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SETTING INCENTIVES: TEMPORARY PERFORMANCE PREMIUMS VERSUS PROMOTION TOURNAMENTS

Volker Meier

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CESifo
Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany
Tel.: +49 (89) 9224-1410
Fax: +49 (89) 9224-1409
e-mail: office@CESifo.de

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Abstract

Two alternative relative compensation schemes are compared with respect to total output that can be generated at a given sum of salaries. While the promotion regime guarantees that any salary increase is permanent, the premium system allows a reduction in the income of an agent to the base salary after one period. It is shown that the optimum promotion tournament system induces a higher total output than the optimum premium system. This result occurs because a promotion regime allows distortion in a contest in favor of winners of previous contests.

Keywords: Tournament, relative compensation, internal labor markets

JEL Classification: D23, J33, J41, M12

Volker Meier
CESifo
(University of Munich and ifo Institute)
Poschingerstr. 5
81679 Munich
Germany
e-mail: meier@ifo.de

1 Introduction

One of the most prominent types of incentive setting in organizations is the use of promotion rules where workers compete against each other. In the personnel management literature since the seminal paper by Lazear and Rosen (1981), this type of incentive scheme is known as a tournament. In such a tournament, the winner, whose output has been perceived to be higher than the output of his competitors, receives a high prize, while the others will get lower prizes. The relative compensation system can induce high effort levels, but is also accused for generating anticooperative behavior among workers (Lazear (1995), ch. 2). Surveys of the tournament literature have been provided by McLaughlin (1988), Prendergast (1999), and Gibbons and Waldman (1999).

A plausible alternative is to base salaries just on performance in the recent past where prizes are paid only on a temporary basis. Such a system shall be introduced, for example, as a compensation scheme for university professors in Germany. Payment depends on an evaluation, where the period between two evaluations shall stretch between five and seven years. A reduction of the salary as a consequence of a negative evaluation is possible at least in some instances (Bundesministerium für Bildung und Forschung (2000)).

This paper addresses the question whether or not changing from the promotion tournament scheme to paying premiums according to recent performance can increase output. The view taken here is that following the principle of payment according to recent performance necessarily means that contests must be fair – at least within a generation. In contrast, the main function of prizes that are paid to the winners of promotion tournaments is to motivate younger workers. Hence, they are generally unrelated to actual present or past productivities, and the contest design may be unjust.

The literature dealing the construction of sequential contests is still quite limited. Rosen (1986) considers an elimination tournament where players are paired in each round and losers have to quit the tournament. He shows that prizes that have to be paid to the winners of matches have to increase over time in order to keep effort constant. The current contribution bears some similarity to Meyer (1992) who analyzes a sequence of two contests between two agents. The message of her paper is that the organization should bias the winning probability in the second round in favor of the winner in the first round. A similar result turns out if the employer wants to find out which worker has a higher level of ability (Meyer (1991)).

In the model presented in section 2, the two alternative compensation schemes are introduced. The premium system is also regarded as a tournament, where, however, prizes are only paid on a temporary basis. Output depends on effort in a stochastic fashion. In every period, three generations of workers are present in the organization. Initially, all workers of a generation are identical. In both systems, the employer chooses the prize structure and the winning probabilities of the workers. Under a promotion system, both can depend on the worker's promotion record.

Section 3 discusses the impacts of parameter changes on workers' efforts. Increasing the differential between the winners' expected salary and the losers' expected compensation always raises the effort of workers in the corresponding contest. In contrast, the impact of raising the probability of winning on effort depends on the structure of the stochastic productivity shock.

It is demonstrated in section 4 that the promotion tournament will induce a higher output at a given sum of salaries. This can be explained as follows. A promotion tournament can always be designed that imitates the optimal premium system. However, it turns out that incentives for young workers can be improved by distorting the contest in the middle generation in favor of those who receive their first promotion early. Since the premium system relies on fair contests within a generation, biasing the contest is only possible by using a promotion system. Section 5 concludes.

2 The model

Following the basic structure of Malcomson's (1984) tournament model, we consider a continuum of workers in every generation. In a given period, three generations of workers are found in the organization, and all workers stay in the organization for three periods. Each generation of workers is of equal size, which is normalized to unity. In his first period a worker receives a salary I_1 and has to decide on his effort e_1. His cost of providing effort, reflecting the disutility of labor, is given by $C_1(e_1)$, where $C_1'(e_1) > 0$ and $C_1''(e_1) > 0$ hold. Thus, the marginal disutility of labor increases in effort. Two incentive regimes are distinguished. In the premium system (system P) the probability π_1 of receiving the higher salary I_{2P} in the next period depends on the observed effort in the current period. However, effort is not perfectly observed. Let X_1 denote observed effort in period 1 where $X_1 = e + Z_1$. The random variable Z_1 is described by the density function $f(z)$. The variable X_1 can be interpreted as an output measure, and Z_1 may be viewed as good luck (high z) or bad luck (low z). All individual values of Z_1 are independently drawn from the same distribution.

The cutoff level which has to be exceeded for obtaining the premium in period 2 is denoted by x^*_{1P}. Assuming that all workers of a generation choose the same effort level, the probability of receiving the higher salary in the second period is given by $\pi_1 = 1 - F(x^*_{1P} - e_{1P})$, with F representing the distribution function of Z_1 and e_{1P} denoting the chosen effort level. Hence, the worker receives I_{2P} in period 2 with probability $\pi_1(e_{1P})$ and I_1 with probability $1 - \pi_1(e_{1P})$. Exerting a higher effort has a positive impact on getting the higher income in the next period, i. e. $\pi'_1(e_{1P}) > 0$. In the second period the worker has to decide on his effort e_2. The associated cost he has to bear is given by $C_2(e_2)$, where again $C'_2 > 0$ and $C''_2 > 0$ hold. No restrictions are set with respect to the evolution of the cost of providing effort. While aging should increase the disutility of effort, implying that $C_1(e) < C_2(e)$ for given e, the accumulation of experience works in the opposite direction.

Again, the observed signal is subject to an idiosyncratic stochastic shock Z_2, where Z_1 and Z_2 are identically and independently distributed. In the third period, the worker receives the salary I_{3P} with probability $\pi_2(e_{2P})$ and the salary I_1 with probability $1 - \pi_2(e_{2P})$ with e_{2P} denoting the chosen effort level. Given that the marginal cost of providing effort in the third period is still positive, every worker chooses the minimum effort level $e_3 = 0$ in the last period. For simplicity, all workers are supposed to be risk neutral. There is no discounting, and the interest rate is set to zero.

In system P, the worker's decision problem in the second period is to maximize

$$V_{2P} = \pi_2(e_2)I_{3P} + (1 - \pi_2(e_2))I_1 - C_2(e_2)$$

with respect to e_2 irrespective of the salary he receives in period 2. The first-order condition for an interior maximum is

$$\pi'_2(e_{2P})(I_{3P} - I_1) - C'_2(e_{2P}) = 0.$$

Due to our assumptions, the first period problem is analogous. Since V_{2P} does not depend on choices in the first period, the first-order condition for an optimum first period effort level reads

$$\pi_1'(e_{1P})(I_{2P} - I_1) - C_1'(e_{1P}) = 0.$$

Since we assume the number of agents to be large, we can ignore strategic interactions. Every worker maximizes expected utility taking the actions of other workers as given. Then we have $\pi_1'(e_{1P}) = f(x_{1P}^* - e_{1P})$. The cutoff level for obtaining the premium is given from the perspective of a worker. Therefore, optimal choices depend on salaries and on the cutoff level. Hence, we have $e_{1P}(I_1, I_{2P}, x_{1P}^*)$ and $e_{2P}(I_1, I_{3P}, x_{2P}^*)$. The sufficient second-order condition for an interior maximum e_{1P} is

$$-f'(x_{1P}^* - e_{1P})(I_{2P} - I_1) - C_1''(e_{1P}) < 0.$$

From the employer's point of view, setting π_i determines x_{iP}^* where $\pi_i = \int_{x_{iP}^* - e_{iP}}^{\infty} f(y)dy$ implies $\dfrac{\partial x_{iP}^*}{\partial \pi_i} = -\dfrac{1}{f(x_{iP}^* - e_{iP})} < 0$. A higher winning probability lowers the cutoff level for receiving the higher prize. Since the random output component Z cannot be influenced, the employer aims at maximizing total effort at a given sum of salaries. Thus, the employer maximizes his objective function

$$W_P = e_{1P}(I_1, I_{2P}, \pi_1) + e_{2P}(I_1, I_{3P}, \pi_2)$$

with respect to I_{2P}, I_{3P}, π_1 and π_2 subject to the budget constraint $B - [\pi_1 I_{2P} + \pi_2 I_{3P} + (3 - \pi_1 - \pi_2)I_1] \geq 0$, where B represents the employer's budget for salaries. The conception of maximizing output subject to a budget constraint seems to be a bit odd for private enterprises since only the profit maximizing budget will be chosen. However, the formulation suits well to other types of employers, as, for example, non-profit organizations or the government.

In order to simplify the analysis, the base salary level I_1 is fixed. Base salary levels are often determined by some institutional arrangement outside the control of the organization, e.g. a centralized wage bargaining procedure. In contrast, it is usually easy to change winning probabilities, premiums and higher income levels. Given that the base salary is sufficiently high, neither participation constraints for workers nor minimum wage requirements need to be discussed.

The first-order conditions for an interior maximum are given by

$$\frac{\partial W_P}{\partial I_{2P}} = \frac{\partial e_{1P}}{\partial I_{2P}} - \lambda_P \pi_1 = 0, \quad (1)$$

$$\frac{\partial W_P}{\partial I_{3P}} = \frac{\partial e_{2P}}{\partial I_{3P}} - \lambda_P \pi_2 = 0, \quad (2)$$

$$\frac{\partial W_P}{\partial \pi_1} = \frac{\partial e_{1P}}{\partial \pi_1} - \lambda_P (I_{2P} - I_1) = 0, \quad (3)$$

$$\frac{\partial W_P}{\partial \pi_2} = \frac{\partial e_{2P}}{\partial \pi_2} - \lambda_P (I_{3P} - I_1) = 0, \quad (4)$$

$$\frac{\partial W_P}{\partial \lambda_P} = B - [\pi_1 I_{2P} + \pi_2 I_{3P} + (3 - \pi_1 - \pi_2) I_1] = 0, \quad (5)$$

where λ_P is the Lagrange multiplier associated with the budget constraint of the organization.

Raising the winners' salaries will affect the effort level of workers in the respective contest. At the same time, the sum of salaries to be paid at given winning probabilities will rise. Increasing any of the two winning probabilities has an impact on the behavior of the employees in the respective contest. However, the sum of premiums to be paid also goes up. The Lagrange multiplier λ_P measures how much effort can be generated if the budget of the organization is increased by one unit.

We suppose that an interior solution to the organization's optimization problem exists. It is not necessary that this solution is unique. Note that an interior solution requires $\frac{\partial e_i}{\partial \pi_i} > 0$ for $i \in \{1, 2\}$.

In the promotion tournament regime (system T) the promotion probability p_1 at the end of the first period depends on e_{1T}, the effort exerted in the first period. As above, $\frac{\partial p_1}{\partial e_{1T}} > 0$ holds. Promoted workers receive I_{2T} in the second period, while the wage of the others remains at I_1. In the second period, the worker chooses his effort according to his respective position. Should he has already been promoted, he exerts an effort level of e_{22}. His conditional probability of getting the highest salary I_{3T} in the third period is $q(e_{22})$ with $q' > 0$. With conditional probability $1 - q(e_{22})$ no second promotion occurs, resulting in a salary I_{2T} in the third period. If the worker receives I_1 in the second period, he chooses his effort e_{21}. His conditional probability of being promoted and obtaining I_{21T} in the third period is $p_2(e_{21})$ with $p_2' > 0$. Otherwise, he will stick to his salary I_1 in the third period. As above, setting incentives in the third period is impossible, which implies $e_3 = 0$ regardless of the promotion record of the worker. The optimal strategy of a worker at given promotion probabilities and salary levels can be derived as follows: If a promotion has occurred in the first period, the objective is to maximize

$$V_{22T} = q(e_{22})I_{3T} + (1 - q(e_{22}))I_{2T} - C_2(e_{22})$$

with respect to e_{22}. The necessary first-order condition in case of an interior solution is

$$(I_{3T} - I_{2T})q'(e_{22}) = C_2'(e_{22}).$$

Again, every worker maximizes expected utility taking the actions of other workers as given. Then we have $q'(e_{22}) = f(x_{3T}^* - e_{22})$, where x_{3T}^* denotes the cutoff output for a second promotion. Since the cutoff level for promotion is given from the perspective of a worker, optimal choices depend on wage rates and the cutoff level. Hence, we have $e_{22}(I_{2T}, I_{3T}, x_{3T}^*)$. The sufficient second-order condition for an interior maximum is

$$-f'(x_{3T}^* - e_{22})(I_{3T} - I_{2T}) - C_2''(e_{22}) < 0.$$

A worker who has not been promoted in the first period maximizes

$$V_{21T} = p_2(e_{21})I_{21T} + (1 - p_2(e_{21}))I_1 - C_2(e_{21})$$

with respect to e_{21}. The optimality condition associated with an interior solution is

$$(I_{21T} - I_1)p_2'(e_{21}) = C_2'(e_{21}).$$

Since every worker maximizes expected utility taking the actions of other workers as given, it follows that $p_2'(e_{21}) = f(x_{2T}^* - e_{21})$. Optimal choices depend on wage rates and the cutoff level for a late first promotion, x_{2T}^*. Hence, we have $e_{21}(I_{21T}, I_1, x_{2T}^*)$. The sufficient second-order condition for an interior maximum is

$$-f'(x_{2T}^* - e_{21})(I_{21T} - I_1) - C_2''(e_{21}) < 0.$$

In his first period, the worker's objective is to maximize

$$V_{1T} = p_1(e_1)(I_{2T} + \tilde{V}_{22T}) + (1 - p_1(e_1))(I_1 + \tilde{V}_{21T}) - C_1(e_1)$$

with respect to e_1 where $\tilde{V}_{22T}$ and $\tilde{V}_{21T}$ are related to maximized values. The first-order condition associated with an optimum e_{1T} is

$$(I_{2T} + \tilde{V}_{22T} - (I_1 + \tilde{V}_{21T}))p_1'(e_{1T}) = C_1'(e_{1T}).$$

Again, $p_1'(e_{1T}) = f(x_{1T}^* - e_{1T})$ holds, with x_{1T}^* representing the crucial level for an early promotion. The effort choice of a young worker is determined by all salary levels and all cutoff levels, that is $e_{1T}(I_1, I_{2T}, I_{21T}, I_{3T}, x_{1T}^*, x_{2T}^*, x_{3T}^*)$. The sufficient second-order condition for an interior maximum is

$$-f'(x_{1T}^* - e_{1T})(I_{2T} + \tilde{V}_{22T} - (I_1 + \tilde{V}_{21T})) - C_1''(e_{1T}) < 0.$$

The budget of the organization, B, is the same under both regimes. Thus,

$$B = I_1(3 - \pi_1 - \pi_2) + \pi_1 I_{2P} + \pi_2 I_{3P}$$

and

$$\begin{aligned} B &= I_1(1 + (1-p_1) + (1-p_1)(1-p_2)) \\ &\quad + I_{2T}(p_1 + (1-q_1)p_1) + I_{21T}p_2(1-p_1) + I_{3T}p_1 q \end{aligned} \tag{6}$$

In the promotion tournament regime, the employer maximizes total effort with respect to $I_{2T}, I_{3T}, I_{21T}, p_1$, p_2, and q. The Lagrangian is given by

$$\begin{aligned} W_T &= e_{1T}(I_1, I_{2T}, I_{21T}, I_{3T}, p_1, p_2, q) + p_1 e_{22}(I_{2T}, I_{3T}, q) \\ &\quad + (1-p_1)e_{21}(I_1, I_{21T}, p_2) + \lambda_T \Big[B - p_1 q I_{3T} \\ &\quad - [p_1 + p_1(1-q)]I_{2T} - (1-p_1)p_2 I_{21T} \\ &\quad - [1 + (1-p_1) + (1-p_1)(1-p_2)]I_1 \Big], \end{aligned} \tag{7}$$

where λ_T denotes the Lagrange multiplier associated with the employer's budget constraint. The first-order conditions in case of an interior solution are

$$\frac{\partial W_T}{\partial I_{2T}} = \frac{\partial e_{1T}}{\partial I_{2T}} + p_1 \frac{\partial e_{22}}{\partial I_{2T}} - \lambda_T [p_1 + p_1(1-q)] = 0, \tag{8}$$

$$\frac{\partial W_T}{\partial I_{3T}} = \frac{\partial e_{1T}}{\partial I_{3T}} + p_1 \frac{\partial e_{22}}{\partial I_{3T}} - \lambda_T p_1 q = 0, \tag{9}$$

$$\frac{\partial W_T}{\partial I_{21T}} = \frac{\partial e_{1T}}{\partial I_{21T}} + (1-p_1)\frac{\partial e_{21}}{\partial I_{21T}} - \lambda_T(1-p_1)p_2 = 0, \tag{10}$$

$$\frac{\partial W_T}{\partial p_1} = \frac{\partial e_{1T}}{\partial p_1} + e_{22} - e_{21} - \lambda_T[qI_{3T} + [1+(1-q)]I_{2T} - p_2 I_{21T} - [1+(1-p_2)]I_1] = 0, \tag{11}$$

$$\frac{\partial W_T}{\partial p_2} = \frac{\partial e_{1T}}{\partial p_2} + (1-p_1)\frac{\partial e_{21}}{\partial p_2} - \lambda_T(1-p_1)(I_{21T} - I_1) = 0, \tag{12}$$

$$\frac{\partial W_T}{\partial q} = \frac{\partial e_{1T}}{\partial q} + p_1\frac{\partial e_{22}}{\partial q} - \lambda_T p_1(I_{3T} - I_{2T}) = 0, \tag{13}$$

$$\frac{\partial W_T}{\partial \lambda_T} = B - p_1 q I_{3T} - [p_1 + p_1(1-q)]I_{2T} - (1-p_1)p_2 I_{21T} - [1+(1-p_1)+(1-p_1)(1-p_2)]I_1 = 0. \tag{14}$$

The conditions can be interpreted as follows. Raising the salary of early promoted workers, I_{2T}, increases both the winning prize in the contest among young workers and the prize of those who fail to achieve a second promotion. At the same time, the sum of salaries increases at given strictly positive probability of early promotion. A higher salary after a second promotion, I_{3T}, increases the winning prize both in the contest among promoted workers and in the contest among young workers. Should both promotion probabilities p_1 and q be strictly positive, the sum of salaries will rise. Increasing the salary after a late first promotion, I_{21T}, raises the prize differential in the contest among the middle-aged losers of the first round, but reduces the prize differential in the contest among young workers. The sum of salaries increases if $p_1 < 1$ and $p_2 > 0$.

Raising the probability of a promotion of a young worker, p_1, has several impacts. First, it changes the effort level of young workers, as captured in $\frac{\partial e_{1T}}{\partial p_1}$. Second, by increasing the number of promoted workers in the second generation, it raises total effort in the second generation by $e_{22} - e_{21}$. Third, at given promotion probabilities q and p_2, the total labor cost rises due to higher number of employees receiving the higher salaries.

Increasing the probability of promotion of previously non-promoted workers, p_2, will induce a higher output by non-promoted workers, captured by $(1-p_1)\frac{\partial e_{21}}{\partial p_2}$, but also change the effort levels of young workers according to

$\frac{\partial e_{1T}}{\partial p_2}$. At the same time, the sum of salaries will increase due to a higher share of old workers receiving I_{21T} rather than I_1. Lowering the probability of a second promotion, q, affects both the effort level of young workers, represented by $\frac{\partial e_{1T}}{\partial q}$, and the effort level of promoted workers in the middle generation, being expressed by $p_1 \frac{\partial e_{22}}{\partial q}$. At the same time, total salaries will decrease since more old workers will receive I_{2T} rather than I_{3T}.

The Lagrange multiplier λ_T expresses how much additional effort will be achieved by increasing the budget of the organization by one unit of money.

3 Comparative statics

Lemma 1 summarizes the workers' reactions to changing incentives under a premium scheme.

Lemma 1 *Effort increases in the probability of obtaining the temporary premium, π_i, $i \in \{1, 2\}$ if and only if $f'(x^*_{iP} - e_{iP}) < 0$ holds. A higher premium always raises effort in the respective contest.*

Proof: Due to the implicit function theorem,

$$\frac{\partial e_{iP}}{\partial \pi_i} = -\frac{\frac{\partial^2 V_{iP}}{\partial e_{iP} \partial x^*_{iP}} \frac{\partial x^*_{iP}}{\partial \pi_i}}{\frac{\partial^2 V_{iP}}{\partial e^2_{iP}}}$$

and

$$\frac{\partial e_{iP}}{\partial I_{i+1P}} = -\frac{\frac{\partial^2 V_{iP}}{\partial e_{iP} \partial I_{i+1P}}}{\frac{\partial^2 V_{iP}}{\partial e^2_{iP}}}$$

holds. Since $\frac{\partial^2 V_{iP}}{\partial e^2_{iP}} < 0$ is valid according to the second-order condition, it follows that

$$sgn[\frac{\partial e_{iP}}{\partial \pi_i}] = sgn[-(I_{i+1P} - I_1) \frac{f'(x^*_{iP} - e_{iP})}{f(x^*_{iP} - e_{iP})}] = -sgn[f'(x^*_{iP} - e_{iP})].$$

Moreover,

$$\frac{\partial^2 V_{iP}}{\partial e_{iP}\partial I_{i+1P}} = f(x^*_{iP} - e_{iP}) > 0.$$

□

A higher winning probability decreases the cutoff level for obtaining the respective premium, $I_{2P} - I_1$ or $I_{3P} - I_1$. Should $f'(x^*_{iP} - e^*_{iP}) < 0$ hold, this raises the marginal benefit of effort. Hence, effort will increase. The opposite turns out if $f'(x^*_{iP} - e^*_{iP}) > 0$. A higher premium raises the marginal benefit of effort. Therefore, the workers will respond by increasing their effort level.

Lemma 2 discusses the impacts of parameters on the effort level of middle-aged workers under the promotion tournament regime.

Lemma 2 *The impact of an increase in the promotion probability q on the effort of a promoted worker is positive if and only if $f'(x^*_3 - e_{22}) < 0$ holds. A higher promotion probability p_2 raises the effort of a non-promoted worker if and only if $f'(x^*_{2T} - e_{21}) < 0$ holds.*

Proof: Due to the implicit function theorem,

$$\frac{\partial e_{22}}{\partial q} = -\frac{\frac{\partial^2 V_{22T}}{\partial e_{22}\partial x^*_{3T}}\frac{\partial x^*_{3T}}{\partial q}}{\frac{\partial^2 V_{22T}}{\partial e^2_{22}}}$$

and

$$\frac{\partial e_{21}}{\partial p_2} = -\frac{\frac{\partial^2 V_{21T}}{\partial e_{21}\partial x^*_{2T}}\frac{\partial x^*_{2T}}{\partial p_2}}{\frac{\partial^2 V_{21T}}{\partial e^2_{21}}}$$

hold. Since $\frac{\partial^2 V_{22T}}{\partial e^2_{22}} < 0$ and $\frac{\partial^2 V_{21T}}{\partial e^2_{21}} < 0$ are valid according to the second-order conditions, it follows that

$$\begin{aligned} sgn[\frac{\partial e_{22}}{\partial q}] &= sgn[-(I_{3T} - I_{2T})\frac{f'(x^*_{3T} - e_{22})}{f(x^*_{3T} - e_{22})}] = -sgn[f'(x^*_{3T} - e_{22})], \\ sgn[\frac{\partial e_{21}}{\partial p_2}] &= sgn[-(I_{21T} - I_1)\frac{f'(x^*_{2T} - e_{21})}{f(x^*_{2T} - e_{21})}] = -sgn[f'(x^*_{2T} - e_{21})]. \end{aligned}$$

□

Again, the impact of a higher promotion probability on effort is positive if and only if the marginal utility of providing effort is raised by reducing the cutoff level for promotion. In addition, it is obvious that effort increases in the respective salary differential, i.e. $\frac{\partial e_{21}}{\partial I_{21T}} > 0$, $\frac{\partial e_{22}}{\partial I_{3T}} > 0$, and $\frac{\partial e_{22}}{\partial I_{2T}} < 0$ hold.

Lemma 3 shows how young workers change their behavior as a consequence of changes in the parameters.

Lemma 3 *Raising the probability of a second promotion, q, or lowering the probability of a late promotion, p_2, increase the effort of a young worker. The impact of a rise in the probability of early promotion, p_1, on effort is positive if and only if $f'(x^*_{1T} - e_{1T}) < 0$ holds. The effort level of a young worker rises in I_{2T} and I_{3T}, but decreases in the salary after a late first promotion I_{21T}.*

Proof: Due to the implicit function theorem,

$$\begin{aligned}
\frac{\partial e_{1T}}{\partial q} &= -\frac{\frac{\partial^2 V_{1T}}{\partial e_{1T}\partial q}}{\frac{\partial^2 V_{1T}}{\partial e_{1T}^2}} \\
\frac{\partial e_{1T}}{\partial p_2} &= -\frac{\frac{\partial^2 V_{1T}}{\partial e_{1T}\partial p_2}}{\frac{\partial^2 V_{1T}}{\partial e_{1T}^2}} \\
\frac{\partial e_{1T}}{\partial p_1} &= -\frac{\frac{\partial^2 V_{1T}}{\partial e_{1T}\partial x^*_{1T}}\frac{\partial x^*_{1T}}{\partial p_1}}{\frac{\partial^2 V_{1T}}{\partial e_{1T}^2}} \\
\frac{\partial e_{1T}}{\partial I_{iT}} &= -\frac{\frac{\partial^2 V_{1T}}{\partial e_{1T}\partial I_{iT}}}{\frac{\partial^2 V_{1T}}{\partial e_{1T}^2}}
\end{aligned}$$

hold, where $i \in \{2, 3, 21\}$. Since $\frac{\partial^2 V_{1T}}{\partial e_{1T}^2} < 0$ is valid according to the second-order condition and taking into account the envelope theorem, it follows that

$$\begin{aligned}
sgn[\frac{\partial e_{1T}}{\partial q}] &= sgn[(I_{3T} - I_{2T})f(x_{1T}^* - e_{1T})] > 0, \\
sgn[\frac{\partial e_{1T}}{\partial p_2}] &= sgn[-(I_{21T} - I_1)f(x_{1T}^* - e_{1T})] < 0, \\
sgn[\frac{\partial e_{1T}}{\partial p_1}] &= sgn[-(I_{2T} + \tilde{V}_{22T} - (I_1 + \tilde{V}_{21T}))\frac{f'(x_{1T}^* - e_{1T})}{f(x_{1T}^* - e_{1T})}] \\
&= -sgn[f'(x_{1T}^* - e_{1T})], \\
sgn[\frac{\partial e_{1T}}{\partial I_{2T}}] &= sgn[(2 - q)f(x_{1T}^* - e_{1T})] > 0, \\
sgn[\frac{\partial e_{1T}}{\partial I_{3T}}] &= sgn[qf(x_{1T}^* - e_{1T})] > 0, \\
sgn[\frac{\partial e_{1T}}{\partial I_{21T}}] &= -sgn[p_2 f(x_{1T}^* - e_{1T})] < 0.
\end{aligned}$$

□

Lemma 3 is easily understood. The effort level increases in the prize differential. Raising the probability of a second promotion implies a higher prize for young winners while reducing the probability of a late first promotion lowers the prize for young losers. As before, a higher probability of promoting young workers will not induce a higher effort level if the marginal utility of effort is not increased. While raising either I_{2T} or I_{3T} increases the prize differential, a higher I_{21T} implies a higher prize for losers of the first contest.

4 Strict dominance of promotion

Given that an interior solution to the problem of finding the optimum premium system exists, Proposition 1 shows that a promotion system can be designed that generates a higher output.

Proposition 1 *The promotion regime yields a higher total effort than the premium regime.*

Proof: Note that the optimum premium regime P^* can be replicated by a promotion regime with $p_1 = \pi_1$, $p_2 = q = \pi_2$, $I_{2T} = I_1 + \frac{I_{2P} - I_1}{2}$, $I_{21T} = I_{3P}$, $I_{3T} = I_{2T} + I_{3P} - I_1$. Increasing q at the expense of p_2 such as to keep the total budget constant, i.e $p_1 dq = -(1 - p_1)\, dp_2$, then yields

$$\left.\frac{\partial W_T(P^*)}{\partial q}\right|_{p_1 dq = -(1-p_1)dp_2} = \frac{\partial e_{1T}}{\partial q} - \frac{p_1}{1-p_1}\frac{\partial e_{1T}}{\partial p_2} + p_1 \frac{\partial e_{22}}{\partial q} - p_1 \frac{\partial e_{21}}{\partial p_2}$$
$$= \frac{\partial e_{1T}}{\partial q} - \frac{p_1}{1-p_1}\frac{\partial e_{1T}}{\partial p_2} > 0$$

because $\frac{\partial e_{22}}{\partial q} = \frac{\partial e_{21}}{\partial p_2}$ holds at P^*, while $\frac{\partial e_{1T}}{\partial q} > 0$ and $\frac{\partial e_{1T}}{\partial p_2} < 0$ are valid according to Lemma 3. □

Since the optimum premium system can be imitated, it is trivial that a promotion tournament can be found that weakly dominates the premium system. The proof demonstrates that a fair contest among middle-aged workers, which is implied by a premium system, does not maximize output. Given any optimum premium scheme, output can be raised by increasing the conditional probability of a second promotion and lowering the conditional probability of a late first promotion such that the total budget remains the same. This variation means that the cutoff level for promoted workers decreases while it rises for the losers of the first round. Hence, it is beneficial for the organization to promote some middle-aged workers who have been promoted before although their current output falls short of the output of other workers in the same generation who will stick to the base salary.

The reason for introducing this bias is straightforward. A higher probability of a second promotion stimulates the effort of young workers. In contrast, an increase in the probability of a late first promotion reduces the incentive for young workers. It is therefore rational to distort the contest for middle-aged workers in favor of winners of the first period. This outcome generalizes a similar result in Meyer (1992) derived in a framework with two agents.

The optimum promotion tournament need not show the property that the conditional probability of a second promotion exceeds the conditional probability of a late first promotion. Introducing a bias in favor of young winners can also be achieved by a variation in the prizes that can be earned in the two contests for middle-aged workers. An alternative proof of Proposition

1 may consider a variation of I_3 and I_{21} where $-(1-p_1)p_2 dI_{21} = p_1 q dI_3$ holds. Starting at the promotion tournament that imitates the optimum premium regime, it follows that

$$\begin{aligned}\left.\frac{\partial W_T(P^*)}{\partial I_3}\right|_{p_1 q dI_3 = -(1-p_1)p_2 dI_{21}} &= \frac{\partial e_{1T}}{\partial I_3} - \frac{p_1 q}{(1-p_1)p_2}\frac{\partial e_{1T}}{\partial I_{21}} \\ &\quad + p_1 \frac{\partial e_{22}}{\partial I_3} - \frac{p_1 q}{p_2}\frac{\partial e_{21}}{\partial I_{21}} \\ &= \frac{\partial e_{1T}}{\partial I_3} - \frac{p_1 q}{(1-p_1)p_2}\frac{\partial e_{1T}}{\partial I_{21}} > 0.\end{aligned}$$

Note that $\frac{\partial e_{22}}{\partial I_3} = \frac{\partial e_{21}}{\partial I_{21}}$ and $q = p_2$ are valid at P^*. At the same time, Lemma 3 states that $\frac{\partial e_{1T}}{\partial I_3} > 0$ and $\frac{\partial e_{1T}}{\partial I_{21}} < 0$. Compared to the optimum premium regime, the output of young workers can be increased if the highest salary, I_3, is raised at the expense of the prize paid to workers receiving a late first promotion. The output loss due to a lower effort of losers of the first round is just offset by a higher output of the promoted middle-aged workers if we consider small variations in salary levels.

It is obvious that the argument in favor of the promotion tournament will also apply in other frameworks. In particular, the outcome will be the same if agents display risk aversion. Moreover, the number of evaluation periods does not matter if there are at least two. The result would clearly also hold if it were possible to set incentives for old workers. A premium at the end of the working life would then correspond to an increase in the pension payment.

The workers will generally prefer a premium scheme. While expected lifetime income is identical under both schemes, expected lifetime effort is higher under the optimum tournament regime. The distribution of effort under the premium regime tends to be efficient. Should $C_1(x) = C_2(x)$ hold for any x, the organization will choose a symmetric solution with $\pi_1 = \pi_2$ and $I_{2P} = I_{3P}$. This would imply $C_1'(e_1) = C_2'(e_2)$, i.e. effort would be provided such as to minimize total effort cost. If effort is provided in an efficient fashion under the premium system, the total effort cost to be borne by an individual must be higher under the optimum tournament scheme.

5 Conclusion

It has been shown that a promotion tournament scheme can always be found that achieves a higher output than any given premium scheme. Hence, changing from a promotion regime to a premium system in order to pay according to performance will generally reduce overall performance. It is just the feature of the premium system to establish fair contests within a generation that is responsible for the output loss. Distorting contests in favor of winners of previous rounds induces stronger incentives for younger members of the organization.

The main result will presumably carry over to a situation in which initial abilities differ provided that the number of evaluation periods is sufficiently high. Since we can expect that groups with identical winning history become more homogenous over time, the problem in the final rounds will be similar to the one outlined in the model.

It is evident that no general conclusions can be drawn with respect to efficiency for the economy as a whole. If the output of the organization is of little value, the main effect of choosing the tournament system is the increase in total effort cost. In contrast, if there are substantial positive externalities, the promotion system will be the efficient choice.

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