Neoclassical Growth Accounting and Frontier Analysis: A Synthesis

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Abstract

The standard measure of productivity growth is the Solow residual. Its evaluation requires data on factor input shares or prices. Since these prices are presumed to match factor productivities, the standard procedure amounts to accepting at face value what is supposed to be measured. In this paper we determine total factor productivity growth without recourse to data on factor input prices. Factor productivities are defined as Lagrange multipliers to the program that maximizes the level of domestic final demand. The consequent measure of total factor productivity is shown to encompass not only the Solow residual, but also the efficiency change of frontier analysis and the hitherto slippery terms-of-trade effect. Using input-output tables from 1962 to 1991 we show that the source of Canadian productivity growth has shifted from technical change to terms-of-trade effects.

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1 Introduction

In this paper we synthesize two strands of productivity analysis, namely neoclassical growth accounting and a frontier approach, known as data envelopment analysis (DEA).[1] In either branch of literature productivity is essentially the output-input ratio and, therefore, productivity growth the residual between output growth and input growth. The difficulty is to implement these concepts when there is more than one output or input. Neoclassical analysis weights them by value shares, a procedure that has been justified for competitive economies by Solow (1957). In such economies inputs are rewarded according to their marginal productivities and outputs according to their marginal revenues. Therefore, the residual measures the shift of the production function. DEA analysis considers the output and input proportions observable in activities (representing various economies and/or various years) and determines for instance how much more output could be produced if the inputs were processed by an optimal combination of all observed activities.

Although each approach tracks changes in the output-input ratio of an economy, the constructions are quite distinct. Neoclassical growth accounting attributes productivity growth to the inputs, say labor and capital. Indeed, Jorgenson and Griliches (1967) have shown that the total factor productivity (TFP-) growth residual equals the growth of the real factor rewards, summed over endowments. In this sense the residual truly represents total factor productivity growth indeed. The frontier approach decomposes productivity growth in a movement of the economy towards the frontier and a shift of the latter. Productivity growth is efficiency change plus technical change. The alternative decompositions inherit the advantages and disadvantages of their respective methodologies. Neoclassical growth accounting imputes productivity growth to factors, but cannot distinguish a movement towards the frontier and a movement of the frontier. This is the contribution of the frontier approach, which, however, is not capable of imputing value to factor inputs.

Practitioners of both approaches are aware of each others' work and sometimes report correlations between the alternative productivity measures (Perelman, 1995). What is missing, however, is a theoretical framework that encompasses the two approaches. This is the purpose of our paper. We reproduce the neoclassical TFP growth formulas, but

[1] In the frontier literature, there is a distinction between deterministic and stochastic frontiers. The former are obtained by linear programming methods such as DEA, the latter are estimated by econometric methods. See Coelli et al. (1998).

in a framework that is DEA in spirit. Unlike Färe, Grosskopf, Lovell and Zhang (1994), we do not determine an economy's frontier by benchmarking on other economies, but by reallocating resources domestically so as to maximize the level of domestic final demand (that is excluding net exports) given input and output proportions and subject to a set of feasibility constraints. The shadow prices of the factor constraints measure the individual factor productivities. Our model is a general equilibrium activity analysis model with multiple inputs and outputs, and intermediate inputs, where the frontier is determined by an economic criterion and not a mechanical expansion factor. A first attempt to model intermediate inputs was made by Färe and Grosskopf (1996).

Our starting point is the Solow residual between output and input growth. We derive the equality of the Solow residual with the growth rates of the factor productivities by differentiating not the national income identity, but the related formula of the main theorem of linear programming applied to the aforementioned frontier program. By the same token, we capture the movement towards the frontier, or efficiency change, in addition to the standard Domar decomposition of TFP, involving technical change at the sectoral levels. A further contribution is the identification of the terms-of-trade effect in productivity analysis. It is well-known that an improvement in the terms of trade is equivalent to technical progress, but the treatment of this effect has been ad hoc so far. For example, Diewert and Morrison (1986) classify commodities a priori as exports or imports. In our frontier program net exports are appropriately endogenous. We model the trade deficit as a factor input, similar to capital or labor.

Although we reproduce the formulas of neoclassical growth accounting and frontier analysis in a consistent way, there are some subtle differences. Compared to traditional growth accounting, the input value shares that enter our Solow residual are no longer the observed ones, but those based on the shadow prices of the frontier program. In perfectly competitive economies, where inputs are rewarded according to their marginal productivities and outputs according to their marginal revenues, it is perfectly legitimate to use the observed value shares in aggregating inputs or outputs. But, observed economies are not perfectly competitive and are not even on their production possibility frontiers. Hence the Solow residual based on observed value shares does not isolate technical change, but also captures variations of the economy about the competitive benchmark, such as changes in market power, returns to scale or disequilibrium in factor holdings. One approach followed in the literature is to correct the Solow residual for such departures from perfect competition, estimating mark-ups over marginal cost, scale elasticities and shadow prices, and modifying the formula for the residual (Morisson, 1988, and Hall, 1990). Our approach, however, is to endogenize commodity and factor prices by finding the frontier of the economy subject to its fundamentals, namely endowments, technology, and preferences. Endowments are represented by the labor force, the accumulated stocks of capital and the trade deficit. Technology is given by the combined inputs and outputs of the various sectors of the economy. Preferences are represented by the commodity proportions of domestic final demand.[2]

Compared to frontier analysis, we use a non-parametric linear programming based technique a la DEA, but our efficiency change is not based on cross-sectional or intertemporal benchmarking, but on sectoral efficiency-improving reallocations of factors of production within a multi-sectoral economy model. Few frontier analysts have modelled efficiency changes in an open DEA economy with interindustry trade.

The paper is organized as follows. In the next section, we set up an activity analysis model to determine an economy's frontier. Then we define TFP-growth in the spirit of Solow (1957) and decompose it into frontier productivity growth and efficiency change. In section 3, frontier productivity growth is further decomposed into technical change and a terms-of-trade effect. In section 4 we measure TFP-growth and its various components for the Canadian economy over the period 1962-1991. Section 5 concludes and the data are described in an appendix.

2 TFP-growth as the sum of frontier productivity growth and efficiency change

We consider an open economy endowed with labor N, capital M (decomposed into various types) and an allowable trade deficit D. The economy is subdivided into a number of sectors, each producing a vector of commodities. Part of the commodities are used as intermediate inputs and the rest flows to final demand (either domestic final demand or exports). The frontier of the economy is defined as the maximal expansion of its vector of final demand f, keeping the relative composition of that vector as fixed.

[2] This is in the spirit of Mohnen, ten Raa and Bourque (1997) who, however, stay in the realm of neoclassical growth accounting and, therefore, do not capture efficiency changes nor the terms-of-trade effect.

The composition reflects preferences. The frontier of the economy can be reached by an optimal allocation of inputs (primary and intermediate) and production across sectors and by an optimal trade of commodity with the rest of the world.

The **frontier** of the economy is defined by the primal program,

$$\max_{s,c,g} e^{\intercal} fc \text{ subject to}$$

$$\begin{aligned} (V^{\intercal} - U)s &\geq fc + Jg =: F \\ Ks &\leq M \\ Ls &\leq N \\ -\pi g &\leq -\pi g^t =: D \\ s &\geq 0\,. \end{aligned} \tag{1}$$

We discuss the objective and the constraints, respectively, and then list all the variables and parameters. The objective is the expansion of the level of domestic find demand, c. Domestic final demand comprises consumption and investment. Investment is merely a means to advance consumption, albeit in the future. We include it in the objective function to account for future consumption. In fact, Weitzman (1976) shows that for competitive economies domestic final demand measures the present discounted value of consumption.[3]

Preserving the proportions of domestic final demand, f, we expand its level by letting the economy produce fc, where scalar c is the expansion factor. $c = 1$ is feasible (as it reflects the status quo), but $c > 1$ represents a movement towards the frontier of the economy. The model maximizes c. This is equivalent to the maximization $e^{\intercal} fc$, where e is the unit vector (with all entries equal to one), $\intercal$ the transposition sign, and f is the given domestic final demand vector. It is important to understand that f is not a variable but an exogenous vector. The positive multiplicative factor in the objective, $e^{\intercal} f$, will control the nominal price level.

The preservation of domestic final demand in finding the frontier of the economy in each year amounts to imposing a Leontief preference structure (fixed consumption pattern).It should be noted that the ray output expansion typical in multi-output DEA (the Farrell

[3]In principle, our methodology could accommodate endogenous investment and the determination of the intertemporal production possibility frontier as in Hulten (1979), but we have not pursued this approach.

(1957) measure of efficiency) implicitly assumes fixed output proportions. It should also be noted that the Leontief specification of production and preferences that we adopt admits a great deal of substitutability as trade is free and, therefore, acts as a valve for factor imbalances between endowments and factor contents of domestic final demands. The economy may even mimic a Cobb-Douglas behavior, as demonstrated in ten Raa (1995).

The first constraint of the linear program (1) is the material balance: net output must cover domestic final demand plus net exports. Then follow the capital and labor constraints.[4] The next to last constraint is the trade balance: net imports valued at world prices may not exceed the existing trade deficit. Finally, sector activity levels must be nonnegative.

The variables (s, c, g) and parameters (all other) are the following [with dimensions in brackets]

s	activity vector [# of sectors]
c	level of domestic final demand [scalar]
g	vector of net exports [# of tradeable commodities]
e	unit vector of all components one
T	transposition symbol
f	domestic final demand [# of commodities]
V	make table [# of sectors by # of commodities]
U	use table [# of commodities by # of sectors]
J	0-1 matrix placing tradeables [# of commodities by of tradeables]
F	final demand [# of commodities]
K	capital stock matrix [# of capital types by # of sectors]
M	capital endowment [# of capital types]
L	labor employment row vector [# of sectors]
N	labor force [scalar][5]
π	U.S. relative price row vector [# of tradeables]
g^t	vector of net exports observed at time t [# of tradeables]
D	observed trade deficit [scalar].

[4] Actually, there is also non-business capital and labor, proportional to the activity level of the non-business sector, that is c. We have included their levels in the capital and labor constraints and their factor rewards in the coefficient of c in the objective function. For reasons of clarity in the exposition, we have not indicated these additional terms related to the non-business sector in (1).

[5] Labor could also be decomposed into various types, but we have not done so in the empirical part.

The linear program basically reallocates activity so as to maximize the level of domestic final demand. Final demand also includes net exports, but they are considered not an end, but a means to fulfill the objective of the economy.[6] This endogenization of trade explains the role of the terms-of-trade in TFP analysis, as we shall see in section 3.

The theory of mathematical programming teaches us that the Lagrange multipliers corresponding to the constraints of the primal program measure the competitive values or marginal products of the constraining entities (the commodities and factors) at the optimum. We will use the Lagrange multipliers in defining productivity growth. They are p (a row vector of commodity prices), r (a row vector of capital productivities), w (a scalar for labor productivity), and ε (a scalar for the purchasing power parity). They are determined by the dual program associated with (1),

$$\min_{p,r,w,\varepsilon\geq 0} rM + wN + \varepsilon D \text{ subject to}$$

$$\begin{aligned} p(V^\mathsf{T} - U) &\leq rK + wL \\ pf &= e^\mathsf{T} f \\ pJ &= \varepsilon\pi \ . \end{aligned} \tag{2}$$

Factor costs are minimized subject to price constraints.[7] The first dual constraint defines competitive shadow prices of the commodities. Value added must be less than or equal to factor costs in each sector. (If it is less than factor costs, the sector will be inactive according to the phenomenon of complementary slackness.) The second dual constraint normalizes the prices.[8] Our commodities are measured in base-year prices and hence observed prices are one. The optimal competitive prices from the linear program will be slightly off, but we maintain the overall price level. The third and last dual constraint aligns the prices of the tradeable commodities with their opportunity costs: the relative U.S. prices. In free trade, the law of one price must hold (in the absence of transaction costs and imperfect information).

Following Solow (1957) we define total factor productivity growth as the residual between the final demand growth and aggregate-input growth, where each of them is a weighted

[6] We make no distinction between competitive and non-competitive imports. (Non-competitive imports are indicated by zeros in the make table.)

[7] Since the commodity constraint in the primal program has zero bound, p does not show up in the objective function of the dual program. For details of the derivation see, for example, ten Raa (1995).

[8] Remember from footnote 4 that there are two more constraint terms featuring c, namely non-business capital and labor. To preserve the price normalization, we have also included in the objective function of the primal program, (1), the base-year expenditure on non-business capital and labor which also shows up on the right-hand side of the second dual constraint.

average of component growth rates and the weights are competitive value shares. The growth rate of domestic final demand for commodity i is $\hat{f}_i = \dot{f}_i/f_i$, where $\dot{}$ denotes the time derivative. The competitive value shares are $p_i f_i/(\Sigma p_i f_i) = p_i f_i/(pf)$. Hence overall output growth amounts to

$$\Sigma[p_i f_i/(pf)]\hat{f}_i = \Sigma p_i \dot{f}_i/(pf) = p\dot{f}/(pf). \tag{3}$$

Inputs are aggregated in the same vein. The growth rate of labor is $\hat{N}$ and its competitive value share is $\beta = wN/(rM + wN + \varepsilon D)$. Likewise, denote the competitive value shares of capital by α (a row vector) and of the trade deficit by γ (a scalar). Then overall input growth can be written as

$$\alpha\hat{M} + \beta\hat{N} + \gamma\hat{D} \tag{4}$$

where each of the terms can be rewritten in time derivatives as we have done for outputs. For example, $\beta\hat{N} = [wN/(rM + wN + \varepsilon D)]\hat{N} = w\dot{N}/(rM + wN + \varepsilon D)$. **Total factor productivity growth** is the residual between (1) and (2):

$$\begin{aligned} TFP &= p\dot{f}/(pf) - (\alpha\hat{M} + \beta\hat{N} + \gamma\hat{D}) \\ &= p\dot{f}/(pf) - (r\dot{M} + w\dot{N} + \varepsilon\dot{D})/(rM + wN + \varepsilon D)\ . \end{aligned} \tag{5}$$

In the growth accounting literature it is customary to assume that the economy is perfectly competitive. Unfortunately this assumption is seldom fulfilled. Observed prices are not perfectly competitive and the economy need not be on its frontier. Also notice that p is not a device to convert nominal values to real values, but the endogenous price vector that sustains the optimal allocation of resources in the linear program.

In the spirit of Nishimizu and Page (1982) and further frontier analysis, such as DEA, for example Färe et al. (1994), we will decompose TFP in a shift of the frontier and a movement towards the frontier:

$$TFP = FP + EC \tag{6}$$

where FP if frontier productivity growth and EC is efficiency change. We will now define each of them.

By the theory of Lagrange multipliers, real shadow prices measure the marginal products of the factors at the optimum. Hence **frontier productivity growth** is the growth rates of the shadow prices of the factors (weighted by relative factor costs) minus the growth rate of the commodity prices.[9]

$$FP = (\dot{r}M + \dot{w}N + \dot{\varepsilon}D)/(rM + wN + \varepsilon D) - \dot{p}f/(pf) \tag{7}$$

Frontier productivity growth so defined corresponds to the dual expression of TFP-growth for perfectly competitive economies elaborated by Jorgenson and Griliches (1967). It imputes productivity growth to factor inputs, which is beyond the scope of standard frontier analysis. The latter, however, is capable of accounting for efficiency change. Inefficiency is measured by the degree to which the economy can be expanded towards its frontier, c. A reduction in c signals an efficiency gain and, therefore, **efficiency change** is defined by

$$EC = -\hat{c}\ (= -\dot{c}/c) \tag{8}$$

We must now demonstrate that frontier productivity growth and efficiency change sum to total factor productivity growth. In other words, we must prove equation (6), given definitions (5), (7) and (8). Now, by the main theorem of linear programming, (1) and (2) have equal solution values, or, substituting the price normalization constraint of (2),

$$pfc = rM + wN + \varepsilon D\ . \tag{9}$$

This is the identity of national product and income, where national income consists of factor costs. Differentiating (9) with respect to time, applying the product rule, rearranging terms, and dividing through by (9) itself, we obtain (6). It is interesting to notice that c cancels out everywhere, except in the denominator under $\dot{c}$, which yields the efficiency change.

3 Frontier productivity growth decomposition into technical change and the terms-of-trade effect

In the previous section we decomposed TFP into frontier productivity growth and efficiency change. This section provides the further decomposition of frontier productivity

[9] Since prices are normalized at unity by the second dual constraint, see (2), the price correction term is a sheer compositional effect. If the composition of domestic final demand, f, is constant, then pf is also constant, by (2), and it follows that $p(f)^{\cdot} = \dot{p}f$ is zero. Otherwise the price correction term corrects marginal factor productivity growth rates for an inflationary effect, which does not reflect a change in the price level (since everything is already specified in real prices) but only a compositional effect.

growth into technical change and the terms-of-trade effect. The latter is not an add-on, but emerges naturally from our linear programming model of TFP. It should not come as a surprise that terms-of-trade and sectoral technical changes arise simultaneously. The trade sector is like a production sector, with multiple inputs (namely exports) and outputs (namely imports). The technology is different though. While production sectors feature no substitutability, the trade sector features perfect substitutability (with the marginal rate of substitution given by the terms of trade).

Our point of departure is TFP as defined in (5). Focus on the numerator of (5), by multiplying with the denominator, or (9). Then we obtain the following TFP numerator,

$$p\dot{f}c - r\dot{M} - w\dot{N} - \varepsilon\dot{D}. \tag{10}$$

By definition of F, see (1), the first term is $p\dot{F} - pJ\dot{g} - pf\dot{c}$. If the factor constraints are binding, then $M = Ks$, $N = Ls$ and $\varepsilon\dot{D} = -\varepsilon\pi\dot{g} - \varepsilon\dot{\pi}g$. The second subterm of the first term, $-pJ\dot{g}$, cancels against the first subterm of the last term, $-\varepsilon\pi\dot{g}$, (because of the third constraint of (2)) and (10) reduces to

$$[p\dot{F} - r(Ks)^{\cdot} - w(Ls)^{\cdot}] + \varepsilon\dot{\pi}g - pf\dot{c}. \tag{11}$$

Dividing by the denominator of TFP, or (9), we reobtain TFP, but now in the following three-way form:[10]

$$TFP = SR + TT + EC \tag{12}$$

where SR is the **Solow residual**,

$$SR = [p\dot{F} - r(Ks)^{\cdot} - w(Ls)^{\cdot}]/(rM + wN + \varepsilon D) \tag{13}$$

TT is the **terms-of-trade effect**,

$$TT = \varepsilon\dot{\pi}g/(rM + wN + \varepsilon D) \tag{14}$$

and EC is the efficiency change, defined earlier in (8). Comparison of TFP decompositions (12) and (6) reveals that effectively we have decomposed the structural change

[10] If the factor constraints are not binding, a fourth term accounts for slack changes. Notice that all components, including the Solow residual, account for the value of trade balance in the numerator. This minor departure from the standard expression in the literature (including Mohnen, ten Raa and Bourque, 1997) is a consequence of our unifying framework that encompasses terms-of-trade effects.

term, frontier productivity growth FP, into the technical change and terms-of-trade effects,

$$FP = SR + TT. \tag{15}$$

In discrete time, the expressions involving differentials are approximated using the identity $x_t y_t - x_{t-1} y_{t-1} = \hat{x}\overline{x_t y_t} + \hat{y}\overline{x_t y_t}$, where discrete time $\hat{x}_t = (x_t - x_{t-1})/\bar{x}_t$ and $\bar{x}_t = (x_t + x_{t-1})/2$, and similarly for $\hat{y}_t$ and $\bar{y}_t$.

4 Application to the Canadian economy

To illustrate our methodology, we examine productivity growth in the Canadian economy during the period from 1962 to 1991 at the medium level of disaggregation, which comprises 50 industries and 94 commodities. The linear program was solved for each year from 1962 to 1991 yielding the optimal activity levels and shadow prices for the TFP-expressions.

Table 1 contains the shadow prices of labor (in 1986 $/hour), of the three types of capital (building, equipment and infrastructure), and of the trade deficit (the latter four are in 1986$/1986$, that is rates of return) from 1962 to 1991. Labor was worth at the margin $16.13 in 1986 prices in 1962. Its productivity followed an increasing trend until 1982 and then a bumpy road ending at $46.13 in 1991. The rate of return on buildings followed a downward trend, dropping to zero in 1982, sharply rebounded in 1984, and then dropped again to reach zero from 1988 on. In other words, there were excess buildings in 1982 and in 1988-1991. Equipment was not fully utilized until 1983 and again in 1988, 1990 and 1991. Comparing the evolutions of their shadow prices, labor seems to be a substitute to building and equipment. Infrastructure had an increasing rate of return until 1974, much greater than the other two types of capital, and then a declining productivity until the end of our period. On average over the 1962-1991 period, a dollar increase in the trade deficit allowed a 64 cents increase in final demand.[11] Its shadow price was pretty stable until 1981 and more volatile and somewhat lower after 1981.

Table 2 shows the decomposition of TFP-growth into a shift of the frontier (frontier productivity growth FP) and a movement towards the frontier (efficiency change EC).

[11] Final demand does not increase by the full dollar because of the need to produce locally nontradeable commodities for a given commodity composition of final demand.

The healthy TFP-growth in the period 1962-1974 reflects frontier productivity growth. The frontier slowed down, in fact contracted, in the period 1974-1981, but this was compensated by efficiency change, yielding a tiny TFP-growth rate. The period 1981-1991 showed no recovery, but an interesting reversal of the components. The frontier moved out, but this effect was nullified by a detoriation in efficiency change. The economy became healthy, but there were severe adjustment problems. The shift of the frontier displays the well-known pattern of the golden 1960s, the slowdown in the 1970s, and the structural recovery of the 1980s.[12]

Table 3 accounts for frontier productivity growth by factor input. The bulk of FP-growth is attributed to labor, next to nothing to the trade deficit, and the remainder to capital. In the first period FP and labor productivity both grow by 2.4%. The 0.2% capital productivity growth is distributed very unevenly over the three types of capital, with infrastructure picking up 1.0%, equipment none, and buildings plummeting by 0.8%. The slowdown in the second period is ascribed to both labor (dropping to 0.4% a year) and capital (turning -1.2% a year). As in the first period, infrastructure is decisive, now explaining all of the negative productivity growth in the second period. The successful FP-growth in the last period is again a labor story. Labor productivity grew at a dramatic 4.8% a year, offsetting a reduction in capital productivity growth of 1.1% a year. Again, the latter is determined by the productivity of infrastructure. The price correction term reflecting a change in final demand composition played a minor role. Demand has always tended to shift towards commodities requiring scarce resources, decreasing, but not by much, the positive effects of individual factor productivities on frontier productivity growth.

While Table 3 shows the composition of FP-growth by factor input, Table 4 decomposes it into the two sources of frontier shift, namely technical change and the terms-of-trade effect. In the first period the bulk of FP-growth (2.4%) is caused by technical change (the Solow residual at shadow prices is 1.7%). The FP-slowdown in the second period is also ascribed to a downturn in technology. The recovery in the last period, however, is due not only to a Solow residual (at shadow prices) increase of one percent, but above all to an improvement in the terms-of-trade effect from 0.5 to 3.8% annually. It might

[12] According to Bergeron, Fauvel and Paquet (1995), Canada hit a recession from January 1975 to March 1975, from May 1980 to June 1980, from August 1981 to November 1982, and from April 1990 to March 1991. We chose the breakpoints before the slump years 1975 and 1982 to compare productivity performances as much as possible over comparable phases of the business cycles.

look strange to have some negative Solow residuals, albeit at shadow prices. How can technology regress? There are at least two serious explanations to it. First, technical progress does not show in the statistics right away. This is the argument raised by David (1990) to explain the productivity paradox. It takes time to absorb the new information technology and to use it to its maximal efficiency, just as it took time to adjust to electricity at the beginning of the century. Second, the negative productivity growth is due to infrastructure, where the benefit might show up in the long run, and not in the short run because of adjustment costs.

It is interesting to contrast our measure of technical change (Table 4, line 1) with the traditional Solow residual, which we have added to Table 4. The main distinction of our productivity measures is the endogeneity of value shares. Prices are marginal productivities and quantities reflect frontier allocations. The Solow residual is a Domar weighted average of sectoral productivity growth rates, see Equation (18), but our Domar weights are different, say from Wolff (1985), by the use of competitive activity levels for sectors and supporting prices for commodities and factor inputs. Table 4 reveals quite dramatic differences. The observed-price based Solow residual is fairly unbiased in the period 1962-1974, but overstates the role of technical change in the periods 1974-1981 and 1981-1991. The terms-of-trade effect was far more important in explaining total factor productivity growth, particularly in the 1980s.

The intended contribution of this paper is to demonstrate that, at least in principle, productivity can be measured without recourse to factor shares or prices. The main reason for this disclaimer is that our model is fairly macro-economic in nature, featuring only one type of labor and three types of capital, with perfect mobility across sectors. More detailed specifications would affect the shadow prices and hence measured TFP. For example, if some type of capital is sector specific, then its constraint separates and each sector yields its own rate of return.

5 Conclusion

Standard measures of TFP-growth hinge on the use of value shares, hence of factor input prices. Since the latter are presumed to match factor productivities, the standard procedure amounts to accepting at face value what is supposed to be measured. In this paper we have demonstrated that factor productivities can be determined as the

Lagrange multipliers to a program that maximizes the level of domestic final demand. The consequent measure of total factor productivity growth encompasses not only the Solow residual, but also the terms-of-trade and the efficiency change effects.

We have applied our new measure of TFP-growth to the Canadian economy in the period from 1962 to 1991. Canadian TFP grew by 1.8% yearly in the 1960s, dropped in the 1970s to 0.1% and stayed put in the 1980s. The healthy TFP-growth in the 1960s reflects an outward shift of the frontier. The frontier then contracted in the 1970s, but this was offset by efficiency change. The 1980s shows a reversal of the two components. The frontier moved again, but efficiency change was negative, reflecting adjustment problems. The shift of the frontier tells the story of the golden 1960s, the slowdown of the 1970s, and the structural recovery of the 1980s. The bulk of this movement in frontier productivity can be ascribed to labor productivity growth. Of the capital stock the infrastructure component is the main driving force. The healthy factor productivity growth in the 1960s and the slowdown in the 1970s were both caused by technical change, but the recovery in the 1980s was due almost wholly to an improvement in the terms of trade.

The Solow residual measures the shift of the production possibility frontier of an economy that is presumed to be on its frontier. When this assumption is not tenable, this paper shows how the frontier can be traced using input-output statistics. The Lagrange multipliers to the program that determines potential GDP measure the factor productivities. The expansion factor of the program is an inverse measure of the efficiency of the economy. By the main theorem of linear programming, factor productivity growth and efficiency change sum to TFP-growth.

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Table 1: Factor productivities (shadow prices).

Year	Labor	Buildings	Equipment	Infrastructure	Trade deficit
1962	16.13	0.32	0.00	0.20	0.71
1963	16.50	0.33	0.00	0.19	0.71
1964	17.46	0.26	0.00	0.22	0.69
1965	17.86	0.28	0.00	0.18	0.69
1966	18.28	0.28	0.00	0.18	0.69
1967	19.31	0.20	0.00	0.18	0.68
1968	20.38	0.21	0.00	0.17	0.67
1969	20.91	0.17	0.00	0.18	0.67
1970	20.40	0.19	0.00	0.23	0.68
1971	21.78	0.14	0.00	0.24	0.66
1972	22.44	0.08	0.00	0.28	0.66
1973	22.96	0.05	0.00	0.32	0.65
1974	23.24	0.01	0.00	0.47	0.61
1975	22.70	0.05	0.00	0.45	0.64
1976	23.61	0.12	0.00	0.37	0.64
1977	24.52	0.08	0.00	0.34	0.65
1978	24.83	0.07	0.00	0.31	0.65
1979	24.85	0.06	0.00	0.32	0.65
1980	24.60	0.07	0.00	0.28	0.66
1981	24.31	0.10	0.01	0.25	0.69
1982	29.66	0.00	0.00	0.18	0.57
1983	12.07	0.62	0.83	0.15	0.82
1984	12.22	0.49	1.03	0.11	0.81
1985	23.11	0.24	0.22	0.16	0.73
1986	20.09	0.18	0.83	0.05	0.72
1987	20.76	0.11	0.99	0.03	0.70
1988	44.11	0.00	0.00	0.01	0.31
1989	22.41	0.00	1.21	0.00	0.63
1990	44.33	0.00	0.00	0.01	0.32
1991	46.13	0.00	0.00	0.01	0.29

Labor productivity is in 1986$ per hour. The shadow prices of capital (buildings, equipment and infrastructure) and the trade deficit are rates of return.

Table 2: Frontier productivity growth (FP) and efficiency change (EC) (Equations (4) and (8), annualized percentages).

	1962-1974	1974-1981	1981-1991
FP	2.4	-0.8	3.6
EC	-0.6	0.9	-3.7
TFP	1.8	0.1	-0.1

Table 3: Frontier productivity growth (FP) by factor input (Equation (7), annualized percentages).

	1962-1974	1974-1981	1981-1991
Buildings	-0.9	0.4	-0.3
Equipment	0.0	0.1	0.3
Infrastructure	1.1	-1.5	-1.2
Capital, total	0.2	-1.0	-1.1
Labor	2.4	0.5	5.0
Deficit	-0.0	0.0	-0.1
Price	-0.2	-0.3	-0.2
FP	2.4	-0.8	3.6

Table 4: Frontier productivity growth (FP) by Solow residual and terms-of-trade effect (Equations (12-14), annualized percentages).

	1962-1974	1974-1981	1981-1991
SR	1.7	-1.3	-0.3
TT	0.7	0.5	3.8
FP	2.4	-0.8	3.6
SR at observed prices and activity levels	1.4	0.5	0.2

APPENDIX: Data

The constant price input-output tables obtained from Statistics Canada are expressed in 1961 prices from 1962 to 1971, in 1971 prices from 1971 to 1981, in 1981 prices from 1981 to 1986, and in 1986 prices from 1986 to 1991. All tables have been converted to 1986 prices using the chain rule. For reasons of confidentiality, the tables contain missing cells, which we have filled using the following procedure. The vertical and horizontal sums in the make and use tables are compared with the reported line and column totals, which do contain the missing values. We select the rows and columns where the two figures differ by more than 5% from the reported totals, or where the difference exceeds $250 million. We then fill holes or adjust cells on a case by case basis filling in priority the intersections of the selected rows and columns, using the information on the input or output structure from other years, and making sure the new computed totals do not exceed the reported ones.

There are three capital types, namely buildings, equipment, and infrastructure.[13] The gross capital stock, hours worked and labor earnings are from the KLEMS database of Statistics Canada, described in Johnson (1994). In particular, corrections have been made to include in labor the earnings of the self-employed, and to separate business and non-business labor and capital. The total labor force figures are taken from Cansim (D767870) and converted in hours using the number of weekly hours worked in manufacturing (where it is the highest). Out of the 50 industries, no labor nor capital stock data exist for sectors 39, 40, 48, 49, 50, and no capital stock data for industry 46. The capital stock for industry 46 has been constructed using the capital/labor ratio of industry 47 (both industries producing predominantly the same commodity).

The international commodity prices are approximated by the U.S. prices, given that 70% of Canada's trade is with the United States. We have used the U.S. producer prices from the U.S. Bureau of Labor Statistics, Office of Employment Projection. The 169

[13] Statistics Canada calls them "building constructions," "equipment" and "engineering constructions." Alternatively we could have modeled capital as being sector-specific, the so-called putty-clay model. We prefer the present hypothesis of sectoral mobility of capital within each group for three reasons. First, to let the economy expand, we would have needed capacity utilization rates which are badly measured and unavailable for a number of service sectors. Second, to relieve a numerical collinearity problem, we would have to relieve the capital constraint on the non-business sector. Third, the combination of 11 non-tradeables and sector-specific capacity expansion limits is too stringent. It would lead to a high shadow price on construction commodities and zero shadow prices almost anywhere else.

commodity classification has been bridged to Statistics Canada's 94 commodity classification. As the debt constraint in (1) is given in Canadian dollars, we convert U.S. prices to Canadian equivalents. We have used, whenever available, unit value ratios, (UVRs, which are industry specific) computed and kindly provided to us by Gjalt de Jong (1996). The UVRs are computed using Canadian quantities valued at U.S. prices. For the other commodities, we have used the purchasing power parities (PPP) computed by the OECD (which are based on final demand categories). The UVRs establish international price linkages for 1987, the PPPs for 1990 in terms of Canadian dollars per U.S. dollar. We hence need two more transformations. First, U.S. dollars are converted to Canadian dollars using the exchange rates taken from Cansim (series 0926/133400). Second, since the input-output data are in 1986 prices, we need the linkage for 1986, which is computed by using the respective countries' commodity deflators: the producer price index for the U.S. (see above) and the total commodity deflator from the make table (except for commodities 27, 93 and 94, for which we use the import deflator from the final demand table) for Canada. Finally, international commodity prices are divided by a Canadian final demand weighted average of international commodity prices to express them in real terms.

Are considered as non-tradeable, services incidental to mining, residential construction, non-residential construction, repair construction, retail margins, imputed rent from owner occupied dwellings, accommodation & food services, supplies for office, laboratories & cafetaria, and travel, advertising & promotion, for which no trade shows up in the input-output tables for most of the sample period.

The structure of some non-tradeability constraints implies the equality of the activity levels of "construction" and final demand, "owner-occupied dwellings" and final demand, and "printing and publishing" and "travel, advertising and promotion." We have forced the activity level of industry 39 (government royalties on natural resources, which essentially pertains to oil drigging in Alberta) to follow industry 5 (crude petroleum and natural gas) to ensure there are no such royalities without oil drigging. A more detailed documentation of the data and their construction is available from the authors upon request.