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Life cycle consumption models with uncertainty within periods

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Abstract

The standard life cycle model with a von Neumann-Morgenstern expected utility function is generalized to avoid deterministic relationships in the multi-good version with intertemporal additive utility. The generalization can be estimated and tested without complicating the econometric analysis. © 1997 Elsevier Science S.A.

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1. Introduction

When considering a life cycle model with intertemporal additive utility and with more than one good per period, the first order conditions not only result in Euler equations, but also in deterministic relationships between marginal utilities pertaining to the same period. Such restrictions are clearly far too strong, since they are not likely to be satisfied in any empirical application. Consequently, the life cycle models must be specified such that these deterministic restrictions are relaxed.

The contribution of this paper is a modification of the life cycle model that will accomplish this in a natural way. The idea is to allow for much more variability across consumers in the way they plan their consumption than usually is assumed. The modification will be such that the resulting life cycle model can be estimated and tested without complicating the econometric analysis. Moreover, the proposed life cycle model is a generalization of the standard way of modelling life cycle models. Consequently, it will also be possible to test whether the proposed extension makes sense.

The remainder of the paper is organized as follows. In Section 2 we discuss a standard formulation of the life cycle model, together with the problem of the deterministic relationships and the standard solution in terms of random preferences. In Section 3 we present our modification and we discuss some consequences of our modification, in particular, the avoidance of deterministic relationships. Finally, Section 4 contains some concluding remarks.

2. A standard life cycle model formulation

We start with a (finite) population S characterizing the consumers. In our model S will denote the

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population at the beginning of the first period. We assume that there are L periods, with $L \in \mathbb{N}$ the maximum lifetime. We restrict attention to two goods per period[1]. The first one is always consumed in positive quantities, the second one may be consumed only now and then. Consumer $s \in S$ is supposed to solve in period 1 the following maximization problem with respect to $q_1 = (q_{11}, q_{21})', \ldots, q_L = (q_{1L}, q_{2L})'$:

$$\text{Max}\, E_{(s)} \left\{ \sum_{\tau=1}^{L} u_\tau(s; q_\tau) \right\}, \quad \text{subject to} \tag{1}$$

$$\sum_{\tau=1}^{L} \left(\prod_{j=1}^{\tau} (1+\rho_j(s))^{-1} \right) p_\tau(s)' q_\tau \leq a_1(s) + \sum_{\tau=1}^{L} \left(\prod_{j=1}^{\tau} (1+\rho_j(s))^{-1} \right) y_\tau(s), \quad \text{and}$$

$$q_{2\tau} \geq 0 \quad \tau = 1, \ldots, L. \tag{2}$$

Here $p_\tau(s)$ is the corresponding two-dimensional price vector of consumer s, $u_\tau(s;.)$ is the utility index of consumer s, $y_\tau(s)$ denotes nominal non-property income in period τ of consumer s, $p_\tau(s)$ is the nominal interest rate in period τ of consumer s, $a_1(s)$ is non-human wealth at the beginning of period 1 of consumer s, and $E_{(s)}$ denotes taking expectation by consumer s at the beginning of period 1, conditional upon all information at that moment. In this model the vectors q_τ are allowed to depend upon the variables contained in period τ's information set. Denote by $v_\tau(s)$ the vector of random variables in period τ's information set of consumer s upon which q_τ is allowed to depend. Included in $v_\tau(s)$ are, for example, $y_t(s)$ and $p_t(s)$, for $t \leq \tau$. Over time, new information will be received, but old information will not be forgotten. Thus the vector $v_\tau(s)$ contains the $v_t(s)$ of all previous periods $t < \tau$.

Denote the optimal solution of optimization problem (1)-(2), as solved by consumer s, by $(q_1(s), \ldots, q_L(s))'$, with $q_\tau(s) = (q_{1\tau}(s), q_{2\tau}(s))'$. Then it is not hard to obtain the following standard Euler equations:

$$E_{(s,1)}\{(\partial u_1/\partial q_{11})/p_{11}(s) - (1+\rho_2(s))(\partial u_2/\partial q_{12})/p_{12}(s)\} = 0, \tag{3}$$

$$E_{(s,1)}\{[(\partial u_/\partial q_{21})/p_{21}(s) - (1+\rho_2(s))(\partial u_2/\partial q_{12})/p_{12}(s)] \times 1_{(0;\infty)}(q_{21}(s))\} = 0, \tag{4}$$

with $\partial u_\tau/\partial q_{i\tau} \equiv \partial u_\tau(s; q_\tau(s))/\partial q_{i\tau}$. Here $E_{(s,1)}$ denotes the expectation of consumer s, conditional upon all information in period 1, i.e., conditional upon (s) and $v_1(s)$. Restrictions (3)-(4) are not the only ones. One can also derive as first order condition

$$[(\partial u_1/\partial q_{11})/p_{11}(s) - (\partial u_1/\partial q_{21})/p_{21}(s)] \times 1_{(0;\infty)}(q_{21}(s)) = 0. \tag{5}$$

Contrary to (3)-(4), equation (5) is deterministic. It is, therefore, clearly far too strong: (5) will generally not be satisfied in any empirical application! This indicates misspecification of the model, before even using (3)-(4).

Of course, result (5) is well known. A standard approach, applied in particular when the possibility of binding nonnegativity constraints is ignored, is to introduce random preferences, see MaCurdy (1983). However, as discussed by Adang and Melenberg (1995), a combination of random preferences and nonnegativity constraints that may be binding is empirically unattractive. In order to avoid (5), without loosing (4), some other approach is required.

[1] Generalization to more goods per period is straightforward.

3. A life cycle model with "uncertainty within periods" and its consequences

To modify the life cycle formulation, first write $v_\tau(s)=(v_{\tau-1}(s), \tilde{v}_\tau(s))$, with $\tilde{v}_\tau(s)$ the variables in the information set of period τ, but not in the information set of period $\tau-1$. If $\tau=1$, we define $\tilde{v}_1(s)=v_1(s)$. All we do is to allow for the possibility that $q_{i\tau}$ depends upon $(v_{\tau-1}(s), \tilde{v}_{i\tau}(s))$, with $\tilde{v}_{i\tau}(s)$ instead of $\tilde{v}_\tau(s)$, where $\tilde{v}_{i\tau}(s)$ at least includes the price $p_{i\tau}(s)$, but not necessarily any other price or any other component of $v_\tau(s)$. Thus, we simply allow for the possibility that different $q_{i\tau}-s$ depend on different $\tilde{v}_{i\tau}(s)-s$, which also may differ across consumers. We shall refer to this modification by stating that we allow for "uncertainty within periods", or, perhaps even better "variation in uncertainty within periods", where the variability applies to goods and consumers.

The motivation is simple. First, it is a straightforward and natural generalization of the standard formulation. Intuition, without formalities, is provided by Adang and Melenberg (1995). Secondly, without it, the standard life cycle model as presented in the previous section will be rejected as a consequence of deterministic relationships. But with this modification, the deterministic relationships will not appear, as we will now show.

Notice first that without variable uncertainty within periods (5) can be obtained using a calculus of variation approach[2], with variations

$$h_{11}(s) = +A(s)\times(1/p_{11}(s))1_{(0;\infty)}(q_{21}(s)), \tag{6}$$

$$h_{21}(s) = -A(s)\times(1/p_{21}(s))1_{(0;\infty)}(q_{21}(s)), \tag{7}$$

where $h_{11}(s)$ corresponds to $q_{11}(s)$ and $h_{21}(s)$ corresponds to $q_{21}(s)$, and where $A(s)$ stands for the left hand side of (5): add $\epsilon h_{11}(s)$ to $q_{11}(s)$ and $\epsilon h_{21}(s)$ to $q_{21}(s)$ and substitute these expressions in the expected utility function for $q_{11}(s)$ and $q_{21}(s)$, respectively; then, by taking the derivative with respect to ϵ, and evaluating the resulting derivative at $\epsilon=0$, we obtain: $E_{(s)}(A(s)^2)=0$, or $A(s)=0$ (with probability one[3]). Of course, this is equation (5). In such a calculus of variations approach, only directions are allowed which depend upon the same components of $v_1(s)$ as the corresponding $q_{i1}(s)$. Without variable uncertainty within periods, both $h_{11}(s)$ and $h_{21}(s)$ are allowed to depend upon the whole of $v_1(s)$. But with variable uncertainty within periods, $h_{11}(s)$ is only allowed to depend on $\tilde{v}_{11}(s)$ and $h_{21}(s)$ is only allowed to depend on $\tilde{v}_{21}(s)$. But then the directions $h_{11}(s)$ and $h_{21}(s)$ given in (6) and (7) are no longer valid directions: $A(s)$ depends on both $\tilde{v}_{11}(s)$ and $\tilde{v}_{21}(s)$, and $q_{21}(s)$ in (7) also depends on $\tilde{v}_{21}(s)$. But if the directions (6) and (7) are not allowed any more, we are also no longer able to obtain the deterministic relationship (5)!

Consider next (3). Choose as variations

$$h_{11}(s) = B(s)/p_{11}(s), h_{12}(s) = -B(s)(1+\rho_2(s))/p_{12}(s), \tag{8}$$

with B(s) the left hand side of (3), but not using $E_{(s,1)}$, but, instead, $E_{(s,11)}$: the expectation conditional upon (s) and $\tilde{v}_{11}(s)$. Using these variations, we obtain (with probability one)

[2]See Hall (1978) for the use of calculus of variations in life cycle models and see Neustadt (1976) for an extensive study on optimization and first order conditions.

[3]Here and in the sequel "with probability one" means with probability one with respect to the probability distribution corresponding to $E_{(s)}$.

$$E_{(s,11)}((\partial u_1/\partial q_{11})/p_{11}(s) - (1+\rho_2(s))(\partial u_2/\partial q_{12})/p_{12}(s)) = 0, \tag{9}$$

Similarly, we obtain as alternative of (4) (with probability one)

$$E_{(s,12)}([(\partial u_1/\partial q_{21})/p_{21}(s) - (1+\rho_2(s))(\partial u_2/\partial q_{12})/p_{12}(s)] \times 1_{(0;\infty)}(q_{21}(s))) = 0, \tag{10}$$

where $E_{(s,12)}$ stands for the expectation conditional on (s) and $\tilde{v}_{12}(s)$.

Thus, we see that the Euler equations (3)-(4) remain valid, with only a slight modification: the conditioning should not be on (s) and the whole of $v_1(s)$, but on (s) and a part of $v_1(s)$, depending upon the Euler equation under consideration. The Euler equations (9)-(10) provide an easy *rule of thumb*. The part of $v_1(s)$ is generally only the price of the first period's good that occurs in the Euler equation: according to our assumptions at least $p_{11}(s)$ is included in $\tilde{v}_{i1}(s)$, and since no other information concerning the correct form of $\tilde{v}_{i1}(s)$ is available, the rule of thumb follows.

4. Concluding remarks

For empirical analyses unconditional moment restrictions can be obtained by using instruments that are functions of s and the admissible part of $v_1(s)$ which follows from the given rule of thumb. Thus, due to within period variable uncertainty, not all variables included in $v_1(s)$ can be used as instruments. In particular, variables like $\rho_1(s)$ and $y_1(s)$ are excluded. This has an important consequence: excess sensitivity with respect to these latter variables need not imply rejection of the life cycle model presented in this paper. In other words, rejection of our version of the life cycle model becomes much harder than when using the standard way of modelling, although the econometric part is not complicated. This is already illustrated by Adang and Melenberg (1995).

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