Equity as a Prerequisite for Stable Cooperation in a Public-Good Economy – The Core Revisited

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Abstract

In this paper we explore the relationship between an equitable distribution of the cost shares in public-good provision on the one hand and the core property of an allocation on the other. In particular we show that it is an inhomogeneous distribution of cost shares that motivates some coalition of agents to separate and to block an initially given Pareto optimal allocation which can be interpreted as the outcome of a negotiation process when all agents form a grand coalition. Distributional equity of the individual burdens of public-good contribution is assessed by a specific sacrifice measure (the "Moulin sacrifice") which is derived from the egalitarian-equivalent concept suggested by Moulin (1987). We also develop a simple core test by which it can be checked whether a given allocation is in the core thus being a possible outcome of a cooperative agreement in the public-good economy.

JEL-Code: C71, D63, H41.

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1. Introduction

Pareto optimal allocations in public-good economies are hard to implement when there is no central agency that has coercive power to enforce cooperation. Since no world government exists this is the typical situation in the international system such that, realistically, it cannot be expected that global public-goods like protection of the world climate will be provided at an efficient level. A major problem in this context is that collective action aiming at Pareto optimality in a public-good economy tends to be unstable which means that even if the partners have signed an agreement it is individually rational for them not to fulfil their obligations and to enjoy a free-ride instead. As intuitive this widespread fear of instability of cooperative ventures in a public-good economy seems to be it is, however, not quite straightforward to pin it down to a distinct and unambiguous stability criterion.

The most common stability concept refers to the effects that are caused when a single agent unilaterally deviates from a cooperative arrangement and thus wants to stand aside as a free-rider. So, as known from the theory of cartel formation by oligopolistic firms (see d'Aspremont et al., 1983), an allocation is called *internally stable* if no participating agent would benefit from departing while all other agents still adhere to cooperation (see, e.g., Carraro and Siniscalco, 1993, Barrett, 1994, or Finus, 2001, on this stability concept which is of particular importance in the context of international environmental agreements). It is, however, not certain that the agents left behind will always react in this way. When other agents, in the simple static version of the game at once or in the dynamic version in subsequent periods, instead completely abandon cooperation after one agent has pulled out this leads to the γ-core concept as applied by Chander and Tulkens (1995). Such a strategy of punishment may effectively deter violation of the agents' duties and thus make cooperative arrangements self-enforcing and stable.

As an alternative to these of stability criteria that refer to *individual* deviations, *collective* deviations of sub-groups of agents can be used as the starting point. This leads back to the traditional core concept (see Foley, 1970), which differs from Chander and Tulkens' (1995) γ-core approach: An allocation is said to lie in the core of an economy if no coalition of agents is able to improve the welfare of all of its members by leaving ("blocking") the original allocation and bringing about a new allocation in which only the endowments being available in the separating coalition are used. Focusing on standalone allocations of sub-groups the core criterion is, by definition, independent of specific assumptions on the behaviour of the remaining agents since their contributions to the public good after separation are assumed to be zero.[1] If, however, these agents are not complete free-riders but still make a positive contribution to the public good the incentive for some coalition to deviate is further increased (see e.g. Kolm, 2006, for a further discussion). So the core property indeed gives a "minimal rationale" for the stability of a cooperative arrangement, which also motivates "the relevance of the core to political reality" (Foley, 1970). For a further discussion of the relationship between the core and the formation of specific coalition structure see Finus (2001) and Ray (2007).

Beyond those stability issues another main question concerning the core of a public-good economy is what specific properties make core allocations distinct from those outside the core. In this context, it has been suggested that there is some relationship between the core on the one hand and some notion of equity on the other. So Moulin (1987, p. 964) maintains that the core may serve as "a meaningful guideline in the search for equitable cost sharing" since it "means that no coalition should be charged more than its 'stand alone' cost". But the remarks on this remain rather cursory and non-conclusive, and no explicit reference to distributional norms and equity criteria is made. The main objective of this paper, therefore, is to examine more closely what the core property of a Pareto optimal public-good allocation (and thus stability of cooperation in a specific sense) has to do with distributional equity. In particular we will show that any core allocation in a public-good economy must offer something like a fair burden-sharing in financing the public good where the equity standard is provided by the agents' levels of 'Moulin sacrifices' being based on Moulin's (1987) famous egalitarian-equivalence concept. Otherwise, if there is an unequal distribution of the Moulin sacrifices, it becomes more likely that a sub-coalition of agents has an incentive to withdraw and to block the initially given allocation. From another perspective quite apart from Moulin's approach, a similar insight has also been put forward by Mas-Colell and Silvestre (1989) and Weber and Wiesmeth (1991) who have shown that the core of a public-good economy is equivalent to the set of 'cost-share equilibria' which are characterized by a specific kind of incentive compatibility. As a by-product of our analysis we also develop a core test by which it can be examined in a simple way whether some given Pareto optimal allocation lies in the core or not.

2. The Framework

We consider a public-good economy consisting of $n \geq 2$ agents (see Bergstrom, Blume and Varian, 1986, and Cornes and Sandler, 1996, as classical references). Each agent i is endowed with

[1] For a discussion of different core concepts see Chander and Tulkens (1997).

y_i units of a private good that can either be used for private consumption x_i or as an input $z_i = y_i - x_i$ for the production of a public good whose total supply then is $G = g(Z)$. Here $Z := \sum_{i=1}^{n} z_i$ is the aggregate input to public-good production being the sum of all individual contributions. The production function $g(Z)$ is assumed to be differentiable, strictly monotone increasing and to have $g(0) = 0$. Thus, the case is included in which there are non-constant returns to scale in providing the public good. As the function $g(Z)$ is invertible, we denote its inverse by $c(G)$ which gives, in units of the private good, the costs of providing the public good G.

Preferences of agent i are defined for all non-negative consumption bundles, consisting of private consumption x_i and public-good supply G. They can be represented by a utility function $u_i(x_i, G)$ which is assumed to have the standard properties, i.e. it is twice partial differentiable, strictly monotone increasing in both arguments and strictly quasi-concave. In addition we suppose that all indifference curves do not intersect the x_i-axis. This assumption facilitates the exposition in the general case and it is also fulfilled by the two types of utility functions which we will use in our illustrative examples, i.e. Cobb-Douglas utility functions on the one hand and quasi-linear utility functions (that are linear in private consumption) on the other.

Let some Pareto optimal allocation $A = (x_1^A, ..., x_n^A, G^A)$ be given. The utility level of agent i in allocation A is denoted by $u_i^A = u_i(x_i^A, G^A)$, and $h_i^A(G) = h_i(G, u_i^A)$ is the indifference curve corresponding to u_i^A as a function of public-good supply G. Agent i's contribution to the public good then is $z_i^A = y_i - x_i^A$, which, in principle, may be positive or negative. First of all, we define what is meant by blocking a given allocation and by the core property.

Definition 1: The given allocation A can be blocked by a subgroup of agents, i.e. a coalition $K \subset N = \{1, ..., n\}$ of $k < n$ agents if there is an allocation $B(K) = (x_1^{B(K)}, ..., x_k^{B(K)}, G^{B(K)})$ for coalition K such that

(1) $$c(G^{B(K)}) + \sum_{i \in K} x_i^{B(K)} = Y_K \quad \text{and}$$

(2) $$u_i^{B(K)} = u_i(x_i^{B(K)}, G^{B(K)}) > u_i^A$$

holds for all agents $i \in K$. A Pareto optimal allocation $A = (x_1^A, ..., x_n^A, G^A)$ lies in the *core* if no coalition exists that is able to block A.

This definition says that a blocking coalition K must be able to obtain a Pareto improvement for its members by leaving the allocation A and providing the public good only by using the total income $Y_K = \sum_{i \in K} y_i$ that is available in subgroup *K*. The allocation $B(K)$ will be called a *blocking standalone allocation* of the deviating coalition K. If an allocation were not Pareto optimal it would, according to the criterion provided by Definition 1, trivially be possible to block it even by the whole group of all agents. To exclude this uninteresting possibility the definition is restricted to Pareto optimal allocations.

In order to characterize the core and to infer equity criteria for the core property we will make use of the egalitarian-equivalent public-good levels as devised by Moulin (1987). **These** are defined as follows.

Definition 2: Given some allocation $A = (x_1^A, ..., x_n^A, G^A)$ agent i´s *egalitarian-equivalent level of public-good supply* $\bar{G}_i^A$ is given by

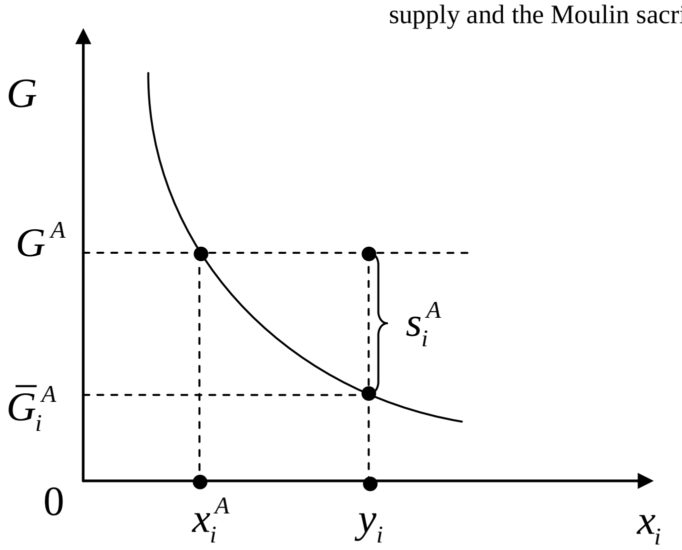
(3) $$u_i(y_i, \bar{G}_i^A) = u_i(x_i^A, G^A) = u_i^A .$$

The *Moulin sacrifice* of agent *i* then is

(4) $$s_i^A := G^A - \bar{G}_i^A .$$

The definition of egalitarian-equivalent public-good level means that agent i has the same utility as in the original allocation A when she consumes $\bar{G}_i^A$ of the public good and makes no contribution to the public good such that her private consumption is equal to her income y_i (see Figure 1). Given the properties of utility functions assumed above, egalitarian-equivalent public-good levels exist and are uniquely determined for all agents i if public-good supply G^A and all private consumption levels x_i^A are strictly positive.

Figure 1: Egalitarian-equivalent public-good supply and the Moulin sacrifice



Agent i´s Moulin sacrifice then can be interpreted as an equivalent by which her public-good contribution $z_i^A = y_i - x_i^A$ is transformed into units of the public good. It is a willingness-to-pay measure and indicates which amount of the public good agent *i*, starting from initial public-good supply G^A, is ready to give up if she could in return increase her private consumption from x_i^A to y_i. Therefore, the Moulin sacrifice of an agent is a subjective burden for the contribution she makes to produce the public good, given allocation *A*. For the same agent the Moulin sacrifice is the higher the higher her individual public contribution is. For different agents that make a public-good contribution of the same size the Moulin sacrifice is lower for the agent with the stronger preference for the public good and/or, given normality of the public good, with the higher income.[2] In the following section the Moulin sacrifice will be used as an equity standard by which the core property can be characterized.

3. Blocking Coalitions of Overburdened Agents

We first of all provide a basic condition which is necessary as well as sufficient for the ability of some coalition K to block a given allocation A. This simple requirement is in the spirit of duality theory (see Cornes, 1992), and it will serve as an instrument in our further considerations.

[2] See Buchholz and Peters (2008) for a detailed analysis how the size of the Moulin sacrifice depends on income and preferences.

Proposition 1: A coalition K can block the allocation A if and only if there is some level of public-good supply $\tilde{G}$ such that

(5) $$\sum_{i\in K} h_i^A(\tilde{G}) + c(\tilde{G}) < Y_K.$$

Proof: (i) Only if: If K can block allocation A by choosing the allocation $B(K)$, condition (2) directly gives that $h_i^A(G^{B(K)}) < x_i^{B(K)}$ must hold for all $i \in K$. Then condition (5) is obvious fulfilled by letting $\tilde{G} = G^{B(K)}$.

(ii) If: According to inequality (5), we have $d := \sum_{i\in K}(y_i - h_i^A(\tilde{G})) + c(\tilde{G}) > 0.$ The allocation $B(K)$ defined by $G^{B(K)} = \tilde{G}$ and $x_i^{B(K)} = h_i^{B(K)}(\tilde{G}) + \frac{d}{k}$ for all $i \in K$ then is feasible, given the total income Y_K of subgroup K. Clearly, $B(K)$ blocks the initial allocation A since all agents in subgroup K are strictly better off in $B(K)$ than in A. QED.

Proposition 1 can now be used to explore the composition of potential blocking coalitions more closely. In this context we first show that, if blocking is possible at all, always a specific type of a blocking coalition exists. The idea is to leave out all agents that would need an income transfer in the blocking coalition and, at the same time, to add those agents that would be willing to make a positive contribution to the public good.

Proposition 2: If an allocation $A = (x_1^A, ..., x_n^A, G^A)$ can be blocked and thus is not in the core, there must be some threshold level $\breve{s}$ for the Moulin sacrifices such that the agents with the highest sacrifice form the blocking coalition $\breve{K} = \{i \in N : s_i^A \geq \breve{s}\}$.

Proof: Assume that some coalition K can block the given allocation A by choosing public-good supply $G^{B(K)}$. Then we first show that the coalition defined by $K' := \{i \in N : G^{B(K)} \geq \bar{G}_i^A\}$ is also able to block the allocation A. As indifference curves are downward sloping we clearly have $K' := \{i \in N : h_i^A(G^{B(K)}) \leq y_i\}$ which implies the following inequalities:

(6) $$\sum_{i\in K'}(y_i - h_i^A(G^{B(K)})) \geq \sum_{i\in K'\cap K}(y_i - h_i^A(G^{B(K)})) \geq \sum_{i\in K}(y_i - h_i^A(G^{B(K)})) > c(G^{B(K)}).$$

The first inequality in (6) holds since $y_i - h_i^A(G^{B(K)}) \geq 0$ for all $i \in K'$ from the definition of K' and, clearly, $K' \cap K \subseteq K'$. The second inequality is obtained since for all agents i that are in K but not in K' we have $y_i - h_i^A(G^{B(K)}) < 0$, and the third inequality is implied by the only-if-part of Proposition 1. The if-part of Proposition 1 then directly gives that coalition K' is also able to block the allocation A. If we now define $\breve{s} := G^A - G^{B(K)}$ as the threshold levels for the Moulin sacrifices the assertion of Proposition 2 follows since obviously $\breve{K} = \left\{ i \in N : s_i^A \geq \breve{s} \right\} = K'$. QED.

The construction made in the proof of Proposition 2 can verbally be explained as follows: We start from some coalition that blocks allocation A and has public-good supply $G^{B(K)}$. Another blocking coalition K' then is obtained

- by having the same level $G^{B(K)}$ of public-good supply also in the blocking allocation that is chosen by coalition K'.
- by eliminating all agents $i \in K$ that only can attain utility u_i^A given public-good supply $G^{B(K)}$ when they receive a positive income transfer (and thus make a negative contribution to the public good in the original blocking allocation),
- by adding all individuals $i \in N$ that could get utility u_i^A by making a non-negative contribution to the public good if public-good supply were $G^{B(K)}$,

The result in Proposition 2 basically says that, if an allocation can be blocked at all, there is always a blocking coalition consisting of those agents whose Moulin sacrifice is higher than that of the other agents outside the blocking coalition. If an allocation is not in the core there must be a group of "overburdened" agents that has an incentive to withdraw leaving behind the agents with low Moulin sacrifices. Those who are excluded from cooperation enjoyed some free-ride at the expense of the 'overburdened' agents who then do better by leaving the original allocation.

It is a further straightforward implication of Proposition 2 that the existence of a blocking coalition requires at least some heterogeneity of the levels of Moulin sacrifices. In this way a basic result obtained by Moulin (1987), which says that egalitarian-equivalent cost sharing and thus equality of the Moulin sacrifices leads into the core, is confirmed in an alternative way. [3]

Proposition 3: The egalitarian-equivalent allocation $(x_1^E,...,x_n^E,G^E)$ in which the Moulin sacrifice is identical for all agents lies in the core.

By a standard continuity argument Proposition 3 entails that any Pareto optimal allocation in which the individual Moulin sacrifices do not diverge too much will also be in the core. That the core property crucially depends on the degree of homogeneity of the Moulin sacrifices is reinforced by the following result which shows that a more balanced distribution of Moulin sacrifices makes it more likely that an allocation belongs to the core and in this sense is stable.

Proposition 4: Assume that in an economy with n agents aggregate income and individual preferences are fixed and let some Pareto optimal allocation A be in the core for some initial income distribution $(y_1,...,y_n)$. Then A remains in the core when a shift of income from one agent to another is made that equalizes the distribution of the Moulin sacrifices but leaves their ranking unchanged.

Proof: Assume that agents are ranked according to their Moulin sacrifices, i.e. $s_1^A \geq ... \geq s_n^A$. Then only by shifting income from an agent j to an agent $l > j$, which decreases the Moulin sacrifice of the donor and increases that of the recipient, leads to a more equal distribution of the Moulin sacrifices if, as assumed, the ranking of the sacrifice levels is to be kept unchanged. Let for any $k = 1,...,n$ aggregate income of the subgroup $K(k) = \{1,...,k\}$ be denoted by Y_k before the income transfer is made and $\tilde{Y}_k$ be aggregate income of this subgroup after the transfer. Obviously, $\tilde{Y}_k = Y_k$ for $k < j$ or $k \geq j$ and $\tilde{Y}_k < Y_k$ for $j \leq k < l$. It then follows from Proposition 1 that **even** after the transfer no subgroup $K(k)$ can block the given allocation A. According to Proposition 3, however, some of these subgroups has to have the ability to block if A can be blocked at all. This shows that the allocation A remains in the core after the transfer has been made. QED.

[3] Note, however, that in other types of economies with a public good an egalitarian-equivalent allocation need not be in the core (see Hahn and Gilles, 1998).

4. An Example

The results of Propositions 3 and 4 can be illustrated by a simple example in which there are two agents $i=1,2$ with identical Cobb-Douglas preferences, i.e. $u(x_i,G)=x_iG$. The production technology for the public good now has constant returns to scale and aggregate income is $Y=1$. Let (y_1,y_2) denote any income distribution and $A=(x_1^A,x_2^A,G^A)$ some Pareto optimal allocation. It directly follows from the Samuelson rule that Pareto optimality requires $G^A=\frac{1}{2}$ and thus $x_1^A+x_2^A=\frac{1}{2}$ in this case. An allocation A is in the core given (y_1,y_2) if each agent's standalone utility $\frac{y_i^2}{4}$ is smaller than the utility $\frac{x_i^A}{2}$ which she has in A. We now depict the set of core allocations in a diagram (see Figure 2) with y_1 at the horizontal and x_1^A at the vertical axis. The lower contour of the core set is given by $x_1^A=\frac{1}{2}y_1^2$, where agent 1 is indifferent between staying in the coalition and choosing her standalone allocation. The upper contour is analogously obtained from the non-blocking condition for agent 2, which gives $\frac{1}{2}-x_1^A=x_2^A=\frac{1}{2}y_2^2=\frac{1}{2}(1-y_1)^2$ and thus $x_1^A=y_1-\frac{1}{2}y_1^2$. The lower and upper contours meet in the origin *O* and the point $M=(0,\frac{1}{2})$ such that the set of core allocation is represented by the lens *OM.*

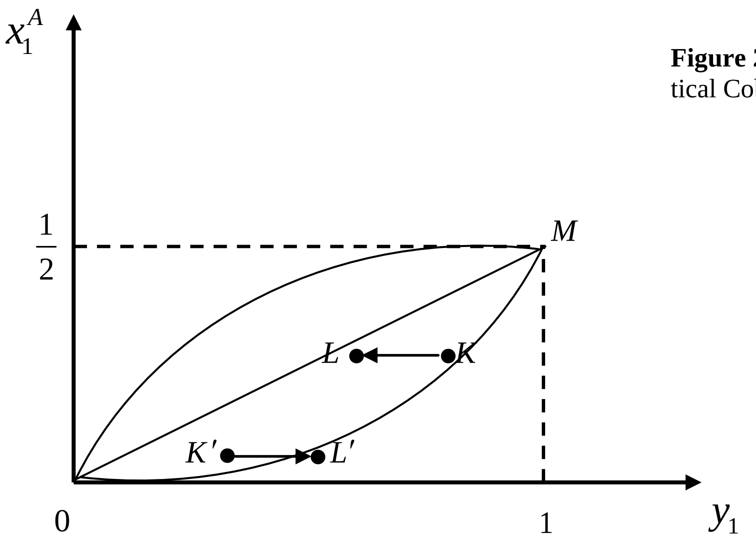


Figure 2: The core in the case of identical Cobb-Douglas preferences

Egalitarian-equivalent public-good supply $\bar{G}_i^A$ of agent $i=1,2$ then is obtained from the condition $x_i^A G^A = x_i^A \frac{1}{2} = y_i \bar{G}_i^A$ as $\bar{G}_i^A = \frac{1}{2}\frac{x_i^A}{y_i}$ which gives $s_i^A = \frac{1}{2}(1-\frac{x_i^A}{y_i})$ as Moulin sacrifice of agent i. Equality of both agents' Moulin sacrifices, i.e. $s_1^A = s_2^A$, is attained if $x_1^A = \frac{1}{2}y_1$ which yields the straight line that connects the points *O* and *M.* This equal Moulin sacrifice line clearly lies within the core lens *OM* which confirms Proposition 3 for the special case. Below (above) the equal sacrifice line agent 1's Moulin sacrifice is larger (smaller) than that of agent 2.

Now let us start from some point K in the lens that lies below the equal sacrifice line and fix the level of agent 1's private consumption x_1^A. An income transfer from agent 1 to agent 2, leading to point L, reduces agent 1's Moulin sacrifice and increases that of agent 2 thus equalizing the distribution of Moulin sacrifices. As long as the original ranking of the Moulin sacrifices is not changed by the transfer, i.e. L still lies to the right of the equal Moulin sacrifice line, L is contained in the core lens. This confirms Proposition 4 for the specific example. Note that, under the otherwise same assumptions, this includes the case where the transfer from agent 1 to agent 2 gives a more equal income distribution. In general, however, equalization of income may lead out of the core. In Figure 2 this would happen if the original point *K'* were close to the origin *O* and income is shifted from agent 2 to agent 1, which is depicted by the move from K' to L'.

5. Determinants of Moulin Sacrifice and the Special Case of Quasi-Linear Preferences

When preferences and income levels are the same for all agents there is clearly a perfect correlation between an agent's Moulin sacrifice and her contribution to the public good. But in the general case the size of the individual Moulin sacrifice is also determined by the level of personal income and individual preferences. So an agent *i*'s Moulin sacrifice not only becomes high (low) for a large (small) contribution to the public good z_i^A but also for a low (high) income level y_i when the public good is non-inferior and a weak (strong) individual preference for the public good (see Buchholz and Peters, 2008). In the special situation when all agent make a public-good contribution of equal size Proposition 2 thus implies that a coalition of agents with low income or weak preferences for the public good will have a rather high incentive to break off.[4]

In order to explain, in the case of homogenous preferences, how different income levels affect the Moulin sacrifice we look again at the example of the previous section where two agents have

[4] For the case of an economy with multilateral externalities a related result concerning different preferences has been suggested by Helm (2001).

the same utility function $u_i(x_i, G) = x_i G$, aggregate endowment is $Y = 1$ and the Pareto optimal allocation $A = (x_1^A, x_2^A, G^A) = (\frac{1}{16}, \frac{7}{16}, \frac{1}{2})$ is given. The income distribution now is taken to be $(\hat{y}_1, \hat{y}_2) = (\frac{1}{7}, \frac{6}{7})$. For the Moulin sacrifices of the two agents we obtain $\hat{s}_1^A = \frac{9}{32} = 0.281$ and $\hat{s}_2^A = \frac{47}{192} = 0.245$ while the contributions to the public good are $\hat{z}_1^A = \frac{9}{112}$ and $\hat{z}_2^A = \frac{47}{112}$. So even though agent 1 makes a much smaller public-good contribution in allocation *A* than agent 2 she has nevertheless the higher Moulin sacrifice. The explanation is that the willingness to pay for the public good is higher for the rich agent 2 than for the poor agent 1 such that, as measured in public-good equivalents, the higher public-good contribution means a lower subjective burden for agent 2. Examples that highlight the effect of a diversity of preferences can also easily be constructed, e.g. by applying different asymmetric Cobb-Douglas utility functions for both agents.

The income effect, i.e. the increase of marginal willingness to pay for the public good when income increases, is excluded when agents have quasi-linear utility preferences given by

(7) $$u_i(x_i, G) = x_i + \varphi(G).$$

where $\varphi'(G) > 0$ and $\varphi''(G) < 0$. [5] Such preferences are of special importance in environmental economics where the private good x_i serves as the numéraire and the benefit of environmental quality *G* is measured in monetary terms, i.e. in units of the numéraire good. For preferences as described in (7) all indifference curve are parallel along the x_i -axis such that equal Moulin sacrifice with equal public-good contributions (see Figure 3 for two agents *j* and *k*) and a higher Moulin sacrifice coincides with a higher public-good contribution even if income differs between the agents.

[5] This case is also considered by Chander and Tulkens (1997) in their analysis of coalition stability.

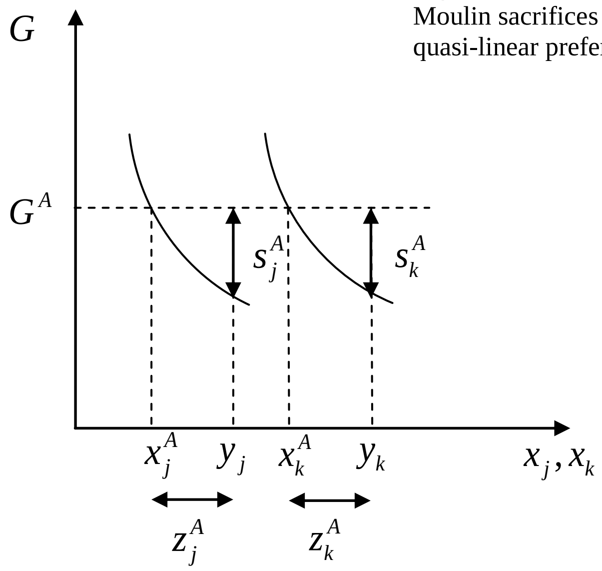


Figure 3: Public-good contributions and Moulin sacrifices in the case of identical quasi-linear preferences.

With quasi-linear preferences it is then possible to take the levels of public-good contributions z_i^A instead of the Moulin sacrifices s_i^A as indicators for the individual burden that result for different agents in allocation *A*. In this case Proposition 3 reads as follows:

Proposition 5: Assume that all agents have the same quasi-linear utility function. If an allocation $(x_1^A,...,x_n^A,G^A)$ is not in the core of the economy then there is some $\hat{z}$ such that the coalition $\breve{K} = \left\{i \in N : z_i^A \geq \hat{z}\right\}$ can block the allocation.

So if in the case of identical quasi-linear preferences an allocation can be blocked at all there must be a group with relatively high public-good contributions that has an incentive to separate.

6. A Simple Core Test

To know whether a certain Pareto optimal allocation *A* lies in the core or not is important since, according to our previous analysis, all core allocations are both equitable and stable and thus are obvious candidates for cooperative solutions in a public-good economy. The core of a public-good economy normally is rather large, even if the number of agents in the public-good economy is high which makes a sharp contrast to economies with public goods. (Example for this major result on the core in a public-good economy are provided, e.g., by Milleron, 1972, Muench, 1972, and more recently by Buchholz and Peters, 2007. Conley, 1994, Wooders, 1997, Vasil'ev, Weber and Wiesmeth, 1995, and Allouch, 2010, however, describe specific situations in which the "Edgeworth conjecture" also holds in the presence of public goods.) Moreover, if one wants to make a specific selection among the core allocations no "natural" solution exists which would make all other core allocations uninteresting and thus render a core test redundant. So it is a well-known fact that not only Moulin's egalitarian-equivalent solution but in the case of a constant returns technology also the Lindahl equilibrium belongs to the core (see Foley, 1970). In most cases, however, both solutions will not coincide. This ambiguity supports the view that all core allocations should "have a claim on our attention as outcomes of plausible political processes" (Cornes and Sandler, 1996, p.303). We therefore now want to develop a procedure by which it can be easily checked which allocations lie in the core.

The proposed core test works as follows: Again we rank the agents according to their Moulin sacrifice levels in a given allocation A, i.e. as in the proof of Proposition 4, we assume

(8) $$s_1^A \geq ... \geq s_n^A$$

and consider the subgroups $K(k)$. For any $k = 1,...,n-1$ we then consider the function $Y_k^A(G) := \sum_{i=1}^{k} h_i^A(G) + c(G)$ which, in the spirit of duality theory (see Cornes, 1992, for a general presentation), gives the minimum aggregate income that is needed to have public-good supply G when each agent in coalition $K(k)$ to attain the utility level u_i^A and public-good supply is G. By $\hat{G}_k^A$ we then denote the public-good level that minimizes $Y_k^A(G)$ (and which is characterized by the Samuelson rule $\sum_{i=1}^{k} \frac{\partial h_i^A}{\partial G}(\hat{G}_k^A) = -c'(\hat{G}_k^A)$)**.** Thus, we obtain the minimum amount of aggregate income which is needed to provide utility u_i^A for each agent $i \in K(k)$

(9) $$\hat{Y}_k^A := \sum_{i=1}^{k} h_i^A(\hat{G}_k^A) + c(\hat{G}_k^A) .$$

It is now easily possible to infer whether a given allocation A is a core allocation or not.

Proposition 6: Given the ranking (8) an allocation is in the core of the public-good economy if and only if

(10) $$\hat{Y}_k^A \geq Y_k := \sum_{i=1}^{k} y_i$$

for all $k = 1,...,n-1$.

Proof: The result directly follows from combining Proposition 1 and Proposition 3. QED.

The criterion for the core property that is provided by Proposition 6 is simple insofar as at most $n-1$ coalitions have to be inspected in order to confirm the core property of the given allocation A. The number of required tests therefore is much lower than the total number of all possible coalitions which is $\sum_{j=1}^{n} \binom{n}{j}$.

7. Conclusion

The core property which means the impossibility to improve by separation for any subgroup is a general condition for the stability of cooperative solutions. In a public-good economy as considered in this paper a Pareto optimal allocation lies in the core if no sub-coalition is able to provide the same utility levels to its members as in the original allocation by only using a smaller amount of resources as being available in this sub-coalition. Using this basic core criterion, it was possible to give an intuitive interpretation of the core property that refers to an equity norm. So it could be shown that just those coalitions whose members bear a high Moulin sacrifice and so are disadvantaged relative to other agents can be expected to have the ability to block. Therefore, equity w.r.t. to the levels of Moulin sacrifices is necessary and sufficient for the core property of an allocation and thus for its stability. In this way we extend the result by Moulin (1987) who has, under additional assumptions, given a core-based characterization of the egalitarian-equivalent solution where Moulin sacrifices are equal.

The Moulin sacrifice concept moreover incorporates both the ability-to-pay and the benefit principle as additional equity norms, whose application requires consideration of the individual characteristics of the participating agents, i.e. their income and their preferences. Thus, in total, a relationship between stability of cooperation according to the core concept and the three classical basic principles for fair burden-sharing (equality of sacrifice, ability-to-pay, benefit principle) could be established. This not only resuscitates a very long and venerable tradition in the public finance literature but also gives an additional and normatively sound motivation for the core concept.

In reality blocking coalitions of overburdened agents often will have the difficulty to identify and organize themselves, and it is not a priori clear which coalition in fact will withdraw (see Ray, 2007, for a general discussion). If such transaction costs are high, splitting-off by a subgroup becomes more difficult and an unfair burden-sharing in a cooperative venture is more likely to persist. Formation of a blocking coalition is impeded less when the distribution of burdens follows an easily discernible geographic pattern such that some regions feel exploited by the others. Then stability of a federation is endangered because the disadvantaged regions may gain from becoming politically independent thus "blocking" the common state. Even though the relationship between interregional distribution and stability is, at a general level, a major topic in the theory of federalism, specific application to public-good provision and reference to the core concept is rare. In this sense, this paper also tries to bring the theory of federalism and standard public-good theory closer together.

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