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MANUFACTURING IN THE 1980S AND 1990S**

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Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s and 1990s

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Abstract

Manufacturing productivity growth recovered during the 1980s and 1990s, while other sectors, particularly services, did not. In the same period U.S. manufacturing has engaged in the "outsourcing" or "contracting-out" of service functions. Has the recovery of manufacturing been accomplished by industrial reorganization - sloughing off sluggish services - rather than technical progress? We analyze this question by reducing service inputs to their constituent elements of material inputs. Service productivity growth is thus imputed to the goods sectors, reducing the recovery of manufacturing productivity growth in the 1980s by one fifth. The recovery lasted through the 1990s, when high productivity performers in manufacturing have been relatively successful at outsourcing sluggish services.

J.E.L. ...eld: 047 Measurement of Economic Growth; Aggregate Productivity.

1 Introduction

The motivation for this study comes from two recent phenomena that have received considerable attention. The first is that manufacturing productivity growth recovered during the 1980s and 1990s, after a protracted period of slowdown during the 1970s, while other sectors, particularly services, did not recover. Statistics calculated from input-output data and shown in Table 1 illustrate this point. Average annual total factor productivity (TFP) growth, defined as the price-weighted sum of changes in input-output coefficients, increased from 0.04 percent over the 1967-77 period to 0.87 percent over the 1977-87 period and 0.72 percent over the 1987-96 period. Construction was the only other major sector which showed a similar recovery over the 1977-87 period but then experienced a relapse in the 1987-96 period. Utilities and other services both underwent a slight recovery in the 1987-96 period.

The second is that during the 1980s U.S. manufacturing has engaged in the "outsourcing" or "contracting-out" of service functions. This process refers to the process of replacing in-house services, such as legal, advertising, accounting, and related business services with services purchased from outside the firm (see, for example, Postner, 1990 for a discussion of this issue from an accounting point of view).¹

Anne Carter in her 1970 book noted early on the rapid increase in total service requirements per unit of manufacturing output between 1947 and 1967. Studies of the U.K. economy indicate that this phenomenon has continued in more recent years. Using U.K. input-output data, Barker (1990) reported that about 20 percent of the growth in service (gross) output between 1979 and 1984 was attributable to changes in manufacturing intermediate demand, and Barker and Forssell (1992) calculated that 22 percent of the growth of business services over the same period was associated with change in input-output coefficients in manufacturing.

Our own calculations from input-output data, presented in Table 2, do suggest that the outsourcing of services has accelerated during the 1980s. There was a gradual increase in the share of total service inputs in gross output (in both current and constant dollars) between 1947 and 1977, with the proportion rising from 9.6 to 13.4 percent in current dollars and from 12.0 to 16.7 percent in constant dollars. Between 1977 and 1982, total service inputs jumped from 13.4 to 16.3 percent of gross output in current dollars and increased from 16.7 to 17.7 percent in constant dollars. Between 1982 and 1996, there was a further rise in the service share in total output, from 16.3 to 18.4 percent in current dollars and from 17.7 to 18.4 percent in constant dollars.

Results are also shown for the share of total services inputs excluding wholesale and retail trade in gross output and the share of total services except trade and utilities in total output. The time trends in these series are quite similar to those for the share of total services in output. One of the most notable changes was in the proportion of business service and finance and insurance inputs in total gross output, which almost tripled between 1947 and 1966 in current dollar terms (from 1.8 to 4.9 percent) and doubled in constant dollar terms (from 3.6 to 7.3 percent).

Our speculation is that part of the recovery of manufacturing productivity growth may be a consequence of the outsourcing of services from manufacturing. The argument is that if these services have lower productivity growth rates than the production of goods within manufacturing, then the outsourcing of previously internally provided services will increase measured productivity growth within manufacturing. Siegel and Griliches (1992) found a correlation coefficient of .13 (significant at .01 level) between acceleration in TFP and the average ratio of purchased services to output. It is of direct interest to determine how much of the change in measured productivity growth in manufacturing is due to changes in intermediate demand for services.

It should be emphasized at the outset that we do not have any direct data about the degree of outsourcing that takes place. This would require information on the firm level of the amount of inputs devoted to internally provided services (such as accounting or advertising) and the replacement of these inputs with externally provided services. However, the degree of increase in intermediate inputs of services used in manufacturing, particularly in constant dollars, during the 1980s is suggestive that contracting-out was a major factor explaining its rise.² Indeed, Abraham (1990) and Abraham and Taylor (1993) found that market mediated work arrangements associated with business service employment increase substantially over the period 1975-1990.

Our analytical technique is based on a consolidation framework initiated by Leontief (1967). In this approach, service inputs into manufacturing are essentially reduced to their constituent elements of labor, capital, and goods inputs. This consolidation framework will allow us to decompose the change in TFP growth within manufacturing into effects emanating from the purchases of services and from the rate of material productivity growth in manufacturing. Services productivity growth is imputed to the consuming sectors. If the recovery of TFP growth in manufacturing during the 1980s is due largely to outsourcing of services, then the increase in the rate of TFP growth computed with this consolidated measure should be smaller than the rate of TFP growth computed from only the direct coefficients.

Our approach avoids many of the problems of measurement normally accounted in measuring the output of service industries (Griliches, 1992). In traditional measures of TFP growth within manufacturing, service inputs are treated in analogous fashion to inputs of goods industries, labor, and capital. Difficulties in measuring service outputs may seriously distort measures of TFP within manufacturing. On the other hand, input measures are quite adequate in service sectors, as in other industries within the economy. Labor, capital, and material inputs are easily identifiable and measurable in services, and are, in principle, no different than in other industries.

The basic data sources for this study are U.S. 85-order input-output tables for 1947, 1958, 1967, 1977, 1987, and 1996. The use-make framework will be exploited in the analysis for the last three tables. The next section develops the model. A description of the data sources and methods is given in Section 3, the results are discussed in Section 4, and concluding remarks and interpretations are made in the last section.

2 Productivity Analysis of Services

Carter (1970) found an increase in the total requirements of service output over the 1947-67 period in the U.S., but could not decompose it into a real interindustry effect of greater specialization and a spurious effect from the reclassification of such service activities from sectors where they are secondary output to sectors where they constitute primary output. By introducing the use-make activity framework, ten Raa and Wolpin (1991) showed that many establishments which produced services in addition to their primary output during the 1960s sloughed off this production during the 1970s. In this paper we want to assess the implication of this process by decomposing total growth of the goods sectors into an own component and a service component.

A main problem is that service output is not tangible. In other words, how do we measure the output of services required for nonservice production and how do we impute factor inputs to this output? The idea proposed in this paper is to circumvent the problem by relating the inputs of services to the outputs of the sectors that use those services. We will do so by elimination of the intermediate services, where the elimination is to be understood in a mathematical sense.

Since outsourced services are intermediate inputs and we want to relate them to manufacturing outputs, we need an input-output framework. The strengths of this approach are the capacity to impute service TFP growth to the goods producing sectors and, more generally, to decompose macro TFP growth into sectoral technical changes. The

weaknesses are the implicit identification of activities and outputs and the use of exogenous factor prices. To overcome the former element we distinguish commodities from sectors and admit secondary products.

We partition the n commodities of the economy in (1) m goods and (2) $n - m$ services. Goods can be dropped on your feet and services are anything else. Goods include manufactures, but also agricultural and other produce. The measurement of TFP growth requires the following data for sector i ; $i = 1; \dots; n$: commodity outputs v_{ij} ; $i = 1; \dots; n$, commodity inputs u_{ji} , labor employment L_i and capital employment K_i : We also need the wage rate w and the rental rate r . Obvious matrix notation for the flows is V ; U ; L , and K . Although V and U are both $n \times n$ -dimensional, V is of dimension sector \times commodity and U commodity \times sector, adhering to the System of National Accounts convention (see Kop Jansen and ten Raa, 1990). L and K are n -dimensional row vectors. w and r are scalars.

The basic idea exposed in this paper is to impute the gross output and the productivity of the services to the goods by relating the goods inputs into the services to the goods outputs of the service consuming sectors. The idea is applicable to the services used in the production of goods (business services), but not to the services which feed final demand (personal services). We therefore define standard TFP growth as the relative real rate of change of the net output of goods to the required factor inputs ratio,

$$\frac{p^1 dy^1}{p^1 y^1} \approx \frac{w dL^g + r dK^g}{w L^g + r K^g}$$

Here y^1 is the m -dimensional subvector of goods components of n -dimensional net output $y = (V^T - U)e$, where e is the unit vector with all entries equal to one. Similarly, p^1 collects the goods components of the row vector of competitive prices, which is defined by $p(V^T - U) = wL + rK$. The amount of labor required for the production of the net output of goods is

$$L^g = \begin{pmatrix} \tilde{A} & \mathbf{1} \\ (I - A)^{-1} & 0 \end{pmatrix} y^1 \quad (1)$$

where input-output coefficients are $A = U(V^T)^{-1}$ and labor coefficients are $\tilde{A} = L(V^T)^{-1}$ (see Kop Jansen and ten Raa, 1990). L^g includes labor employment in the service sectors needed for the fulfillment of intermediate demand in the goods sectors. $(I - A)^{-1}$ is the $n \times n$ -dimensional row vector of total labor coefficients, measuring the labor contents of units of goods and services. The amount of capital required for the production of the net output of goods is defined similarly.

Our strategy is to evaluate the labor and capital contents in the lower dimensional space of goods, using the Leontief inverse of consolidated input-output coefficients, $m \times m$ -dimensional matrix A^g , and consolidated labor and capital coefficients, m -dimensional row vectors λ^g and k^g . In the appendix (statement A) we show that

$$L^g = \lambda^g (I - A^g)^{-1} y^g \quad (2)$$

where A^g is Leontief's (1976) matrix of consolidated input-output coefficients,

$$A^g = A_{11} + A_{12} (I - A_{22})^{-1} A_{21} \quad (3)$$

and consolidated labor coefficients (as well as capital coefficients) are defined similarly,

$$\lambda^g = \lambda^1 + \lambda^2 (I - A_{22})^{-1} A_{21} \quad (4)$$

The m -dimensional subvector of labor coefficients for the goods, λ^1 , is augmented by a row vector that features the direct service requirements of goods, A_{21} , transformed to total service requirements by the Leontief inverse of services own coefficients, A_{22} , and further to labor contents by the services labor coefficients, λ^2

There are two ways to decompose standard TFP growth. As a preliminary, we show in the appendix (statement B) that the denominators in the expression for goods TFP growth are equal: $p^1 y^1 = w L^g + r K^g$. Now the first way is a direct Wolpin (1985) decomposition into sectoral TFP growth rates,

$$\% \dot{y}_i = \sum_{j=1}^n \frac{\tilde{A}_{ij} x_j}{p_j d_{ji} + w d_i^g + r d k_i^g} p_i \quad (5)$$

where index i runs through the goods and the services. Standard TFP growth is a weighted average of all the sectoral TFP growth rates. The weights are proportional to the competitive value shares of the gross outputs of goods and services that sustain the final demand for goods. The weights sum to the gross-to-net-output competitive value ratio by the Domar aggregation rule.³

The second way is a similar decomposition into consolidated TFP growth rates,

$$\% \dot{y}_i^g = \sum_{j=1}^n \frac{\tilde{A}_{ij} x_j^g}{p_j d_{ji}^g + w d_i^g + r d k_i^g} p_i \quad (6)$$

where i runs through the goods only. Standard TFP growth is a weighted average of the consolidated TFP growth rates over goods. The weights remain $(p_i x_i^g) = (p x^g)$, where x^g is the goods subvector of the full Leontief inverse of $\begin{pmatrix} y^1 \\ 0 \end{pmatrix}$, or, equivalently, the

consolidated Leontief inverse of y^1 . The equivalence is demonstrated in the appendix (statement C). In this paper we analyze sectoral TFP growth rates, particularly for manufacturing. Manufacturing consolidated TFP growth is the weighted average of the manufacturing γ_i^m 's, where the weights are the competitive value shares of consolidated manufacturing output, $(p_i x_i^m) = (p x^m)$. Note that comparison with macro-economic TFP growth would require inflation of our measures by the Domar factor.

A first decomposition of consolidated TFP growth rates (6) is obtained by separating the consolidated coefficient reductions according to (3) and (4) and sorting out the leading and following terms:

$$\gamma_i^m = \gamma_{2i} + \gamma_{3i} \quad (7)$$

where

$$\gamma_{2i} = \frac{\tilde{A}_i x_i^m}{p_i} \sum_{j=1}^m p_j da_{ji} + w d_i + r dk_i \quad (8)$$

and γ_{3i} is defined similarly, capturing the reductions in the following terms of (3) and (4). γ_{2i} measures the reduction in direct materials, labor, and capital inputs. γ_{3i} measures the reduction in indirect materials, labor, and capital inputs, as embodied in the service inputs. γ_{2i} is not a standard sectoral TFP growth rate, for the summation is through m only, ignoring the service inputs, $m + 1; \dots; n$. However, it is not difficult to forge the relationship. It amounts to a further decomposition.

For this purpose, return to the decomposition of standard TFP growth using (5). Ignoring the Domar factor, standard TFP growth amounts, using (5) and (8),

$$\begin{aligned} & \sum_{i=1}^n \frac{p_i x_i^m}{p x^m} \gamma_i^m \\ &= \sum_{i=1}^m \frac{p_i x_i^m}{p x^m} \gamma_{2i} + \sum_{i=m+1}^n \frac{p_i x_i^m}{p x^m} \gamma_{2i} \\ &= \sum_{i=1}^m \frac{p_i x_i^m}{p x^m} \left(\sum_{j=1}^m p_j da_{ji} + w d_i + r dk_i \right) p_i + \sum_{i=m+1}^n \frac{p_i x_i^m}{p x^m} \gamma_{2i} \\ &= \sum_{i=1}^m \frac{p_i x_i^m}{p x^m} \gamma_{2i} + \sum_{j=m+1}^n p_j da_{ji} p_i + \sum_{i=m+1}^n \frac{p_i x_i^m}{p x^m} \gamma_{2i} \end{aligned} \quad (9)$$

Comparison with (7) shows that the service components of consolidated TFP growth, γ_{3i} , pick up not only the reductions of the service inputs, which we denote by

$$\gamma_{3i}^1 = \sum_{j=m+1}^n p_j da_{ji} p_i \quad (10)$$

but also services TFP growth, γ_{m+1} through γ_n , the last terms of (9). The amount of services TFP growth imputed to good i is denoted by γ_i^2 and is determined residually by

$$\gamma_i = \gamma_i^1 + \gamma_i^2 \quad (11)$$

This equation amounts to the further reduction of consolidated TFP growth, (7). A rearrangement of terms is illuminating. For this purpose, note that the terms under the first summation sign on the right hand side of (9) contain the standard TFP growth rates,

$$\gamma_i = \gamma_i + \gamma_i^1 \quad (12)$$

It follows from equations (7), (11), and (12) that consolidated TFP growth equals standard goods TFP growth plus imputed services TFP growth,

$$\gamma_i^{\text{con}} = \gamma_i + \gamma_i^2 \quad (13)$$

If some standard TFP growth rate is great, but the corresponding consolidated TFP growth is modest, it must be that γ_i^2 is negative. The sector performs well, but service productivity drag is imputed to it. Such a sector looks "smart" in the sense that outsourced and external services productivity drag does not show in its sectoral TFP growth rate. Here we model outsourcing by imputing all services to the goods, even when they were not provided in-house by manufacturing originally. This model does not lay claim to outsourcing as a historic process, but reduces the services "superstructure" to the materials "substructure."

Does manufacturing look "smart"? In other words, has the recovery of manufacturing productivity been accomplished by industrial reorganization – sloughing off sluggish services – rather than technical progress? A macro answer is given by the sign of services TFP growth imputed to manufacturing, that is the manufacturing γ_i^2 's weighted by value shares. If it is negative, then outsourcing of services is a source of manufacturing productivity growth. A micro answer is provided by the correlation between imputed service TFP growth, γ_i^2 , and standard TFP growth, γ_i , across manufacturing sectors. If it is negative, good performers are imputed relatively much service drag and outsourcing would be a determinant of the distribution of TFP growth within manufacturing.

3 Data Sources and Methods

Our basic data source consists of U.S. input-output dollar flow tables, which were originally obtained from the Bureau of Economic Analysis on the 87-sector level for years 1947, 1958 and 1967 in single-table format, and on the 85-sector level for years 1967, 1977, 1987, and 1996 in the dual use-make table format. The single-table format relies on the so-called BEA transfer method. In this method, the transaction matrix is constructed on an industry by industry basis. A secondary product produced by industry i which is primary to industry j is recorded as a purchase made by industry j from industry i . The actual sales of the secondary product produced in i are then "transferred" to the sales row of industry j .⁴

The 1967, 1977, 1987, and 1996 data are available in separate make and use tables.⁵ The 1977 and 1987 tables use the same accounting conventions. However, there are four important changes between the 1967 tables and those of the later years. First, two dummy sectors, business travel and entertainment and office supplies, are present in the 1967 table but were eliminated in the 1977, 1987, and 1996 tables. We follow the later convention and distribute the output of the two dummy sectors to the appropriate using industries. Second, in the 1977, 1987, and 1996 tables, the restaurant sector was separated from the trade sector, while in the 1967 table the two are aggregated into a single sector. It was not possible to separate the restaurant sector from the trade sector in the 1967 data. As a result, we have aggregated the two sectors in the 1977, 1987, and 1996 data for consistency with the earlier year.⁶

Third, in the 1967 table, a portion of the wholesale and retail trade activity and real estate (rental) activity engaged in by the various sectors were recorded as a secondary product of these sectors, whereas in the later years these transactions were recorded as primary to the trade and real estate sectors, respectively. For consistency with the later years, we transferred these secondary outputs to their primary sector.⁷ Fourth, in the 1967 table, comparable imports are recorded as if purchased by the industry producing the comparable domestic commodity and then added to that industry's output for distribution to the actual purchasing industries. In the later tables, comparable imports are recorded as directly purchased by the using industry from the comparable domestic industry. We follow the later convention in our work.

Charles Bowman of the U.S. Bureau of Labor Statistics graciously provided us with a consistent set of labor coefficients, capital coefficients, and sectoral price deflators through 1996. The labor and output data are an updated version of the Bureau of

Labor Statistics' Historical Output and Employment Data Series.⁸ The capital stock data are derived from the U.S. Bureau of Economic Analysis' net stocks of plant and equipment by industry series through 1996. Capital stock figures are based on chain-type quantity indexes for net stock of fixed capital in 1992\$, year-end estimates. For technical details, see Katz and Herman (1997).

Five sectors – research and development, business travel and office supplies, scrap and used goods, and inventory valuation adjustment – appeared in some years but not in others (the earlier years for the first three sectors and the later years for the last two sectors). In order to make the accounting framework consistent over the years of analysis, we eliminated these sectors from both gross and final output. This was accomplished by distributing the inputs used by these sectors proportional to either the endogenous sectors which purchased the output of these five sectors or to final output.¹⁰

One additional refinement, suggested by Leontief (1941), was made. Instead of treating noncompetitive imports as exogenous, an endogenous column of exports was incorporated in the A matrix to balance the row of noncompetitive imports. In this way, imports can also be thought of as being "produced" domestically by the exports required to sell in exchange for them. The final output vector was adjusted for this. (See Wolpin, 1985, and ten Raa and Wolpin, 1991, for more details.)

All matrices were deflated to 1992 dollars using the sectoral price deflators. Productivity growth rates for 1947-1958 and 1958-1967 are calculated using the single-table basic framework (and making use of the 1967 single table data). Productivity growth rates for 1967-1977 and 1977-1987, and 1987-1996 are calculated using the use-make framework (and relying on the 1967 dual table data). Because of alignment difficulties between the various input-output years (several industries are collapsed in the 1987 and 1996 tables, in particular) and with the employment and capital stock data, productivity growth estimates are available for only 51 industries, including 29 manufacturing industries.

We divided the original 85 industries into two groups, goods and services. The goods industries include: agriculture (1-4)¹¹; mining (5-10), construction (11-12), manufacturing (13-64), transportation (65), communications (66-67), and utilities (68). Services include: trade (69), finance, insurance, and real estate (70-71), government services (78, 79, and 82), and all other services (72-77).

4 Results

Table 3 shows our central results, on the decomposition of manufacturing TFP growth into a material inputs component and a service input component. Line 1 shows the trend in standard commodity TFP growth in manufacturing, which falls from 1.14 percent per year in 1958-1967 to 0.04 in 1967-1977 and then recovers to 0.87 in 1977-1987, and 0.72 in 1987-1996. Line 2 presents results on TFP growth for the consolidated manufacturing industries, $\frac{1}{4}^{\text{a}}$, which incorporates both the direct productivity growth of the manufacturing industries as well as the productivity growth of service industries supplying the manufacturing sectors.

The figures also show a fall-off in consolidated TFP growth between the 1958-1967 and 1967-1977 periods, from 1.20 to 0.16 percent per year, and then a recovery in the 1977-1987 period to 0.81 percent per year which was maintained in the 1987-1996 period. These results clearly support the central premise of the paper that the recovery in standard manufacturing TFP growth between 1967-1977 and 1977-1987 (0.83 percentage points) was greater than the recovery in consolidated TFP growth (0.65 percentage points). These findings are consistent with the argument that the outsourcing of services was partly responsible for the recovery of productivity growth in manufacturing during the 1980s.

A decomposition is shown of consolidated manufacturing TFP growth, $\frac{1}{4}^{\text{a}}$, into $\frac{1}{2}$, TFP growth in these industries attributable to a reduction in direct materials, labor, and capital inputs; and into $\frac{3}{4}$, TFP growth in goods industries attributable to a reduction in indirect materials, labor, and capital inputs as embodied in the service inputs into goods industries. The main contributor to consolidated TFP growth was the reduction in the use of direct material inputs within manufacturing (including labor and capital). The average annual rate of change in $\frac{1}{2}$ was 1.05 percent over the 1947-1958 and 1958-1967 periods, 0.26 percent over the 1967-1977 period, 0.91 percent over the 1977-1987 period, and 0.88 over the 1987-1996 period. Though the time path is similar for $\frac{1}{2}$ as for $\frac{1}{4}^{\text{a}}$; the changes between periods are smaller for $\frac{1}{2}$ than for $\frac{1}{4}^{\text{a}}$. Even in the productivity slowdown period, 1967-1977, TFP growth from the change in direct material inputs was still proceeding at a respectable 0.3 percent per year.

The major difference was in the time trend of $\frac{3}{4}$, the rate of TFP growth due to the material inputs embodied in direct manufacturing service inputs. During the 1958-1967 period, this component added 0.15 percentage points per year to consolidated TFP growth in manufacturing. However, during the 1967-1977, 1977-1987, and 1987-1996

periods, $\frac{3}{4}$ was negative, at a steady -0.1 percent per year. These results indicate that while (direct) materials, labor, and capital inputs per unit of output in manufacturing industries fell over the three time periods, their indirect materials, labor, and capital inputs (as embodied in their service inputs) per unit of output increased over both the 1967-1977 and 1977-1987 periods, thus creating a drag on productivity growth within the manufacturing industries. Manufacturing was more successful in reducing its direct material, labor, and capital inputs than in decreasing its indirect inputs from the service sectors. The U.S. economy suffered from the Baumol (1967) disease, that is the low productivity growth of the service sectors.

In Section 2, we broke down $\frac{3}{4}$ into $\frac{3}{4}^1$, productivity growth in the consolidated goods industries emanating from a reduction in direct service inputs, and $\frac{3}{4}^2$, productivity growth in the consolidated goods industries emanating from technical change in the service industries which supply the goods industries, is again revealing. During the 1947-1958 period, of the 0.11 percentage points decrease in $\frac{3}{4}$, almost all was due to the increased use of direct service inputs into manufacturing ($\frac{3}{4}^1 = -0.08$). During the 1958-1967 period, of the 0.15 percentage points increase in $\frac{3}{4}$, 0.09 percentage points came from a reduction in direct service inputs into manufacturing and 0.06 percentage points from positive TFP growth within the service industries. Between 1967 and 1977, of the -0.09 percentage points change in $\frac{3}{4}$, -0.22 percentage points was attributable to the increased use of direct service inputs in manufacturing ($\frac{3}{4}^1$) and 0.13 percentage points to the modest gains of TFP growth within the service industries. Over the 1977-1987 period, of the -0.10 percentage points change in $\frac{3}{4}$, -0.03 percentage points came from increased use of service inputs and -0.06 percentage points from the negative TFP growth of the service industries supplying manufacturing.

The results show that the TFP growth of service industries which supply manufacturing declined from 0.13 percent per year in the 1967-1977 period to -0.06 percent per year in the 1977-1987 period. This decline of intermediate services TFP growth indicates that manufacturing kept the Baumol disease at bay in the 1977-1987 period. A manufacturing firm looks "smart" in terms of TFP growth if it outsources these stagnant service activities and concentrates its activities on high productivity fabrication and assembly operations. During the 1980's, part of the recorded gains in TFP growth in manufacturing is thus attributable to the outsourcing of services. Table 4 shows that "smart" sectors in the recovery period of 1977-1987 are printing and publishing ($\frac{1}{4}_i = 0.52$ but $\frac{3}{4}_i^2 = 0.16$), drugs, cleaning & toilet preparations ($\frac{1}{4}_i = 1.52$, $\frac{3}{4}_i^2 = -0.36$), and aircraft and parts ($\frac{1}{4}_i = 0.41$ and $\frac{3}{4}_i^2 = -0.18$).

To examine the relationship between TFP growth in manufacturing and the supplying service industries, we compute the correlation between standard TFP growth $\frac{1}{4}_i$ and $\frac{3}{4}_i^2$; the TFP growth of the supplying services. A negative correlation means that goods sectors with rapid rates of standard TFP growth are more likely, on average, to purchase services from sectors with low productivity growth. This is an indirect indication that these manufacturing industries have been successful in outsourcing services with low productivity growth.

The correlation coefficients, shown in the last line of Table 4, indicate that this selection mechanism was not the driving force behind the manufacturing recovery in the 1977-87 period (0.039), but in fact happened in the recent 1987-96 period ($\hat{\rho}$ 0.483). The latter period is the period of major outsourcing of U.S. manufacturing indeed. It is interesting to notice that high productivity performers have been relatively successful at outsourcing sluggish services. The industrial reorganization of the economy between manufacturing and services is a new factor that explains the continuation of the manufacturing productivity recovery.

A related study by Siegel and Griliches (1992) looked at the relation between manufacturing productivity growth in 1973-1979 and 1979-1986 and the outsourcing of services, measured as the average ratio of purchased services within manufacturing to manufacturing output in 1977 and 1982. They found very weak correlations between the latter measure and manufacturing productivity growth in each of the two periods as well as the change in productivity growth between these two periods across 392 manufacturing industries. We obtain similar results for the 1977-1987 and 1987-1996 periods (correlations of 0.00 and 0.10, respectively).

5 Concluding Remarks

The results of this paper support our major hypothesis that the recovery of standard TFP growth in manufacturing in the 1977-1987 period was related to an increased use of inputs purchased from service industries. TFP growth in manufacturing, after falling from 1.14 percent per year in the 1958-1967 period to 0.04 in the 1967-1977 period, jumped to 0.87 percent per year in the 1977-1987 period and 0.72 percent per year in the 1987-1996 period. Over the same period, the share of total service inputs in gross output in constant dollars increased moderately, from 15.0 percent in 1958 to 16.7 percent in 1977 and 18.4 percent in 1996.

Our most compelling evidence in support of our hypothesis is that the recovery in annual

manufacturing TFP growth between 1967-1977 and 1977-1987 (0.83 percentage points) was greater than the increase in consolidated TFP growth (0.65 percentage points). In other words, if we include the material inputs embodied in the services purchased by manufacturing in the manufacturing industry's input structure, the measured increase in TFP growth is smaller than conventionally measured TFP growth, in fact by one ...fth in the 1977-1987 period. The U.S. economy suffered from the Baumol (1967) disease. The recovery lasted in the 1987-1996 period, when high productivity performers in manufacturing have been relatively successful at outsourcing sluggish services.

Both sets of ...ndings are consistent with the argument that the outsourcing of services was partly responsible for the recovery of conventionally measured TFP growth in manufacturing during the 1980s. The results also support the argument that manufacturing industries have been successful at externalizing the slow productivity growth service activities.

It is difficult to say whether the increased service inputs into manufacturing industries were due strictly to outsourcing – that is, the substitution of externally produced output for internally provided inputs – since we have no way of assessing the production of intermediate services within the industry. However, the increase of service inputs within manufacturing during the 1977-1987 period was probably a result of a combination of outsourcing and the substitution of service activities in general for material inputs.

What is clear is that service inputs into manufacturing industries have increased in importance over time and have created an increasingly large drag on both their consolidated productivity growth and even their direct productivity growth. Manufacturing industries have been much more successful in reducing their direct material, labor, and capital inputs per unit of output than in reducing their service inputs.

Footnotes

^a Tilburg University, the Netherlands, and New York University, respectively. We would like to express our gratitude to the C.V. Starr Center for Applied Economics at New York University, CentER at Tilburg University, and the Alfred P. Sloan Foundation which provided financial support for this work. We also acknowledge useful suggestions by two anonymous referees.

1. Actually, in the modern literature this idea can be traced back to Stigler (1951), who defined "externalization" or "unbundling" as referring to the portion of intermediate demand for services which is supplied by service firms, rather than in the production unit itself.
2. If the increasing share of service inputs in gross output was due only to the rising relative prices of services (relative to goods), then we would expect that the service share would increase in current dollar terms but remain unchanged in constant dollar terms. Since the service share rises in constant dollars as well, this reflects a real shift in input structure, rather than a change in relative prices. Quality adjustment would reinforce this observation.
3. According to this rule, standard TFP growth amounts $\prod_{i=1}^n \frac{p_i x_i^\alpha}{p^1 y^1}^{1/\alpha}$. For a derivation see Wolpin (1985). The weights sum to $\frac{p x^\alpha}{p^1 y^1}$, the Domar factor. The calculus of TFP growth holds for non-constant returns to scale, provided that the index of returns to scale is constant. For an analysis see ten Raa (1995, chapter 12).
4. See, for example, U.S. Industry Economics Division (1974), for a discussion of methodology and for a listing for the sectors. This method creates artificial transactions. A formula for the transfer based input-output coefficient is given by Kop Jansen and ten Raa (1990) who also show that the method distorts the material and financial balance equations of input-output analysis. As a result, the method can distort the measurement of productivity growth in both industries i and j . Moreover, it can also affect the measurement of linkages between sectors. The bias in TFP growth associated with alternative coefficients constructions is small in ten Raa and Wolpin (1991), where the nature of the distortions is exposed.
5. A description of the 1967 tables can be found in U.S. Interindustry Economics Division (1974); of the 1977 tables in U.S. Interindustry Economics Division (1984),

and of the 1987 tables in Lawson and Teske (1994). The 1967 data were not published as separate make and use tables, but the raw data for them are available on computer tape, which Paula Young of BEA graciously supplied to us. No published documentation is yet available for the 1996 tables.

6. We refer to the aggregated sector as the trade sector.
7. To balance the flow tables, we adjusted the value added of the trade sector so that its total inputs equalled its new output total and adjusted both the value added of the real estate sector and the real estate input row so that the value of total output and inputs of the real estate sector matched.
8. Data on hours worked by sector, though the preferable measure of labor input to employment, are not available by sector and year and therefore could not be incorporated.
9. In addition, the deflator for transferred imports was calculated from the NIPA import deflator, that for the Rest of the World industry was calculated as the average of the NIPA import and export deflator, and the deflator for the inventory valuation adjustment was computed from the NIPA change in business inventory deflator. The source is U.S. Council of Economic Advisers (1999).
10. The allocation of the scrap sector was handled differently in the make-use framework of the 1967, 1977, and 1987 tables. See ten Raa and Wolpin (1991) for details.
11. Sector numbers refer to the standard BEA 85-sector classification scheme. See, for example, U.S. Interindustry Economics Division (1984) for details. Although transportation, communications, and utilities are traditionally classified as services, their output is more easily measurable than that of other services and their productivity performance over time more closely mirrors that of the other goods industries rather than the other services. See Baumol, Blackman, and Wolpin (1989), Chapter 6, for further discussion.

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Table 1. Sectoral Total Factor Productivity Growth by Major Sector, 1947-1996^a (Percent per Annum)

	1947-1958	1958-1967	1967-1977	1977-1987	1987-1996	1947-1996
Agriculture	1.82	0.87	1.78	1.36	1.42	1.47
Mining	-0.95	0.12	-1.75	-2.42	1.19	-0.82
Construction	0.08	0.92	-0.85	0.58	-0.43	0.05
Manufacturing	0.96	1.14	0.04	0.87	0.72	0.74
Transportation	-0.22	3.29	2.62	1.34	1.10	1.57
Communication	2.09	1.94	2.31	-1.35	0.10	1.04
Utilities	3.32	1.61	-1.11	-1.16	0.36	0.64
Wholesale and Retail Trade	0.34	1.25	1.58	1.74	1.14	1.19
Finance, Insurance Real Estate	1.61	0.65	2.28	-2.23	-0.59	0.38
Other services	-0.09	-0.21	-0.22	-0.33	0.40	-0.10
Weighted Average ^b	0.73	0.93	0.39	0.33	0.53	0.58
GDP ^c	1.41	1.77	0.71	0.58	0.96	1.08

a. Periodization and calculations are based on U.S input-output data. See equation (5) for the definitions of sectoral TFP growth.

b. Weighted average of sectoral rates of TFP growth, where current dollar gross output (GDO) by sector is used as the weight.

c. Economy-wide TFP growth is computed by weighting sectoral TFP growth by GDO/GDP (as GDP represents current dollar net output only).

Table 2. Ratio of Service Inputs to Gross Output in U.S. Manufacturing, 1947-1996^a

Year	All Service Inputs ^b		All Service Inputs Except Trade		All Service Inputs Exc. Trade & Utilities	
	Ratio in Current \$	Ratio in 1992 \$	Ratio in Current \$	Ratio in 1992 \$	Ratio in Current \$	Ratio in 1992 \$
	1947	0.096	0.120	0.071	0.099	0.064
1958	0.119	0.150	0.085	0.120	0.076	0.104
1963	0.113	0.138	0.084	0.113	0.073	0.095
1967	0.122	0.150	0.093	0.125	0.082	0.106
1972	0.139	0.161	0.099	0.126	0.088	0.107
1977	0.134	0.167	0.091	0.124	0.075	0.102
1982	0.163	0.177	0.110	0.120	0.086	0.095
1987	0.154	0.162	0.103	0.110	0.085	0.093
1992	0.181	0.181	0.121	0.121	0.102	0.102
1996	0.184	0.184	0.118	0.123	0.103	0.109

- a. Calculations are based on U.S. input-output data. See Section 3 for a discussion of data sources and methods.
- b. Service inputs are defined as (sector numbers refer to the standard 85-sector BEA classification scheme): (65) transportation and warehousing; (66) communications; (67) broadcasting; (68) utilities; (69) wholesale and retail trade; (70) finance and insurance; (71) real estate and rentals; (72) hotels and repair services; (73) business service; (74) eating and drinking establishments; (75) auto services and repairs; (76) amusements and entertainment; (77) medical and educational services; (78) federal government enterprises; (79) state and local government enterprises; (82) government industry.

Table 3. Decomposition of Manufacturing TFP Growth Into Material Inputs and Service Inputs, 1947-1996.

(Average annual growth in percentage points)

	1947- 1958	1958- 1967	1967- 1977	1977- 1987	1987- 1996
1. Standard TFP Growth [$\frac{1}{4}$]	0.96	1.14	0.04	0.87	0.72
2. Consolidated TFP Growth [$\frac{1}{4}^a$]	0.94	1.20	0.16	0.81	0.81
a. Change in Material Inputs [$\frac{1}{2}$]	1.05	1.05	0.26	0.91	0.88
b. Change in Consolidated Service Inputs [$\frac{3}{4}$]	-0.11	0.15	-0.09	-0.10	-0.07
1. Change in Use of Service Inputs [$\frac{3}{4}^1$]	-0.08	0.09	-0.22	-0.03	-0.16
2. Change in TFP of Service Industries [$\frac{3}{4}^2$]	-0.02	0.06	0.13	-0.06	0.09

Table 4. Standard TFP Growth ($\frac{1}{4}$) and TFP Growth of Service Industries Supplying Manufacturing Industries ($\frac{3}{4}^2$) by Manufacturing Industry, 1958-1987
(Average annual growth in percentage points)

	1947-1958		1958-1967		1967-1977		1977-1987		1987-1996	
	$\frac{1}{4}_i$	$\frac{3}{4}_i^2$	$\frac{1}{4}_i$	$\frac{3}{4}_i^2$	$\frac{1}{4}_i$	$\frac{3}{4}_i^2$	$\frac{1}{4}_i$	$\frac{3}{4}_i^2$	$\frac{1}{4}_i$	$\frac{3}{4}_i^2$
Ordance and accessories	-2.15	-0.18	0.94	0.06	-0.20	0.12	2.48	-0.10	-0.16	0.11
Food and kindred products	1.17	-0.00	0.81	0.06	0.25	0.11	0.59	-0.07	0.49	0.10
Tobacco manufactures	0.54	-0.10	0.41	0.05	-0.46	0.12	-1.13	-0.22	-2.39	0.17
Textile mill products	1.58	-0.01	1.42	0.04	0.86	0.09	1.32	-0.03	0.61	0.08
Apparel and other textile products	2.18	-0.00	1.20	0.05	0.95	0.09	1.35	-0.03	2.93	0.03
Paper and allied products	0.91	-0.02	0.95	0.05	0.05	0.12	0.42	-0.02	0.39	0.08
Printing and publishing	-0.68	-0.12	0.90	0.06	-0.24	0.31	0.52	-0.16	0.17	0.09
Industrial chemicals, agricultural fertilizers and chemicals	2.36	-0.03	1.34	0.04	-1.75	0.13	1.15	-0.10	-1.14	0.10
Plastics and synthetic materials	3.13	-0.07	2.34	0.02	1.77	0.11	1.04	-0.07	0.72	0.09
Drugs, cleaning & toilet preparations	4.27	-0.17	1.76	0.15	1.96	0.29	1.52	-0.36	-3.31	0.25
Paints and allied products	1.37	-0.03	-0.45	0.08	0.48	0.17	1.81	-0.06	-0.25	0.09
Petroleum and coal products	1.73	-0.03	1.69	0.02	-2.04	0.07	1.77	-0.04	-0.78	0.05
Rubber and miscellaneous products	0.91	0.06	1.78	0.07	-0.32	0.12	0.95	-0.08	1.60	0.08
Leather and leather products	1.50	-0.05	0.03	0.05	0.07	0.15	-0.09	-0.06	-0.16	0.07

Lumber and wood products	-1.29	-0.03	1.83	0.06	-1.07	0.10	0.80	-0.01	-0.33	0.09
Furniture and ...xtures	0.67	-0.03	0.53	0.08	0.81	0.15	0.79	-0.09	0.97	0.12
Stone, clay, and glass products	0.05	-0.04	0.96	0.05	-0.49	0.11	0.51	-0.08	1.80	0.06
Primary metal industries	0.47	-0.01	1.06	0.04	-1.48	0.11	-0.02	-0.03	1.68	0.11
Fabricated metal products	0.07	-0.03	0.83	0.05	-0.32	0.13	0.32	-0.04	1.01	0.10
Industrial machinery and equipment, except OCA	0.18	-0.03	1.00	0.07	0.98	0.14	3.11	-0.03	0.69	0.11
Computer and office equipment	1.33	-0.06	0.50	0.11	5.20	0.16	6.82	-0.07	6.36	0.09
Other electronic and electric equipment	1.11	-0.06	1.65	0.08	1.08	0.16	0.65	-0.07	1.27	0.13
Audio, video, and communication equipment	2.77	-0.06	3.10	0.06	0.14	0.19	2.00	-0.08	0.79	0.12
Electronic components and accessoires	2.29	-0.08	2.86	0.07	2.80	0.14	2.63	-0.06	3.33	0.09
Motor vehicles - cars & trucks	0.59	-0.05	1.22	0.06	0.51	0.09	-1.38	-0.02	0.21	0.10
Aircraft and parts	1.88	-0.04	0.64	0.04	-0.13	0.16	0.41	-0.18	2.01	0.06
Other transportation equipment	-0.49	-0.02	1.51	0.06	0.37	0.14	-0.46	0.00	1.53	0.09
Instruments and related products	0.95	-0.06	0.51	0.08	1.87	0.15	2.56	-0.12	0.96	0.11
Miscellaneous manufacturing industries	0.30	-0.01	1.15	0.09	0.83	0.18	0.21	-0.20	1.97	0.14
Correlation ^a between $\frac{1}{4}$ and $\frac{3}{4}$ ²		0.003		-0.088		0.243		0.039		-0.483

a. Correlation across 29 manufacturing sectors. This is a pure number.

Appendix

First let us collect the statements in section 2 that remain to be proved:

A. $L^a = \int_0^1 (I - A^a)^{-1} y^1$

B. $p^1 y^1 = wL^a + rK^a$

C. $\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} I & A \\ 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} y^1 \\ 0 \end{pmatrix} = \begin{pmatrix} I & A^a \\ 0 & 0 \end{pmatrix}^{-1} y^1$

We will do so using

Lemma. $\begin{pmatrix} I & A \\ 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} = \begin{pmatrix} I & \\ 0 & (I - A_{22})^{-1} A_{21} \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1$; where in $\begin{pmatrix} I & \\ 0 & 0 \end{pmatrix}^{-1}$; the entries are the unit and zero matrices of dimensions m and $n - m$; respectively.

Proof of Lemma. It suffices to show that

$$\begin{pmatrix} I & A \\ 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} = \begin{pmatrix} I & \\ 0 & (I - A_{22})^{-1} A_{21} \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1$$

Now the left hand side is

$$\begin{aligned} & \begin{pmatrix} I & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} I & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} I & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix}^{-1} \begin{pmatrix} I \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} I & A^a \\ 0 & 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1 \end{aligned}$$

which is the right hand side indeed.

Q.E.D.

Proof of A. By definition of L^a ; (1), and the lemma,

$$\begin{aligned} L^a &= \int_0^1 \begin{pmatrix} I & A \\ 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} y^1 \\ 0 \end{pmatrix} = \int_0^1 \begin{pmatrix} I & A \\ 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} y^1 \\ 0 \end{pmatrix} = \\ &= \int_0^1 \begin{pmatrix} I & \\ 0 & (I - A_{22})^{-1} A_{21} \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1 = \\ &= \int_0^1 \begin{pmatrix} I & A^a \\ 0 & 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1 = \int_0^1 \begin{pmatrix} I & A^a \\ 0 & 0 \end{pmatrix}^{-1} (I - A^a)^{-1} y^1 \end{aligned}$$

by definition of \tilde{p}^a ; (4).

Q.E.D.

Proof of B. Competitive prices are defined by

$$p(V \mid U) = wL + rK$$

Division by $V \mid$ yields

$$p(I \mid A) = w + rk$$

Hence

$$\begin{aligned} p^1 y^1 &= p \begin{matrix} \tilde{A} \\ \mid \\ 0 \end{matrix} y^1 = (w + rk)(I \mid A)^{i-1} \begin{matrix} \tilde{A} \\ \mid \\ 0 \end{matrix} y^1 = \\ &= (w + rk)(I \mid A)^{i-1} \begin{matrix} \tilde{A} \\ \mid \\ 0 \end{matrix} y^1 = wL^a + rK^a \end{aligned}$$

by definition of L^a ; (1), and similar for capital.

Q.E.D.

Proof of C. The left hand side equals

$$(I \mid 0)(I \mid A) \begin{matrix} \tilde{A} \\ \mid \\ 0 \end{matrix} y^1 = (I \mid 0)(I \mid A^a)^{i-1} y^1$$

by the lemma.

Q.E.D.