PIRACY AND COMPETITION

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Abstract

The effects of (private, small-scale) piracy on the pricing behavior of producers of information goods are studied within a unified model of vertical differentiation. Although information goods are assumed to be perfectly horizontally differentiated, demands are interdependent because the copying technology exhibits increasing returns to scale. We characterize the Bertrand-Nash equilibria in a duopoly. Comparing equilibrium prices to the prices set by a multiproduct monopolist, we show that competition drives prices up and reduces total surplus.

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1 Introduction

Over the last decade, the fast penetration of the Internet and the increased digitization of information have turned piracy of information goods (in particular music, movies and software) into a topic of intense debate. Not surprisingly, economists have recently shown a renewed interest in information goods piracy.[1] The recent contributions revive the literature on the economics of copying and copyright, which was initiated some twenty years ago.[2] The seminal papers mainly discussed the effects of photocopying and examined, among other things, how publishers can appropriate indirectly some revenues from illegitimate users (Novos and Waldman, 1984, Liebowitz, 1985, Johnson, 1985, and Besen and Kirby, 1989). The economics of intellectual property (IP) protection was then addressed more generally by Landes and Posner (1989) and Besen and Raskind (1991). Both papers discuss the following trade-off between *ex ante* and *ex post* efficiency considerations. From an *ex ante* point of view, IP protection preserves the incentive to create information goods, which are inherently public (absent appropriate protection, creators might not be able to recoup their potentially high initial creation costs). On the other hand, IP rights encompass various potential inefficiencies from an *ex post* point of view (protection grants de facto monopoly rights, which generates the standard deadweight losses; also, by inhibiting imitation, IP rights might limit the creators' ability to borrow from, or build upon, earlier works, and thereby increase the cost of producing new ideas). A third wave of papers paid closer attention to software markets and introduced network effects in the analysis. Conner and Rumelt (1991), Takeyama (1994), and Shy and Thisse (1999) share the following argument: because piracy enlarges the installed base of users, it generates network effects that increase the legitimate users' willingness to pay for the software and, thereby, potentially raises the producer's profits.

To the best of our knowledge, most models address the *ex-post* issue of piracy in industries with monopolies. In this context, Bae and Choi (2003) interestingly demonstrate that the threat of piracy obliges the monopoly to lower its price, implying that the firm's *ex-post* profit falls whereas the usage of authorized copy increases. This result provides a sharp contrast to the common claim of copyright holders, who assert that piracy reduces the demand for a legal copy. As a result, piracy has positive *ex-post* welfare implications. *Ex-ante* welfare implications crucially depend on how the monopoly's *ex-post* losses caused by piracy affect the *ex-ante* incentives to provide the goods, their quality or their diversity.

This literature on the economics of copying obviously abstracts away the strategic interaction among producers of information goods. It is often argued

[1] See the excellent survey by Peitz and Waelbroeck (2003) and the references therein.

[2] With the notable exception of Plant (1934). For a recent survey (and extension) of this literature, see Watt (2000).

that the degree of horizontal differentiation between information goods (like CDs or books) is so large that one can assume that the demand for any particular good is independent of the prices of other goods. An exception is Johnson (1985): his 'fixed cost model' considers a copying technology that involves an investment in costly equipment. As the author emphasizes, "[a]n interesting feature of this model is that the demand for any particular work is affected *indirectly* by the prices of other works since they affect a consumer's decision to invest in the copying technology". However, because the focus is mainly on the welfare implications of copying, Johnson (1985) does not fully explore the effects of the strategic interaction induced by the fixed cost of copying.

The aim of the present paper is to study the strategic interactions among the producers of information goods. On the one hand, in the spirit of Mussa and Rosen (1978), originals and copies are vertically differentiated: copies are seen as lower-quality alternatives to originals (i.e., if copies and originals were priced the same, all consumers would prefer originals). Copies are produced by users with increasing returns to scale. On the other hand, different information goods are assumed to be perfectly horizontally differentiated: information goods here have independent content as it can be the case of software applications for games and word processor, or CD recordings of classic and pop music. For the sake of the exposition, the paper focuses on duopolies and on cheap copying technologies: copying involves a relatively small fixed cost to users. To our view, this is the simplest set-up allowing us to highlight the most interesting aspects of the interaction between producers of independent information goods.[3]

In this paper, we qualify the traditional results and insights about the impact of copying technology obtained in one-good monopoly settings (e.g., Bae and Choi, 2003 and Yoon, 2002). When there exist more than one information good, increasing returns to scale in the copying technology creates an interdependence between the demands for information goods, which are genuinely independent. Basically, the threat of copying is conveyed through the behaviour of 'switching users', i.e., those users who are likely to change their choice of purchasing originals for the acquisition of a copy machine. As a first result, we show that a multiproduct monopoly may set different prices for its goods. Indeed, when copying fixed costs are low enough, switching users hesitate between buying the two goods and copying both of them. Hence, only the price of the bundle matters which gives some leeway to the pricing of each good. The bundle price is nevertheless set as to accommodate those switching users.

In a two-good duopoly, the interaction between firms leads to interesting properties. A firm's best-response to the price set by the competitor can depict up to four different attitudes. First, when the competitor sets a sufficiently low price, users are not enticed to acquire the copying technology and copying is

[3] In Belleflamme and Picard (2004), we provide a broader analytical perspective by letting the number of producers and the cost of copying take any value.

not a threat. The firm is then able to extract the full surplus by setting a high price. Next, when the competitor sets a higher price, some low valuation users are enticed to switch to copying. The firm can then either avoid these switching users and concentrate on higher valuation users by increasing its price, or it can accommodate the switching users by lowering its price. It turns out that the first strategy is the most profitable one when the competitor's price is not too high. Hence, there exits a range of prices such that prices are strategic complements (i.e., the best response function is increasing in the competitor's action). Moreover, such prices exceed the price set when the competitor quoted a low price in the first instance above. When the competitor further rises its price, the second strategy prevails: the firm reduces its price in order to accommodate the switching users and prices become strategic substitutes. Finally, when the competitor sets quite large prices, the marginal consumer of the firm's good is no longer a switching user who is indifferent between buying the good and buying the copying technology. Instead, the marginal consumer becomes a 'resolute copier', who is resolutely decided to purchase and use the copying technology to copy the competitor's good. The firm can charge only a low price to accommodate such a resolute copier. Because the nature of the marginal buyer suddenly changes as the competitor's price rises, the best response function shows discontinuities and equilibria cannot be guaranteed.

As a second result, we show that equilibria in pure strategies do not exist in duopolies when the fixed cost of copying is low enough. Intuitively, the inexistence of equilibria stems from firms' free-riding behavior with respect to the threat of piracy. If all firms takes this threat seriously and quote low prices to accommodate consumers, then they set too low a price and there exists an opportunity for any individual firm to raise its price while keeping a sufficiently large demand and making a larger profit. Technically, increasing returns to scale in the copying technology introduce non-convexities in the profit functions and undermine the existence of market equilibrium.

Still, an equilibrium in pure strategies exists when the fixed cost of copying is not too small. At this equilibrium, both firms accommodate switching users and quote identical prices. However, we show that the prices in the duopoly are larger than the (average) price of a multiproduct monopoly. Indeed, as presented above, firms loose less by setting higher prices to concentrate on high valuation users rather than by lowering their prices to keep the low valuation users who wish to switch to the copy technology. Furthermore, we show that for some non empty set of parameters, the duopoly prices are larger than the price of a monopolist who faces no threat of piracy. The externality that firms impose on each other can therefore be quite important and it can drastically reduce the demand for legal copies.

Also, we show that a symmetric equilibrium in mixed strategies exists when the fixed cost of copying is small. Each firm quotes two prices with positive probabilities. Interestingly, the expected price in the mixed-strategy equilibrium, though smaller than the price that would prevail in the pure-strategy equilibrium, remains above the average price set by a multiproduct monopolist.

Finally, we perform a welfare analysis. Considering first *ex post* efficiency, we stress that industry concentration is welfare improving in the present context. As is the case in Cournot industries with complementary products, prices are higher in a duopoly than in a multiproduct monopoly. Moreover, we show that policy measures aiming at strengthening IP protection contribute to increase further the welfare gap between monopoly and duopoly. Considering next *ex ante* efficiency, we compare our framework with an economy where only a single information good is available. This exercise allows us to measure the (gross) incentives to create a new information good. Comparing those incentives for an incumbent firm and for an entrant, we reach an ambiguous conclusion: creation incentives are higher for an entrant when the cost of copying is sufficiently small and are higher for an incumbent otherwise. Intuitively, the entrant's incentive is reduced by the free-riding effect observed in a duopoly, whereas the incumbent's incentive is reduced by a cannibalization effect (copying becomes more attractive as the number of goods increases). So, when copying is sufficiently costly, industry concentration is welfare-enhancing both in the short and in the long run.

To sum up, our main message is the following. The interactions between producers of information goods under the threat of piracy dramatically alter the equilibrium outcome compared to the outcome obtained under a one-good monopoly setting. Equilibrium prices in pure strategies may not exist and, if they do, they may be higher than those in the one-good monopoly case. Inferences about dynamics and welfare implications are not obvious anymore in oligopolistic industries. For instance, industry concentration may be welfare improving.

The rest of the paper is organized as follows. In Section 2, we lay out the model and we derive the demand schedule for a particular original. In Section 3, we characterize the two-good monopoly case. In Section 4, we present the two-good duopoly case. In Section 5, we perform a welfare analysis. We conclude and propose an agenda for future research in the last section.

2 Demand for originals

There is a continuum of potential users who can consume at most two information goods. These information goods are assumed to be perfectly (horizontally) differentiated and equally valued by the users. In particular, users are characterized by their valuation, θ, for any information good. We assume that θ is uniformly distributed on the interval $[\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} > 0$.

Each information good is imperfectly protected and thus "piratable". As a result, users can obtain each information good in two different ways: they can either *buy* the legitimate product (an "original") or acquire a *copy* of the product. It is reasonable to assume that all users see the copy as a lower-quality alternative to the original.[4] Therefore, in the spirit of Mussa and Rosen (1978), we posit some vertical (quality) differentiation between the two variants of any information good: letting s_o and s_c denote, respectively, the quality of an original and a copy, we assume that $0 < s_c < s_o$.[5]

As for the relative cost of originals and copies, we let p_i denote the price of original i ($i = 1, 2$) and we assume that users have access to a copying technology with increasing returns to scale. To keep things simple, we assume that to be able to copy, consumers must incur a fixed cost $K > 0$. Finally, we normalize to zero the utility of not consuming an information good.

Table 1 expresses consumer θ's net utility in the nine possible combinations of usages for the two information goods.

	Good 2		
Good 1	Purchased (P)	Copied (C)	Not used (O)
Purchased (P)	$\theta s_o - p_1 + \theta s_o - p_2$	$\theta s_o - p_1 + \theta s_c - K$	$\theta s_o - p_1$
Copied (C)	$\theta s_c - K + \theta s_o - p_2$	$2\theta s_c - K$	$\theta s_c - K$
Not used (O)	$\theta s_o - p_2$	$\theta s_c - K$	0

Table 1: Net utility of a typical consumer

Note that as $\theta s_c > 0\ \forall \theta$, the options (O,C) and (C,O) are dominated by option (C,C). Indeed, increasing returns to scale in copying imply that if a user finds it profitable to copy one original, she is even better off copying both originals.

For the sake of the exposition, we further assume that the fixed cost K is sufficiently low.

A1: *Low copying costs:* $K < \underline{\theta} s_c$.

Under this assumption, all users prefer copying a single original over not using any information good ($\theta s_c - K > 0\ \forall \theta$). This assumption greatly simplifies the exposition while it retains the main properties of the model. Indeed, it follows that options (P,O), (O,P) and (O,O) are dominated for any user. In

[4] This assumption is common (see, e.g., Gayer and Shy, 2003) and may be justified in several ways. In the case of analog reproduction, copies represent poor substitutes to originals and are rather costly to distribute. Although this is no longer true for digital reproduction, originals might still provide users with a higher level of services, insofar as that they are bundled with valuable complementary products which can hardly be obtained otherwise.

[5] Similar models are used by Koboldt (1995) to consider commercial copying and by Yoon (2002) and Bae and Choi (2003) to analyze the market for a single information good.

other words, all users will always consume both goods, either by purchasing the original or by copying it. By eliminating the non-users, the last assumption allows us to restrict the analysis to the four options (P,P), (P,C), (C,P) and (C,C).

Furthermore, we claim that this assumption fits the current characteristics of copying technologies for information goods. It seems indeed that the fraction of users we choose not to consider (i.e., those for whom $\theta s_c - K < 0$) is constantly narrowing, as copying devices become widely and cheaply available and as the "moral barrier" to illegal copying is increasingly fading. The widespread use of copied music and software in least developed countries corroborates this assumption.

The demand function for good i is derived as follows. One can write that a user with type θ buys original i iff

$$\theta s_o - p_i + \max\{\theta s_o - p_j; \theta s_c - K\} \geq \max\{\theta s_o - p_j + \theta s_c - K; 2\theta s_c - K\}. \quad (1)$$

This inequality compares user θ's value of purchasing original i, and either purchasing or copying good j, to the best option available given that he/she does not purchase original i, namely, copying good i whereas either buying or copying good j. This inequality can take three different forms, each form corresponding to a specific category of users.

First, for high valuation users such that $\theta (s_o - s_c) \geq p_j$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_o - p_j \geq \theta s_o - p_j + \theta s_c - K \iff \theta \geq \frac{p_i - K}{s_o - s_c}.$$

Because these users purchase the other original whether they purchase good i or copy it, we call them *'buyers'*. The maximum price they are willing to pay for original i is equal to

$$p_i^b(\theta) = \theta (s_o - s_c) + K.$$

That is, they are willing to pay up to the extra value that an original brings on top of a copy, augmented by the cost of the copying technology (which they save once they decide to buy i instead of copying it).

Second, for intermediate valuation users such that $p_j - K \leq \theta (s_o - s_c) \leq p_j$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_o - p_j \geq 2\theta s_c - K \iff \theta \geq \frac{p_i + p_j - K}{2 (s_o - s_c)}.$$

For these users, the best use of one good depends on the best use of the other good: if they purchase good i, they also purchase good j; if they copy good i, they also copy good j. We therefore call them *'switchers'*. How much are switchers willing to pay for good i? Going from two copies to two originals, they earn twice the extra value of an original compared to a copy, and they trade the cost of the copying technology for the price of the other original. So, their maximum price is given by:

$$p_i^s(\theta, p_j) = 2\theta(s_o - s_c) + K - p_j.$$

Finally, for low valuation users such that $\theta(s_o - s_c) \leq p_j - K$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_c - K \geq 2\theta s_c - K \iff \theta \geq \frac{p_i}{s_o - s_c}.$$

Because these users copy good j no matter what they decide about good i, we call them *'resolute copiers'*. What they are willing to pay for good i is just the extra value of an original compared to a copy (for they have already sunk the cost of the copying technology). Their maximum price is thus equal to:

$$p_i^c(\theta) = \theta(s_o - s_c).$$

The three price functions are depicted in Figure 1. We observe that depending on the price of good j, the price function for good i can have up to two kinks. The price function is (increasing and) concave in θ in the neighborhood of $p_j/(s_o - s_c)$ (which separates switchers from buyers), and (increasing and) convex in θ in the neighborhood of $(p_j - K)/(s_o - s_c)$ (which separates resolute copiers from switchers).

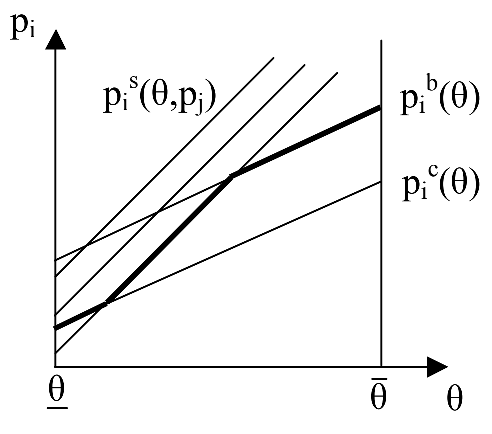

Figure 1: Prices as functions of θ.

As a result, the demand function for good i has three segments and two kinks:

$$D_i(p_i, p_j) = \frac{1}{\overline{\theta} - \underline{\theta}} \times \begin{cases} \left(\overline{\theta} - \frac{p_i - K}{s_o - s_c}\right) & \text{if } p_j + K \leq p_i \text{ (buyers)} \\ \left(\overline{\theta} - \frac{p_i + p_j - K}{2(s_o - s_c)}\right) & \text{if } p_j - K \leq p_i < p_j + K \text{ (switchers)} \\ \left(\overline{\theta} - \frac{p_i}{s_o - s_c}\right) & \text{if } p_i < p_j - K \text{ (resolute copiers)} \end{cases} \tag{2}$$

When p_j is very small, the marginal users are buyers. When p_j is very high, the marginal users always choose to copy good j and they are resolute copiers. When p_j is close to p_i, the marginal users choose to copy according to the value of the bundle, $p_i + p_j$. The demand function has the same concave and convex kink as the price function depicted above.

3 Multiproduct monopoly

The monopoly chooses prices p_1 and p_2 so as to maximize profits:

$$\max_{p_1,p_2} \pi_m = p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2).$$

where demands are given by (2) and where the firm is assumed to have zero production cost.

Suppose w.l.o.g. that $p_1 \leq p_2$. Then, the monopolist gets the following profits according to whether its two prices significantly differ or not:[6]

$$\begin{aligned} \text{either } \max_{p_1,p_2} \pi_m^{(1)} &= p_1\left(\overline{\theta} - \frac{p_1}{s_o - s_c}\right) + p_2\left(\overline{\theta} - \frac{p_2 - K}{s_o - s_c}\right) \text{ s.t. } p_1 \leq p_2 - K \\ \text{or } \max_{p_1,p_2} \pi_m^{(2)} &= (p_1 + p_2)\left(\overline{\theta} - \frac{p_1 + p_2 - K}{2(s_o - s_c)}\right) \text{ s.t. } p_1 \geq p_2 - K. \end{aligned}$$

When prices differ by more than the fixed cost K, the firm faces two groups of consumers: resolute copiers and buyers. When prices are close, the firm faces only switchers. As recorded in the following proposition, it turns out that the monopolist always chooses the latter option.

Proposition 1 *Under low copying cost (**A1**), the multiproduct monopolist sets any price (p_1, p_2) such that $(p_1 + p_2)/2 = p_m \equiv \overline{\theta}(s_o - s_c)/2 + K/4$ and $p_2 - K < p_1 \leq p_2$.*

Proof. The unconstrained solution to the first problem is $p_1 = \overline{\theta}(s_o - s_c)/2$ and $p_2 = \overline{\theta}(s_o - s_c)/2 + K/2$. This solution does not meet the constraint because $p_1 > p_2 - K$. The solution of the first problem is thus the corner solution with $p_1 = p_2 - K$ and profit equal to

$$\pi_m^{(1)} = \frac{\overline{\theta}}{2}\left(\overline{\theta}(s_o - s_c) + K\right).$$

The second problem is equivalent to

$$\max_p \pi_m = 2p\left(\overline{\theta} - \frac{2p - K}{2(s_o - s_c)}\right)$$

[6] Profits are actually multiplied by the constant $\left(\overline{\theta} - \underline{\theta}\right)$, which we forget from now on as it does not affect optimal decisions.

where $p \equiv p_1 + p_2$. Optimal price and profit are easily found as

$$p_m = \frac{\overline{\theta}}{2}(s_o - s_c) + \frac{K}{4} \text{ and } \pi_m^{(2)} = \frac{\left(2\overline{\theta}(s_o - s_c) + K\right)^2}{8(s_o - s_c)}.$$

Noting that $\pi_m^{(2)} - \pi_m^{(1)} = K^2/(8(s_o - s_c)) > 0$ completes the proof. The second problem includes an infinity of prices (p_1, p_2) such that $(p_1 + p_2)/2 = p_m$ subject to the contraint set in this second problem: $p_1 \geq p_2 - K$. ■

The monopolist sells the two goods at prices such that marginal buyers are switchers. Prices are limited upward to avoid that marginal users become resolute copiers. Prices are not unique neither symmetric.

It must be noted that, by assumption **A1**, the monopoly sets an average price p_m which is smaller than the price it would set for each good if there were no threat of copy, namely $\overline{\theta}s_o/2$. Profits are also smaller.

We now turn to the study of the duopoly.

4 Duopoly

Under a duopoly, each information good $i \in \{1, 2\}$ is produced and sold by a separate firm. We proceed in two steps: first, we derive firm i's best reponse and then we compute the Bertrand-Nash price equilibria.

4.1 Best response function

Best response functions are derived from the demand functions (2). Because the demand functions are piece-wise linear and include a convex kink, firms' best response functions are expected to be discontinuous. In fact, the point of discontinuity will take place when marginal users shift from being switchers to resolute copiers. We now characterize the portion of the best response of firm i below and above the discontinuity.

Targeting buyers or switchers? The optimal price and profit on the buyers of good i are equal to

$$\begin{aligned} p_i^{b*} &\equiv \arg\max_{p_i} p_i\left(\overline{\theta} - \frac{p_i - K}{s_o - s_c}\right) = \frac{1}{2}\left(\overline{\theta}(s_o - s_c) + K\right), \\ \text{and } \pi_i^{b*} &= \frac{1}{4}\frac{\left(\overline{\theta}(s_o - s_c) + K\right)^2}{s_o - s_c}. \end{aligned}$$

Firm i's best response is to set $p_i = p_i^{b*}$ as long as the competitor's price does not to entice the marginal consumer to become a switcher. Using (2), this is so as long as

$$p_j \leq p_i^{b*} - K \iff p_j \leq p^f \equiv \frac{1}{2}\left(\overline{\theta}(s_o - s_c) - K\right).$$

Note that $p^f > 0 \iff K < \overline{\theta}(s_o - s_c)$.

For $p_j > p^f$, some low valuation users are enticed to switch to copying. Firm i can either accommodate these switching users by lowering its price, or it can avoid them and concentrate on higher valuation users by increasing its price. On the one hand, when p_j is low enough, firm i sets a 'limit price' to 'deter' switchers. By (2), it sets a price equal to

$$p_i^D(p_j) = p_j + K$$

(or just a small amount below this price) and achieves a corresponding profit of $\pi_i^D(p_j)$. This price is an increasing function of p_j. Since more users tend to switch to the copying technology when the competitor raises its price p_j, firm i must raise its price p_i to avoid the switchers. Hence, there exits a range of prices such that prices are strategic complements.

When p_j gets larger, firm i has no other choice but to accommodate switchers. It sets a price equal to $p_i^{s*}(p_j)$ where

$$p_i^{s*}(p_j) \equiv \arg\max_{p_i} p_i \left(\overline{\theta} - \frac{p_i + p_j - K}{2(s_o - s_c)}\right) = \overline{\theta}(s_o - s_c) + \frac{K - p_j}{2},$$

and where the corresponding profit is equal to

$$\pi_i^{s*}(p_j) = \frac{1}{8}\frac{\left(2\overline{\theta}(s_o - s_c) + K - p_j\right)^2}{s_o - s_c}.$$

This price is a decreasing function of the competitor's price; prices are then strategic substitutes in this range of prices.

The transition between deterrence and accommodation of switchers takes place at the price p^d such that deterence and accommodation of switchers yield the same profit and thus the same price: $p_i^D\left(p^d\right) = p_i^{s*}\left(p^d\right)$, or equivalently

$$p^d = \frac{1}{3}\left(2\overline{\theta}(s_o - s_c) - K\right) > p^f.$$

Note that $p^d > 0 \iff K < 2\overline{\theta}(s_o - s_c)$.

Targeting resolute copiers? Because of the convex kink in the demand function, the shift from switchers to copiers has to be analyzed by comparing profit levels. The optimal price and profit on resolute copiers are equal to:

$$\begin{aligned} p_i^{c*} &\equiv \arg\max_{p_i} p_i \left(\overline{\theta} - \frac{p_i}{s_o - s_c}\right) = \frac{1}{2}\overline{\theta}(s_o - s_c), \\ \text{and } \pi_i^{c*} &= \frac{1}{4}\overline{\theta}^2 (s_o - s_c). \end{aligned}$$

We readily get that

$$\pi_i^{s*}(p_j) > \pi_i^{c*} \iff p_j < p^e \equiv \left(2 - \sqrt{2}\right)\overline{\theta}(s_o - s_c) + K.$$

The regime including accommodation of switchers is part of the best response function as long as

$$p^d < p^e \iff K > \frac{3\sqrt{2}-4}{4}\overline{\theta}\left(s_o - s_c\right). \tag{3}$$

In this case there exits a downard jump at $p_j = p^e$.

Otherwise, accommodation of switchers is not part of the best response function and the latter has a downward jump from deterrence of switchers to accommodation of resolute copiers for another price $p_j = p^{e\prime}$, where $\pi_i^D\left(p^{e\prime}\right) = \pi_i^{c*}$, which is equivalent to

$$p^{e\prime} = \frac{1}{2}\left(\overline{\theta}\left(s_o - s_c\right) - K\right) + \frac{1}{2}\sqrt{K\left(2\overline{\theta}\left(s_o - s_c\right) + K\right)}.$$

Partial market coverage. The above results apply when the best response does not entail full market coverage. That is, when the user with the lowest valuation is not served by any firm. In other words, the solutions to the above problems must be interior solutions. The following lemma provides the condition under which the above results remain valid.

Lemma 1 *The optimal prices p_i^{b*}, $p_i^{s*}\left(p_j\right)$ and p_i^{c*} are interior solutions and do not lead to full market coverage if*

$$K < \left(\overline{\theta} - 2\underline{\theta}\right)\left(s_o - s_c\right). \tag{4}$$

Proof. The price p_i^{b*} is an interior solution iff $p_i^{b*} > p_i^b(\underline{\theta}) \iff K < \left(\overline{\theta} - 2\underline{\theta}\right)\left(s_o - s_c\right)$. The price $p_i^{s*}\left(p_j\right)$ with $p_j \geq 0$ is an interior solution iff $p_i^s\left(\underline{\theta}, p_j\right) < p_i^{s*}\left(p_j\right) \iff K < p_j + 2\left(\overline{\theta} - 2\underline{\theta}\right)\left(s_o - s_c\right)$ The price p_i^{c*} is an interior solution iff $p_i^{c*} > p_i^c\left(\underline{\theta}\right) \iff \overline{\theta} - 2\underline{\theta} > 0$. It is easy to see that the condition for p_i^{b*} implies the two other conditions. ■

Suppose that condition (4) holds. Then, under condition (3), the best response function is given by

$$p_i^*\left(p_j\right) = \begin{cases} p_i^{b*} & \text{if } p_j \leq p^f, \\ p_i^D\left(p_j\right) & \text{if } p^f \leq p_j \leq p^d, \\ p_i^{s*}\left(p_j\right) & \text{if } p^d \leq p_j \leq p^e, \\ p_i^{c*} & \text{if } p_j > p^e. \end{cases}$$

Otherwise it is given by

$$p_i^*\left(p_j\right) = \begin{cases} p_i^{b*} & \text{if } p_j \leq p^f, \\ p_i^D\left(p_j\right) & \text{if } p^f \leq p_j \leq p^{e\prime}, \\ p_i^{c*} & \text{if } p_j > p^{e\prime}. \end{cases}$$

Figure 2 displays these functions (in black for firms 1 and in grey for firm 2) for 'high' and 'low' fixed cost of copying (resp. in the left- and right-hand panel).

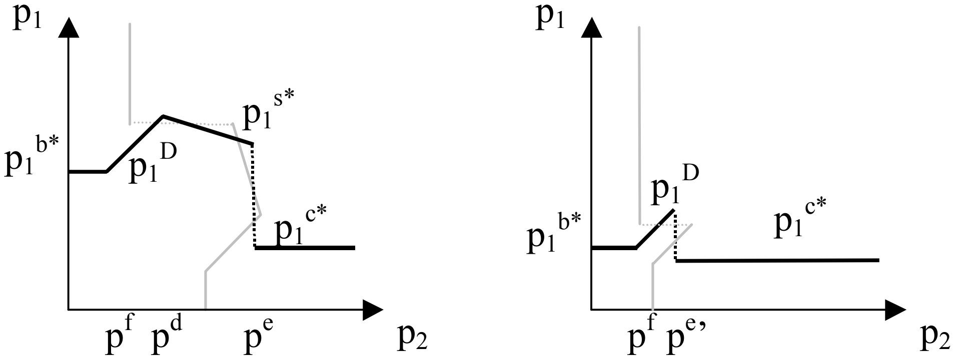

Figure 2. Best Response Functions.

When condition (4) does not hold, the shape of the best response functions slightly differ. It can readily be observed that the prices set by a firm that is constrained by full market coverage are never lower than the unconstrained prices p_i^{b*}, $p_i^{s*}(p_j)$ and p_i^{c*} that we computed above. Indeed, the marginal consumer that is targeted by the unconstrained prices has a type below $\underline{\theta}$. Since this marginal consumer does not exist, the firm increases its prices up to the lowest possible valuation $\underline{\theta}$. As a consequence, some segments of the best response function may be shifted upwards. Also, some segments may have different shapes. Furthermore, when the lowest valuation $\underline{\theta}$ is large enough, the discontinuities may vanish. To abstract away such complexities, we assume that condition (4) is met in the analysis that follows.

4.2 Existence of equilibria in pure strategies

Because of discontinuities in the best response functions, equilibria in pure strategies might fail to exist. Intuitively, the possible inexistence of equilibria stems from firms' free-riding behavior with respect to the threat of copying. If both firms take this threat seriously and quote low prices to accommodate resolute copiers, then there exists an opportunity for either firm to raise its price while keeping a sufficiently large demand and making a larger profit. This situation is shown in the right-hand panel of Figure 1 where best response functions do no intersect. By contrast, the left-hand panel shows the situation where firms reach an equilibrium as their best response function intersect at a symmetric equilibrium. More formally, we can state the following proposition.

Proposition 2 *Suppose that assumption **A1** and condition (4) hold. There exists a unique symmetric Nash equilibrium in which both firms focus on switchers and set the price* $p_S \equiv \frac{1}{3}\left(2\overline{\theta}\left(s_o - s_c\right) + K\right)$ *if*

$$K > \widehat{K} \equiv \frac{3\sqrt{2}-4}{2}\overline{\theta}\left(s_o - s_c\right). \tag{5}$$

Otherwise, there is no Nash equilibrium in pure strategies.

Proof. See Appendix 1. ■

The market fails to reach an equilibrium for small fixed costs of copying because the price p_S and the profit associated to this strategy decrease with K. For a low enough value of K, profits under accomodation of resolute copiers become more attractive and firms tend to cut their price to p_i^{c*}. As a result, the absence of duopoly equilibria for low fixed costs of the copying technology casts some doubts about traditional analyses of the threat of copying in one-good monopoly settings.

4.3 Properties of symmetric equilibria

We now focus on the situation in which the market reaches a symmetric equilibrium. One observes that

$$p_S = \frac{1}{3}\left(2\overline{\theta}\left(s_o - s_c\right) + K\right) > p_m = \frac{1}{4}\left(2\overline{\theta}\left(s_o - s_c\right) + K\right).$$

Corollary 4.1 *The price set by duopolists at the pure-strategy equilibrium is* higher *than the average price set by the multiproduct monopolist.*

The intuition is simple. At equilibrium, the duopolists focus on switchers; the copying technology constitutes thus a common substitute for their original good. The presence of this common substitute turns goods i and j (which are a priori perfectly horizontally differentiated) into complementary goods. The multiproduct monopolist has an incentive to decrease prices further than the duopolists do because he realizes that decreasing the price for one good increases demand for the other good by making copying less attractive. This is just the same argument as for Cournot's model of complementary products.

The externality that each firm imposes on the other can be quite important. Indeed, for some range of admissible parameters, the duopolists end up setting prices larger than the price they would set under no copying threat. To see this, note that a user $\theta \in [\underline{\theta}, \overline{\theta}]$ who purchases the original good i gets a utility of $\theta s_o - p_i$. The demand for this good is simply equal to $D_i(p_i) = \left(\overline{\theta} - p_i/s_o\right) / \left(\overline{\theta} - \underline{\theta}\right)$ and is independent of the demand for the other good. Firms in duopoly and monopoly thus set the same price, the monopoly price: $p_M = \frac{1}{2}\overline{\theta}s_o$. Comparing it to p_S yields the following corollary.

Corollary 4.2 *The price set by duopolists at the pure-strategy equilibrium is* higher *than the monopoly price under no threat of piracy if and only if*

$$K \geq \frac{1}{2}\overline{\theta}\left(4s_c - s_o\right). \tag{6}$$

This condition defines a non empty set of parameters under ***A1****, (4) and (5).*

Proof. It is easy to check that $p_S \geq p_M$ is equivalent to (6). To prove that the set of parameters supporting this price is not empty, we set K to the binding level of assumption **A1**: $K = \underline{\theta} s_c$. Using this, conditions (4), (5) and (6) can be written as functions of $\overline{\theta}/\underline{\theta}$ and s_o/s_c. So we can make the normalization $\underline{\theta} = s_c = 1$ and thus $K = 1$. Conditions (4), (5) and (6) write as

$$1 \leq \left(\overline{\theta} - 2\right)(s_o - 1)\,; \qquad 1 \geq \frac{3\sqrt{2}-4}{2}\overline{\theta}\,(s_o - 1)\,; \qquad 1 \geq \frac{1}{2}\overline{\theta}\,(4 - s_o)$$

Any values $\overline{\theta} \in (2.28, 2.45)$ and $s_o = \left(\overline{\theta} - 1\right) / \left(\overline{\theta} - 2\right)$ fulfill these conditions. ■

The last two corollaries qualify the argument that the threat of piracy forces firms to lower their prices and that the usage of authorized copies increases with piracy. Instead, our model gives some evidence to the common claim of copyright holders, who assert that piracy reduces the demand for legal copies. These results also cast some doubt about the social benefits of stronger competition in information good markets that are subject to potential piracy. The two last corollaries indeed suggest that a more concentrated industry is better equipped to provide surplus both to legal consumers and to producers.

4.4 Properties of mixed strategy equilibria

When $\widehat{K} > K$, there exist no equilibria with pure strategies. Nevertheless, by Glicksberg (1952), there is a mixed-strategy Nash equilibrium because profits are continuous. Moreover, as in Boccard and Wauthy (1997, 2003), the piecewise linearity of the demand function allows us to show that firms do not use continuous densities. We present here a simple and intuitive class of mixed strategy equilibria in which firms play two prices with the same probability distributions. The following proposition shows that such equilibria exist provided that fixed costs are not too small. Let $\widehat{K}' = 0.0274\overline{\theta}\,(s_o - s_c) \in (0, \widehat{K})$.

Proposition 3 *When $\widehat{K} > K > \widehat{K}'$, there exists an equilibrium where firms randomize between the prices*

$$p_a = \frac{2\overline{\theta}\,(s_o - s_c) + xK}{4 - x} \qquad \textit{and} \quad p_b = \frac{2\overline{\theta}\,(s_o - s_c) + (x+1)\,K}{x+3}$$

with probabilities x and $1 - x$. The probability x is equal to zero when K is equal to $\widehat{K}$, it increases when K increase above $\widehat{K}$ and it is equal to $x = 0.3603$ when K tends to $\widehat{K}'$. Prices are such that $p_b > p_a + K$.

Proof. See Appendix 2. ■

Unfortunately the probability x has no explicit expression. Numerical simulations show that for any admissible set of parameters, the probability x monotonically increases when K falls from $\widehat{K}$ to $\widehat{K}'$.

Because $p_b > p_a + K$, the two price mixed strategies equilibrium yields ex-post realizations that include the three regimes with a positive probability. Each firm faces switchers when its price realization is equal to the other firm's realization; it faces buyers when it quotes the lowest price and resolute copiers when it quotes the highest price. It is easy to show that firms randomize with prices that lie between the price p_i^{c*} (charged to resolute copiers) and the price p_S (corresponding to the pure-strategy equilibrium of Proposition 2).

Corollary 4.3 *Prices are ranked as follows:* $p_i^{c*} < p_a < p_a + K < p_b < p_S$.

Hence, the average price in this mixed-strategy equilibrium is smaller than the price in the pure-strategy equilibrium. Intuitively, firm i has no incentive to set prices below p_i^{c*} because, if it does, it gets a positive marginal revenue irrespective of firm j's mixed strategy. Similarly, it gets a *negative* marginal revenue whenever it sets a price *above* p_S.

When $K < \widehat{K}'$, symmetric mixed strategies with two prices are not equilibria; firms randomize over a larger number of prices. The characterization of such equilibria goes beyond the scope of this paper. Note that the previous corollary still applies.

5 Welfare analysis

As indicated in Section 1, the economics of IP protection discusses the trade-off between *ex ante* and *ex post* efficiency considerations: to remedy the long-run underproduction problem that might arise from insufficient incentives to create, the law grants exclusive rights to creators, which entail a short-run underutilization problem.

Our simple framework allows us to shed some new light on this policy debate. First, in a short-run perspective, we can perform comparative statics exercises to assess the effects of stronger IP protection; we can also compare the welfare performances of two market structures, namely a multiproduct monopoly versus a duopoly. Second, in a long-run perspective, we can measure incentives to create and compare again the relative merits of monopoly and duopoly.

5.1 Ex post efficiency considerations

In many discussions, the protection of intellectual property (IP) rights calls for an increase in the cost of piracy (Novos and Waldman (1984), Yoon (2002), etc). In this model, this would call for two policy measures: first, one can apply a tax on the reproduction devices so that the fixed cost of the copying technology, K, increases; second, one can take actions to decrease the value of a copy s_c.

When a pure-strategy equilibrium exists, we can easily analyze the effects of a marginal strengthening of IP rights. Let $\theta_S \equiv \left[4\overline{\theta}\left(s_o - s_c\right) - K\right] / \left[6\left(s_o - s_c\right)\right]$ be the type of the switching user at the equilibrium prices p_S. This is the lowest type amongst the consumers who purchase an original good. Users with type $\theta \in [\theta_S, \overline{\theta}]$ purchase an original whereas, by assumption **A1**, users with type

$\theta \in [\underline{\theta}, \theta_S)$ make use of copies. Hence, on the one hand, a rise in the copying cost ($dK > 0$) implies an upward parallel shift of the demand by switching users. It then increases equilibrium prices ($dp_S > 0$) and increases the set of consumers of original goods ($d\theta_S < 0$). The number of copiers falls. On the other hand, a deterioration of the value of copies ($ds_c < 0$) implies a rotation of the demand by swiching users, thereby diminishing the elasticity of this demand. The deterioration of the value of copies then leads to an increase in price ($dp_S > 0$) and to a reduction of the set of consumers buying original goods ($d\theta_S > 0$). The number of copiers increases.

Welfare properties can be obtained as follows. An increase in K induces variations in profits through effects on price and demand. Changes in the surplus of the two producers can be written as

$$dPS = 2\left(\overline{\theta} - \theta_S\right) dp_S - 2p_S d\theta_S$$

which, as just shown, is positive for $dK > 0$ but is ambiguous for $ds_c < 0$. It is readily verified that $dPS/ds_c > 0$ iff $K > 2\overline{\theta}\,(s_o - s_c)$. So, producers' surplus increases when copies are damaged if K is sufficiently small.

The consumers' surplus obtained from the use of both information goods includes four effects: the negative effect on illegal copiers because of the increase in the copying cost K; the negative effect of the deterioration of copies on the copying users; the negative effect of larger prices on legal consumers; and finally the effect on the switching users who move from copying to purchasing an original. At the price p_S, the latter effect on switching users is nil because switching users are indifferent between copying and purchasing the orginals. Hence,

$$dCS = -\left(\theta_S - \underline{\theta}\right) dK + \left(\theta_S - \underline{\theta}\right)^2 ds_c - 2\left(\overline{\theta} - \theta_S\right) dp_S + 0 * d\theta_S$$

which is negative because all terms are non positive ($dK > 0, ds_c < 0, dp_S > 0$). Consumers are negatively affected by both policy measures.

Finally total surplus writes as

$$dTS = dPS + dCS = -\left(\theta_S - \underline{\theta}\right) dK - \left(\theta_S - \underline{\theta}\right)^2 ds_c - 2p_S d\theta_S.$$

The latter expression is negative if the additional costs imposed on all copiers is smaller than the additional revenue from having an additional legal consumer. This is the case when the policy measure consists of deteriorating the copies ($ds_c < 0$). When the policy measure consists of an increase in the fixed cost of copying ($dK > 0$), the sign of the welfare change is ambiguous. These welfare results are consistent with Bae and Choi (2003).

Proposition 4 *In the region of parameters where a pure-strategy equilibrium exists, a rise in K raises the equilibrium prices* and *the demand for originals, while a fall in s_c raises the equilibrium prices* but *decreases the demand for* *originals. Consumers' surplus decreases with both actions. Producers' surplus increases with a rise in K whereas it also increases with a fall in s_c only if the copying cost is sufficiently small. Finally, total surplus decreases with a fall in s_c while a rise in K have ambiguous effects.*

Proof. We just need to show that the direction of total surplus is ambiguous under $dK > 0$. One can show that $dTS < 0$ decreases iff $K < \frac{2}{5}(s_o - s_c)\left(4\overline{\theta} - 9\underline{\theta}\right)$. This condition is compatible with assumption **A1** and conditions (4) and (5). Take for instance the values $\underline{\theta} = s_c = K = 1$ and take tuples $\left(s_o, \overline{\theta}\right)$ that are convex combinations of $(3, 2.5)$, $(4.5, 2.3)$ and $(4.2, 2.3)$. Also, $dTS > 0$ is compatible. Take for instance the values $\underline{\theta} = s_c = K = 1$ and take tuples $\left(s_o, \overline{\theta}\right)$ that are convex combinations of $(3.5, 2.6)$, $(2, 3)$ and $(2, 8)$. ■

The reader will also observe that the above conclusion about total surplus heavily hinges on the assumption of inefficient taxation. Indeed, if we assume that tax proceeds are efficiently redistributed to consumers through lump sum transfers, taxation has no direct cost and the terms in dK vanish in the above expressions. As a result, while consumers' surplus still falls with larger K, total surplus does increase with it: taxation on the copying technology improves welfare. This remark is not specific to the firms' interaction and we conjecture that it is valid for other models.

The above welfare analysis can easily be replicated for the multi-product monopolist where p_S is simply replaced by p_m and θ_S by $\theta_m \equiv (2p_m - K) / [2(s_o - s_c)] = \frac{1}{4}\left(2\overline{\theta}(s_o - s_c) - K\right) / (s_o - s_c)$. Note that when K rises ($dK > 0$) and s_c falls ($ds_c < 0$), the monopoly price and demand varies in the same direction as in the duopoly ($dp_m > 0$ and $d\theta_m < 0$). As a result the above welfare analysis remains valid for multi-product monopolists.[7] The welfare effects of a tax on the copying technology and of a deterioration of the value of copies have the same direction. Still, those effects do not have the same amplitudes.

As far as the comparison between monopoly and duopoly is concerned, we have already indicated in the previous section that the multiproduct monopolist always sets lower prices than duopolists do. As the monopolist also achieves higher profits, total surplus is undoubtedly higher under monopoly. It can further be shown that the two policy measures analyzed here contribute to increase further the welfare gap between monopoly and duopoly. Indeed, a few lines of computations establish that

$$\Delta TS \equiv TS_{monopoly} - TS_{duopoly} = \frac{7}{144}\frac{\left(2\overline{\theta}(s_o - s_c) + K\right)^2}{s_o - s_c} > 0.$$

On the one hand, it is obvious that $d\Delta TS/dK > 0$. On the other hand, $d\Delta TS/ds_c < 0$ provided that $K < 2\overline{\theta}(s_o - s_c)$, which is clearly implied by

[7] One can also check that a rise in K has ambiguous effects on welfare in a multi-product monopoly. Take $\underline{\theta} = s_c = K = 1$ and $\left(s_o, \overline{\theta}\right) = (1.2, 10)$ for a welfare decrease and $\left(s_o, \overline{\theta}\right) = (1.3, 10)$ for a welfare increase.

condition (4). In other words, a strengthening of IP rights makes the case for a concentrated market structure even more appealing.

5.2 Ex ante efficiency considerations

Comparing our framework with an economy where only a single information good is available allows us to measure the (gross) incentive to create a new information good. In the previous model, if only one information good is available instead of two, it is easy to see that under assumption **A1**, the producer of this good only faces 'buyers'. The demand function is thus given by $D_i(p_i) = \left(\overline{\theta} - \frac{p_i - K}{s_o - s_c}\right) / \left(\overline{\theta} - \underline{\theta}\right)$, and the optimal price and profit are given by p_i^{b*} and π_i^{b*}, which we recall here:

$$p_i^{b*} = \frac{1}{2}\left(\overline{\theta}\left(s_o - s_c\right) + K\right) \quad \text{and} \quad \pi_i^{b*} = \frac{\left(\overline{\theta}\left(s_o - s_c\right) + K\right)^2}{4\left(s_o - s_c\right)}.$$

There are two cases to consider when going from one to two goods. A first possibilitly is that the new good is created by an incumbent firm that already produces the extant good; the ex post economy is then organised as a multiproduct monopoly. From Proposition 1, we know that the multiproduct monopolist's optimal (average) price and profit are given by

$$p_m = \frac{1}{4}\left(2\overline{\theta}\left(s_o - s_c\right) + K\right) \text{ and } \pi_m = \frac{\left(2\overline{\theta}\left(s_o - s_c\right) + K\right)^2}{8\left(s_o - s_c\right)}.$$

One readily observes that, under condition (4), we have that $\pi_m > \pi_i^{b*}$, meaning that an incumbent firm has a gross incentive to introduce a second good. Still, although goods are genuinely independent, the profit per good decreases when the number of goods rises: $\pi_m < 2\pi_i^{b*}$. *The monopolist indeed jeopardizes the sales of the first good when it introduces the second good.* Copying becomes more attractive when the number of goods is larger and the firm is compelled to reduce the average price of originals.

Alternatively, the new good could be created by an entrant firm, turning the ex post economy into a duopoly. Supposing for simplicity that the condition of Proposition 2 is met, prices and profits at the pure-strategy equilibrium are given by:

$$p_S \equiv \frac{1}{3}\left(2\overline{\theta}\left(s_o - s_c\right) + K\right) \text{ and } \pi_S = \frac{\left(2\overline{\theta}\left(s_o - s_c\right) + K\right)^2}{18\left(s_o - s_c\right)}.$$

Let us now compare the two scenarios. Comparing prices, it is easily checked that condition (4) implies the following ranking: $p_m < p_i^{b*} < p_S$. Therefore, *the average price decreases when the new good is introduced by an incumbent firm, whereas it increases when the new good is introduced by an entrant.* This is another illustration of the negative externality independent producers impose on each other, and on consumers, in the presence of copying.

Next, comparing profits, we can gauge the (gross) incentive to create in the two settings. We say that the incumbent has higher incentives to create if it increases more its profit by introducing the second good than would an entrant do. That is, if $\pi_m - \pi_i^{b*} > \pi_S$. Under no threat of piracy, goods are independent and incentives for the incumbent and the entrant are exactly equal. However, under piracy, the free-riding problem between firms may harm more the entrant than the above canibalization effect hurts the incumbent. As both free-riding and canibalization effects on profits increase in K, the comparison between the incumbent's and entrant's incentives to create is a priori ambiguous. As recorded in the next proposition, there exist configurations of parameters for which the incumbent has more to gain from the introduction of a new good than an entrant does. Industry concentration may therefore yield higher incentives to create new varieties and, therefore, be more efficient both in the short and in the long run.

Proposition 5 *In the region of parameters where a pure-strategy equilibrium exists in the duopoly game, an incumbent firm has higher incentives to introduce a second good than an entrant firm if and only if*

$$K \geq \frac{3\sqrt{10}-8}{13}\overline{\theta}\left(s_o - s_c\right). \tag{7}$$

This condition defines a non empty set of parameters under ***A1****, (4) and (5).*

Proof. First, it is easy to check that $\pi_m - \pi_i^{b*} \geq \pi_S$ is equivalent to (7). To prove that this condition is compatible with the other restrictions, we set K to the binding level of assumption **A1**: $K = \underline{\theta} s_c$. Using this, conditions (4), (5) and (7) can be written as functions of $\overline{\theta}/\underline{\theta}$ and s_o/s_c. So we can make the normalization $\underline{\theta} = s_c = 1$ and thus $K = 1$. Conditions (4), (5) and (7) write as

$$1 \leq \left(\overline{\theta}-2\right)\left(s_o - 1\right); \quad 1 \geq \frac{3\sqrt{2}-4}{2}\overline{\theta}\left(s_o - 1\right); \quad 1 \geq \frac{3\sqrt{10}-8}{13}\overline{\theta}\left(s_o - 1\right)$$

Any values $\overline{\theta} \geq 2.28$ and $s_o = \left(\overline{\theta}-1\right)/\left(\overline{\theta}-2\right)$ fulfill these conditions. ■

6 Conclusion

In this paper, we qualify the traditional results and insights about the impact of piracy obtained in a one-good monopoly setting. When there exist more than one information goods, increasing returns to scale in the copying technology create an interdependence between the demands for information goods, which are genuinely independent. We first show that a multiproduct monopoly may set different prices for its goods. We then show that two-product duopolies are subject to free-riding behaviors with respect to the threat of piracy. If the two firms take this threat seriously by quoting low prices, then there exists an opportunity for a firm to take advantage of this situation and to raise its price. This can yield to the absence of an equilibrium in pure strategies if the fixed cost of copying is low enough. In this case, firms may randomize between two prices. To the best of our knowledge, this is the first contribution showing that price dispersion in information good industries can be generated by the presence of piracy. When the fixed cost of copying is not too small, the market can yield a symmetric equilibrium with prices that are larger than the (average) price of the multiproduct monopoly. Furthermore, those prices can even become larger than the price of a monopoly which faces no threat of piracy. The externality that firms impose on each other can therefore be quite important and it can drastically reduce the demand for legal copies. Hence, the interactions between producers of information goods under the threat of piracy dramatically alter the equilibrium outcome compared to the outcome obtained under a one-good monopoly setting. Still, short run welfare implications for symmetric equilibria are close to the ones obtained for the one-good monopoly. Finally, we show that under particular conditions, multi-product monopolies may provide better incentives to create. To sum up, industry concentration can be welfare improving under the threat of piracy.

The present model suggests several avenues of future research. First, the current study is limited to the production of two perfectly differentiated information goods. It would be worthwhile to explore the pricing decisions and welfare aspects under piracy threat in a setting with more numerous and less differentiated varieties. Second, by assuming exogenous production and pricing of the copying technology, the current model sets aside the strategic issue of integration between the creators (or distributors) of information goods and the sellers of copying devices. It seems natural to investigate about the competition and welfare implications of such integration processes.

Appendix 1. Proof of Proposition 2

Each best response function $p_i^*(\cdot)$ and $p_j^*(\cdot)$ can have four segments. Removing symmetric configurations, we need to check the existence of a pure strategy equilibria for the 10 following configurations. For some configurations we will need to distinguish equilibrium conditions in which the switcher's branch $p_j^{s*}(\cdot)$ exists (i.e. $p^d \leq p^e$ or condition (3)) or in which it does not (i.e. $p^d > p^e$ or the reverse of condition (3)).

1. The configuration $\left(p_i^{b*}, p_j^{b*}\right)$ cannot be an equilibrium because $p_j^{b*} > p^f$ and thus the best response of i cannot be equal to $p_i^{b*} : p_i^*\left(p_j^{b*}\right) \neq p_i^{b*}$.

2. The configuration $\left(p_i^D(\cdot), p_j^D(\cdot)\right)$ cannot be an equilibrium because the system $p_i = p_i^D(p_j)$ and $p_j = p_j^D(p_i)$ has no solution.

3. The configuration $\left(p_i^{s*}(\cdot), p_j^{s*}(\cdot)\right)$ is an equilibrium if and only if $K > \widehat{K} \equiv \frac{3\sqrt{2}-4}{2}\overline{\theta}(s_o - s_c)$. Indeed, solving the system $p_i = p_i^{s*}(p_j)$ and $p_j = p_j^{s*}(p_i)$, we find $p_i = p_j = p_S \equiv \frac{1}{3}\left(2\overline{\theta}(s_o - s_c) + K\right)$. It is a best response for both firms to set $p_i = p_i^{s*}(p_j)$ if and only if $p^d \leq p_S \leq p^e$. The first inequality is clearly met, whereas the second is met provided that $p_S < p^e \iff K > \frac{3\sqrt{2}-4}{2}\overline{\theta}(s_o - s_c)$.This last condition is compatible with $p^d \leq p^e$.

4. The configuration $\left(p_i^{c*}, p_j^{c*}\right)$ cannot be an equilibrium because one can check that $p_j^{c*} < p^e$ and $p_j^{c*} < p^{e\prime}$. Hence, $p_i^*\left(p_j^{c*}\right) \neq p_i^{c*}$.

5. The configuration $\left(p_i^{b*}, p_j^D(\cdot)\right)$ cannot be an equilibrium because $p_j^D\left(p_i^{b*}\right) = p_i^{b*} + K > p^f$ and thus $p_i^*\left[p_j^D\left(p_i^{b*}\right)\right] \neq p_i^{b*}$.

6. Similarly, the configuration $\left(p_i^{b*}, p_j^{s*}(\cdot)\right)$ cannot be an equilibrium because $p_j^{s*}\left(p_i^{b*}\right) = \frac{3}{4}\overline{\theta}(s_o - s_c) + \frac{K}{4} > p^f$, and hence $p_i^*\left[p_j^{s*}\left(p_i^{b*}\right)\right] \neq p_i^{b*}$.

7. The configuration $\left(p_i^{b*}, p_j^{c*}\right)$ cannot be an equilibrium because when $p^d \leq p^e$, one can easily check that $p_i^{b*} < p^e$ so that $p_j^*\left(p_i^{b*}\right) \neq p_j^{c*}$. Also, when $p^d > p^e$, we get $p_i^{b*} < p^{e\prime}$ iff $K < \frac{2}{3}\overline{\theta}(s_o - s_c)$ which is always true. So, $p_j^*\left(p_i^{b*}\right) \neq p_j^{c*}$ when $p^e < p^d$.

8. The configuration $\left(p_i^D(\cdot), p_j^{s*}(\cdot)\right)$ cannot be an equilibrium because solving for $p_i = p_i^D(p_j)$ and $p_j = p_j^{s*}(p_i)$, we get $p_j = \widetilde{p}_j \equiv \frac{2}{3}\overline{\theta}(s_o - s_c) > p^d$, meaning that $p_i^*(\widetilde{p}_j) \neq p_i^D(\widetilde{p}_j)$.

9. The configuration $\left(p_i^D(\cdot), p_j^{c*}\right)$ cannot be an equilibrium because when $p^d \leq p^e$, we have $p_i^D\left(p_j^{c*}\right) = p_j^{c*} + K = \frac{1}{2}\overline{\theta}(s_o - s_c) + K < p^e$. When $p^d > p^e$, we have $p_i^D\left(p_j^{c*}\right) = \frac{1}{2}\overline{\theta}(s_o - s_c) + K < p^{e\prime}$ iff $K < \frac{1}{4}\overline{\theta}(s_o - s_c)$, which is always true. Therefore $p_j^*\left[p_i^D\left(p_j^{c*}\right)\right] \neq p_j^{c*}$.

10. The configuration $\left(p_i^{c*}, p_j^{s*}(\cdot)\right)$ cannot be an equilibrium because, for this to be an equilibrium, we should have (a) $p_j^{s*}\left(p_i^{c*}\right) = \frac{3}{4}\overline{\theta}(s_o - s_c) + \frac{K}{2} \geq p^e \iff K \leq 0.328\overline{\theta}(s_o - s_c)$, and (b) $p_i^{c*} \geq p^d \iff K \geq 0.5\overline{\theta}(s_o - s_c)$, which is incompatible with (a).

Appendix 2. Proof of Proposition 3

Denote $s_o - s_c$ by s. Firm i's profit is equal to $\pi(p_i, p_j)$ where

$$\pi(p_i, p_j) = \begin{cases} \pi^b(p_i) = p_i\left(\overline{\theta} - \frac{p_i - K}{s}\right) & \text{if } p_j \in [0, p_i - K) \\ \pi^s(p_i, p_j) = p_i\left(\overline{\theta} - \frac{p_i + p_j - K}{2s}\right) & \text{if } p_j \in [p_i - K, p_i + K), \\ \pi^c(p_i) = p_i\left(\overline{\theta} - \frac{p_i}{s}\right) & \text{if } p_j \in [p_i + K, \infty) \end{cases}$$

Each section of the profit function is concave in p_i.

We consider mixed-strategy equilibria that include two price atoms p_{ai} and p_{bi} played by player i with probabilities x_i and $1 - x_i$. Firm i's expected profit is equal to

$$\begin{aligned} \Pi_i &= x_i x_j \pi(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi(p_{ai}, p_{bj}) \\ &\quad + (1 - x_i)x_j\pi(p_{bi}, p_{aj}) + (1 - x_i)(1 - x_j)\pi(p_{bi}, p_{bj}). \end{aligned}$$

We look for a symmetric mixed-strategy equilibrium. This exists if we can find probabilities and prices such that $(x, p_a, p_b) = (x_i, p_{ai}, p_{bi})$, $i = 1, 2$ and

$$(x_i, p_{ai}, p_{bi}) = \arg \max_{(x_i, p_{ai}, p_{bi})} \Pi_i \text{ s.t. } (x_j, p_{aj}, p_{bj}) = (x, p_a, p_b), \ i = 1, 2.$$

First, suppose that the symmetric equilibrium is such that $p_b \leq p_a + K$. For (x_i, p_{ai}, p_{bi}), $i \in \{1, 2\}$ close enough to (x, p_a, p_b), firm i's payoff is given by the function

$$\begin{aligned} \Pi_i &= x_i x_j \pi^s(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi^s(p_{ai}, p_{bj}) \\ &\quad + (1 - x_i)x_j\pi^s(p_{bi}, p_{aj}) + (1 - x_i)(1 - x_j)\pi^s(p_{bi}, p_{bj}). \end{aligned}$$

Because π^s is strictly concave, it is easy to show that there is a unique symmetric equilibrium with $p_S = p_{ai} = p_{bi}$, $i \in \{1, 2\}$. This is the equilibrium with pure strategies found in Proposition 2.

Second, suppose that the symmetric equilibrium is such that $p_b > p_a + K$. For (x_i, p_{ai}, p_{bi}), $i \in \{1, 2\}$ close enough to (x, p_a, p_b), firm i's expected payoff writes as

$$\begin{aligned} \Pi_i &= x_i x_j \pi^s(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi^c(p_{ai}) \\ &\quad + (1 - x_i)x_j\pi^b(p_{bi}) + (1 - x_i)(1 - x_j)\pi^s(p_{bi}, p_{bj}). \end{aligned}$$

There are three first order conditions for equilibrium:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_{ai}} &= 0 \iff x_i = 0 \quad \text{or} \quad x_j \pi^s_{p_i}(p_{ai}, p_{aj}) + (1 - x_j)\pi^c_{p_i}(p_{ai}) = 0 \\ \frac{\partial \Pi_i}{\partial p_{bi}} &= 0 \iff x_i = 1 \quad \text{or} \quad x_j \pi^b_{p_i}(p_{bi}) + (1 - x_j)\pi^s_{p_i}(p_{bi}, p_{bj}) = 0 \\ \frac{\partial \Pi_i}{\partial x_i} &= 0 \iff \left\{ \begin{array}{c} x_j \pi^s(p_{ai}, p_{aj}) + (1 - x_j)\pi^c(p_{ai}) \\ -x_j \pi^b(p_{bi}) - (1 - x_j)\pi^s(p_{bi}, p_{bj}) \end{array} \right\} = 0 \end{aligned} \tag{8}$$

where the subscript p_i denotes a partial differentiation w.r.t. to p_i. These three conditions guarantee a maximum because $\frac{\partial^2 \Pi_1}{\partial p_{a1}^2} < 0$, $\frac{\partial^2 \Pi_1}{\partial p_{b1}^2} < 0$ and the Hessian determinant is zero:

$$|H| = \begin{vmatrix} \frac{\partial^2 \Pi_i}{\partial p_{ai}^2} & 0 & 0 \\ 0 & \frac{\partial^2 \Pi_i}{\partial p_{bi}^2} & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

We now determine the symmetric equilibrium by setting $(x, p_a, p_b) = (x_i, p_{ai}, p_{bi})$, $i = 1, 2$ and $x_i = x \in (0, 1)$. We successively get

$$p_a = \frac{2\overline{\theta}s + xK}{4 - x}, \quad p_b = \frac{2\overline{\theta}s + (x + 1) K}{x + 3}$$

and

$$x = \frac{\pi^s (p_b, p_b) - \pi^c (p_a)}{\pi^s (p_a, p_a) + \pi^s (p_b, p_b) - \pi^c (p_a) - \pi^b (p_b)}.$$

One can check that $p_b > p_a + K$ which is consistent with the condition $p_{bi} > p_{ai} + K$, $i \in \{1, 2\}$.

Equilibrium profits can be computed as the following functions of x:

$$\begin{aligned}
\pi^c (p_a) &= \left((2 - x) \overline{\theta}s - xK\right) \frac{\left(xK + 2\overline{\theta}s\right)}{(x - 4)^2 s}, \\
\pi^s (p_a, p_a) &= \frac{1}{2} \left(2 (2 - x) \overline{\theta}s + (4 - 3x) K\right) \frac{\left(xK + 2\overline{\theta}s\right)}{(x - 4)^2 s}, \\
\pi^s (p_b, p_b) &= \frac{1}{2} \left((x + 1) 2\overline{\theta}s + (1 - x) K\right) \frac{\left(2\overline{\theta}s + (x + 1) K\right)}{(x + 3)^2 s}, \\
\pi^b (p_b) &= \left(\overline{\theta}s (x + 1) + 2K\right) \frac{\left(2\overline{\theta}s + (x + 1) K\right)}{(x + 3)^2 s}.
\end{aligned}$$

After some substitutions, x solves :

$$x = \frac{\left[\begin{array}{c} \left(K^2 + 4\overline{\theta}sK\right) x^4 + \left(20K^2 - 2\overline{\theta}sK + 8\overline{\theta}^2 s^2\right) x^3 \\ + \left(3K^2 + 38\overline{\theta}sK - 12\overline{\theta}^2 s^2\right) x^2 + \left(-8K^2 + 20\overline{\theta}^2 s^2\right) x \\ +16K^2 - 8\overline{\theta}^2 s^2 + 64\overline{\theta}sK \end{array} \right]}{2K (x + 3) (4 - x) \left(x^2 K + 2\overline{\theta}sx - \overline{\theta}s - 2K\right)}. \tag{9}$$

Hence x is the solution of a polynomial with degree 5. There is at least one real solution. We have found no analytical solution.

Two cases can readily be studied. On the one hand, when $K = \widehat{K} \equiv \frac{3\sqrt{2}-4}{2} s\overline{\theta}$, expression (9) implies that $x = 0$. Furthermore, one can check that $\left[\frac{\partial^2 \Pi_i}{\partial x_i \partial x_j}\right]_{x_j=0} < 0$ and that $\left[\frac{\partial^2 \Pi_i}{\partial x_i \partial K}\right]_{x_j=0} < 0$ so that $\left[\frac{\partial x_j}{\partial K}\right]_{x_j=0} = \left[\frac{\partial x}{\partial K}\right]_{x=0} < 0$. A smaller K increases the probability x above zero. Hence, mixed strategy equilibria occur for $K < \widehat{K}$ and pure strategy equilibria occur otherwise. Furthermore, when $x = 0$ and $K = \widehat{K}$, we have that $\Pi_i = \pi_i^{c*}$ and that

$d\Pi_i/dK = \partial\Pi_i/\partial K + (dx/dK)(\partial\Pi_i/\partial x) = (-10+\sqrt{2})\overline{\theta} < 0$. Therefore, expected profits under symmetric mixed strategy increase above π_i^{c*} as K decreases below $\widehat{K}$. For K smaller and close enough to $\widehat{K}$, symmetric mixed strategy dominate the price strategy p_i^{c*}.

On the other hand, when $K \to 0$, we have that $\pi_1^s(p_a,p_a) = \pi_1^s(p_b,p_b) = \pi_1^c(p_a) = \pi_1^b(p_b)$. So the RHS of expression (9) indefinite. To solve this problem, we approximate expression (9) by dropping terms in K of order larger than one and we get

$$K\left(x^4 - x^3 - x^2 + 3x + 8\right) + \frac{1}{2}\overline{\theta}s\,(2x-1)\left(x^2 - x + 2\right) = 0$$

which yields the unique solution $x = 1/2$ when $K \to 0$. Applying this result, we get that the expected profits is equal to $\pi_i^{c*} - \overline{\theta}s/196$. Therefore, the symmetric strategy is dominated by the strategy p_i^{c*} when $K \to 0$.

The previous paragraphs suggest that the two-price symmetric mixed strategy is a local maximum and that it can be dominated by the one-price strategy p_i^{c*} for small enough K. To check when the symmetric mixed strategy is a global maximum, let us fix firm j's strategy as $(x_j, p_{aj}, p_{bj}) = (x, p_a, p_b)$ where $x > 0$, and we sketch firm i's expected profit as a function of the single price p_i:

$$\begin{aligned}\Pi_i &= x_i x \pi(p_i, p_a) + x_i(1-x)\pi(p_i) \\ &\quad + (1-x_i)x\pi(p_i) + (1-x_i)(1-x)\pi(p_i, p).\end{aligned}$$

It is readily shown that $\Pi_i(p_i)$ is a piece-wise quadratic and concave function. Consider p_i increasing from zero. One can check that the first section of $\Pi_i(p_i)$ is either increasing or bell-shaped with a maximum at p_i^{c*}; the second section is bell-shaped with maximum at $p_i = p_a$; the third section is monotonically increasing; the fourth section is bell-shaped with maximum at $p_i = p_b$ and the last section is monotonically decreasing. There are three candidates for a global maximum: $p_i = p_a, p_b$ and p_i^{c*}. We know that expected profits are equal at $p_i = p_a$ and $p_i = p_b$ so that firm i is indifferent between the two prices. The expected profit under symmetric mixed strategy can be evaluated by setting $x_i = 1$ and $p_{ai} = p_a$:

$$\Pi_i^* \equiv x\pi^s(p_a, p_a) + (1-x)\pi^c(p_a) = \frac{2s\overline{\theta} + (2-x)Kx}{2s(4-x)^2}$$

We now compare this profit level with the one obtained under strategy p_i^{c*}. Some computations show that $\Pi_i^* \geq \pi_i^{c*}$ is equivalent to

$$2x(2-x)K^2 + 8\overline{\theta}s(2-x)K - xs^2\overline{\theta}^2 \geq 0$$

Hence, $K \geq K_1(x) \equiv \overline{\theta}s\left(2(4-2x) + (4-x)\sqrt{4-2x}\right)/[2x(x-2)]$. Note that the probability x depends on K. To get the probability x that makes this inequality binding and that is simultaneously compatible with a mixed strategy equilibrium, we insert $K_1(x)$ in expression (8), we evaluate at the symmetric mixed strategy equilibrium to get

$$32 - 4(4-x)\sqrt{4-2x}\left(1+x^2\right) + x\left(48 + x\left(-1 + x\left(-8 + x\left(-1+2x\right)\right)\right)\right) = 0$$

This equation has a unique solution in the interval $x \in (0,1)$ which is equal to $\widehat{x}' = 0.3603$. The associated level of fixed cost of copying is equal to $\widehat{K}' = 0.02739 s\overline{\theta} < \widehat{K}$.

References

[1] Bae, S.H. and Choi, J.P. (2003). A model of piracy. Mimeo. Michigan State University.

[2] Belleflamme, P. and Picard, P. (2004). Competition over piratable goods. CORE Discussion Paper 2004/55, Université catholique de Louvain, Belgium.

[3] Besen, S.M. and Kirby, S.N. (1989). Private copying, appropriability, and optimal copying royalties. *Journal of Law and Economics* **32**: 255-280.

[4] Besen, S.M. and Raskind, L.J. (1991). An introduction to the law and economics of intellectual property. *Journal of Economic Perspectives* **5**: 3-27.

[5] Boccard, N. and Wauthy, X. (1997). The Hotelling model with capacity precommitment. CORE Discussion paper 9783, Université catholique de Louvain, Belgium.

[6] Boccard, N. and Wauthy, X. (2003). Optimal quotas, price competition and products' attributes. *The Japanese Economic Review* **54**: 395-408.

[7] Conner, K.R. and Rumelt, R.P. (1991). Software copying: An analysis of protection strategies. *Management Science* **37**: 125-139.

[8] Gayer, A. and Shy, O. (2003). Copyright protection and hardware taxation. *Information Economics and Policy* **15**: 467-483.

[9] Glicksberg, I. (1952). A further generalization of the Kakutani fixed point theorem with application to Nash equilibrium points, *Proceedings of the American Mathematical Society* **3**: 170–174.

[10] Johnson, W.R. (1985). The economics of copying. *Journal of Political Economy* **93**: 158-174.

[11] Koboldt, C. (1995). Intellectual property and optimal copyright protection. *Journal of Cultural Economics* **19**: 131-155.

[12] Landes, W.M. and Posner, R.A. (1989). An economic analysis of copyright law. *Journal of Legal Studies* **38**: 325-363.

[13] Liebowitz, S.J. (1985). Copying and indirect appropriability: Photocopying of journals. *Journal of Political Economy* **93**: 945-957.

[14] Novos, I.E. and Waldman, M. (1984). The effects of increased copyright protection: An analytical approach. *Journal of Political Economy* **92**: 236-246.

[15] Mussa, M., and Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory* **18**: 301-317.

[16] Peitz, M. and Waelbroeck, P. (2003). Piracy of digital products: A critical review of the economics literature. CESifo Working Paper No. 1071.

[17] Plant, A. (1934). The economic aspects of copyright in books. *Economica* **1**: 167-195.

[18] Shy, O. and Thisse, J.-F. (1999). A strategic approach to software protection. *Journal of Economics and Management Strategy* **8**: 163-190.

[19] Takeyama, L.N. (1994). The welfare implications of unauthorized reproduction of intellectual property in the presence of network externalities. *Journal of Industrial Economics* **62**: 155-166.

[20] Watt, R. (2000). *Copyright and Economic Theory. Friends or Foes?* Edward Elgar Publishing, Cheltenham (UK) and Northampton (Mass.).

[21] Yoon, K. (2002). The optimal level of copyright protection. *Information Economics and Policy* **14**: 327-348.

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