation discount rate when lifetimes are fixed and equal to T, i.e. r_{F} solves $h_{\mathrm{F}}(r) = \{1 - (1 + r)^{-T}\}/r = B$. Let r_{S} denote the reservation discount rate when lifetime t follows a distribution function f(·) with $\mathrm{E}t = T$, *i.e. r_{S} solves $h_{\mathrm{S}}(r) = \{1 - \mathrm{E}(1 + r)^{-t}\}/r = B$. Then $r_{\mathrm{F}} \geq r_{\mathrm{S}}$.*

Proof. Using Jensen's inequality we have $0 < (1 + r)^{-\mathrm{E}t} \leq \mathrm{E}(1 + r)^{-t} < 1$. As a consequence, $h_{\mathrm{F}}(r) \geq h_{\mathrm{S}}(r)$, for all $r > 0$. Since $h_{\mathrm{F}}(r)$ and $h_{\mathrm{S}}(r)$ are monotonically decreasing (see proof of the lemma), it follows that $r_{\mathrm{F}} \geq r_{\mathrm{S}}$. □

Note that no restrictions on $f(\cdot)$ are imposed other than $\mathrm{E}t = T$.

3. Implications

Consider a population of consumers who are homogeneous except with respect to their subjective discount rates. Suppose that $q\%$ are observed to purchase the low-efficiency version. Assuming deterministic lifetimes, one would conclude that $q\%$ of the consumers have a subjective discount rate exceeding r_F. Assuming random lifetimes, one would conclude that $q\%$ of the consumers have a subjective discount rate only exceeding r_S ($\leq r_F$). Thus the assumption of a deterministic lifetime would result in an upward bias of the estimated average discount rate. The intuitive explanation is that the possible benefit of a late failure does not offset the possible loss incurred at an early failure because 'late' is discounted more heavily than 'early'.

Hausman's (1979) widely cited estimate of 26.4% was determined by solving r with a fixed lifetime of 9.94 years).[1] Assuming a hazard rate that increases linearly such that $\mathrm{E}t = 9.94$, Hausman's parameter estimates imply an annual discount rate of 23.5%. Assuming that the hazard rate is constant over time, so that t follows the geometric distribution, with $\mathrm{E}t = 9.94$, the implied discount rate is 19.2%, much closer to the interest rate for personal loans at the end of the 1970s.[2]

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[1] 9.94 was the estimated mean lifetime of air-conditioners. Thus the calculation of the discount rate was based on discounted costs evaluated at the expected lifetime rather than on expected discounted costs.

[2] In case of a constant hazard, $\mathrm{E}(1+r)^{-t} = (1+Tr)^{-1}$, which allows r_S to be solved from (3) analytically as $r_S = B^{-1} - T^{-1}$. In the case of a linearly increasing hazard rate, the rate of increase is chosen such that $\mathrm{E}t = T$. Next r_S is solved from (3) numerically, with $\mathrm{E}(1+r)^{-t}$ being evaluated numerically at each trial value of r_S.