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Harrie A.A. Verbon
Cees A. Withagen

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Abstract

In the standard setting a system of tradable permits is effective and cost-efficient in attaining the policy objective of pollution reduction. This outcome is challenged in case of a tradable permit system in a federal system/constitution with individual states having discretionary power regarding environmental policy and where pollution is transboundary across states. This paper explores the opportunities of the central authority to influence the effectiveness and efficiency of the system, under various institutional arrangements, through the initial allocation of permits.

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Harrie A.A. Verbon
Department of Economics
Tilburg University and Center
PO Box 90153
NL – 5000 LE Tilburg
The Netherlands
h.a.a.verbon@uvt.nl

Cees A. Withagen
VU University of Amsterdam,
Tilburg University and Center
PO Box 90153
NL - 5000 LE Tilburg
The Netherlands
c.withagen@uvt.nl

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1. Introduction

In a closed economy, where the government sets the optimal amount of emissions and allocates the corresponding total number of permits to the individual cost-minimizing firms in an arbitrary way, a tradable permits system generates efficiency if perfect competition prevails on all relevant markets[2]. In such an efficient equilibrium the marginal abatement costs are equalized among firms. For a federation-like system of tradable permits Shiell [6] shows that efficiency can also be obtained but then the federal government should be able to set the optimal total amount of pollution as well as the correct initial allocation of permits among the national states. In other words, an arbitrary allocation of permits by the federal government does not generally achieve Pareto efficiency. In Shiell's model national states are atomistic agents that determine national production, taking goods and permits prices as given, and do not have their own tax policy. This is in contrast with Santore et al. [5] who considered national states that acknowledge that, by setting a local environmental tax, they have an impact on the permits price. They prove that, just as in Shiell's model, an arbitrary allocation of permits does not generally lead to Pareto efficiency. However, the existence of a permits allocation generating Pareto-efficiency is not investigated. Unlike Shiell and Santore et al. Ogawa and Wildasin [4], recently claimed that with perfect competition on all markets any arbitrary distribution of permits with welfare maximization by individual nation states will lead to Pareto efficiency.

In this paper we qualify both the inefficiency result by Santore et al. and Shiell and the efficiency result by Ogawa and Wildasin (and Shiell). We highlight the effect of assumptions on the nature of pollution and the degree of competition on both the goods market and the permits market. Regarding the former assumptions, in both Shiell and Ogawa and Wildasin

[2] See [1] and [3] for early results. Hahn [2] shows that with one firm having market power on the permits market, efficiency of a TEP system is violated in a world where firms aim at cost minimization. This can be corrected by the initial permit distribution, an idea we will use in the sequel as well.

spillovers are of a symmetric global nature, while in the model of Santore et al. spillovers are asymmetric among heterogeneous states. We consider a model where individual states, as in Santore et al. and Ogawa and Wildasin, maximize state welfare by setting an optimal local emission tax, (possibly) taking the effect of their decision on the permits price into account. The federal government issues the emission permits and aims to maximize federal welfare, composed of states' welfare. We compare the outcome of this process with the efficient solution, where the federal government can allocate consumption, production and abatement to the individual states as well as impose transfers. We investigate the conditions under which the efficient allocation is realized in the interaction between consumers, producers, states and the federal government. Firms may have an abatement technology at their disposal and can have market power on the goods market. The latter assumption is made to investigate which deviations from atomistic behavior by both firms and states are allowed without harming the federal government's capacity to attain the first best. State welfare is specified as quasi-linear where welfare is linear in money and non-linear in the final good and pollution from production in the own state and abroad. Apart from the analytical convenience it delivers this specification implies that the issue of the socially efficient allocation of production, consumption and pollution can be separated from the issue of redistribution between states. More specifically, transfers between states, organized by the federal government, do not affect the efficient allocation. This can be compared with Shiell [6] who considers lump-sum transfers between states to be infeasible.

Given this set up we show that, provided the federal government has enough information on states' preferences and the states' production technologies, it can set the permit allocation to the states in such a way that the federal first-best is realized. This result, therefore, confirms the claim made by Shiell that a Pareto-efficient allocation of permits can be found. However, this result holds in more general circumstances than those considered by

Shiell. In particular, it also holds for the case of asymmetric pollution spillovers and national states that act strategically on the permits markets, the model considered by Santore et al. Moreover, perfect competition on the goods market is not a necessary prerequisite for this result either.

However, given the possibly sizable amount of information the federal government needs to have in order to replicate the first-best by issuing permits in the proper way, we explore next whether the initial permit allocation does matter. In this exercise we consider how (imperfect) competition on the goods and permits market, having an abatement technology or not and different specifications of the pollution damage affect our results. Ogawa and Wildasin assumed that pollution within a state is determined by emissions generated within the state itself and the federation-wide amount of emissions, wherever it originates. Moreover, they assumed the absence of an abatement technology. For this case we can show that if the number of states gets large, so that each government (and, as a consequence, each firm) is small relative to the size of the permits market and the goods market, as assumed by Ogawa and Wildasin, then the permits allocation becomes irrelevant in the limit and the efficient allocation is realized. If firms can abate pollution the same result holds. Notice that Shiell in a more simple setting than ours claimed that an arbitrary allocation will not lead to Pareto-effciency, the reason for this result being that the Samuelson-condition for the optimal production of the polluting good will not automatically be satisfied if national states do not impose a pollution tax in their won state.

For more general damage functions, e.g., the case where the emission damage is asymmetric among states, such as with SO2 emissions, however, we demonstrate that although the permits allocation is again irrelevant for a large number of states, the first-best is not guaranteed. This holds whether abatement by individual firms is possible, or not.

In the sequel we start in Section 2 by presenting a model that captures the essential features of the models used by Ogawa and Wildasin [4] and Santore et al. [5]. Then we derive several general results. In section 3 we consider some special cases. Section 4 concludes.

2. Efficiency

2.1 The model

Consider an economy consisting of n $(n > 1)$ states. In each state i there is a firm producing a consumer good y_i, at a cost $C_i(y_i)$. Interstate trade of the consumer good is allowed for but aggregate federal net exports are zero. The amount of pollution generated in state i depends on production. For simplicity it is taken equal to production itself. Net emissions from production in jurisdiction i, denoted by ξ_i, can be lower due to abatement, $y_i - \xi_i \geq 0$. Abatement costs in state i are $H_i(y_i - \xi_i)$. Consumers in each state have preferences defined over consumption z_i of the good, reflected in $U(z_i)$, net emissions, reflected in the damage function $D_i(\xi_1, \xi_2, ..., \xi_n)$, and money m_i, which equals the value of net exports minus production costs and abatement costs plus net revenues from selling permits and transfers received from (or paid to) the federal government: $m_i = py_i - pz_i - C_i(y_i) - H_i(y_i - \xi_i) + \tau(x_i - \xi_i) + T_i$. Here p is the market price of the final commodity, τ is the permit price, x_i is the amount of permits allocated to jurisdiction i, and T_i is the transfer to state i from the federal government. The functions involved are assumed to obey the usual conditions such as concavity/convexity and differentiability. Welfare is decreasing in net pollution from all states and increasing in both other arguments. It is additively separable[3] in the three arguments: $W_i = m_i + U(z_i) - D_i(\xi_1, \xi_2, ..., \xi_n)$. State welfare can then be written as:

$$(1) \quad \begin{aligned} W_i &= \{U(z_i) - pz_i + \varphi_i \xi_i\} + \{py_i - C_i(y_i) - H_i(y_i - \xi_i) + \tau(x_i - \xi_i) - \varphi_i \xi_i\} \\ &- D_i(\xi_1, \xi_2, ..., \xi_n) + T_i. \end{aligned}$$

where φ_i is the state pollution tax, which is fully recycled to the consumers. Hence state social welfare consists of the sum of consumer surplus and producer surplus minus pollution damage plus transfers.

2.2. State welfare maximization

State i's consumers maximize utility subject to their budget constraint, leading to

$$(2) \quad U_i^{'}(z_i) = p$$

Define $y = \Sigma_i y_i$. It follows from (2) that $z_i = z_i(p)$ and hence $y(p) = \sum_i z_i(p)$. We can therefore write $p = p(y)$. Firm profits in state i are equal to $\Pi_i = p(y)y_i - C_i(y_i) - \varphi_i \xi_i - \tau(\xi_i - x_i) - H_i(y_i - \xi_i)$. On the final goods market firms compete in a Cournot fashion. With an interior solution necessary conditions for profit maximization read

$$(3) \quad p'(y)y_i + p(y) = C_i^{'}(y_i) + H_i^{'}(y_i - \xi_i)$$

$$(4) \quad H_i^{'}(y_i - \xi_i) = \tau + \phi_i$$

where primes denote derivatives. Interiority includes positive abatement: $y_i - \xi_i > 0$. We assume that either this holds for all firms, or that abatement technologies are absent altogether. In the latter case, called the *no abatement case*, we have

[3] Ogawa and Wildasin [4] don't assume separability, but this higher degree of generality is not needed to qualify their results. Quasi-linearity of the welfare functions is also an assumption made for convenience. With more general preferences the point we want to make holds a fortiori.

(5) $p'(y)y_i + p(y) = C_i^{'}(y_i) + \tau + \phi_i$

The federal government issues a total of $\hat{\xi}$ permits. Hence, equations (3)-(4) together with $\hat{\xi} = \Sigma_i \xi_i$ constitute $2n+1$ equations. The unknowns are $(z_i, y_i, \xi_i, \varphi_i)\ i = 1,2,...,n$ and τ. Under mild regularity conditions we can use the implicit function theorem to write all variables as functions of the state pollution taxes: $y_i(\varphi_1, \varphi_2, ..., \varphi_n)$, $\xi_i(\varphi_1, \varphi_2, ..., \varphi_n)$, $p(\varphi_1, \varphi_2, ..., \varphi_n)$, $\tau(\varphi_1, \varphi_2, ..., \varphi_n)$, $z_i = z_i(\varphi_1, \varphi_2, ..., \varphi_n)$ for $i = 1,2,...,n$, the latter from $U_i^{'}(z_i) = p(y)$. For the no abatement case we get a similar result.

In pursuing optimal state welfare government i maximizes W_i, defined in equation (1), by choosing an optimal emission tax, thereby taking the emission taxes by all other states, as well as consumer and producer behavior described by (3)-(4) or (5) as given. The first-order condition for state welfare maximization reads:

$$(U_i^{'} - p)\frac{\partial z_i}{\partial \varphi_i} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial \varphi_i}(y_i - z_i) + p\frac{\partial y_i}{\partial \varphi_i} - C_i^{'}\frac{\partial y_i}{\partial \varphi_i} - H_i^{'}(\frac{\partial y_i}{\partial \varphi_i} - \frac{\partial \xi_i}{\partial \varphi_i})$$

(6) $$-\tau\frac{\partial \xi_i}{\partial \varphi_i} - \frac{\partial \tau}{\partial \varphi_i}(\xi_i - x_i) - \sum_{j=1}^{n} D_{ij}\frac{\partial \xi_j}{\partial \varphi_i} = 0$$

where $D_{ij} = \partial D_i / \partial \xi_j$.

2.3 Social welfare maximization

We investigate the conditions under which state behaviour, described by equation (6), implies efficiency. As *a benchmark* we consider the case where the federal government can determine the allocation of consumption, production, net emissions and money to the individual states taking into account that total production equals total consumption and that net money transfers should be zero. Aggregate federal welfare is an increasing function of state welfare: $W(W_1, W_2, ..., W_n)$ with

$$W_i = U_i(z_i) - D_i(\xi_1, \xi_2, ..., \xi_n) - C_i(y_i) - H_i(y_i - \xi_i) + T_i$$

Social welfare W is maximized subject to $\Sigma_i y_i = \Sigma_i z_i$ and $\Sigma_i T_i = 0$. The latter condition implies that the transfers account for all interstate monetary transfers. Suppose that the maximization has an interior solution. Then it is straightforward to see that the following holds, with λ the ratio of the Lagrangian parameters corresponding with the output constraint and the net transfers.

(7) $U_i^{'}(z_i) = \lambda$

(8) $\lambda = C_i^{'}(y_i) + H_i^{'}(y_i - \xi_i)$

(9) $H_i^{'}(y_i - \xi_i) = \sum_{j=1}^{n} D_{ji}(\xi_1, \xi_2, ..., \xi_n)$

So, the sum of marginal production and abatement cost is equal across states and this sum equals marginal utility. Moreover, for each state marginal abatement costs equal marginal total damage inflicted. Note that it is not necessarily true that marginal abatement costs are equalized across states. They will be equal, for example, if the damages depend on aggregate emissions so that the right-hand side of (9) independent of i (see Shiell [6]). We denote the solution by $(\hat{z}_i, \hat{y}_i, \hat{\xi}_i), i = 1, 2, ..., n$ and define $\hat{z} = \Sigma_i \hat{z}_i$, $\hat{y} = \Sigma_i \hat{y}_i$ and $\hat{\xi} = \Sigma_i \hat{\xi}_i$. In the sequel this allocation will be called the efficient allocation, assuming it is unique. We avoid the expression first-best, because the distribution of welfare over the states still has to be taken care of by the transfers[4].

[4] The first-best is obtained if money is redistributed across states until the marginal social welfare of transfers is equalized, i.e. $\partial W / \partial W_i = \partial W / \partial W_j$.

Equation (6) shows a relationship between state consumption, production and emissions, on the one hand, and permits allocation, on the other hand, as a result of welfare maximization at the state level. In certain circumstances the federal government can now solve (6) for the unique distribution of permits, $\hat{x}_i$ say, that replicates the efficient allocation. This result is in line with Shiell's result in a more simple setting than ours where national states are price takers in all markets [6].

Notice, however, that in order to replicate the efficient allocation the federal government is in need of a sizable amount of information on cost, utility and damage functions and on the characteristics of markets. Another qualification regarding equation (6) concerns the assumption that states will pass on all their allocated permits to the firms within their state. If they withhold a certain amount of permits, the federal government is no longer able to reach the efficient allocation. In particular, the federal government is then no longer able to affect local taxes in the desired way by the distribution of permits. It can be shown that only if the states have market power on the product market, they may have an incentive to withhold a certain amount of permits (proof available upon request).

Obviously, when an arbitrary allocation of permits leads to the efficient allocation the information burden for the federal government will be greatly diminished. Moreover, the first-best distribution of welfare can then be taken care of by the permits allocation. Therefore, in the next section we address the question when permit allocations matter.

3. The (ir)relevance of the permits allocation

In this section we consider several cases that may lead to a simplification of equation (6) and we explore especially whether in these cases an arbitrary allocation of permits will generate efficiency. A necessary condition to that end is that total net emission is set at its efficient value $\xi = \hat{\xi}$, which is henceforth assumed. Note at the outset that although we leave open the possibility of imperfect competition on the output market, from the analysis below it will appear that in all cases firms are not able to affect prices on the final goods market nor on the permits market. If no abatement technology is available the former result follows from the fact that if the federal government fixes the total amount of permits, the total production and, therefore, consumption is set as well. This, in turn, determines the price through equation (2). It might be that firms are unaware that the price is set by the decisions of the federal government, so that they still can behave as Cournot competitors on the goods market, even though they cannot affect the goods price *de facto*. In the presence of abatement price-taking behavior is necessary for the efficient allocation to be implemented. If we compare (3) and (4) with (7) and (8), we notice that imperfect competition on the goods market and efficiency together require $y_i = 0$ for all i, which cannot be efficient. In the case of abatement we therefore have to assume perfect competition on the goods market from the outset. The assumption of price-taking behavior by firms on the permits market is a necessary consequence of the possibility of strategic behavior by the states on this market. We assume that the firms take state behavior and, therefore, prices on the permits market as given.

3.1. No abatement and a special damage function

Suppose there is no abatement technology: $H_i \equiv 0, i = 1,2,...,n.$ Hence $y_i \equiv \xi_i, i = 1,2,...,n$ and $y = \hat{\xi}$, implying that no state government, nor any individual firm can *de facto* manipulate the final goods price p. But, as indicated above, we still allow for firms *perceiving* imperfect competition. Assuming away abatement surely leads to a more tractable version of the allocation mechanism, but is not enough for getting the result that the permit allocation is immaterial in reaching efficiency. So, we make another simplifying assumption: the damage function can be written as $D_i(\xi_1, \xi_2, ..., \xi_n) = \tilde{D}_i(\xi_i, \xi)$ with $\xi = \Sigma_i \xi_i$.[5] This is what we mean by symmetry. Define $\kappa_i = 1/\{p'(\hat{\xi}) - C_i^{''}\}$, $i = 1,2,...,n$ and $\kappa = \Sigma_i \kappa_i$. It follows from (5) with $y = \hat{\xi}$ that $0 = d\hat{\xi} = \Sigma_i dy_i = \Sigma_i \kappa_i (d\varphi_i + d\tau)$. Therefore $\frac{\partial y_i}{\partial \varphi_i} = \frac{\kappa_i}{\kappa}(\kappa - \kappa_i)$ and $\frac{\partial \tau}{\partial \varphi_i} = -\frac{\kappa_i}{\kappa}$.

Inserting these (perceived) effects of the local tax into the first-order condition (6) and taking account of the fact that the state knows that it cannot affect the second argument of the damage function, $\hat{\xi}$, gives:

(10) $$x_i = y_i + (p(\hat{y}) - C_i^{'} - \tau - \tilde{D}_{i1})(\kappa - \kappa_i)$$

The prices on the goods market and the permits market can be solved from (10) by using $y = \Sigma_i y_i = \Sigma_i \hat{x}_i = \hat{\xi}$. It follows that

$$p - \tau = \frac{\sum_{i=1}^{n}(C_i^{'} + \tilde{D}_{i1})(\kappa - \kappa_i)}{(n-1)\kappa}$$

We insert this back into (10) to arrive at

(11) $$x_i = y_i + (\kappa - \kappa_i)\left\{\frac{\sum_{j=1}^{n}(C_j^{'} + \tilde{D}_{i1})(\kappa - \kappa_j)}{(n-1)\kappa} - (C_i^{'} + \tilde{D}_{i1})\right\}$$

So, even in this very simple setting the permit allocation matters, due to the possibility of individual states to manipulate the permits market. A specific permit allocation is then necessary to restore efficiency. Consider then the case where the number of states goes to infinity. This implies that states are no longer able to manipulate the price on the permits

[5] Ogawa and Wildasin assume that deposits within state i can be written as $d_i = a(1-\beta)\xi_i + a\beta\Sigma_j \xi_j$ where the constants a and β are not state specific.

market so that perfect competition prevails on both goods and permits market. As $n \to \infty$ we have $\kappa = \sum_i \kappa_i \to \infty$, and, therefore, from (10)[6]:

(12) $\quad 0 = p - C_i^{'} - \tau - \tilde{D}_{i1}$, $i = 1,2,...,n$

Hence $C_i^{'} + \tilde{D}_{i1} = C_j^{'} + \tilde{D}_{j1}$, $i \neq j$, which has to hold under efficiency. All variables are at their efficient values and the permits price is such that the externality caused by the international spillover is corrected, i.e. $\hat{\tau} = \Sigma_j \tilde{D}_{j2}(\hat{\xi}_j, \hat{\xi})$. That $\tau = \hat{\tau}$ can be shown as follows. Suppose, without loss of generality, that $y_1 = \xi_1 > \hat{y}_1 = \hat{\xi}_1$. Then $p - \tau = C_1^{'}(y_1) + \tilde{D}_{i1}(y_1, \hat{\xi}) > C_1^{'}(\hat{y}_1) + \tilde{D}_{i1}(\hat{y}_1, \hat{\xi})$. But then it follows that also $p - \tau = C_j^{'}(y_j) + \tilde{D}_{ij}(y_j, \hat{\xi}) < C_j^{'}(\hat{y}_j) + \tilde{D}_{ij}(\hat{y}_j, \hat{\xi})$ for some $j \neq 1$. This contradicts that $C_j^{'}(\hat{y}_j) + \tilde{D}_{ij}(\hat{y}_j, \hat{\xi}) = C_h^{'}(\hat{y}_h) + \tilde{D}_{ih}(\hat{y}_h, \hat{\xi})$ for all j and h. Given that in addition $\hat{y}$ is efficient, we conclude that the permit allocation doesn't matter.

This confirms Ogawa and Wildasin's result that the federal government needs only to determine the 'proper' aggregate amount of emissions, to achieve the efficient allocation [4]. This total amount leads to the appropriate emission price τ, whereas individual states 'repair' the local externality through their own emission tax and, moreover, cannot free ride. Notice that we do get an efficient allocation, as defined in section 2.3, and, due to the quasi-linearity of the welfare functions this is the allocation of consumption, production and emissions for all possible weights in the social welfare function that the federal government might attach to the individual states. However, there is only one set of weights that allows for zero transfers. Hence, for arbitrary weights the social optimum still requires lump sum transfers.

We can now understand why Shiell [6] found that if national states take all prices as given only one distribution of permits leads to Pareto-efficiency. First, in contrast with Ogawa

[6] An alternative way to this result is to take $\partial \tau / \partial \phi_i = 0$, $i = 1,2,...,n$.

and Wildasin and our model the national states in Shiell do not have a pollution tax policy of their own. Given symmetric pollution, national tax policy in Ogawa and Wildasin provides the correction to the spillovers. The second difference is that Shiell assumes that lump-sum transfers between states are infeasible. That means that the allocation of the permits has the double function of providing efficiency and equity. Even if the weights in the social welfare are such that no transfers are needed, the efficient allocation might not be attainable. This is the case if the first-best permit allocation includes negative permits for some states. Shiell [6] and Santore et al., [5] consider this infeasible, because negative permits are considered as a lump sum transfer. As we will see later, one can come across cases where negative permit allocations cannot be excluded. Notice that in the case studied by Ogawa and Wildasin permits are interpreted as production factors [4]. Obviously this assumption rules out a non-positive allocation of permits, so that efficiency may no longer be achieved in their case as well.

3.2 Abatement and a special damage function

Let us maintain the damage function of the previous example but allow for abatement. As demonstrated before, in this case perfect competition on the goods market has to be assumed from the outset. However, contrary to the previous example, the final goods price is not solely determined through the total permit allocation $\hat{\xi}$ and can therefore be manipulated by the individual states[7], like the permit price. But already in the previous example we needed the number of states to go to infinity in order for the states to take the permit price as given. In the case at hand this assumption is obviously needed again to arrive at the allocation of permits being immaterial for efficiency so that p and τ can be assumed as given by states and firms alike. The first-order condition for state welfare maximization reads:

[7] Note that perfect competition as perceived by firms does not yet imply that states are price takers.

$$p\frac{\partial y_i}{\partial \varphi_i} - C_i^{'}\frac{\partial y_i}{\partial \varphi_i} - H_i^{'}(\frac{\partial y_i}{\partial \varphi_i} - \frac{\partial \xi_i}{\partial \varphi_i}) - \tau\frac{\partial \xi_i}{\partial \varphi_i} - \tilde{D}_{i1}\frac{\partial \xi_i}{\partial \varphi_i} = 0$$

Using profit maximization by firms, in particular $p = C_i^{'} + H_i^{'}$, the first-order condition for the state reduces to $H_i^{'} - \tau - \tilde{D}_{i1} = 0.$ This yields again efficiency with the permits price set at the efficient value. To see this, suppose $d\tau = \tau - \hat{\tau} < 0$ and, without loss of generality, that $d\xi_1 = \xi_1 - \hat{\xi}_1 < 0.$ Then $H_1^{''}(dy_1 - d\xi_1) = d\tau + \tilde{D}_{111}d\xi_1 < 0$, where $\tilde{D}_{111}$ is the second derivative of the damage function with respect to the first argument. Therefore $dy_1 < 0.$ Moreover $dp = C_1^{''}dy_1 + H_1^{''}(dy_1 - d\xi_1) < 0$. Hence $dz_i > 0$ for all i. So, $\Sigma_i dy_i > 0$ and $dy_2 + dy_3 ... + dy_n > -dy_1 > -d\xi_1 = d\xi_2 + d\xi_3 ... + d\xi_n$ from which it follows that $d(y_2 - \xi_2) + d(y_3 - \xi_3) ... + d(y_n - \xi_n) > 0$. Suppose, without loss of generality $d(y_2 - \xi_2) > 0$. Then $0 > dp = C_2^{''}dy_2 + H_2^{''}d(y_2 - \xi_2)$. Hence $dy_2 < 0$ and $d\xi_2 < 0$. But $0 < H_2^{''}d(y_2 - \xi_2) = d\tau + D_{211}d\xi_2 < 0$, a contradiction. Therefore $d\xi_i = \xi_i - \hat{\xi}_i \geq 0$ for all i, but this can hold only with equality. Hence $\tau = \hat{\tau}$. So, allowing for abatement under the special damage function, price taking coupled with an efficient amount of total permits issued leads to efficiency.

3.3. General damage functions with abatement and perfect competition on all markets

In the case of a general damage function an arbitrary allocation of permits will not lead to the efficient allocation if abatement is possible even if perfect competition on both markets prevails. Perfect competition implies that the states perceive p and τ not to depend on their action φ. Hence, equation (6) reduces to

$$(p - C_i^{'} - H_i^{'})\frac{\partial y_i}{\partial \varphi_i} - (H_i^{'} - \tau - D_{ii})\frac{\partial \xi_i}{\partial \varphi_i} - \sum_{j \neq i} D_{ij}\frac{\partial \xi_j}{\partial \varphi_i} = 0$$

The first term vanishes due to profit maximization. Moreover, we have $\partial \xi_j / \partial \varphi_i = 0$ for all $i \neq j$. This follows from the fact that from (3) and (4) we have $d\xi_i = -\mu_i d\tau - \mu_i d\varphi_i$, with $\mu_i = (1/C_i^{"}) + (1/H_i^{"})$ and $\mu = \Sigma_i \mu_i$, implying from $d\tau = 0$ and $\Sigma_i d\xi_i = 0$ that $d\xi_i = -\mu_i d\varphi_i$. Hence $H_i^{'} - D_{ii} = \tau$ for all i. This can only yield efficiency if $\tau = \sum_{j \neq k} D_{jk}(\hat{\xi}_1, \hat{\xi}_2, ..., \hat{\xi}_n)$ for all k which would be a coincidence. Hence, the special functional form of the damage function is indispensable.

3.4. General damage: a numerical example

If no abatement is allowed, an arbitrary allocation of permits under a general damage function will not lead to the efficient allocation either. We illustrate this with a specific example where social welfare equals the sum of states welfare and where $C_i = \frac{1}{2} c y_i^2$ for all *i* and $D_i = \frac{1}{2} \beta y_1^2$ for all *i>1* with $D_1 = 0$. So, we have identical cost functions, but asymmetric pollution damage with state 1 the only state that is emitting. For this specific case the efficient production values obey $\hat{y}_1 < \hat{y}_i$ for $i > 1$. Notice that the total efficient pollution fixes total production and, therefore, given individuals' preferences the goods price. Nevertheless, firms assume that they can affect the price as they act like Cournot competitors with *p'≠0.* We have $\kappa_i = \kappa_j \equiv \kappa^* < 0$ for all i and *j*. For state 1 equation (6) reduces to $x_1 = \hat{y}_1 \left(1 + (n-1)\kappa^* \beta \{(n-1)^2 - 1\} / n\right)$. Notice that, provided that $n > 2$, state 1 will get less permits than its efficient production. Obviously, for all other states the reverse holds: they get more permits than their first-best production level. As a result, they will sell the permits that are on top of their efficient production levels to state 1. Moreover, given $n > 2$, if β is large enough, it is optimal to allocate a negative amount of permits to state 1. Hence, state 1 is forced to buy all the permits it needs, but in addition it has to pay an 'entrance fee' before it can enter the permits market. The state government determines the tax rate by maximizing state welfare, given the allocated amount of permits. However, as the federal government sets the permit allocation such that efficient production and prices are realized, the tax rate can be calculated from the necessary condition for profit maximization, i.e. from $\varphi_i = p'(\hat{y})\hat{y}_i + p(\hat{y}) - C_i^{'}(\hat{y}_i) - \hat{\tau}$. For state 1 this reduces to $\varphi_1 = p'(\hat{y})\hat{y}_1 + (\hat{x}_1 - \hat{y}_1)/\{(n-1)\kappa^*\}$. This expression is positive under perfect competition on the goods market, or if the price effect $p'(\hat{y}) < 0$ is small enough. For the other states the optimal tax rate reads $\varphi_i = p'(\hat{y})\hat{y}_i +$ $\left((\hat{x}_i - \hat{y}_i) - \beta\kappa^* \hat{y}_1\right)/\{(n-1)\kappa^*\}$ and this will certainly be negative (remember $\hat{y}_i < \hat{x}_i$). So, these states subsidise production, while the polluting state may tax domestic production. It does so in order to restrict pollution which will lead to a cost saving due to less demand for permits. If the number of states goes to infinity, state behaviour is again independent of the permits allocation as we saw in earlier cases. From state welfare maximization $p(\hat{y}) - C_i^{'}(\hat{y}_i) - \hat{\tau} \to 0,$ so that all states, in the limiting case of perfect competition on the permits market and goods market , will not impose any tax on home production, i.e. $\varphi_i = 0, i = 1,2,...,n,$ whatever the amount of emitted pollution by state 1. Obviously, this cannot generally lead to the first-best allocation.

3.5. Equal states

If states are identical in all respects it immediately follows from (6) that the only permit allocation that yields the first-best outcome is $x_i = \hat{\xi}_i, i = 1,2,...,n$. Hence, the permit allocation matters.

4. Conclusion

We have developed a simple model of emission control. It has been shown that only in very special circumstances the permit allocation is immaterial for reaching the efficient outcome in a federal state. For instance, take the case where pollution has the uniformly mixing characteristic of global warming. A world-wide tradable permits market with an arbitrary allocation of permits across states will generate first-best social welfare only if perfect competition prevails on all markets and the states are neither able to manipulate the product price nor the permits price. If one of these conditions is not met a first-best allocation is not warranted. Global warming implies a very special damage function as the damage is independent of the origin of emissions. This does not hold for other forms of pollution. Acid rain is an example of pollution that implies a damage function that does not only depend on the total amount of pollution, but also on the location of pollution. If in such a case, states are manipulating the price on the permits market and/or the goods market, and the federal government has all the necessary information on cost functions and market characteristics, the efficient allocation can be attained by the correct permit allocation, provided the states issue all the permits they get from the central government. However, an arbitrary allocation of permits will generally not lead to efficiency even if perfect competition on all markets prevails and the states take both the product and the permits price as given.

We derived our results by assuming that the federal government is able to determine the optimal total amount of emissions. If the federal government issues an arbitrary total amount of emissions social welfare maximization with the arbitrary number of permits as a constraint, leads to amended efficiency conditions. Performing the same analysis as in section 3 we can derive that 'permits constrained' efficiency (see Santore et al. who introduced the concept [5]) is obtained under our special damage function $D_i(\xi_1, \xi_2, ..., \xi_n) = \tilde{D}_i(\xi_i, \xi)$ only with price taking consumers, producers and states. Asymmetric damage functions or imperfect competition on the goods market or the permits market, generally does not imply permits constrained efficiency.

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