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ENDOGENOUS GROWTH IN AN OPEN ECONOMY AND THE REAL EXCHANGE RATE

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Abstract

This paper is a step in the direction of a larger research project aimed at determining the long run equilibrium value of the euro/dollar real exchange rate. Given this value, one could then give a precise meaning to the notion of undervaluation or overvaluation of the euro, and calculate its misalignment. The problem however arises of how to assess the reliability of such misalignment calculations. In our opinion, we must have a benchmark (namely a period in which we exactly know *from outside sources* the misalignment itself), against which we can test the validity of the model underlying our calculations. This of course is not (yet) possible for the euro, so that all the calculations of the misalignment of the euro that have been made can only be compared with one another, without knowing which is the good one. Hence, before building a model to be applied to the euro/dollar, we tested our ideas incorporating them in a basic model to be applied to the lira/dollar in a period in which we do exactly know the actual misalignment of the lira from outside sources.

Keywords: NATREX, equilibrium exchange rates, international capital flows, misalignment.

JEL Classification: F43, F31, C39, C59.

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1 Introduction

This paper is a step in the direction of a larger research project aimed at determining the long run equilibrium value of the euro/dollar real exchange rate. Given this value, one could then give a precise meaning to the notion of undervaluation or overvaluation of the euro, and calculate its misalignment. The problem however arises of how to assess the reliability of such misalignment calculations. In our opinion, we must have a benchmark (namely a period in which we *exactly* know *from outside sources* the misalignment itself), against which we can test the validity of the model underlying our calculations. This of course is not yet possible for the euro, so that all the calculations of the misalignment of the euro that have been made (for a survey see Stein, 2001) can only be compared with one another, without knowing which is the good one. Hence, before building a model to be applied to the euro/dollar, we tested our ideas incorporating them in a basic model to be applied to the lira/dollar in a period in which we do exactly know the actual misalignment of the lira from outside sources.

This model consists of three main ingredients:

1) the NATREX model (see the next section) to determine the long-run real exchange rate. This model, however, does not account for growth, which leads us to

2) the introduction of growth. Since in real life growing economies do not seem to converge to a steady state in which all magnitudes grow at the same rate, the kind of growth to be considered is *endogenous growth.* However, endogenous growth models are mostly referred to a closed economy; the few that deal with endogenous growth in open economies are usually of the "pure trade theory" or "barter" type, where the exchange rate plays no role. Notwithstanding their theoretical interest and sophistication, they are unable to deal with actual problems, where the exchange rate plays a prominent role. A bridge can be built if we concentrate on the real exchange rate.

3) continuous time econometrics. The NATREX model and endogenous growth models are built and analysed in continuous time (differential equations). However, when it comes to estimation, discrete time approximations are used which may turn out to incorrectly represent the original model. The tools of continuous time econometrics enable us to rigorously estimate the parameters of dynamic continuous time models.

2 The NATREX Approach

While the theory of nominal exchange rate determination is still in an unsatisfactory state, more encouraging results have been achieved in the analysis of the real exchange rate (RER hereafter), especially when the time horizon is taken to be the long run. Among the theories that have been developed in the recent past (for a survey see MacDonald and Stein, 1999), the NATREX (acronym of *NAT*ural *R*eal *EX*change rate) approach to the equilibrium RER determination, originally formulated by Stein (1990), has been able to satisfactorily explain the medium-to-long run dynamics of the RER in several industrial countries: USA (Stein 1995a, 1995b), Australia (Lim and Stein, 1995), Germany (Stein and Sauernheimer, 1996), France (Stein and Paladino, 1998), Italy (Stein and Paladino, 1998; Gandolfo and Felettigh, 1998), Belgium (Verrue and Colpaert, 1998).

In these models, however, growth is not explicitly considered. What we aim to do is to introduce the simplest form of endogenous growth into the NATREX model that we have developed (Gandolfo and Felettigh, 1998). For this purpose we briefly summarize the NATREX approach.

The NATREX theory explains the dynamics of the medium-to-long run equilibrium RER. It is not a single model but rather a class of models each tailored to the particular features of the economy under study. In this section we briefly recall the theoretical framework of the NATREX approach: for a complete treatment, the reader should consult Allen (1995) and Stein (1995a). The NATREX is the intercyclical equilibrium real exchange rate that ensures the balance of payments' equilibrium in the absence of cyclical factors, speculative capital movements and movements in international reserves. In other words, the NATREX is the equilibrium real exchange rate that would prevail if the above-mentioned factors could be removed and the GNP were at capacity. Since it is an equilibrium concept, the NATREX should guarantee both the internal and the external equilibrium, the focus being on the long run. The long-run internal equilibrium is achieved when the economy is at capacity output, that is when the GNP is at its potential level. The long-run external equilibrium is achieved when the long-term accounts of the balance of payments are in equilibrium. Short term (speculative) capital movements and movements in official reserves are bound to be short term transactions, since they are unsustainable in the long run. In the long-run equilibrium they must average out at zero; hence, the excess of national (private plus public) investment over national saving must be entirely financed through international long term borrowing.

Under these conditions long term capital inflows and excess national investment over saving coincide, so that also the real market long-run equilibrium condition and the long-term external equilibrium condition coincide:

$$S - I = CA, \tag{1}$$

where CA is the balance of payments' current account, the private and public sectors having been aggregated into a single one. Relation (1) is intended in real terms: the model assumes neutrality of money and that monetary policy keeps inflation at a level compatible with internal equilibrium (at least in the long run). Therefore, the focus being on the real part of the economy, there is no need to model the money market. Perfect international capital mobility is assumed: the real interest rate is driven by the portfolio equilibrium condition or real interest parity condition (supposed to hold instantaneously), possibly with a risk premium.

The system is assumed to be self-equilibrating (hence the adjective *natural* in the acronym NATREX). Take for example an initial position of full equilibrium ($S - I = CA = 0$) and suppose an exogenous shock leads to a situation where $S - I < 0$. Given the perfect international capital mobility, the interest rate cannot play the role of the adjustment variable; rather, the difference between national investment and national saving originates a corresponding inflow of long-term capital. The RER appreciates accordingly, leading to a deterioration in the current account. The capital inflow also causes an increase in the stock of foreign debt, which in turn determines (see below, Eq.(3)) a decrease in consumption and hence an increase in saving, until equilibrium is restored. In conclusion, the RER is the adjustment variable in equation (1).

The hypothesis of perfect foresight is rejected. Rather, rational agents that efficiently use all the available information will base their intertemporal decisions upon a sub-optimal feedback control (SOFC) rule (Infante and Stein, 1973; Stein, 1995a). Basically, SOFC starts from the observation that the optimal solution derived from standard optimization techniques in perfect-knowledge perfect-foresight models has the saddle-path stability property, hence the slightest error in implementing the stable arm of the saddle will put the system on a trajectory that will diverge from the optimal steady state. Actual optimizing agents know that they do not possess the perfect knowledge required to implement the stable arm of the saddle without error, hence it is rational for them to adopt SOFC, which is a closed loop control that only requires current measurements of a variable, not perfect foresight, and will put the economy on a trajectory which is asymptotic to the unknown perfect-foresight stable arm of the saddle.

The consumption and investment functions are derived accordingly, through dynamic programming techniques with feedback control. No difference is made between the private and the public decisional process. The model can be solved for its medium run and long run (steady state) solutions. Any perturbation on the real fundamentals of the system pushes the equilibrium RER on a new medium-to-long-run trajectory. Since cyclical, transitory and speculative factors are considered noise, averaging out at zero in the long run, the actual RER converges to the equilibrium trajectory. The PPP theory turns out to be only a special case of the NATREX approach: "the issue is not whether or not the real exchange rate is stationary over an arbitrary period, but whether it reflects the [real] fundamentals." (Stein, 1995a, p. 43).

3 The Theoretical Model

We start with a SOE (Small Open Economy) model. Admittedly, this does not allow to consider spillover effects (or "foreign repercussions", as they were once called), but is sufficient for the purpose at hand. This model will have to be enlarged in the future with the endogenization of the ROW (Rest of the World) in a two-country context.

The main point in our model is that all variables adjust with a certain lag to their desired (or partial equilibrium) level, according to the dynamic disequilibrium modelling approach in continuous time (see, for example, Gandolfo, 1981; Barnett, Gandolfo and Hillinger eds., 1996).

The basic equations are described below. In what follows the symbol D denotes the operator d/dt.

Investment is the sum of private plus public investment; the same is true as regards consumption. Population is assumed to be constant, hence it is indifferent to use total or per capita magnitudes.

$$\begin{aligned} &\mathrm{D}I = \alpha_1(\widehat{I} - I), \\ &\text{where} \\ &\widehat{I} = f_1[(MPK - R)], \quad \text{sgn } f[...] = \text{sgn } [...], \quad f_1' > 0. \end{aligned} \tag{2}$$

Real net fixed investment adjusts with a mean time lag $1/\alpha_1$ to its desired or partial equilibrium level $\widehat{I}$, the lag being due to adjustment costs (see, for example, Patrat, 1999, Ch. 7). $\widehat{I}$ is a sign-preserving function of the difference between the marginal productivity of capital (MPK) and the long-run real interest rate (R).

This investment function is derived in the context of an intertemporal optimization problem in which agents apply a suboptimal feedback control (SOFC) rule. The standard optimal control problem gives rise to the rule according to which the marginal productivity of capital must be equal to the sum of the growth rate and discount rate. In the neighbourhood of the steady state the optimal control is that the rate of investment must be proportional to the gap between actual and steady-state capital intensity. The implementation of this control requires, amongst other, the knowledge of the steady-state capital intensity (which also enters into the coefficient of proportionality). The slightest mistake would cause the system to diverge.

It can be shown (Infante and Stein, 1973, Table 1; Stein, 1995a, pp. 52-3) that a SOFC rule which

(i) requires only current measurements of the marginal product of capital,

(ii) is guaranteed to drive the system to the unknown steady-state capital intensity, and

(iii) is robust to perturbations,

is that (sub)optimal investment is positively related to the marginal productivity of capital less the discount rate. Now, "Let the real long-term interest rate substitute for the discount rate. The SOFC law states that one should focus upon the current marginal product of capital less the real long term rate of interest." (Stein, 1995a, p. 53).

As regards consumption, the NATREX approach assumes that saving (hence consumption) decisions are made independently of investment decisions. This is equivalent to assuming that optimizing agents are functionally separated into two categories, those who take investment decisions (firms) and those who take consumption-saving decisions (consumes). It can be shown (Stein and Sauernheimer, 1996, 109-110) that an appropriate optimization process will give rise to a function $\widehat{C} = f_2(Y, F)$, so that

$$\begin{aligned} &\mathrm{D}C = \alpha_2(\widehat{C} - C), \\ &\text{where} \\ &\widehat{C} = f_2(\underset{+}{Y}, \underset{-}{F}). \end{aligned} \tag{3}$$

Real consumption adjusts with a mean time lag $1/\alpha_2$ to its partial equilibrium or desired value $\widehat{C}$, a positive function of real domestic product (Y) and a negative function of the real stock of net foreign debt (F, where a negative F means foreign assets). The assumed signs of the partial derivatives are shown below each variable. The intuition behind the formulation of $\hat{C}$ is the following. From standard optimizing theory, consumption is proportional to wealth or, equivalently, to permanent income, where the coefficient of proportionality depends on the parameters of the utility function and the rate of time preference. Permanent income is based upon the expectation that current income will grow at a given expected rate; future expected income flows are discounted at the rate of time preference. Hence consumption ultimately depends on current income. The presence of the term F is due to a feedback control on the part of the government (recall that C is private + public consumption). Part of the government's debt is foreign held. The optimal long run value of foreign debt is zero (see Sect. 3.1). When the government realizes that its foreign debt is positive ($F > 0$), it changes its policy by decreasing current expenditure.

It would be possible to express Y per capita in terms of K per capita via a production function, but at the moment we do not want to commit ourselves to a precise production function, postponing the question until later, hence we keep the form (3).

The NATREX theory accepts the standard assumptions in international economics, that real exports depend positively on the ROW's real GDP (Y^*) and negatively on the real exchange rate , while imports depend positively on the home country's real GDP and positively on the real exchange rate. The balance of trade BT adjusts with a mean time-lag $1/\alpha_3$ (due to transport times etc.) to its partial equilibrium value:

$$\begin{aligned}&\mathrm{D}BT = \alpha_3(\widehat{BT} - BT),\\&\text{where}\\&\widehat{BT} = f_3(\underset{-}{E}, \underset{-}{Y}, \underset{+}{Y^*}).\end{aligned} \tag{4}$$

We now have the equation for the real interest rate

$$\begin{aligned}&\mathrm{D}R = \alpha_4(\widehat{R} - R),\\&\text{where}\\&\widehat{R} = R^* + \rho,\\&\rho = f_4(FY - FY_1^*).\end{aligned} \tag{5}$$

The basis for this equation is RIP (real interest rate parity) corrected with a risk premium (ρ). If investors take their decisions in real rather than nominal terms, then portfolio equilibrium in an open economy requires equal expected rates of return in real terms, possibly with a risk premium. Our model assumes that the foreign and domestic real interest rates satisfy the RIP condition with risk premium. This condition, however, does not hold instantaneously, but rather is achieved with a certain delay, due to market imperfections and to the corresponding sluggishness in the re-equilibrating process. Hence the domestic real interest rate adjusts with a mean time-lag $1/\alpha_5$ to its partial equilibrium value, which equals the foreign real interest rate plus a risk premium.

As regards the modelling of the risk premium, it must be noted that the literature on the subject has not yet found an agreement. We found useful hints for the estimation of the risk premium in Dooley and Isard (1983) and

Hooper and Morton (1982). In the end, we have modelled the risk premium ρ as a function of the difference between FY (the ratio of the domestic net foreign debt to *GDP)* and variable FY_1^*, defined as follows:

$$FY_1^* = FY^* \text{ if } FY^* > 0; FY_1^* = 0 \text{ if } FY^* \leq 0, \tag{6}$$

where FY^* is the ratio of the rest-of-the-world net foreign debt to foreign *GDP.* A plausible rationale behind our risk premium is the following: if ROW has a net creditor foreign position, ROW bonds are considered by the market as a safe asset, without risk. In this case, the risk index for domestic assets is related to the ratio of the domestic net foreign debt to *GDP.* If ROW also has a net foreign debt, then the risk index is related to the difference between FY and the corresponding ROW ratio (FY^*).

One of the points made by the endogenous growth literature is that the assumption of decreasing returns to factors (decreasing marginal productivity) should be dropped. The simplest production function with these properties is

$$Y^S = AK, \tag{7}$$

where A is a positive constant that reflects the technological level. For obvious reasons this function has come to be known in the recent literature as the "*AK*" production function (Barro and Sala-i-Martin, 1995), but its use in growth theory dates back at least to Harrod (1939) and Domar (1946). More sophisticated forms (including other factors of production) could be considered, but on the basis of the parsimony principle we decided to start with the simplest possible form.

The labour supply is fixed inelastically.

Aggregate demand is

$$Y^D = C + I_T + BT, \tag{8}$$

where I_T is total investment (fixed investment plus other items). We assume that output adjusts with a lag to excess demand, hence

$$\mathrm{D}Y = \alpha_5(Y^D - Y^S). \tag{9}$$

We now have the definitional equations

$$\mathrm{D}K = I, \tag{10}$$

$$CA = BT + NFI,$$

where NFI is net factor income from abroad; neglecting net labour income from abroad we have $NFI = -RF$ (net interest payments), hence

$$CA = BT - RF \tag{11}$$

defines the current account balance. The stock of net foreign debt F is in turn defined as

$$F(t) = F_0 - \int_0^t CA(\tau)\mathrm{d}\tau, \tag{12}$$

which derives from the balance-of-payments accounting identity

$$CA - \mathrm{D}NFA = 0, \tag{13}$$

where $\mathrm{D}NFA$ is the change in the stock of *net foreign assets* NFA. To avoid confusion, let us recall that the accounting principles of the balance of payments (IMF, 1993, p. 7) require that a decrease in foreign liabilities or an increase in foreign assets ($\mathrm{D}NFA > 0$) should be recorded as a negative figure (debit), and, conversely, that an increase in foreign liabilities or a decrease in foreign assets ($\mathrm{D}NFA < 0$) should be recorded as a positive figure (credit). Since

$$F \equiv -NFA, \quad \mathrm{D}F = -\mathrm{D}NFA, \tag{14}$$

from (13) have

$$\mathrm{D}F = -CA. \tag{15}$$

If we integrate (15) and assume that the arbitrary constant of integration is F_0, we obtain (12).

3.1 Equilibrium Growth, R&D and NATREX

Long-run equilibrium requires absence of any risk premium. In addition, we are interested in a NATREX equilibrium, in which the current account is in equilibrium given output growth at capacity. This last requires $Y = Y^D = Y^S$.

Absence of risk premium implies $\hat{R} = R = R^*$, while the production function (7) implies $MPK = A$. This shows the crucial role of A and R^*, since it is the difference between them that enhances or hinders investment according to Eq. (2). Investment will be positive if $A > R^*$. This will cause capital growth, and hence growth of output.

Thus we have reached the conclusion that equilibrium growth depends on what the foreign real interest rate happens to be.

To avoid this unpleasant conclusion we should endogenize A. For this purpose we assume that the productivity of capital is not a constant, but that its *increase* depends on investment in R&D, which in turn is positively related to the investment ratio, namely

$$\mathrm{D}A = \varphi(\frac{I}{K}), \varphi' > 0. \tag{16}$$

This is an admittedly *ad hoc* formulation, introduced to circumvent the lack of reliable data on R&D investment. Alternatively we could consider vintage capital, learning by doing etc., but at this stage we preferred to choose the simplest possible form.

Current account equilibrium, $CA = 0$, implies $F = F_0$ given Eq. (12). The constancy of F in turn implies $\mathrm{D}F = 0$. With $CA = 0$ and $\mathrm{D}F = 0$ the balance of payments is in equilibrium.

What about the NATREX? We first observe that, with F constant and $R = R^*$ constant, $CA = 0$ implies $BT = R^* F_0$ given Eqs. (20) and (12), where $BT = \widehat{BT}$ since we are in equilibrium. It can clearly be seen that, with Y^* exogenously given and Y endogenously determined at its capacity level, E can be determined from the implicit function $f_3(E, Y^S, Y^*) - R^* F_0 = 0$. If the appropriate invertibility conditions on the Jacobian are satisfied, we have

$$E_N = \phi(Y^S, Y^*, R^* F_0), \tag{17}$$

where the subscript N stands for NATREX. It also turns out that the system is conditionally stable (see the Appendix, where comparative dynamics and the problem of convergence are also considered).

This concerns the long-run equilibrium growth path. In the medium run, however (and this is what most concerns us in real life), the requirements of no risk premium (hence $R = R^*$) and no capital flows, are a bit too stringent. A more plausible alternative is to allow for $R \neq R^*$ and non-zero capital flows, while keeping the basic requirements of NATREX, namely $CA = 0$ with $Y = Y^S$. In this formulation the NATREX turns out to be

$$E_N = \psi(Y^S, Y^*, RF), \tag{18}$$

which is the formulation that we shall use in the empirical analysis.

4 Estimation Results

4.1 Introduction

For purposes of estimation we have slightly changed the investment, consumption, and current account equations.

As regards investment, we have added Y as an explanatory variable for the following reason. The variable I considered is private+public fixed investment. Given the peculiarities of the Italian fiscal system and policies, public I in the sample period is not determined by any optimizing criterion (profit maximization or other) but is a positive function (as all government expenditure) of Y. Hence we have the function

$$\widehat{I} = f_1(Y, A - R).$$

This does not imply the prevalence of Y in the explanation of investment. It is true that Y grows, but it is also true that A is not stationary since it grows according to Eq. (16). Hence the term $A - R$ maintains its importance in the explanation.

As regards consumption, the price variable P (implicit deflator of GDP) has been added to account for possible phenomena of money illusion, that may be present in the Italian economy. Let us start from

$$\widehat{C}_n = f_2(Y_n, F_n, P),$$

where the subscript $_n$ denotes nominal values. The function f_2 is assumed to be homogeneous, hence $\widehat{C}_n/P = \widehat{C} = (P^{-1})^{\mu} f_2(Y_n/P, F_n/P, 1)$, where $\mu > 0$ is the degree of homogeneity. In the case of no money illusion, $\mu = 0$, hence $\widehat{C} = f_2(\underset{+}{Y}, \underset{-}{F})$. In the presence of money illusion, the function is no longer homogeneous, and P has an own effect, that may be any, i.e.,

$$\widehat{C} = f_2(\underset{\pm}{P}, \underset{+}{Y}, \underset{-}{F}).$$

As regards the trade balance, exports and imports may have different adjustment speeds, hence they have been separately considered, rather than directly considering their balance, that is to say

$$\begin{array}{ll} D\log XGS = \alpha_3' \log(\widehat{XGS}/XGS), & \widehat{XGS} = f_3'(Y^*, E) \\ D\log MGS = \alpha_3'' \log(\widehat{MGS}/MGS), & \widehat{MGS} = f_3''(Y, E) \\ BT = XGS - MGS. & \end{array} \tag{19}$$

In empirical studies we must take account that CA also includes net unilateral transfers (UT) and net labour income from abroad (W), hence

$$CA = BT - RF + W + UT. \tag{20}$$

It only remains to specify the various functional forms. We have chosen them on the basis of previous empirical studies on the Italian economy (Gandolfo and Padoan, 1990; Gandolfo and Felettigh, 1998), hence the model to be estimated is the following (all parameters are assumed to be positive, unless otherwise indicated):

$$
\begin{aligned}
&S - I = CA, \\
&\quad \text{where } S \equiv Y - C, \quad CA = (XGS - MGS) - RF + W + U \\
&\mathrm{D}I = \alpha_1 \left(\hat{I} - I \right), \\
&\quad \text{where } \hat{I} = \gamma_0 + \gamma_1 Y + \gamma_2 (A - R), \gamma_0 \gtreqless 0, \\
&\mathrm{D}C = \alpha_2 \left(\hat{C} - C \right), \\
&\quad \text{where } \hat{C} = \gamma_3 e^{\beta_1 P} Y^{\beta_2} F^{-\beta_3}, \beta_1 \gtreqless 0, \\
&\mathrm{D} \log XGS = \alpha_3 \log(\widehat{XGS}/XGS), \\
&\quad \text{where } \widehat{XGS} = \gamma_4 E^{-\beta_4} Y^{*\beta_5}, \\
&\mathrm{D} \log MGS = \alpha_4 (\widehat{MGS}/MGS), \\
&\quad \text{where } \widehat{MGS} = \gamma_5 Y^{\beta_6} E^{\beta_7}, \\
&\mathrm{D}R = \alpha_5 (\hat{R} - R), \\
&\quad \text{where } \hat{R} = R^* + \beta_8 \left[(FY) - (F^* Y^*)_1 \right], \\
&\mathrm{D}Y = \alpha_6 \left(Y^D - Y^S \right), \\
&\quad \text{where } Y^D = C + I + (XGS - MGS) + CF, Y^S = AK, \\
&\mathrm{D}A = \alpha_7 I/K, \\
&\mathrm{D}F = -(XGS - MGS) + RF - W - UT, \\
&\mathrm{D}K = I.
\end{aligned}
\tag{21}
$$

4.2 Parameter estimates

Model (21) is a nonlinear differential equation system that can be estimated in continuous time and without need for linearisation by using the computer program ESCONA developed by C.R. Wymer (1993, 1995). We used quarterly data (for details see Data Appendix) from 1976:Q1 to 1992:Q4 for estimation, and from 1993:Q1 to 1995:Q4 for out-of-sample analysis. The resulting parameter estimates, together with their asymptotic standard errors and t-ratios (this is a shorthand notation for the ratio parameter estimate/asymptotic standard error, which is not distributed like a Student t but is asymptotically normal) are given in the table. Since the α's are adjustment speeds (with the exception of α_7), their reciprocals $1/\alpha$ can be interpreted as mean time-lags, namely the time required for about 63% of the discrepancy between the actual and desired value of the variable concerned to be eliminated by the adjustment process incorporated into the partial adjustment equation (Gandolfo, 1981).

PARAMETER ESTIMATES

Parameter	Point estimate	Standard error	t-ratio	Mean time-lag
α_1	0.198309	0.000702	282.30	5.04
α_2	0.016610	0.000127	130.44	60.20
α_3	0.002073	0.000011	186.60	482.62
α_4	0.018359	0.000099	184.67	54.47
α_5	0.080743	0.032603	2.48	12.38
α_6	0.952652	0.025140	37.89	1.05
α_7	0.000196	0.000001	138.15	
γ_0	-0.916439	0.010489	87.37	
γ_1	0.270593	0.002247	120.38	
γ_2	4.556514	0.087334	52.17	
γ_3	0.685129	0.006225	110.06	
γ_4	0.274452	0.002513	109.20	
γ_5	0.538720	0.003344	161.07	
β_1	0.074256	0.001405	52.84	
β_2	1.262833	0.018948	66.64	
β_3	0.027702	0.000217	127.28	
β_4	0.985452	0.022944	42.95	
β_5	2.237705	0.030132	74.26	
β_6	0.301429	0.001178	255.71	
β_7	0.140317	0.001838	76.33	
β_8	0.003854	0.000199	19.35	

Although our estimates are still preliminary, we note that all parameters have the correct sign and are highly significant. Hence the theory behind the NATREX is fully consistent with the data. First, the SOFC investment rule, according to which investment is positively related to the marginal productivity of capital less the real long term rate of interest, is confirmed ($\gamma_2 > 0$). Second, the negative dependence of total consumption on the real stock of foreign debt is also confirmed ($-\beta_3 < 0$). Also note that the presence of money illusion in the consumption function is confirmed (β_1 is significantly different from zero).

Elasticities have plausible values: for example, the sum of the absolute value of the export and import elasticities with respect to the real exchange rate, $\beta_4 + \beta_7$, is 1.126 (with a standard error 0.024, hence the sum is significantly greater than unity) so that the Marshall-Lerner condition is satisfied. This corresponds to the observation of the dramatic improvement in the Italian balance of trade after the huge depreciation of the Italian lira following the September 1992 crisis, an improvement that would not have taken place without the satisfaction of the Marshall-Lerner condition.

We also note that our modelling of the risk premium in the interest rate equation is consistent with the data (β_8 is significantly positive).

The result that does not convince us is that adjustment speeds look too low. Previous studies on the Italian economy carried out with a 24-differential equation continuous-time model (Gandolfo and Padoan, 1990; Gandolfo et al., 1996) did find that macroeconomic variables have relatively low adjustment speeds, but not so low as those found in the present model. The reason, in our opinion, is that adjustment speeds are variable through time, hence constraining them to be constant throughout the sample period is bound to give rise to economically implausible (although statistically excellent) results. This will be investigated in future work.

We now come to the results concerning endogenous growth. First, our findings support the AK modelling of the production function and its predictions regarding the dynamic returns to capital accumulations. Second, the endogenous determination of the productivity of capital (the A coefficient) is fully consistent with the data, since the crucial parameter α_7 turns out to be positive and highly significant. We thus feel that our unified treatment of investment in capital accumulation and investment in R&D is a step in the right direction for a full understanding of endogenous growth.

4.3 Predictive performance

To complete the study of the fit of the model we computed the in-sample and out-of-sample root mean square errors (RMSE) of *dynamic* forecasts. In discrete-time models the distinction is made between one-period (or *static*) and multi-period (or *dynamic*) forecasts. The former are those calculated using the actual observed values of any lagged endogenous variables present as explanatory variables in the model. The latter are those calculated using, for the lagged endogenous variables, the values generated by the model in the appropriate period. It is well known that dynamic forecasts are generally poorer than the static ones, since errors cumulate.

An equivalent distinction in continuous time models can be made according as the solution of the differential equation system used to produce the values of the endogenous variables is (a) calculated afresh in each period, or (b) calculated once and for all. In case (a) the differential equation system is re-initialized and solved n times (where n is the number of periods): in each time t the observed values of the endogenous variables at time t are used as initial values to solve the system and obtain forecasts for time $t+1$. This is equivalent to static forecasts in discrete-time models. In case (b) the observed values of the endogenous variables at the initial time are used as initial values in the solution of the differential equation system, which is then used to calculate the values of the endogenous variables throughout the sample period. This is equivalent to dynamic forecasts in discrete-time models.

Static forecasts can be used in both ex post forecasting (for example, to examine the in-sample predictive performance) and ex ante forecasting for one-period ahead. But if we wish to produce ex ante forecasts for several future periods and/or for a time interval different from the observation interval, we must use dynamic forecasts. Dynamic forecasts can of course also be used for ex post forecasting.

Although dynamic forecasts are generally poorer than static forecasts, they give a much better idea of the forecasting ability of the model. In the following table we give the in-sample and out-of-sample RMSE of dynamic forecasts, which were produced with reference to the same time interval inherent in the data (the quarter) for reasons of comparability. The errors have been expressed as a proportion of the observed values, hence the RMSE gives the average error as a percentage of the actual level of the endogenous variable.

In-sample and out-of-sample RMSE

Variable	In-sample	Out-of-sample
I	0.005014	0.016171
C	0.012613	0.022260
XGS	0.042879	0.048731
MGS	0.048393	0.030817
R	0.000756	0.000889
Y	0.028498	0.033390
A	0.008138	0.010924
F	0.020742	0.021600
K	0.006343	0.011148

Let us begin with in-sample RMSE. All variables have errors below 5%, and for some of them the result is really good. We point out that, although well below the 5% limit, the relatively highest errors are present in real exports and imports, the variables which play a crucial role in determining the NATREX. Out-of-sample errors are generally higher (except for MGS) than the corresponding in-sample ones, which is a normal occurrence, but in our case they are only slightly worse, and remain below the 5% limit.

4.4 The NATREX

To determine the NATREX we carried out a simulation of the estimated model with the appropriate modifications. These simply amount to imposing the condition $CA = 0$ in model (21), and to replacing Y with capacity output Y^S.

Rather than giving the numerical results in tabular form, we show them in a diagram (Fig. 1), where both the actual real exchange rate (E) and its NATREX equilibrium value ($NATREX$) are plotted. To avoid possible misunderstanding we stress that the NATREX does *not* aim at tracking the actual real exchange rate, but is, on the contrary, a measure of the long-run equilibrium real exchange rate, the benchmark against which we can measure the misalignment of the actual real exchange rate. Thus expressions like "the domestic currency is weak", "the domestic currency is strong", "the domestic currency is undervalued", "the domestic currency is overvalued", etcetera, which are often used in a vague sense, can be given a precise meaning. Let us remember that the real exchange rate (and hence the NATREX) has been defined in such a way that an *increase* means a (real) *appreciation* of the domestic currency. Hence when the actual real exchange rate is *lower* (*higher*) than the NATREX, it follows that the domestic currency is *undervalued* (*overvalued*).

The NATREX shown in the figure presents a certain quarterly volatility. This is an unavoidable consequence of working with quarterly data, since the conditions of internal and external equilibrium appear too stringent when assumed to hold in quarterly terms. However, apart from this volatility, the results that we obtain appear quite sensible and consistent with what is known from other sources on the real misalignment of the Italian lira in the period under consideration.

Better to compare our results with stylized facts, we define a misalignment index $\mu = 100(E/NATREX)$ (Fig. 2). Thus an undervaluation (overvaluation) of the domestic currency implies $\mu \lesseqgtr 100$ respectively.

4.4.1 The lira misalignment according to our model

In the evaluation of the real misalignment of the Italian lira three periods must be distinguished. In the second half of the Seventies the Italian lira is undervalued in real terms. In the third quarter of 1980 the actual real exchange rate (RER) jumps above its long-run equilibrium level and remains overvalued until the second quarter of 1993. After the currency crisis and the temporary abandonment of the EMS in September 1992, Italy experienced a regime of flexible exchange rates. In this period the actual real exchange rate moves (though not monotonically) towards its long-run equilibrium value, reaching it in the third quarter of 1993 and remaining in equilibrium until the first quarter of 1994, after which misalignment is again present.

What does our model say about the 1992 currency crisis? It clearly shows that the lira was overvalued at the outburst of the crisis. A few remarks are however in order on this point. The real overvaluation of the lira shown by our model is of the order of 5-10%, a number that may seem too low to warrant the crisis. Actually, the crisis was not (or not only) triggered by the overvaluation, but (mainly) by speculation caused by the liberalization of capital movements and destabilising expectations, as we have shown elsewhere (Gandolfo and Padoan, 1992).

4.4.2 Stylized facts

The analysis made possible by our simulations is in agreement with both the literature that has focused on the Italian economic policies during the period under consideration and the evaluation of the real misalignment of the Italian lira given over time by our monetary authorities.

It is a generally held opinion that during the seventies the Italian monetary authorities gave up the target of inflation control in favour of a policy aimed at re-equilibrating international transactions, allowing firms to defend their profit margins mostly through the depreciation of the exchange rate; hence a real undervaluation of the lira (Ciccarone and Gnesutta, 1993, p. 44; Bertocco, 1991, p. 208). In the eighties there was a U-turn in the policy stance: the monetary authorities now aimed at slowing down the inflation rate using the exchange rate among the tools (Bertocco, 1991, p. 220); hence a real overvaluation of the lira.

The then Governor of the Bank of Italy Carlo Azeglio Ciampi explicitly acknowledges the overvaluation of the Italian lira during the first half of the Eighties: "The central bank chose to deploy the exchange rate in the fight against inflation in the belief that balance-of-payments adjustment had to be achieved through alterations in both the conditions of production and demand and the criteria for setting prices and wages. Had this task been entrusted to exchange devaluation, the benefits would have been short-lived. [...]. In short, adoption of an accommodating exchange rate policy would have only postponed the choices that had to be made." (Ciampi, 1988, p. 59).

During the currency crisis of September 1992 Ciampi confirms that the real overvaluation had continued until the beginning of the Nineties: "The exchange rate is not an objective in itself; it is one of the instruments in the management of the economy. [...] The Bank of Italy pursued this line of conduct with determination [...] despite the deficit in the current account of the balance of payments and the accumulation of a net foreign debt ..." (Ciampi, 1992, p. 85*; our translation). This statement is fully consistent with the findings of previous studies (e.g. Stein and Paladino, 1998, who find that the Italian lira was overvalued in real terms with respect to the German mark at the time of the currency crisis).

The dynamics of the real misalignment after 1992 suggested by our model are explicitly confirmed by the Governor of the Bank of Italy Antonio Fazio: "The second half of 1993 can be seen as a period in which the effective exchange rate of the lira and the interest rates returned to a situation of acceptable equilibrium. [...] The lira nonetheless continued to weaken, falling considerably further than could be justified on the grounds of the competitiveness of goods and services. [...] A more careful assessment by market participants of the underlying conditions of the Italian economy could lead to a recovery of the lira." (Fazio, 1995, pp. 83, 87). It is worthwhile to stress the complete agreement between Governor Fazio's words and our analysis. In particular, our simulations do show an almost perfect coincidence of the actual RER with its equilibrium value during the second half of 1993.

5 Conclusion

We have built and estimated (with reference to the Italian economy) a new model based on three main ingredients, namely the NATREX notion of long run equilibrium real exchange rate, endogenous growth, and continuous time econometrics. The aim of testing the model's ability to correctly determine the periods of over- or under-valuation of the Italian lira (periods that we exactly know from outside sources) has been achieved. This warrants the next step, that consists of applying it (with appropriate modifications) to the euro/dollar exchange rate.

6 Data Appendix

A = productivity of capital, defined as Y^S/K. The unobserved value Y^S is calculated applying the output gap (source: OECD) to Y.
C = social internal real consumption. Source: ISTAT.
E = real multilateral exchange rate (an increase denotes an appreciation of the domestic currency). Source: IMF.
F = net real foreign debt. Source: OECD.
FY = ratio of net foreign debt to GDP. Source: authors' calculation.

F^*Y^* = ratio of German net foreign debt to GDP. Source: authors' calculation from OECD data.
I = social net real fixed investment. Source: EUROSTAT.
K = real capital stock. Source: authors' calculation cumulating social net real fixed investment.
MGS = real imports of goods and services (excluding factor income). Source: ISTAT.
P = implicit GDP deflator. Source: authors' calculation from ISTAT data.
R = real long term interest rate. Source: IMF.
RF = real net capital income from abroad. Source: Bank of Italy.
UT = real unilateral transfers. Source: OECD.
W = real net labour income from abroad. Source: OECD.
Y = real GDP. Source: ISTAT.
Y^* = index of real GDP of OECD countries. Source: OECD.
Y^S = capacity output. Authors' calculation applying the output gap series (source: OECD) to Y.
XGS = real exports of goods and services (excluding factor income). Source: ISTAT.

Note: "real" means "at 1990 prices".

7 Mathematical Appendix

Differently from traditional growth models, where the variables converge to constant (per capita) values, endogenous growth models of the AK variety give rise to unlimited growth (Romer, 1986; Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998). In endogenous growth models of the AK variety all the (per capita) variables grow at the same rate, in particular, as is obvious, $\mathrm{D}\ln Y = \mathrm{D}\ln K$.

In our model output grows faster than capital, because of the increase in A. In fact, $\mathrm{D}\ln Y = \mathrm{D}\ln K + \mathrm{D}\ln A$. It is however possible to show that the rate of growth of output asymptotically approaches that of the capital stock.

To show this, let us consider the equilibrium growth path, where the variables are equal to their desired values (namely, the actual and hatted variables are equal, $I = \widehat{I}$ etc.) and examine the production side of the economy

$$\begin{aligned} Y &= AK, \\ I &= f_1[A - R^*], \\ \mathrm{D}A &= \varphi(I/K), \\ \mathrm{D}K &= I. \end{aligned} \tag{22}$$

If we take a linear approximation of the function φ and let $\varphi'() = \alpha_7$ (as has already been done for estimation purposes), from the third and fourth equations in (22) we obtain $\mathrm{D}A = \alpha_7(\mathrm{D}K/K)$ and hence, by integrating,

$$A = \alpha_7 \ln K + A_0, \tag{23}$$

where the arbitrary constant of integration can be normalized to zero.

From $Y = AK$ and (23) we have

$$\mathrm{D}\ln Y = \mathrm{D}\ln A + \mathrm{D}\ln K = \mathrm{D}\ln K/\ln K + \mathrm{D}\ln K = \mathrm{D}\ln K(1 + 1/\ln K). \tag{24}$$

This gives rise to an equilibrium rate of growth of output which is higher than the rate of growth of the capital stock. However, given $I > 0$, K will grow and hence the term $1/\ln K$ will become smaller and smaller. Thus the rate of growth of output asymptotically approaches that of the capital stock.

As regards the NATREX, if we have non-convergent endogenous growth at home we can assume that we also have it abroad, hence $Y^*(t)$ is non-convergent as well. With both $Y(t)$ and $Y^*(t)$ non-convergent (presumably at similar rates, a typical feature of two-country models), the NATREX is well defined and arguably non-divergent. A detailed examination of this point would however require an explicit two-country model, which will be the subject of future research.

Let us now consider a simple *comparative dynamics exercise*, for example a (favourable) domestic productivity shock. This stimulates investment and output, and hence saving less investment. This in turn causes a decrease in foreign debt and hence in interest payments: a lower balance-of-trade surplus will be required to keep $CA = 0$, which entails an appreciation of the NATREX.

We finally come to stability. We first recapitulate the dynamic structure of the model around the growth equilibrium (where $Y = Y^D = Y^S = AK$), which is

$$\begin{array}{ll} \mathrm{D}I = \alpha_1(\widehat{I} - I), & \widehat{I} = f_1[A - R^*], \\ \mathrm{D}C = \alpha_2(\widehat{C} - C), & \widehat{C} = c\widehat{Y} = cAK, \\ \mathrm{D}BT = \alpha_3(\widehat{BT} - BT), & \widehat{BT} = f_3(E, Y, Y^*) = R^* F_0, \\ \mathrm{D}R = \alpha_4(R^* - R), & \\ \mathrm{D}A = \alpha_7 I/K, & \\ \mathrm{D}K = I. & \end{array} \tag{25}$$

The coefficient matrix M of the linear approximation to system (25) is

$$M = \begin{bmatrix} -\alpha_1 & 0 & 0 & 0 & \alpha_1(f_1')_0 & 0 \\ 0 & -\alpha_2 & 0 & 0 & \alpha_2 c(K)_0 & \alpha_2 c(A)_0 \\ 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha_4 & 0 & 0 \\ \alpha_7/(K)_0 & 0 & 0 & 0 & 0 & -\alpha_7(I/K^2)_0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where the subscript $()_0$ denotes the linearisation point.

A simple inspection of the matrix shows that $\lambda_1 = -\alpha_2, \lambda_2 = -\alpha_3, \lambda_3 = -\alpha_4$ are three (stable) real roots of the characteristic equation of M. The remaining three are given by the roots of the matrix M_1

$$M_1 = \begin{bmatrix} -\alpha_1 & \alpha_1(f_1')_0 & 0 \\ \alpha_7/(K)_0 & 0 & -\alpha_7(I/K^2)_0 \\ 1 & 0 & 0 \end{bmatrix}$$

and are obtained solving the characteristic equation

$$\lambda^3 + \alpha_1\lambda^2 - [\alpha_1(f_1')_0\alpha_7/(K)_0]\lambda + \alpha_1(f_1')_0\alpha_7(I/K^2)_0 = 0. \tag{26}$$

The succession of signs in Eq. (26) is $++-+$ hence Descartes' theorem tells us that, if the roots are all real, one will be negative and two positive.

The other possibility is that one root is real and two are complex conjugate. This eventuality is slightly more complex to analyse, but it can be shown that the real root is negative while the complex roots have positive real part.

Let λ_4 denote the real root and $\lambda_{5,6} = \theta \pm i\omega$ the two complex roots. Well known relations between the roots and the coefficients (Gandolfo, 1997, p. 220) tell us that

$$\begin{aligned} -\lambda_4(\theta^2+\omega^2) = -\lambda_4\lambda_5\lambda_6 &= [\alpha_1\,(f_1')_0\,\alpha_2 c(A)_0] > 0, \\ 2\theta\lambda_4 + \theta^2 + \omega^2 = \lambda_4\lambda_5 + \lambda_4\lambda_6 + \lambda_5\lambda_6 &= -\alpha_1 < 0. \end{aligned} \tag{27}$$

The first inequality in (27) shows that $\lambda_4 < 0$. The second inequality, given $\lambda_4 < 0$, implies $\theta > 0$.

Both cases are quite tractable from the mathematical point of view. In fact, given a first-order differential system in normal form with distinct characteristic roots, partly stable and partly unstable (a *conditionally stable* system), we can always make the system stable provided that we can choose as many initial conditions as there are unstable roots (Gandolfo, 1997, Chap. 18, Sect. 18.2.2.3, Theorem 18.3). In our model, the presence of the government in the consumption and investment equations ensures that it is possible to choose two initial conditions so as to make the system stable. Stability will be monotonic, since all stable roots are real.

8 References

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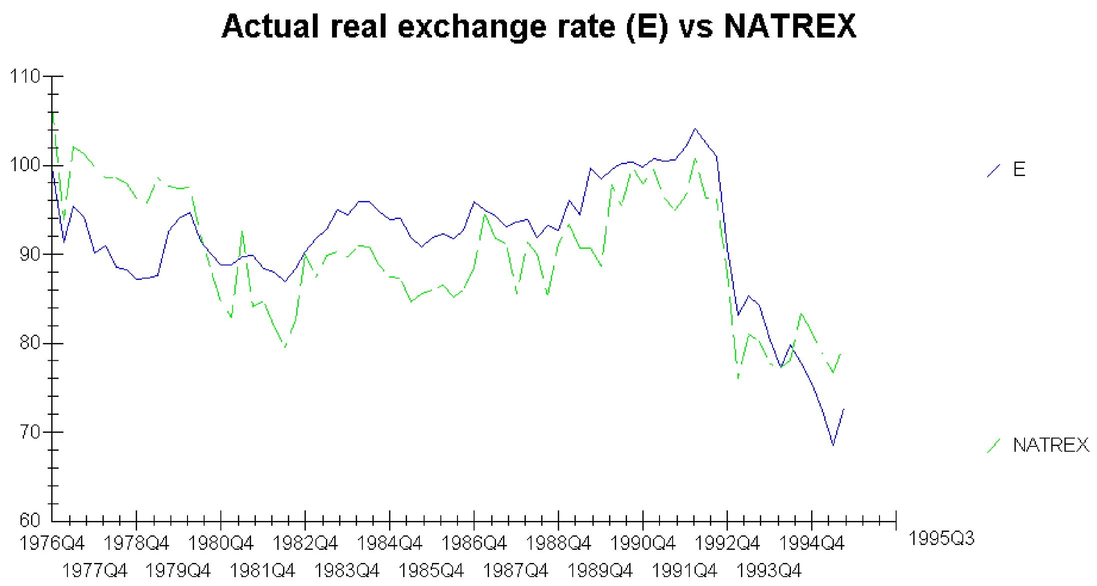
Actual real exchange rate (E) vs NATREX
110
100
90
80
70
60
1976Q4
1978Q4
1980Q4
1982Q4
1984Q4
1986Q4
1988Q4
1990Q4
1992Q4
1994Q4
1977Q4
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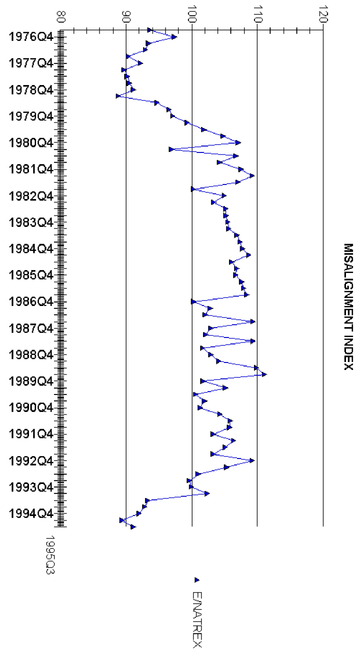
1993Q4
1995Q3
E
NATREX

Figure 1:


MISALIGNMENT INDEX
80
90
100
110
120
1976Q4
1977Q4
1978Q4
1979Q4
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1982Q4
1983Q4
1984Q4
1985Q4
1986Q4
1987Q4
1988Q4
1989Q4
1990Q4
1991Q4
1992Q4
1993Q4
1994Q4
1995Q3
E/NATREX

Figure 2: