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HOUSING MARKET DYNAMICS: ON THE CONTRIBUTION OF INCOME SHOCKS AND CREDIT CONSTRAINTS*

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Abstract

This paper presents a dynamic theory of housing market fluctuations. It develops a life-cycle model where households are heterogeneous with respect to income and preferences, and mortgage lending is restricted by a down-payment requirement. The market interaction of young credit-constrained households with older or richer unconstrained households generates the following results. (1) Current income of young credit-constrained households affects housing prices independently of aggregate income. (2) Housing prices and the number of housing transactions are positively correlated. (3) Housing prices over-react to income shocks. (4) A relaxation of the down-payment constraint triggers a boom-bust cycle. These results are consistent with patterns observed in the US and the UK.

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Two features distinguish residential real estate from financial assets: households' consumption demand for a dwelling and the indivisibility of properties. Because of credit market imperfections, the indivisibility implies that the purchase of a home requires a significant amount of liquid wealth up front. Furthermore, the only homeownership option for households with limited resources is to buy a starter home as a first step on the property ladder. It is only by continuing to accumulate wealth that households can eventually trade up. The performance of their highly leveraged investment in the starter home is key to their ability to accumulate wealth and hence affects the timing of their move up the property ladder.

This paper builds upon these features of housing consumption to address two fundamental issues concerning housing market dynamics: the large predictable housing price swings and the co-movement of prices and transactions. The volatility of housing prices is a well known and studied phenomenon, not unlike fluctuations in financial asset prices.[1] However, whereas transactions tend to rise in response to both positive and negative price changes on financial markets, housing transactions tend to increase during booms and decrease in downturns.[2]

To address these issues, we investigate the transmission of income shocks in a stochastic life-cycle model where heterogeneous households face a credit constraint in their attempt to climb the property ladder. The first contribution of the paper is to point out the critical role of the income of young households to housing market fluctuations. Since the income of young households is more volatile than per capita income, the variable typically used in the empirical housing literature, this goes some way towards explaining "excessive" price volatility. The second contribution of the paper is the characterization of a mechanism by which credit constraints not only amplify income shocks, but also affect the timing of households' moves in a way that explains the co-movement of prices and transactions. The response of housing prices to income shocks in our model depends on the distribution of debt within the population in ways which enable us to rationalize city level evidence. The richness of our life-cycle framework further enables us to confront the model's predictions with data on the behavior of various age groups in response to both income shocks and credit market liberalization.

[1] See for example Englund and Ioannides (1997).

[2] The correlation of housing prices and transactions has long been known to market practitioners. Stein (1995) reports evidence for the U.S. The same regularity is observed in the U.K. (Ortalo-Magné and Rady 2000).

The key to our results lies in our modeling of the housing market interaction of households with different wealth levels, as well as different preferences for various types of accommodation. In equilibrium, housing prices ensure that the number of homeowners equals the number of homes. Given everyone's preference for a dwelling of their own, only the poorest households do not own one. With starter homes being the cheapest available properties, by definition, their price must be high enough to keep enough households out of homeownership so as to guarantee that total demand for owner-occupied dwellings equals total supply. The price of starter homes is therefore closely linked to the wealth of the poorest potential first-time buyers, hence to the income of young households. When this income rises, the price of starter homes must follow so as to maintain the up front cost of a starter home unaffordable to enough potential first-time buyers.

At the other end of the wealth distribution, rich households in the model can afford to choose their dwelling across all available types on the basis of relative preferences and relative costs of ownership. The relative price of any two dwellings must reflect the relative preferences of the households who are just indifferent between them, given relative costs of ownership. This arbitrage across property types implies that the price of every dwelling can be broken down into two components: the price of a starter home, and the market value of the utility premium the specific dwelling provides relative to a starter home. An immediate corollary is that fluctuations in the price of starter homes affect the price of all dwellings. This implies a direct relationship between the income of young marginal buyers and the housing price index. This relationship is independent of (and complementary to) the standard income effect whereby households whose income rises spend more on everything, housing included.

A third group of households in the model is key to fluctuations in the number of transactions: the households whose first purchase was constrained by lack of liquid wealth and who, therefore, own a starter home. They are eager to accumulate wealth in order to move into a more expensive property as soon as possible. The timing of their move up the property ladder depends on the pace at which they can accumulate wealth. When incomes rise, so do property prices; homeowners enjoy capital gains in addition to higher earnings. In equilibrium, this faster accumulation of wealth allows constrained owners to precipitate their move up the property ladder. On the other hand, when earnings drop, so do housing prices. Homeowners suffer capital losses which constrain them to delay their move up the property ladder. These endogenous fluctuations in the timing of moves up the property ladder generate a positive correlation between housing transactions and prices.

The fact that some households' first purchase is a constrained step toward the purchase of a more expensive property implies that past housing prices and incomes affect current housing demand in the model. For all cohorts except the very youngest, accumulated earnings are lower when the economy is growing toward a high income level than after successive periods of the same high income. However, the capital gains realized by current owners during a period of growth more than compensate for their lower accumulated earnings. The aggregate demand for housing is therefore higher when income has recently jumped to a higher level than after successive periods at that same high level. As a result, the same is true for housing prices. Capital gains by current owners thus amplify the effects of income shocks on housing prices, generating price overshooting. The symmetric reasoning applies when income decreases: prices fall to a lower level than if income had been low for two successive periods. A direct corollary of this price overshooting result is that a mere slowdown in income growth may lead to a downturn on the housing market.[3]

The overshooting prediction of our model is in agreement with the empirical findings of Lamont and Stein (1999). They estimate a housing price regression based on city level data. Simulating the effect of a permanent income shock, they obtain a price response which rises progressively before decreasing to a new steady state level. In Section 6, we obtain the same pattern in the model. Our modeling framework is also able to account for the dramatic boom-bust episodes which seem to follow credit market liberalizations. The overlapping generation structure allows us to rationalize the concomitant cohort specific changes in mortgage demand and owner-occupancy rates. In particular, the model sheds light on the causes of the U.K. housing boom-bust cycle which followed the credit market liberalization of the early Eighties.

The traditional approach to housing prices treats dwellings like any other financial asset. A dwelling is assumed to provide units of housing services. The optimizing behavior of a representative investor ensures that the equilibrium price of a unit of housing services depends solely on the user cost of capital, rent, and some form of supply cost, and so is a forward-looking price.[4] This approach has led empirical researchers to estimate housing price equations based on construction cost, rent, permanent income (typically measured by per capita real disposable income), interest rates, and demographic variables.[5] Overall, these studies reveal that current per capita income

[3]These price dynamics are driven by capital gains, hence different from expectations-driven overshooting as modeled by Zeira (1999).

[4]E.g., Poterba (1991) and Wheaton (1999).

[5]E.g., DiPasquale and Wheaton (1994), Muellbauer and Murphy (1997), Malpezzi (1999).

and demographic variables, in particular the size of the young cohort, are key determinants of short run housing price fluctuations. They fail to explain the magnitude of observed booms and busts without assuming some departure from rational expectations. Short run changes in housing prices and excess returns appear to be positively serially correlated. However, this does not mean that moving households could realize excess returns by delaying the timing of their move.[6] Similar features will emerge in our model: predictable prices, but no excess returns that could be realized by moving households.

This failure to account for the observed price volatility has encouraged the search for alternative theoretical approaches. Focusing on the search and matching feature of the housing market, Wheaton (1990) presents a model where housing prices are determined by the expected time it takes a dissatisfied owner to find a suitable property to move into. Small fluctuations in demand or supply generate large fluctuations in prices through their impact on the equilibrium vacancy rate. This model only addresses repeat purchases.

Closer to the present paper, Stein (1995) develops a model of owners with some exogenous initial distribution of dwellings and debt. These owners enjoy moving, but are subject to a down-payment requirement, which implies a locally upward sloping demand for housing. The resulting possibility of multiple equilibria is key to Stein's explanation of housing price volatility. The main difference with the present paper is that Stein's model is static and requires the proportion of highly indebted agents to be very large in order to generate multiple equilibria. In contrast, the overshooting result we obtain stems from the analysis of the dynamics of our model, and holds even for small (and plausible) proportions of credit-constrained agents in the population. Furthermore, in our dynamic model, both the beginning and end of period distribution of debt and dwellings are equilibrium distributions. Finally, in contrast to both Wheaton (1990) and Stein (1995), our model considers not only repeat buyers, but also first-time buyers. They play a critical role in the dynamics of the housing market.

Microeconomic evidence provides support for the building blocks of our model. First, various studies of household level data strongly support the idea that credit constraints significantly restrict the housing consumption of a sizable proportion of

[6] This point was first made by Case and Shiller (1989). Cho (1996) provides a useful survey of the literature since.

households.[7] Linneman and Wachter (1989) find that the down-payment requirement imposed by mortgage lenders restricts households' access to credit more than the income constraint which precludes monthly principal, interest, property tax and mortgage insurance payments (known as PITI) above a given fraction of income. Accumulating sufficient wealth takes time and effort, for example, lower consumption or greater spousal labor force participation. The literature presents various estimates of how long it takes first-time buyers to accumulate the wealth required for their purchase. Survey data collected by the Chicago Title and Trust Company (1984-96) implies an average between 2 and 3 years, with a rising trend over time. Engelhardt and Mayer (1998) find an average of 3.5 years for their sample. Caplin et al. (1997) present numerical examples of how time to save varies with annual income and factors such as education debt. In this accumulation phase, only a minority of first-time buyers receive significant help from their family. For example, Engelhardt (1996) reports that only one-fifth of U.S. first-time buyers benefit from such help.

Second, household surveys indicate that own savings and housing equity are by far the two major sources of funds for repeat buyers. For example, in the years 1984-96, proceeds from the sale of a previous property represented between 30.7 and 57.1 percent of repeat buyers' down payments in major U.S. metropolitan areas (Chicago Title and Trust Company 1984-96). During housing market recessions, low equity in one's home seems a significant hurdle to moving (Genesove and Mayer 1997). During housing market booms, proceeds from the sale of a previous property typically make up a large proportion of down payments, helping households move up the property ladder, a feature our model will reproduce in equilibrium. For example, the 57.1 percent figure above was reached in the year 1987, following two years of strong price growth.

A third major building block of our model is the assumption that at least some households climb a property ladder over the course of their lives. The typical life-cycle pattern of housing consumption does involve lumpy adjustments along the property ladder with jumps toward bigger dwellings when young.[8] U.S. Census data shows the median price of first-time purchases at about 75 percent of the median price of repeat purchases. First-time buyers tend to be in their early thirties, whereas repeat buyers are generally in their early forties. Similar evidence is available in the U.K. where first-time buyers tend to be younger. Evidence from housing surveys both in the U.S.

[7]E.g., Linneman and Wachter (1989), Jones (1989), Ioannides (1989), Zorn (1989), Duca and Rosenthal (1994), Engelhardt (1996), Haurin, Hendershott and Wachter (1996, 1997).

[8]Lumpy adjustments can be explained by transaction costs that have to be paid when a dwelling is traded; cf. Grossman and Laroque (1990).

and the U.K. demonstrates that some households move to a more expensive property within a few years of their first purchase.

Finally, to generate the co-movement of transactions and prices, the model requires that the number of owners moving up the property ladder be allowed to fluctuate in equilibrium. This can only happen if the number of homeowners who remain in an expensive home also fluctuates. Hence, an endogenously changing number of them must want to move down the property ladder. There is some evidence that housing consumption declines with age for the elderly, but this is still a debated issue in the literature.[9] Our results do not presuppose that elderly households reduce housing consumption. What is critical is that every period, *some* households move to a home cheaper than the one they sell. This is supported by survey evidence in both the U.S. and the U.K.[10]

The rest of the paper is organized as follows. Section 1 presents the simplest version of our model. Section 2 provides an analytical characterization of equilibrium prices and transaction volume. Section 3 examines the response of the model to permanent income shocks; this clarifies the basic mechanism at work in the model. Section 4 generalizes the analysis to a Markov process for income. Section 5 presents an extension of the simple model. Section 6 discusses the empirical implications of the model. Section 7 contains concluding remarks. All technical details are relegated to an appendix.

1 The Model

A model in which the above ideas can be explored must be rich enough to capture the interaction on the housing market of households eager to climb the property ladder but credit constrained, with wealthier households who choose their home according to preferences and costs. The concept of a property ladder requires at least two types of dwelling. Climbing this minimal property ladder requires at least three periods, one period to buy, one to trade up, and one to sell. For income shocks to affect the pace at which *some* agents climb the property ladder, agents must differ in terms of their wealth. As to those agents who trade without restrictions imposed by wealth, there are

[9] See Merrill (1984), Mankiw and Weil (1989), Venti and Wise (1990, 1991, 2000), Sheiner and Weil (1992), Green and Hendershott (1996), Megbolugbe, Sa-Aadu and Shilling (1997), Jones (1997).

[10] See U.S. Bureau of Census (various years) and U.K. Department of the Environment, Transport and the Regions (1994/5). In the U.K., for example, of the people who bought their previous house in 1990 or later and moved in 1994/95, 23 percent moved to a cheaper property. This proportion is higher for people around age 65 (55 percent), and lower for people below age 45 (11 percent).

many options available for their representation. Here, we choose to add an extra period of life, a period when one's wealth is sufficiently high so that credit constraints no longer bind, no matter what type of property is considered. Of course, if not all wealthy agents are to hold the same dwelling, they must have heterogeneous preferences. The challenge is therefore to design a stochastic model that incorporates this double heterogeneity of wealth and preferences and still allows a tractable determination of equilibrium prices and transaction volume.

We consider an economy with a numeraire consumption good and two types of dwellings: starter homes ("flats" hereafter) and larger dwellings ("houses" hereafter). Each dwelling can accommodate only one agent, who must be an owner-occupier. The introduction of a rental market does not affect the nature of the results and will be discussed later. Flats and houses are available in fixed quantities, S^F and S^H. The assumption of a perfectly inelastic supply is not critical to our results; they go through as long as supply is not perfectly elastic, which will hold given an upward sloping supply of land. Of course, everything else equal, the response of prices to shocks would be the smaller, the higher the price elasticity of supply.

A household who wishes to buy a flat or house at price q^h $(h \in \{F, H\})$ can do so only if its wealth exceeds a fixed fraction γ of this price. For simplicity, we will refer to the amount γq^h as the "down payment", but it can be thought of as including other items such as moving or closing costs as well.[11] The no-homeownership option, $h = \emptyset$, is free: $q^{\emptyset} = 0$. Agents can borrow and save at the same exogenously fixed positive interest rate r.

Agents live for four periods. Each period, a measure one is born without any assets. Within each period, agents first receive their income, second, they trade, and third, they consume the numeraire good. Housing is enjoyed in between periods. Over their lifetime, agents thus have three opportunities to make a housing purchase (at ages 1, 2 and 3) and three opportunities to derive utility from housing (from age 1 to 2, from age 2 to 3, and from age 3 to 4). This means in particular that age 4 agents will sell any property they own.

[11] In actual housing markets, the amount of liquid assets needed at the time of a purchase depends on a household's choice from the menu of mortgage contracts on offer. In the U.S., for example, households can reduce down payments to 5 percent if they buy private mortgage insurance, and 3 percent if they obtain a Federal Housing Administration (FHA) loan. In either case, this involves costs in form of higher PITI payments because of insurance premia or higher interest rates, which in turn restricts the maximal available loan size. Caplin et al. (1997, p. 31) therefore conclude that "it is almost impossible for a household to buy a home without available liquid assets of at least 10% of the home's value". It is this effective down-payment constraint that we wish to model here.

Within each cohort, agents are distributed uniformly over the unit square. Each agent is identified by the indices $(i, m) \in [0,1] \times [0,1]$ which determine her income stream and housing preferences, respectively. The income received at time t by agents of age j is a random variable $w_t(i,j)$ that depends continuously on the income index i. At age 1 and 2, agents with a higher index i have a higher endowment: $w_t(i_1, j) < w_t(i_2, j)$ whenever $i_1 < i_2$ and $j = 1, 2$. Further, the accumulated earnings of the age 2 agents with index i are always greater than the earnings of the age 1 agents with the same index i: $w_t(i,2)+w_{t-1}(i,1)(1+r) > w_t(i,1)$ for all i. Age 3 agents represent households whose housing choices are not restricted by the down-payment constraint. They are modeled as receiving an endowment $w_t(i,3)$ large enough to render both housing alternatives affordable for all i and all t. (This will be made more precise later.) Differences in housing choices at age 3 arise from the heterogeneity of preferences. Age 4 income is irrelevant as long as it is non-negative.

The preferences of agent (i,m) are described by the utility function $\sum_{j=1}^{4} c(j) + \sum_{j=1}^{3} U(h(j), m, j)$ where $c(j)$ is the non-negative amount of the numeraire good consumed in the jth period of life, and $h(j) \in \{F, H, \emptyset\}$ the type of housing held at the end of that period. All agents strongly prefer owning a house or a flat to not owning a property: $U(h, m, j) \gg U(\emptyset, m, j)$ for $h \in \{F, H\}$, $m \in [0,1]$ and $j = 1, \ldots, 3$. (Precise conditions as to how large these utility differences have to be will be given later.) Agents' utility premia for a house over a flat are determined by a strictly increasing and continuous function $u : [0,1] \to I\!R$ as follows. During the first two periods of life, all agents have the same positive utility premium: $U(H,m,j) - U(F,m,j) = u(\frac{1}{2}) > 0$ for $m \in [0,1]$ and $j = 1, 2$. At age 3, agents learn their index m. The utility premium of agents with preference index m is $U(H,m,3) - U(F,m,3) = u(m)$. The change in utility premium from the "median" level $u(\frac{1}{2})$ to $u(m)$ when agents reach age 3 means that some of them may have a purely preference-driven reason to move in their third period of life. The direction of these moves can be up the property ladder as well as down.

Given a positive interest rate r and stochastic processes of earnings $\{w_t(i,j)\}$, flat prices $\{q_t^F\}$ and house prices $\{q_t^H\}$, agent (i,m) of age a coming into period t with savings $s(a-1)$ and a dwelling $h(a-1)$ solves

$$\max_{\{c(j),h(j),s(j)\}_{j=a}^{4}} \mathrm{E}\left[\sum_{j=a}^{4} c(j) + \sum_{j=a}^{3} U(h(j), m, j)\right] \tag{1}$$

subject to the budget constraint

$$c(j) + q_{t+j-a}^{h(j)} + s(j) \leq w_{t+j-a}(i,j) + q_{t+j-a}^{h(j-1)} + (1+r)\, s(j-1), \tag{2}$$

the credit constraint[12]

$$s(j) \geq (\gamma - 1)\, q^{h(j)}_{t+j-a}, \tag{3}$$

and the non-negativity constraint

$$c(j) \geq 0 \tag{4}$$

at all $j = a, \ldots, 4$. The expectation in (1) is taken over future property prices and, at age 1 and 2, over the yet unknown preference index m.

As there is no time to enjoy a property acquired at age 4 and no utility to be derived from a bequest, the solution to (1) will always have $h(4) = \emptyset$ and $s(4) = 0$. Given that the interest rate is positive, the specification of the utility function together with the non-negativity constraint (4) yields a simple consumption plan for the numeraire good: $c(1) = c(2) = c(3) = 0$, that is, all such consumption is postponed until the end of life. While this feature is not attractive in itself, we accept it as a necessary evil to keep the model analytically tractable, in particular with respect to the equilibrium law of motion of the distribution of dwellings and savings. We shall discuss later how introducing a trade-off between instantaneous numeraire and housing consumption might affect the response of the model to shocks.

2 Equilibrium Prices and Transaction Volume

We fix a positive interest rate r and exogenous bounded income processes $\{w_t(i,j)\}$. Given an initial distribution of dwellings and wealth across agents, an equilibrium in this economy consists of stochastic processes of property prices $\{q_t^F, q_t^H\}$ and consumption, savings and housing decisions $\{c_t(i,m,j), s_t(i,m,j), h_t(i,m,j)\}$ for all agents such that, at those prices, these allocations solve each agent's constrained utility maximization problem (1)–(4), and the flat and house markets clear.

Market clearing requires that the measure of agents who own a flat be equal to the supply of flats, and likewise for houses. As we mentioned in the previous section, it is

[12] We formulate this constraint in a way that is stronger than in reality and stronger than we need for our results. In particular, as it is written, constraint (2) implies that banks make margin calls if the price of a property drops. A formulation closer to actual practices would require (2) to hold only when a property transaction is made ($h(j) \neq h(j-1)$) or when the individual wants to increase the size of her loan ($s(j) < s(j-1)$ and $s(j) < 0$); this would not involve margin calls. In the equilibrium that we will consider below, this point will be moot, so we formulate the credit constraint in the simplest possible way.

never optimal for an age 4 agent to own a home. In order to preclude the uninteresting case where all other agents own a property in equilibrium, we impose the restriction that $S^F + S^H < 3$. For reasons to become obvious below, we also assume that $\frac{1}{2} < S^H < 1$ and $S^F + S^H > \frac{5}{2}$.

We wish to consider an equilibrium where all age 3 agents are wealthy enough to be able to afford the down payment on a house.[13] In such an equilibrium, all agents will indeed own a property by the end of their third period of life since they strongly dislike the option of not owning a home. An age 3 agent will own a house if the utility premium she derives from a house relative to a flat exceeds the expected user cost difference between holding a house and holding a flat from the current period to the next. Since the function $u(m)$ describing the distribution of utility premia is strictly increasing in m, the distribution of dwellings among age 3 households splits in two subsets: agents with index m close to 0 are in a flat, those with higher m are in a house.[14] One group of agents is at the margin between the two subsets, namely those whose utility premium just equals the expected difference in the cost of holding the two types of properties. This equilibrium cutoff point along the preference dimension, denoted m_t^H, is defined by the equation

$$u(m_t^H) = \left[(1+r)\, q_t^H - \mathrm{E}_t q_{t+1}^H\right] - \left[(1+r)\, q_t^F - \mathrm{E}_t q_{t+1}^F\right], \tag{5}$$

with E_t denoting the conditional expectation operator given the information available at time t. Since wealth is irrelevant to the housing decision at age 3, all agents with the same preference index m will own the same dwelling at that age.

Note that given a supply of houses $S^H > \frac{1}{2}$, the equilibrium index m_t^H cannot exceed $\frac{1}{2}$. For otherwise no age 1 or 2 agent would want to hold a house even if she could afford the down payment. This is because all age 1 or 2 agents have the same current utility premium for a house as the median (along the preference dimension) age 3 agents – if the latter did not want a house, the former would not want one either. The aggregate demand for houses would then be lower than $\frac{1}{2}$, which would contradict market clearing. The equilibrium distribution of dwellings within the age 3 cohort will therefore look in general as in Figure 1 where the index i increases from 0 to 1 as we move right, and the index m, as we move up.

[13]Precise conditions that ensure an equilibrium of this type are spelled out in Appendix A.

[14]When we talk about the distribution of dwellings, we always refer to the end-of-period distribution; i.e., the distribution reached after trading takes place.

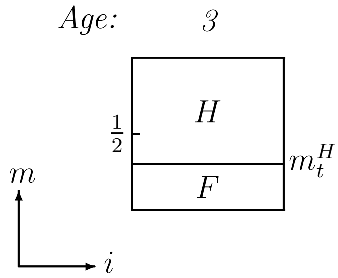

Figure 1: Housing choice at age 3

Age 1 and 2 agents have the same preferences as the median age 3 agents. Like these age 3 agents, they prefer a house over a flat at equilibrium prices. Hence, any age 1 or 2 agent rich enough not to face a binding credit constraint also owns a house in equilibrium. The poorest house owners in each of the age 1 and 2 cohorts are therefore those agents who can just afford the down payment on the house. Anyone with lower wealth who can afford the down payment on a flat owns a flat since all agents strongly prefer this type of dwelling to not owning a property at all. The poorest flat owners are the agents who can just afford this down payment. So the only agents who do not own a home are those who cannot afford a down payment on a flat.

Given our assumption that for every history of income shocks each age 2 agent has higher accumulated income than the age 1 agent with the same index i, the group of agents who do not own any property is larger at age 1 than at age 2. Moreover, since by assumption the total supply of dwellings is smaller than the measure of age 1 to 3 agents, there will always be a positive measure of age 1 agents who do not own a dwelling. The monotonicity of earnings at age 1 then implies a monotonic distribution of dwellings at age 1: the poorest agents own nothing, richer ones own a flat, and the richest may own a house. (That there are homeowners at age 1 follows from the assumption that $S^F + S^H > \frac{5}{2}$.)

The distribution of dwellings at age 2 is of the same form as long as there is no "leap-frogging" whereby the wealth ranking between certain agents is reversed from one period to the next. Such a reversal could occur after large drops in incomes and prices. In fact, someone who bought a flat at age 1 could suffer capital losses severe enough to prevent her from trading up to a house, while someone who was not able to buy anything at age 1 could benefit from low prices to acquire a house right away at age 2. We will restrict our analysis to stochastic income processes without such dramatic recessions.[15] Then, the equilibrium distribution of dwellings within the age 1 and 2 cohorts is monotonic, the richer agents being in the more expensive type of dwelling. Moreover, the agents who can just afford a flat respectively house at age 2 have a lower income index i than their counterparts at age 1.

For ease of exposition, the income heterogeneity will be assumed such that in equilibrium no age 1 agent can afford a house while all age 2 agents can afford at least a flat.[16] The income indices of the poorest age 1 flat owners and age 2 house owners are denoted $i_t^F(1)$ and $i_t^H(2)$, respectively. This yields the distribution of dwellings within the age 1 and 2 cohorts shown in Figure 2.

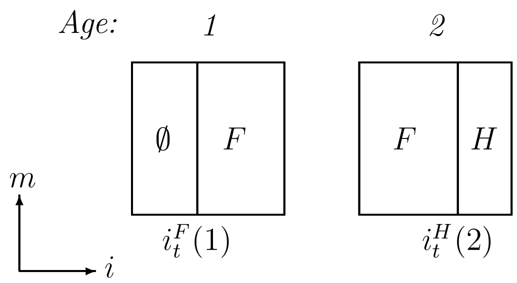

Figure 2: Housing choices at ages 1 and 2

Having defined the cutoff indices that characterize the distribution of dwellings within cohorts, we can now state the market clearing conditions for flats and houses:

$$1 - i_t^F(1) + i_t^H(2) + m_t^H = S^F, \tag{6}$$

$$1 - i_t^H(2) + 1 - m_t^H = S^H. \tag{7}$$

Combining these two equalities yields

$$i_t^F(1) = 3 - S^H - S^F. \tag{8}$$

The income index of the marginal flat buyers of age 1 is invariant over time. This is due to the specific configuration we consider, in particular to the fact that only age 1 agents do not own a home. This simplification is not crucial to our results and will be relaxed later.

As $S^H < 1$ and $m_t^H \leq \frac{1}{2}$ for all t, equation (7) implies $i_t^H(2) > \frac{1}{2}$ for all t. And given the assumption that $S^F + S^H > \frac{5}{2}$, equation (8) implies $i_t^F(1) < \frac{1}{2}$ for all t. So the

[15] The reader may want to think of income processes that fluctuate within a sufficiently small distance from a steady state. What happens in the model when large (in the above sense) price and income drops occur will be discussed later.

[16] This is achieved by making incomes at age 2 sufficiently large relative to incomes at age 1 (precise conditions are given in Appendix A). We will relax this assumption in Section 5.

marginal house buyers at age 2 always enter the period with a flat; i.e., $i_t^H(2) > i_{t-1}^F(1)$ for all t. This feature of the distribution of dwellings implies that via the down-payment constraint, capital gains on flats have a direct impact on the demand for houses.

To solve for the equilibrium price of flats, it is enough to characterize the income of the age 1 marginal flat buyers. By definition, these are the agents who receive an endowment just sufficient to pay the down payment on a flat; i.e.,

$$w_t(i_t^F(1), 1) = \gamma q_t^F. \tag{9}$$

Substituting $i_t^F(1)$ as given in (8) yields the equilibrium price of flats:

$$q_t^F = \frac{w_t(3 - S^F - S^H, 1)}{\gamma}. \tag{10}$$

Since the income index of the marginal flat buyers of age 1 is invariant over time, the flat price moves proportionally with the income of these marginal agents.

The equilibrium price of flats depends on the current income of the marginal constrained age 1 agents. As this income is bounded, so is the price of flats. In the same way, the house price must be bounded because it cannot exceed $1/\gamma$ times the wealth of the richest agent. This rules out bubbles, so we can solve the marginal unconstrained house buyers' indifference condition (5) forward to obtain the house price

$$q_t^H = q_t^F + \mathrm{E}_t\left[\sum_{s=0}^{\infty} (1+r)^{-s-1}\, u(m_{t+s}^H)\right]. \tag{11}$$

Equations (10)–(11) provide a simple intuition for the *fundamental determinants of housing prices* in the model. The price of a house is equal to the price of a flat plus the present discounted value of the stream of extra utility a house provides over and above the flat.[17] The relevant utility premia are those of the marginal unconstrained owners. In a sense, these agents provide arbitrage across property types. They ensure that the relative price of properties reflects the difference in services they provide. But these unconstrained agents do not provide arbitrage between housing and savings. This is because of their consumption demand for a home: the trade-off they consider in their decision-making process is not whether to own or not, but what type of property to own and enjoy.

This same consumption demand drives the bottom end of the market. Young agents are so eager to acquire their own dwelling that they prefer owning something smaller

[17]This readily generalizes to a property ladder with more than two types of housing: The price of any property is equal to the price of the cheapest home plus the present discounted value of the stream of extra utility the property provides over and above the cheapest home.

than what they would like at current prices (a flat instead of a house) rather than owning nothing at all. They own a home as soon as they can afford to – hence the direct link between their current income and the price of the properties at the bottom of the housing ladder.

The house price equation (11) highlights how house prices are affected by current and expected future states of the market. Hidden in this formula is the fact that house prices are also backward-looking: the preference index of the marginal unconstrained house buyers, m_t^H, is determined jointly with the income index of the marginal constrained house buyers, $i_t^H(2)$. These are the agents who compete for houses on the market.

The value of $i_t^H(2)$ depends on current and past incomes and flat prices because accumulated earnings and capital gains on a flat determine the ability of age 2 agents to afford the down payment on a house. The marginal age 2 house buyers are the agents whose wealth is just enough to cover the down payment on a house:

$$W_t(i_t^H(2)) - (1+r)\, q_{t-1}^F + q_t^F = \gamma q_t^H. \tag{12}$$

where $W_t(i)$ stands for the accumulated earnings (before any property transactions) of age 2 agents with income index i; i.e., $W_t(i) = (1+r)\, w_{t-1}(i,1) + w_t(i,2)$. One can solve for $i_t(2)$ in (12) and m_t^H in (5) by inverting the functions $W_t(i)$ and $u(m)$ which are continuous and strictly increasing by assumption. The market clearing condition (7) for houses then yields a second-order difference equation for the house price:

$$\begin{aligned} 2 - S^H \;=\; & W_t^{-1}\left(\gamma q_t^H + (1+r)\, q_{t-1}^F - q_t^F\right) \\ & + u^{-1}\left(\left[(1+r)\, q_t^H - \mathrm{E}_t q_{t+1}^H\right] - \left[(1+r)\, q_t^F - \mathrm{E}_t q_{t+1}^F\right]\right). \end{aligned} \tag{13}$$

This equation can be visualized as follows. Given previous and current flat prices q_{t-1}^F and q_t^F as well as price expectations $\mathrm{E}_t q_{t+1}^F$ and $\mathrm{E}_t q_{t+1}^H$, the maximal price at which a flat owner of age 2 and income index i can afford to buy a house is

$$Q_t^{H,2}(i) = \frac{W_t(i) - (1+r)\, q_{t-1}^F + q_t^F}{\gamma}. \tag{14}$$

This is also the price at which a measure $1-i$ of age 2 agents demand a house, as long as $i > i_{t-1}^F(1)$, so that all these agents are indeed flat owners. Similarly, the maximal price at which an age 3 agent with preference index m is willing to buy a house is

$$Q_t^{H,3}(m) = q_t^F + \frac{u(m) + \mathrm{E}_t[q_{t+1}^H - q_{t+1}^F]}{1+r}. \tag{15}$$

Equivalently, $Q^{H,3}(m)$ is the price at which a measure $1-m$ of age 3 agents demand a house. In equilibrium, we must have $Q_t^{H,2}(i) = Q_t^{H,3}(m)$ and, from the market clearing condition (7), $i+m = 2-S^H$. This jointly determines the current house price q_t^H and the cutoff indices $i_t^H(2)$ and m_t^H. Graphically, the solution is found by drawing the two inverse demand curves as in Figure 3 where we measure i from the left along the horizontal axis and m from the right, and where the distance between the two origins is $2-S^H$. The $Q^{H,2}$ curve is valid for flat owners only, so it starts at the income index $i_{t-1}^F(1)$, which we know to be somewhere to the left of $i=\frac{1}{2}$. This graphic representation will be useful when we discuss equilibrium price responses to income shocks. Note in particular that the slopes of these inverse demand curves are directly proportional to the gradients of the wealth and preference distribution, respectively.

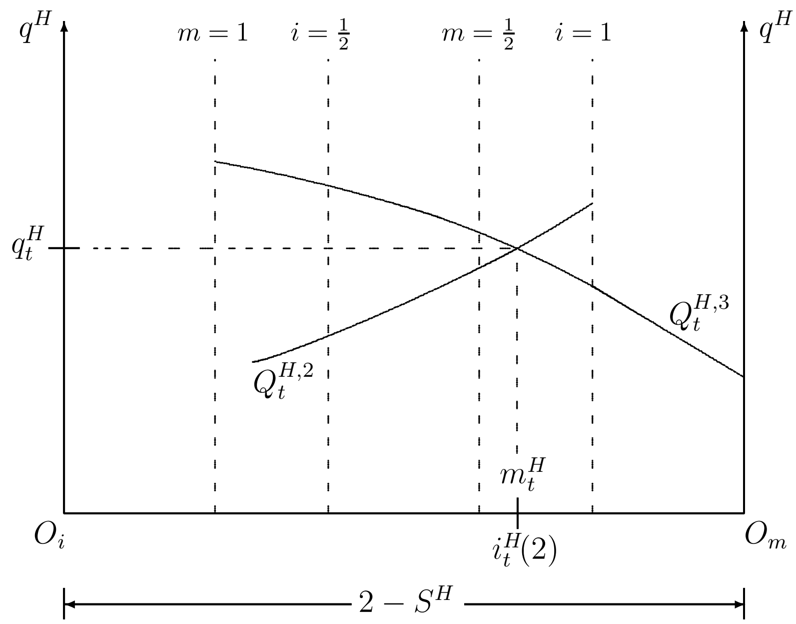

Figure 3: Equilibrium in the market for houses

The number of transactions in any given period depends on the changes in the distribution of dwellings and so on fluctuations of the same cutoff indices that matter for prices. Referring back to Figures 1 and 2, the measure of age 1 agents who buy a flat at time t is $1-i_t^F(1)$. In the age 2 cohort, it is $i_{t-1}^F(1)$ (all those agents who could not afford a flat one period earlier); and in the age 3 cohort, it is $[1-i_{t-1}^H(2)]m_t^H$ (all those agents who lived in a house and now decide to move to a flat). Since the index

$i_t^F(1)$ is constant in equilibrium, the total number of transactions of flats simplifies to

$$n_t^F = 1 + [1 - i_{t-1}^H(2)]\, m_t^H. \tag{16}$$

A measure $1 - i_t^H(2)$ of agents buys a house at age 2, and a measure $i_{t-1}^H(2)\,(1 - m_t^H)$ does so at age 3. The number of transactions in the market for houses is therefore

$$n_t^H = 1 - i_t^H(2) + i_{t-1}^H(2)\,(1 - m_t^H). \tag{17}$$

Through its dependence on both the current and lagged income index of the marginal age 2 house buyers, the number of transactions depends on the income history of length three: current income plus its first two lags.

Simple manipulations using the market clearing condition for houses show that the total number of property transactions is given by

$$n_t = 2 - m_{t-1}^H + 2\,(S^H - 1 + m_{t-1}^H)\, m_t^H. \tag{18}$$

(see Appendix B for details).

We are now in a position to study impulse responses to unanticipated permament income shocks. In a second step, we will consider rational expectations equilibria under Markovian income processes.

3 Response to Income Shocks

Consider time-invariant incomes $w(i,j)$ for $i \in [0,1]$ and $j = 1,\ldots,4$ leading to a steady state equilibrium allocation of dwellings as in Figures 1 and 2 with prices q^F and q^H and cutoff indices $i^F(1)$, $i^H(2)$ and m^H. At time $t = s$, incomes are hit by an unanticipated permanent shock. Agents do not expect any further changes in incomes from time s onward.

For simplicity of exposition, we focus on proportional income shocks where each agent's current income is increased by the same proportion. Thus, for $t \geq s$ incomes are given by $\overline{w}(i,j) = (1+\sigma)w(i,j)$ where $\sigma > 0$ is assumed small enough to preserve the equilibrium configuration depicted in Figures 1 and 2. Our results are easily extended to more general shocks. These generalisations and all proofs are presented in Appendix C.

From the characterization of the equilibrium in the previous section, we know that the price of flats adjusts immediately to the new steady state level $\overline{q}^F = (1+\sigma)q^F$.

Households foresee that the price of houses will reach its new steady state level $\overline{q}^H$ one period later, at time $t = s+1$, once the accumulated earnings of the age 2 cohort have reached their new steady state level. The number of transactions then reaches its new steady state level in the following period, $t = s+2$.

Proposition 3.1 (Comparison of steady states) *The steady state with higher incomes is characterized by higher property prices, a higher difference between the price of houses and the price a flats, a higher number of age 2 house buyers, and a lower number of age 3 house buyers.*

To see the intuition for this result, consider the age 2 agents who are just able to acquire a house in the initial steady state, i.e., whose ability to pay for a house is precisely equal to the initial equilibrium house price. In the new steady state, these same agents' wealth (and hence their borrowing capacity) is a fraction σ higher. This means that their ability to pay for a house increases by σ times the initial house price. The highest steady state price that an age 3 agent is willing to pay for a house, on the other hand, increases by exactly the price difference for flats, which is σ times the initial price of flats. Since the initial house price exceeds the initial price of flats, the wealth effect on the age 2 cohort is clearly stronger than the substitution effect on the age 3 cohort. This means that the house price must rise by more (in absolute terms) than the flat price; if it rose by less, there would be excess demand for houses. In particular, the difference between prices of the two types of property is higher in the new steady state. Now, given that the house price rises by more than the price of flats, fewer age 3 agents decide to own a house. So more houses are bought by age 2 agents.

We now turn to the transition from the initial to the new steady state.

Proposition 3.2 (Overshooting of the house price) *The house price in the period of the shock is higher than in the initial steady state. The house price overshoots its new steady state if*

$$w(i^H(2), 1) < q^F, \tag{19}$$

i.e., if in the initial steady state the price of flats exceeds the age 1 income of those agents that are the marginal house buyers at age 2.

The first part of this proposition is easy to explain. Note first that the flat owners that reach age 2 at the time of the shock have a dual advantage over their counterparts in the previous cohort: they earn higher incomes in the second period of their lives and they enjoy capital gains on their flats. This implies a higher demand for houses by age 2 agents at the time of the shock. Second, the higher flat price at the time of the shock and the expectation of a higher price differential between houses and flats in the future increases the demand for houses by age 3 agents. So the house price must rise at the time of the shock.

As to the overshooting result, condition (19) is not in terms of model parameters directly, but has the advantage of relaying the right intuition. Note first that the change in the house price from time s to $s+1$ is entirely driven by the wealth of age 1 and 2 agents. This is because age 3 agents' willingness to pay for a house (which depends on the current flat price and expectations about future property prices) is the same in these two periods. Now consider a flat owner with income index $i \leq i^H(2)$ reaching age 2 in period $s+1$. In period s, such an agent earned $\overline{w}(i,1)$ and bought a flat at the price $\overline{q}^F$. Contrast this with an agent who has the same income index but was born one period earlier, i.e., reaches age 2 already at time s. In period $s-1$, this agent earned $w(i,1) < \overline{w}(i,1)$ and bought a flat at the price $q^F < \overline{q}^F$. Under condition (19), the endowment disadvantage of the agent born one period earlier is more than compensated by the price advantage: since both the income and the price of flats rise by the same *proportion*, and the income is smaller than the price of flats, the price rises by more than the income *in absolute terms*. Given that the price of flats is the same in periods s and $s+1$, the agent born earlier is thus richer when she reaches age 2. As this translates into a higher ability to pay for a house, the equilibrium price must be higher at the time of the shock: $q^H_s > q^H_{s+1} = \overline{q}_H$. Put more simply, flat owners that reach age 2 as the shock occurs enjoy capital gains, while those one period later do not. Under condition (19), these capital gains outweigh the endowment disadvantage, hence lead to higher demand and a higher house price.

Condition (19) stipulates that those agents who are just able to afford a house in the second period (the poorest credit-constrained repeat buyers) were not able to pay cash for the flat that they acquired a period earlier. This is highly plausible.[18]

[18]Even so, it is instructive to provide a sufficient condition for overshooting that is formulated in terms of the model parameters. We can use the characterisation of the flat price in equation (10) to translate condition (19) into the inequality $w(i^H(2), 1) < w(3 - S^F - S^H, 1)/\gamma$, a sufficient condition for which is $w(1, 1) < w(3 - S^F - S^H, 1)/\gamma$ by the monontonicity of the age 1 income profile. Thus, we will have overshooting in the house price as long as the initial age 1 income profile is not too steep, i.e., as long as the earnings of the richest age 1 agents are not too different from those of the marginal flat buyers.

Figure 4 summarizes the results we have obtained so far. It depicts the inverse demand functions for houses at $t = s - 1$, s and $s + 1$ as defined in (14) and (15). Age 3 households' ability to pay for a house shifts upward once at the time of the

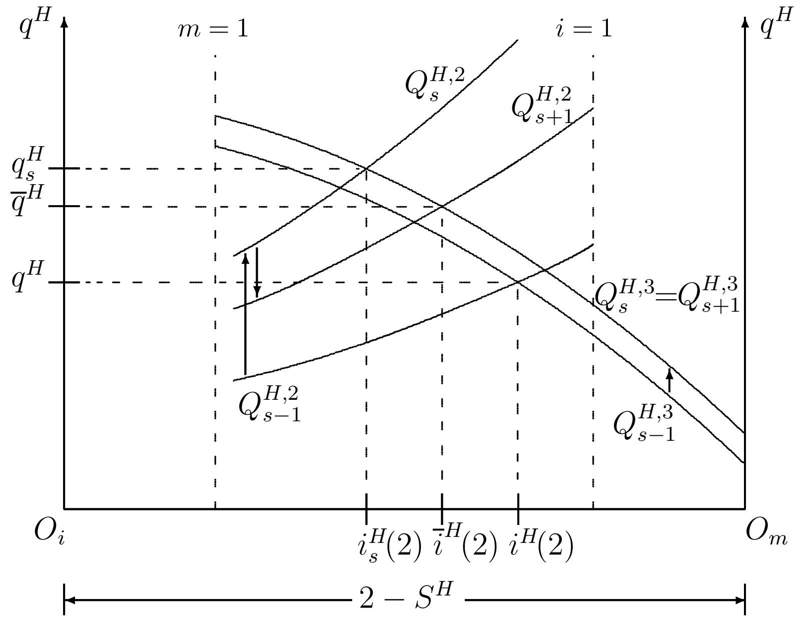

Figure 4: Transition from initial to new steady state

shock and then stays unchanged. Under condition (19), age 2 households' ability to pay for a house rises at the time of the shock and then falls back because of the lack of further capital gains on flats. As a consequence, the house price overshoots its new steady state. That age 2 households rely primarily on the capital gains on their flat to purchase a house in the period of the shock is in agreement with the empirical evidence mentioned earlier: in a boom, the proportion of the down payment coming from the sale of the previous property is higher whereas the proportion coming from own savings is lower.

By how much the house price must rise to clear the house market after the income shock depends on each potential house buyer's relative willingness to hold a flat instead of a house. For the flat owners attempting to move up the property ladder, a rise in the house price means an increase in the required down payment. The resulting increase in the number of flat owners who remain constrained to live in a flat depends on the wealth gradient of the concerned cohorts. On the other side of the house market, preference traders make their decisions purely based on relative user cost and relative utility. For a given house price rise, *ceteris paribus*, the increase in the number of house owners who prefer to move to a flat depends on the preference gradient within the unconstrained cohorts. The steeper the preference or wealth gradients, the smaller is the effect that a given price rise has on the number of households demanding a house, and the larger is therefore the increase in the house price required to balance the market. As a direct corollary of this result we note that the steeper the gradients, the more volatile the house price will be relative to the price of flats.[19]

As Figure 4 makes evident, the higher ability of age 2 households to pay for a house at the time of the shock implies an increase in the equilibrium number of age 2 house buyers. This yields the following result.

Proposition 3.3 (Transactions in the period of the shock) *Under the overshooting condition* (19)*, the number of transactions in the period of the shock is higher than in the initial steady state. This higher number of transactions is due to an increase in the number of repeat purchases by buyers of age 2.*

In fact, there are three effects. There is an increase in the number of agents acquiring a house at age 2, an increase in the number of agents who move from a house to a flat at age 3, and a decrease in the number of agents who move from a flat to a house at age 3. The proof shows that the third effect dominates the second (so there are fewer repeat purchases by age 3 households overall), and that the first effect dominates the combination of the other two. Note that the increase in the number of houses owned by the young does not mean a one-for-one increase in the number of older households trading down from a house to a flat. Instead, the shift in the distribution of houses in favor of the young is partially accomodated by those age 3 households who were planning to move to a house for preference reasons but now choose to remain in their flat because houses have become more expensive.

Returning to Figure 4 and our discussion of wealth and preference gradients, it is easy to see that steeper gradients, while implying larger fluctuations of the house price, mean smaller fluctuations in the cutoff indices at age 2 and 3, and thus smaller fluctuations in the number of transactions.

Propositions 3.2 and 3.3 immediately lead to a result that is a central prediction of our model.

[19] There is some empirical evidence both in the U.S. and the U.K. that more expensive properties experience larger price swings; for the U.S., see Poterba (1991) and Mayer (1993), for the U.K., see Earley (1996). However, these papers are concerned with particular price swings in particular markets, not with the effect of pure income shocks.

Proposition 3.4 (Co-movement of prices and number of transactions) *Under the overshooting condition* (19)*, property prices and the number of transactions all rise in the period of the income shock.*

In summary, the results of this section show how an unexpected permanent increase in income causes property prices and the number of transactions to rise in response. Prices of starter homes rise because the increase in income enables young (and thus potentially credit-constrained) households to afford higher down payments. Prices of homes further up the property ladder rise because the income increase and the capital gains on their current home enable homeowners to afford larger down payments toward the purchase of a more desirable property. This is also the reason for the increase in transaction volume: benefiting from higher incomes and capital gains, young households move up the property ladder faster, thereby generating a higher turnover. The capital gains effect is so strong that it makes the property price index overshoot its new steady state level. With an unexpected permanent *decrease* in income, these effects reverse direction: property prices undershoot and the volume of transaction drops as fewer young households trade up. Everything else equal, the magnitude of these price and volume responses depends on the wealth and preference gradients: the steeper these gradients, the larger will be the fluctuations in the price index and the smaller the fluctuations in the number of transactions.

4 A Markov Model for Incomes

The previous section emphasized the backward-looking factors of housing market dynamics. Income shocks were completely unexpected. We turn now to the analysis of the dynamics of the model assuming Markovian processes for income. This enables us to address the role of (rational) expectations and to analyse the joint fluctuations of prices and transactions in a richer setting. Solving for the dynamics of the model in this stochastic environment also enables us to better confront the model with empirical evidence in Section 6.

Suppose that incomes are determined by a stationary Markov process with a finite state space $\mathcal{S} = \{1, 2, 3, \ldots, |\mathcal{S}|\}$. For $\kappa, \lambda \in \mathcal{S}$, the probability of a transition from κ to λ is denoted by $\rho_{\kappa\lambda}$. To avoid degeneracies, we assume that all transition probabilities are strictly positive. Each state $\kappa \in \mathcal{S}$ is associated with (deterministic) income profiles for each cohort: $w_\kappa(i, j) > 0$ is the current income in state κ of all agents who have income index i and are of age j. As before, this income is continuous and strictly increasing in the index i. As we want to focus on income processes that always subject the two youngest cohorts to income changes in the same direction, we assume that the states are ordered in the following way: if $\kappa, \lambda \in \mathcal{S}$ and $\kappa < \lambda$, then $w_\kappa(i,j) < w_\lambda(i,j)$ for all i and $j = 1, 2$. In the following, the restriction to proportional income shocks would not simplify the exposition significantly, so we drop it and merely assume that the income difference $w_\lambda(i,j) - w_\kappa(i,j)$ is increasing in i for $j = 1, 2$. We further assume that the income process is always compatible with an equilibrium allocation of dwellings as in Figures 1 and 2. More specifically, we think of the income process as a sufficiently small perturbation of time-invariant income profiles for which the steady state equilibrium is as in Figures 1 and 2.

Equation (10) implies a Markov chain with $|\mathcal{S}|$ possible values q^F_κ for the equilibrium price of flats. These values are such that $q^F_\kappa < q^F_\lambda$ whenever $\kappa < \lambda$. By equation (13), the equilibrium house price q^H_t depends on time $t-1$ income variables through the flat price q^F_{t-1} and the accumulated earnings profile $W_t(i) = (1+r)\, w_{t-1}(i,1) + w_t(i,2)$. To describe the equilibrium house price, therefore, we have to consider $|\mathcal{S}|^2$ income histories of length two, denoted $\kappa\lambda$, where κ is the previous income state and λ the current one. Accumulated earnings profiles at age 2 are then defined in the obvious way: $W_{\kappa\lambda}(i) = (1+r)\, w_\kappa(i,1) + w_\lambda(i,2)$.

We focus our attention on stochastic equilibria where the house price and the allocation of properties in any period depend only on the income history of length two.[20] By equation (13), a collection of $|\mathcal{S}|^2$ house prices $q^H_{\kappa\lambda}$ (one for each history $\kappa\lambda$) is part of such an equilibrium if and only if

$$\begin{aligned} 2 - S^H \;=\;& W^{-1}_{\kappa\lambda}\left(\gamma q^H_{\kappa\lambda} + (1+r)\, q^F_\kappa - q^F_\lambda\right) \\ &+ u^{-1}\left(\left[(1+r)\, q^H_{\kappa\lambda} - \sum_{\mu\in\mathcal{S}} \rho_{\lambda\mu} q^H_{\lambda\mu}\right] - \left[(1+r)\, q^F_\lambda - \sum_{\mu\in\mathcal{S}} \rho_{\lambda\mu} q^F_\mu\right]\right) \end{aligned} \quad (20)$$

for all $\kappa, \lambda \in \mathcal{S}$. For continuously differentiable functions $W_{\kappa\lambda}$ ($\kappa, \lambda \in \mathcal{S}$) and u with strictly positive first derivatives, the implicit function theorem implies that these $|\mathcal{S}|^2$ equations have a unique solution in a neighborhood of the "unperturbed" steady state.

[20] In other words, we want to study a stochastic steady state (invariant equilibrium) where the set of possible prices and allocations is the same finite (and minimal) set in each period. Of course, an initial allocation outside this set will imply equilibrium prices outside this set at first. But in a rational expectations equilibrium, the stochastic steady state will actually be reached after finitely many periods.

By (5) and (12), the cutoff indices $i^H_{\kappa\lambda}(2)$ and $m^H_{\kappa\lambda}$ that describe the allocation of houses after history $\kappa\lambda$ are then determined by the equations

$$u(m^H_{\kappa\lambda}) = \left[(1+r)\, q^H_{\kappa\lambda} - \sum_{\mu\in\mathcal{S}} \rho_{\lambda\mu} q^H_{\lambda\mu}\right] - \left[(1+r)\, q^F_{\lambda} - \sum_{\mu\in\mathcal{S}} \rho_{\lambda\mu} q^F_{\mu}\right] \tag{21}$$

and

$$W_{\kappa\lambda}(i^H_{\kappa\lambda}(2)) - (1+r)\, q^F_{\kappa} + q^F_{\lambda} = \gamma q^H_{\kappa\lambda}. \tag{22}$$

Finally, the number of transactions depends on the income history of length three, $\kappa\lambda\mu$, where κ denotes the income state two periods ago, λ the income state last period, and μ the current state. By (18), the number of transactions after history $\kappa\lambda\mu$ is

$$n_{\kappa\lambda\mu} = 2 - m^H_{\kappa\lambda} + 2\,(S^H - 1 + m^H_{\kappa\lambda})\, m^H_{\lambda\mu}. \tag{23}$$

We start the analysis of stochastic equilibria with a result that extends the overshooting result of Proposition 3.2. Proofs of all the results in this section can be found in Appendix D.

Proposition 4.1 (Capital gains versus earnings) *For $\kappa < \lambda$ and arbitrary μ, the house price after income history $\kappa\mu$ exceeds that after history $\lambda\mu$ if and only if*

$$w_{\lambda}(i^H_{\lambda\mu}(2),\, 1) - w_{\kappa}(i^H_{\lambda\mu}(2),\, 1) < q^F_{\lambda} - q^F_{\kappa}. \tag{24}$$

In this case, the income history that implies the higher house price also implies a higher number of house buyers of age 2 and a lower number of house owners of age 3.

In the special case where $\mu = \lambda$, this is essentially the same "over-reaction" result as Proposition 3.2. With $\kappa < \lambda$, a transition from $\kappa\lambda$ to $\lambda\lambda$ means that incomes first rise and then stay the same, whereas the house price rises and then falls back to a lower level.[21]

[21] Note the slight difference between Proposition 3.2, where we gave a sufficient condition, and Proposition 4.1, where the stated condition is necessary as well as sufficient. The counterpart of (24) in the impulse response setting with a proportional shock is the inequality $q^F > w(\bar{i}^H(2), 1)$. As $\bar{i}^H(2) < i^H(2)$, this is weaker than condition (19), but the latter has the advantage of allowing an easier interpretation – in fact, one that is entirely in terms of the initial steady state. This is not possible in the Markovian setting where we do not have an *a priori* ranking of the age 2 cutoff indices for all possible histories.

The intuition for Proposition 4.1 is the same as in the case of the impulse response. Since the expectations of age 3 agents are the same after both histories under consideration, changes in the house price are again driven entirely by changes in the wealth of the age 2 cohort. On the one hand, flat owners reaching age 2 enjoy larger capital gains (or suffer smaller capital losses) after history $\kappa\mu$ than after history $\lambda\mu$. On the other hand, their accumulated earnings are lower after history $\kappa\mu$. Under condition (24), the more favorable evolution of the price of flats outweighs the endowment disadvantage, hence leads to higher demand and a higher house price.

Condition (24) holds for all $\kappa, \lambda \in \mathcal{S}$ if income changes are small proportional changes to an "unperturbed" steady state where the marginal house buyers of age 2 cannot pay cash for the flat they purchase at age 1. More generally, the flat price equation (10), the inequalities $i^H_{\lambda\mu}(2) < 1$ and the monotonicity of income profiles imply that condition (24) holds for all $\kappa, \lambda \in \mathcal{S}$ if

$$|w_\kappa(1,\,1) - w_\lambda(1,\,1)| < \frac{|w_\kappa(3 - S^F - S^H,\,1) - w_\lambda(3 - S^F - S^H,\,1)|}{\gamma} \tag{25}$$

for all $\kappa \in \mathcal{S}$, so that state transitions do not spread or compress age 1 incomes too drastically.

The previous proposition compares histories ending in the same income state, so that expectations about the future can be held fixed. This is no longer possible when we consider histories that end in different states, because then expectations matter for the willingness of age 3 agents to pay for a house. With $\kappa < \lambda$ and arbitrary μ, for instance, it is possible that expectations about next period's difference between the house and the flat price are so much more pessimistic in state λ that the house price $q^H_{\mu\lambda}$ is smaller than $q^H_{\mu\kappa}$, despite the unambiguosly higher wealth of age 2 agents after history $\mu\lambda$. Two sorts of conditions ensure that the effect of expectations on house prices is dominated by the effect of wealth. One alternative is to assume sufficient persistence of the Markov chain, that is, to generalize the results of Section 3 for transition matrices sufficiently close to the identity matrix. Since this does not add much to the insights gained from the impulse response case, we will not pursue this route any further here. Instead, we turn to the second alternative, which consists in specifying conditions under which the effect of expectations on house prices is small irrespective of the transition probabilities.

Proposition 4.2 (Wealth versus expectations) *For $\kappa < \lambda$ and arbitrary μ, the house price after income history $\mu\lambda$ exceeds that after history $\mu\kappa$ if*

$$\frac{u(\frac{1}{2}) - u(1 - S^H)}{(1 + r)\, r} < q^F_\lambda - q^F_\kappa, \tag{26}$$

and the number of house owners of age 2 is higher after history $\mu\lambda$ if

$$\frac{u(\frac{1}{2}) - u(1 - S^H)}{(1 + r)\, r} < \frac{w_\lambda(\frac{1}{2}, 2) - w_\kappa(\frac{1}{2}, 2) + (1 - \gamma)\, [q^F_\lambda - q^F_\kappa]}{\gamma}. \tag{27}$$

Condition (26) *(resp.* (27)*) also ensures that the house price (resp. number of house owners of age 2) is higher after history $\lambda\lambda$ than after history $\kappa\kappa$.*

The role of conditions (26) and (27) is to bound the effect of expectations on house prices. Recall that by equation (11), the price of a house equals the price of a flat plus the present discounted value of the stream of extra utility that a house provides to the marginal unconstrained house owners. As the preference index of the marginal house owners of age 3 lies between $1 - S^H$ and $\frac{1}{2}$, the difference between the price of a house and the price of a flat is always bounded above by $u(\frac{1}{2})/r$ and bounded below by $u(1 - S^H)/r$. If the difference $u(\frac{1}{2}) - u(1 - S^H)$ is small, then so is the extent to which expectations about future price differences between houses and flats can vary with the current income state.

Under condition (26), age 3 agents are willing to pay more for a house in state λ than in state κ. At the same time, age 2 flat owners are wealthier (hence can afford to pay more for a house) after history $\mu\lambda$ than after history $\mu\kappa$. As a consequence, the house price is higher after history $\mu\lambda$.

Condition (27) is weaker than (26). It just ensures that in the comparison of history $\mu\kappa$ with $\mu\lambda$, any possible upward shift in age 3 agents' willingness to pay for houses is smaller than the upward shift in age 2 agents' ability to pay (this allows for the case where age 3 agents' willingness to pay is actually lower in state λ). So more agents of age 2 (and fewer agents of age 3) own a house after history $\mu\lambda$.

Finally, if we compare histories $\kappa\kappa$ and $\lambda\lambda$, the difference in the wealth of age 2 agents is even larger, while nothing changes for age 3 agents. So the arguments of the previous two paragraphs carry over.

Proposition 4.1 shows that under a very mild condition, capital gains on flats make the number of repeat purchases by potentially credit-constrained young households move with the house price in a transition from history $\kappa\lambda$ to $\lambda\lambda$. Proposition 4.2 shows that with a sufficiently small expectations effect, the current wealth of potentially credit-constrained agents ensures that in a comparison of histories $\mu\kappa$ and $\mu\lambda$ the numbers of repeat purchases by young households are ranked the same way as house prices. In contrast to the simpler exercise of the previous section, it is not clear whether this component of transaction volume is large enough to induce a co-movement of the house price with the total number of transactions. The reason is that this number also depends on the income state two periods ago. To study changes in transaction volume, one thus has to consider income histories of length four. To gain some insight into this problem, we first analyze it in a model with two income states.

Proposition 4.3 (Co-movement of prices and volume) *For two income states, i.e.,* $\mathcal{S} = \{1, 2\}$*, conditions* (25) *and* (26) *imply house prices* $q^H_{12} > q^H_{22} > q^H_{11} > q^H_{21}$ *and age 3 preference cutoffs* $m^H_{12} > m^H_{22} > m^H_{11} > m^H_{21}$*. Given such an ordering of house prices and preference cutoffs, twelve of the sixteen possible income histories of length four imply a movement of the property price index and the number of transactions in the same direction. Two histories imply a change in the number of transactions without any change in prices. Two histories are compatible both with a drop and with a fall in the number of transactions.*

Proposition 4.3 shows that for sufficiently strong capital gains and wealth effects, property prices and the number of transactions are very likely to rise or fall together. Through the down-payment constraint, capital gains and increased earnings strongly raise young households' ability to pay for a property of their own. This drives up property prices and at the same time accelerates these households' moves up the property ladder. As this generates additional transactions, price and volume tend to move together. Exceptions arise in those cases where the history implies a particularly strong offsetting effect on the repeat purchases by unconstrained households.

A similar exercise with three income states shows that 52 of the 81 possible income histories of length four imply a movement of the property price index and the number of transactions in the same direction. Numerical simulations confirm the positive correlation of prices and volume for a higher number of states.

In summary, this section has shown that our main results, price overshooting and the co-movement of prices with volume, do not just hold for impulse responses, but also for rational expectations equilibria where incomes are driven by a finite Markov chain with sufficient persistence or sufficiently small expectations effects. Our next aim is to extend the model further so that it can capture more, and more realistic, life-cycle patterns. In a second step, we will then confront the predictions of the extended model with empirical observations.

5 Extending the Simple Model

Previous sections have focused on the analysis of the simplest possible framework that enabled us to characterize the transmission of income shocks to the housing market and to reproduce the effects we mentioned in the introduction. For analytical tractability and ease of exposition, we presented a model where households spend at most two periods accumulating the down payment for a starter home, are able to move up the property ladder after one or two periods in a starter home, and trade at most once more without being subject to a credit constraint.

The aim of the present section is to check the robustness of our results in a more realistic setting where each of these three phases in a household's life consists of several periods. In such an extended model, a period may represent the length of time it takes a household to find a property to its liking and complete the transaction. A year would seem a reasonable period length in this respect.

A more realistic model of households' life cycles should be consistent with the following observations. First, accumulating a down payment takes time. Second, it typically takes a few years for owners of starter homes to accumulate sufficient net worth to move up the property ladder. Third, most households, and especially middle-aged and older ones, do not face a binding down-payment constraint.

To account for these observations, we increase the length of our agents' lives. We then look for parameters such that it takes more than one period to accumulate a sufficient down payment to purchase a flat, and to purchase a house once one owns a flat. In addition, we make sure that for several periods towards the end of life households' housing consumption is not restricted by wealth.

The main consequence of these extensions of the simple framework is an increase in the number of lags of income that matter for flat prices and an increase in the number of lags of the flat price and income that matter for house prices. Flat prices depend on the income histories of the marginal credit-constrained flat buyers. So the number of income lags relevant to the price of flats is given by the age of the oldest credit-constrained flat buyers. For their backward-looking element, house prices depend on the income history of the oldest credit-constrained house buyers as well as the flat prices that all constrained house buyers paid at the time of their purchases.

A possible equilibrium configuration of such an extended model is depicted in Figure 5. In the first period of life, no agent can afford to buy any property. Some buy a flat at age 2, the others cannot afford the down payment until age 3. No agent can afford a house until age 4. Between age 4 and age 6, an increasing number of agents can afford to move into a house; some remain credit-constrained and hence in a flat until they reach age 7 even though they would prefer a house. From age 7 to age $6 + k$ with $k > 1$, they all have enough wealth to afford what they prefer.[22] In this configuration, the three most recent income shocks affect the equilibrium price of flats. Credit-constrained house buyers are up to six periods old. Consequently, the most recent six income shocks affect the demand for houses by constrained agents. Since no constrained house buyer bought a flat more than three periods before their house purchase, no earlier income shocks affect the current house price.

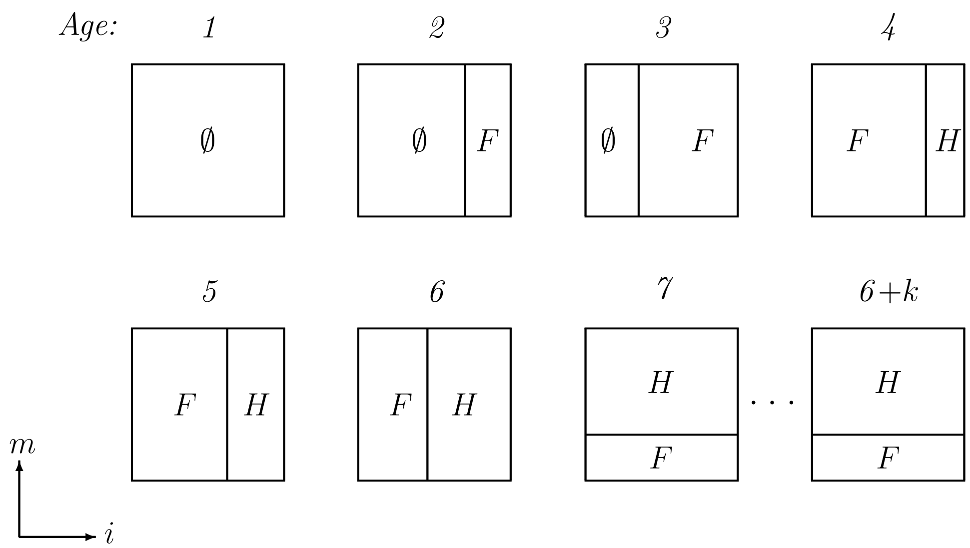

Figure 5: Housing choices in an extended model

Mathematically, the difference between such an extended model and the earlier simpler one is the greater order of the system of difference equations that characterizes

[22]It is understood here that agents have learned their preference index m by the time they reach age 7. Given the quasilinear preference structure, early revelation of m does not matter for housing choices. If this index were learned later, we would see periods where all members of a cohort own a house. This would not alter the results either.

housing price dynamics. This makes analyzing the impulse response to a permanent income shock more difficult, but by no means impossible – one can still proceed along the lines of Section 3. With Markovian income processes, our ability to derive analytical results is limited by the much larger state space required given that longer income histories matter. Still, the price of any property is determined exactly as before: it is the price of a flat plus the expected utility premium the property provides over and above the flat for the marginal unconstrained owners in all future periods.

As before, the price of flats depends only on the current wealth of young marginal first-time buyers. The only difference here is that this wealth depends not only on current but also on a few lagged incomes. Hence, it takes more periods for income shocks to work their way through to flat prices, so the response to a permanent shock is more progressive than in the simple model. Income shocks still induce capital gains for flat owners and hence affect their ability to move into a house. Although the response of the housing price index to an unexpected one percent permanent income shock is more progressive, the overshooting remains.

Figure 6 displays the impulse response to an income shock for two parametrizations of the model that both yield the configuration of Figure 5.[23] The two panels show the "trade-off" explained in Section 3 between price and transaction fluctuations. With a flatter preference gradient, a greater number of unconstrained house owners are willing to downsize for any given rise in the relative price of houses. As a consequence, the rise in the house price necessary to clear the market in the wake of a positive income shock is smaller, while the initial rise in the number of transactions is larger.

Does the positive correlation between housing prices and transactions remain when we subject this model to stochastic incomes? In the previous section, we showed that prices and transactions move in the same direction most of the time when we subject the simple model to a two-state Markov process for income. Here, we solve the model numerically for a three-state Markov process. Relative to state 2, we assume that income is one percent lower in state 1 and one percent higher in state 3. We assume that the state remains the same with probability 0.6 and switches to either of the other states with equal probabilities of 0.2. The two panels of Figure 7 display income, the housing price index and the number of transactions for the same sample income path and different preference gradients. The positive correlation between income, price index

[23]The details of the parametrization used for the simulation in Figure 6 as well as the computation code both for these and the following simulations are available from the authors upon request. The computations rely on a standard routine for systems of linear difference equations.

and transactions is obvious. Again, the relative volatilities of these variables depend on the preference gradients.

The consensus among housing economist seems to be that American households are reluctant to reduce their housing consumption unless they are forced to by financial, health or other exogenous changes in circumstances. This indicates that only dramatic relative price changes could induce a significant number of unconstrained owners to downsize. Within our model, such an empirical feature would be captured by a very steep preference gradient, which would strengthen the degree of price overshooting in response to income shocks. While it is difficult to link the preference gradient in our model directly to any particular piece of empirical evidence, our analytical investigation and the above simulations imply a cross-sectional restriction that is testable.[24]

The extension of the model also allows us to check the robustness of our results to changes in the proportion of unconstrained homeowners. This is particularly important since existing models where credit constraints play a role in the transmission and amplification of shocks often require the proportion of constrained agents to be large in order to obtain significant effects.[25] Within our framework, increasing the proportion of preference traders can simply be achieved by increasing the number of periods where households are unconstrained (i.e., the parameter k in the configuration of Figure 5).[26] Doing this has exactly the same effects as lowering the preference gradient while holding k fixed. This is because either way, there are more unconstrained owners within a given difference in house utility premium from each other. Thus, our findings do not require that a large fraction of households be liquidity constrained. Lowering this fraction changes the relative magnitude of price and volume fluctuations, but does not affect their co-movement or overshooting.

In all, the analytical results we obtain with the simple model survive extending the model to a more plausible configuration. Possible relaxation of additional modeling assumptions will be discussed in our concluding remarks (Section 7). First, we wish to confront some of the implications of our model with empirical observations.

[24]It is worth noting here that the volatility of housing prices relative to that of GDP is higher in the U.K. than in the U.S., while the opposite ranking is observed for the volatility of transactions relative to that of GDP.

[25]In the multiple equilibria example of Stein (1995), for instance, only 1 percent of households are never liquidity constrained; in the simulations of Kiyotaki and Moore (1997), two thirds of the asset is held by credit-constrained agents.

[26]More precisely, we increase the number of periods agents live as unconstrained preference traders and add the appropriate number of flats and houses so as to keep the steady state equilibrium essentially unchanged.

6 Empirical Implications

One of the main predictions of our model, the co-movement of prices and transaction volume, is clearly consistent with the evidence reported for example by Stein (1995). In the present section, we want to discuss other empirical implications of the model: the role of current income of potential first-time buyers, the overshooting of prices, and the effects of financial liberalization.

6.1 Aggregate Price Fluctuations

The model predicts that the income of young households is a key determinant of housing prices. We derived above that the crucial income fluctuations are those of the poorest first-time buyers. In the model, this is the income of a given percentile of the youngest cohort when the cohort is ranked according to household wealth. Among available income time series, the median income of 25-34 year old households seems best suited to proxy for the income of these marginal first-time buyers.

Typically, per capita income is the variable incorporated in housing price regressions to account for the income elasticity of housing demand (e.g., Poterba 1991; Muellbauer and Murphy 1997). To get a sense of whether the income of young households matters for housing price fluctuations independently of per capita income, we graph housing prices, per capita disposable income and the median income of 25-34 year old households (Figure 8).[27] The volatility of housing prices appears closer to that of the income of young households than to the much smoother per capita income series. The trend of housing prices is somewhere in between the trends of the two income series. A simple linear regression on the log-transformed variables, using yearly data from 1975 to 1997, confirms the visual impression. Together, the variables explain 74 percent of housing price variations. Both are highly significant.[28] This evidence is consistent with our theoretical results. It suggests that housing prices embody the fluctuations in both per capita income and income of the young.

[27] Data sources: Freddie Mac national housing price index (1975-1997) extrapolated to 1970 using the Census Bureau median price of existing home index, real average disposable income from the Economic Report of the President, and median 25-34 year-old household income from the Census Bureau.

[28] The estimated coefficients for income per capita and the median income of 25-34 year old households are 0.42 (0.049) and 0.64 (0.194), respectively; standard errors are in parentheses.

6.2 Price Response to Income Shocks

In response to a surprise permanent income shock, the model predicts an instantaneous housing price rise followed by a drop to a new steady state level. This overshooting pattern remains when incomes are assumed to follow a Markov process.

As mentioned in the introduction, Lamont and Stein (1999) use yearly city level data to analyze the response of housing prices to income shocks. In a first step, they estimate the main determinants of housing price fluctuations to be the contemporaneous change in current per capita income, the one-period lagged change in housing price and the one-period lagged price income ratio. Based on their regression, they simulate the dynamic response of housing prices to a one-percent permanent income shock. They find that housing prices rise progressively above the new steady state over a few years before converging to it monotonically. As explained in the previous section and shown in Figure 6, the model produces a similar pattern.

A further finding of Lamont and Stein is that housing prices react more to a permanent income shock in cities where the proportion of households with a high loan-to-value ratio is high. In these cities, a permanent income shock leads to a faster and larger housing price rise than in cities with few highly leveraged households. Moreover, following the initial rise, prices converge to their new steady state at a faster rate.

The proportion of households with a high loan-to-value ratio is of course determined endogenously. In the model, changes in the distribution of debt and housing across households can be obtained through changes in various parameters. Cities with a low proportion of highly indebted households can for example be modeled as cities with a small proportion of young households. One way to add wealthy households in our model is to extend the length of agents' lives as explained in the previous section. Alternatively, one can fix the length of agents' lives but add a flow of unconstrained households to the model economy in any given period. In either case, we have the impulse response result from the previous section: the lower the proportion of households with a high loan-to-value ratio (i.e., the greater the proportion of unconstrained homeowners), the smaller the price rise and the slower the convergence to the new steady state.[29]

[29] Alternatively, we could have simulated long time series for income and prices with the stochastic model of the previous section and then estimated the same equation as Lamont and Stein on the simulated data. The same results would have emerged.

In summary, the predictions of our model are consistent with the evidence reported in Lamont and Stein. This is true not only with respect to the overall response of prices to income but also with respect to the effect of the households' debt on the response of housing prices to income shocks.

6.3 Financial Liberalization

Several countries that liberalized mortgage markets subsequently experienced a strong rise in property prices followed by a downturn. There is also evidence of a rise in the number of transactions during the boom and a drop during the ensuing bust. Examining the reaction of our model to an unanticipated permanent reduction in the down-payment requirement, we find that financial liberalization *per se* contributes to such boom-bust dynamics.

First, compare two steady states with different levels of the financial constraint. In the steady state with the lower down-payment requirement, the flat price must be higher so as to keep enough potential first-time buyers out of the housing market. A higher flat price in the new steady state implies an increased willingness of unconstrained households to pay for a house. On the other hand, flat owners do not need to accumulate as much down payment to purchase a house and so can accelerate their move. The cost of holding a flat for one period is higher but this effect is secondary to the direct effect of the increase in the allowable loan-to-value ratio on a flat owner's ability to purchase a house. Increased steady state demand for houses from both constrained and unconstrained households results in a higher steady state level for the house price. Typically, the steady state house price actually rises by more in absolute terms than the steady state flat price, implying that more young constrained households end up owning a house. Note that this reallocation of properties improves welfare as young households derive more utility from houses than the marginal unconstrained house owners. The formal argument underlying this reasoning is similar to the proof of Proposition 3.1 and is therefore omitted.

As to the transition dynamics following the credit market shock, the capital gains enjoyed by young flat owners boost their ability to pay for a house. As a result, the house price overshoots the new steady state just as it does in response to an income shock. Again, we find a co-movement of property prices and the number of transactions.

Liberalization *per se* was often not the only force at play behind recent boom-bust episodes that followed a loosening of credit market constraints in several countries. For example, the liberalization of the U.K. mortgage market in the early Eighties coincided with the start of a period of strong demographic pressure on the housing market and strong income growth.[30] Interestingly, the breadth of available evidence for the U.K. has enabled us to confront a variety of predictions of our modeling framework with the data. In Ortalo-Magné and Rady (1999), we use a version of our model that includes a rental sector for flats to show that changes in owner-occupancy rates of young households over the period from the mid-1980s to the mid-1990s can be rationalized as a reaction to the liberalization of the early 1980s. In agreement with the data, the model predicts that the largest growth in mortgages occurs with the youngest cohorts.

With regards to transaction fluctuations, by design, the model focuses only on moves up and down the property ladder. In Ortalo-Magné and Rady (2000), we analyze the transaction fluctuations over the past three decades in the U.K. Combining data on whether buyers are first-time or repeat buyers and survey evidence on moving motivations, we find that transactions up and down the property ladder are the main driver of overall transaction fluctuations. Moves due to job relocation and personal/family reasons fluctuate only very little with the economic cycle. The first main reason for the boom in transactions in the Eighties (from 1 million in 1980 to 1.8 million in 1988 for England and Wales) appears to be repeat buyers bringing forward their moves up the property ladder. They are also in large part responsible for the drought of transactions in the early Nineties (less than 1 million in 1992 and 1993). They withdrew from the market when the prices collapsed (minus 25 percent between 1990 and 1993 after 88 percent growth over the previous 8 years). The second main driver for the boom in transactions was the transfer of properties from the private rental sector to the owner-occupied sector, another observation in support of the version of the model that we developed for this application.

Overall, the Eighties witnessed a one-off adjustment of the U.K. housing market to greater availability of credit. Households were suddenly given greater access to mortgages. This prompted a major adjustment of the distribution of debt and housing across households, hence a period of exceptionally many transactions. Once the economy had adjusted, transactions dropped back to levels close to their long term trend, not too different from pre-liberalization levels. This explains a major puzzle for U.K.

[30] Meen (1990) provides a detailed account of the liberalization of the U.K. mortgage market in the Eighties.

mortgage lenders in recent years: why, given the strength of the housing market in 1997-99, transaction levels have remained around their pre-Eighties levels.

In summary, our modeling framework helps reconcile and rationalize price, transactions and cohort behavior over the past three decades in the U.K. The evidence provides support for a focus on moves up and down the property ladder in our search for a better understanding of overall transaction fluctuations.

7 Concluding Remarks

This paper provides a first step toward a dynamic theory of housing prices and transactions. We developed a life-cycle model where households are heterogeneous with respect to income and preferences, and mortgage lending is restricted by a down-payment requirement. The main findings concern the critical role of young households and a characterization of the transmission mechanism by which shocks translate into price and transaction fluctuations.

Extending the model to more than two types of property complicates the analysis by introducing additional margins, but does not alter our main results. The focus on configurations as in Figures 1–2 or 5 and on small shocks does not restrict the validity of our findings either. It just ensures a small number of margins to be tracked and so helps us derive analytical results. The modeling framework is perfectly able to handle more complicated configurations and arbitrary income shocks, but this additional generality will tend to push one in the direction of computational procedures when investigating model dynamics. For example, a large negative shock may force some members of a cohort to delay their move up the property ladder to such an extent that younger households can "overtake" them, having not suffered large capital gains. Interestingly, such leap-frogging actually occurred in the South East of England with the bust of the early Nineties (Earley 1996).

The introduction of a rental market for flats changes the results quantitatively, but not qualitatively. It dampens the response of prices to income shocks. The extent to which prices respond less depends on the ease with which flats can be converted from owner-occupied to rented accommodation. Easier conversion means smaller price and larger volume fluctuations. At the extreme, if rented dwellings are an altogether different commodity than owner-occupied dwellings, the results of this paper apply without change. Whenever conversions are possible, they introduce an independent source of transaction fluctuations because income shocks (and even more so financial liberalization) bring changes in the endogenous owner-occupancy rate. This specific extension of our framework is presented in Ortalo-Magné and Rady (1998, 1999).

A non-linear representation of preferences would enable us to capture the flexibility households have in allocating their resources across investment and consumption opportunities over time. Díaz-Giménez and Puch (1998) confirm the intuition that faced with the prospect of rising prices, potential first-time buyers have strong incentives to cut down non-housing consumption in order to get a foot on the property ladder before starter homes become too expensive. With non-linear utility, we might therefore obtain even stronger amplification of income shocks as potential first-time buyers shift resources toward housing when income and prices are rising.

While the focus of this paper was on income shocks, our model obviously lends itself to the analysis of other relevant shocks. Changes in interest rates, for example, have similar consequences to changes in income. A drop in interest rates yields increases in housing prices and transactions; a rise in interest rates has the opposite effects. Decoupling saving and borrowing rates does not add any further insights.

Demographic changes matter in the model to the extent that they affect the margins that are relevant for the housing market. Hence, predicting the effect of demographic changes requires understanding their consequences for the wealth distribution within the young cohorts and the preference distribution across wealthier households. For example, increased immigration by poor households has only a limited impact. Wealthy newcomers, on the other hand, can generate significant price swings. The impact of immigration from Hong Kong on the Vancouver housing market is a case in point here (Bulan et al. 2000).

The model is rich in empirical predictions. Some have already been addressed by the empirical housing literature. Others deserve further investigation. In terms of individual household behavior, the key question raised by our theory is the extent to which capital gains (and other wealth shocks) affect the timing of repeat buyers' moves. In terms of aggregate market behavior, we presented evidence from both the U.S. and the U.K. housing markets. Much remains to be learned from variations in institutional setups and housing market fluctuations across countries, regions or cities.

Appendix

A Sufficient Conditions for the Equilibrium Configuration of the Simple Model

Maintaining the assumptions on the supply of dwellings and the preference structure made in Sections 1 and 2, we want to spell out conditions that ensure a steady state equilibrium allocation of dwellings as shown in Figures 1 and 2. We suppress time indices whenever we deal with steady state variables, and write $W(i) = (1 + r)\, w(i, 1) + w(i, 2)$.

Proposition A.1 (Unique steady state equilibrium) *Suppose that income profiles are constant over time with*

$$W(0) \geq w(3 - S^F - S^H, 1), \tag{A.1}$$

$$W(\tfrac{1}{2}) \geq \max\{w(1,1),\ W(3 - S^F - S^H)\} + \frac{r}{\gamma} w(3 - S^F - S^H, 1), \tag{A.2}$$

$$w(0,3) \geq W(\tfrac{1}{2}) - (1+r)W(0), \tag{A.3}$$

and preferences satisfy

$$u(\tfrac{1}{2}) \geq \frac{(1+r)r}{\gamma}[W(1) - w(3 - S^F - S^H, 1)], \tag{A.4}$$

$$u(1 - S^H) < \frac{r}{\gamma}\left[W(1) - \left(1 + \frac{r}{\gamma}\right) w(3 - S^F - S^H, 1)\right] \tag{A.5}$$

as well as

$$U(F, m, 1) - U(\emptyset, m, 1) \geq u(\tfrac{1}{2}) + \frac{(1+r)r}{\gamma}[(2+r)w(3 - S^F - S^H, 1) - W(\tfrac{1}{2})], \tag{A.6}$$

$$U(F, m, 2) - U(\emptyset, m, 2) \geq u(\tfrac{1}{2}) - u(1 - S^H) + \frac{(1+r)^2 r}{\gamma} w(3 - S^F - S^H, 1). \tag{A.7}$$

Then the model of Sections 1 and 2 has a unique steady state equilibrium, and the allocation of dwellings in this equilibrium is as in Figures 1 and 2.

Proof: As $0 < 3 - S^F - S^H < 1$ by assumption, we can define $q^F = w(3 - S^F - S^H, 1)/\gamma$. Given our other assumption about supply, $\frac{1}{2} < S^H < 1$, conditions (A.4) and (A.5) now ensure that the system of equations

$$[W(i) - rq^F]/\gamma = q^F + u(m)/r, \tag{A.8}$$

$$i + m = 2 - S^H \tag{A.9}$$

has a unique solution $(i^H(2), m^H) \in [0,1] \times [0,1]$, and this solution satisfies the inequalities $\frac{1}{2} < i^H(2) < 1$ and $0 < m^H < \frac{1}{2}$. Define $q^H = [W(i^H(2)) - rq^F]/\gamma = q^F + u(m^H)/r$.

Consider a time-invariant allocation of dwellings as in Figures 1 and 2 with $i^F(1) = 3 - S^H - S^F$ and the above indices $i^H(2)$ and m^H. By construction, this allocation is market clearing. It remains to show for each household that the allocated consumption path is feasible and achieves maximal utility given the income stream and the above property prices.

Starting with the age 3 cohort, it is easy to see that condition (A.3) ensures that all households of this cohort can afford to own a house. Thus income is irrelevant to their choices, and all households with preference index $m \geq m^H$ find it optimal to own a house (since $u(m) \geq r\,[q^H - q^F]$), while those with $m < m^H$ prefer to live in a flat (since $u(m) < r\,[q^H - q^F]$).

Condition (A.4) guarantees that all households of age 2 would like to acquire a house rather than a flat at the given prices: $u(\frac{1}{2}) \geq (1+r)r[q^H - q^F]$, i.e., the extra utility from occupying a flat outweighs the loss in terminal consumption that arises from the higher cost of holding a house for one period. (The housing decision at age 2 does not influence the decision at age 3, so the latter does not enter the calculation here.) Condition (A.7) ensures that $U(F, m, 2) - U(\emptyset, m, 2) > (1 + r)rq^F$, so all age 2 households are better off acquiring a flat at the given price rather than purchasing no property at all.

By the construction of $i^H(2)$ and the definition of q^H, all households of age 2 that enter the current period with a flat and have income index $i \geq i^H(2)$ can afford a house. Acquiring a house is thus both feasible and optimal for these households. Households of age 2 that enter the current period with a flat but have income index $i \geq i^H(2)$ cannot afford a house. Staying in a flat is the best available option to them.

Condition (A.1) guarantees that $W(0) \geq \gamma q^F$, so even the poorest households of age 2 can afford a flat. Condition (A.2), on the other hand, implies $W(i^F(1)) < \gamma q^H$. Acquiring a flat is therefore the best available choice for all age 2 households that were not able to buy a flat in the previous period, that is, those with income index $i < i^F(1)$.

Condition (A.2) also implies that $w(1, 1) < \gamma q^H$, so no household can afford a house at age 1. Condition (A.6) ensures that at the given prices, all households are better off owning a flat for two periods than owning no property at age 1 and a house at age 2. That is, $U(F, m, 1) - (1 + r)^2 rq^F + U(F, m, 2) - (1 + r)rq^F$ exceeds $U(\emptyset, m, 1) + U(H, m, 2) - (1 + r)rq^H$. Thus, all households that can afford a flat at age 1 will actually find it optimal to acquire one.

This completes the proof that at the prices q^F and q^H, the allocation of dwellings solves each household's constrained optimisation problem.

Proving uniqueness of the steady state is straightforward. The given preferences, wealth distribution and supply of properties first imply that the price of flats must be $q^F = w(3 - S^F - S^H, 1)/\gamma$. Given q^F, then, the steady state house price is uniquely determined as well. ■

Conditions (A.1)–(A.7) suggest a straightforward way to find parameter combinations that support the steady state configuration of Figures 1 and 2. First, fix any age 1 income profile. Second, choose an age 2 income profile that is high and steep enough to satisfy conditions (A.1)–(A.2). Third, choose an age 3 income profile high enough to satisfy (A.3). Fourth, choose a profile of age 3 utility premia $u(m)$ whose level and slope are such that (A.4)–(A.5) hold. Fifth, choose utility premia $U(F, m, 1) - U(\emptyset, m, 1)$ and $U(F, m, 2) - U(\emptyset, m, 2)$ high enough to satisfy (A.6)–(A.7).

The existence and uniqueness of stochastic steady states in a neighborhood of a deterministic one follows by the implicit function theorem as indicated in the main text.

B Total Number of Transactions

From equations (16) and (17), the total number of transactions at time t is

$$\begin{aligned} n_t &= 2 - i_t(2) + [1 - i_{t-1}(2)]\, m_t^H + i_{t-1}(2)\,(1 - m_t^H) \\ &= 2 - i_t(2) + i_{t-1}(2) + [1 - 2i_{t-1}(2)]\, m_t^H. \end{aligned}$$

The market clearing condition for houses, (7), implies that $-i_t(2) + i_{t-1}(2) = m_t^H - m_{t-1}^H$ so the total number of transactions can be written as

$$n_t = 2 - m_{t-1}^H + 2[1 - i_{t-1}^H(2)]\, m_t^H.$$

Substituting $i_{t-1}(2) = 2 - S^H - m_{t-1}^H$ from market clearing yields the expression given in Section 2.

C Response to Income Shocks

C.1 Proofs

Let $\bar{i}^F(1)$, $\bar{i}^H(2)$ and $\overline{m}^H$ denote the cutoff indices in the new steady state equilibrium, and $\overline{W}(i) = (1+r)\,\overline{w}(i,1) + \overline{w}(i,2)$.

Proof of Proposition 3.1: Consider age 2 agents with wealth index $i^H(2)$. Since $i^H(2) > i^F(1)$, these agents will be flat owners in both the old and the new steady state. The increase in their wealth from one steady state to the other is therefore $\overline{W}(i^H(2)) - rq^F - [\overline{W}(i^H(2)) - r\overline{q}^F] = \sigma[W(i^H(2)) - rq^F]$, which translates into an increase in their ability to pay for a house by $\sigma[W(i^H(2)) - rq^F]/\gamma = \sigma q^H$. The highest steady state price that an age 3 agent is willing to pay for a house, on the other hand, increases by exactly the price difference for flats, $\overline{q}^F - q^F = \sigma q^F$.[31] As $q^H > q^F$, the wealth effect on the age 2 cohort is stronger than the substitution effect on the age 3 cohort. This means that the new steady state equilibrium must be such that the house price rises by more (in absolute terms) than the flat price: $\overline{q}^H - q^H > \overline{q}^F - q^F$, or equivalently, $\overline{q}^H - \overline{q}^F > q^H - q^F$. Given that the house price rises by more than the flat price, fewer age 3 agents buy a house: $\overline{m}^H > m^H$. Market clearing then requires that more houses be bought by age 2 agents: $\bar{i}^H(2) < i^H(2)$. ■

Proof of Proposition 3.2: Consider the inverse demand functions (14) and (15) for $t = s-1, s, s+1$. For $i > i^H(1)$, $Q^{H,2}_{s-1}(i) = [W(i) - rq^F]/\gamma$ and $Q^{H,2}_{s+1}(i) = [\overline{W}(i) - r\overline{q}^F]/\gamma$ correspond to the old and new steady state, respectively, while $Q^{H,2}_{s}(i) = [(1+r)w(i,1) + \overline{w}(i,2) - (1+r)q^F + \overline{q}^F]/\gamma$ at the time of the shock. Clearly, $Q^{H,2}_{s}(i) > Q^{H,2}_{s-1}(i)$ for all $i > i^H(1)$. In the age 3 cohort, we have $Q^{H,3}_{s-1}(m) = q^F + [u(m) + q^H - q^F]/(1+r)$ (corresponding to the old steady state) and $Q^{H,3}_{s}(m) = Q^{H,3}_{s+1}(m) = \overline{q}^F + [u(m) + \overline{q}^H - \overline{q}^F]/(1+r)$ (reflecting the new steady price of flats and the new expectations about the future price of houses). By Proposition 3.1, $\overline{q}^H - \overline{q}^F > q^H - q^F$, which implies immediately that $Q^{H,3}_{s}(m) > Q^{H,3}_{s-1}(m)$ for all m. So both the $Q^{H,2}$ curve and the $Q^{H,3}$ curve shift upward at the time of the shock, and the price of houses must rise: $q^H_s > q^H_{s-1} = q^H$. This proves the first part of the proposition.

Next, a simple computation shows that $Q^{H,2}_{s}(i) - Q^{H,2}_{s+1}(i) = \sigma\,(1+r)[q^F - w(i,1)]/\gamma$. If $w(i^H(2),1) < q^F$, therefore, all age 2 agents with wealth index between $i^F(1)$ and $i^H(2)$ have a higher demand for houses at time s than at time $s+1$. Given identical demand from age 3 agents at times s and $s+1$, this implies that $q^H_s > q^H_{s+1} = \overline{q}^H$. ■

Proof of Proposition 3.3: We know from the previous proof that condition (19) implies $Q^{H,2}_{s}(i) > Q^{H,2}_{s+1}(i)$ for all i between $i^F(1)$ and $i^H(2)$, hence in particular for $i^H_{s+1}(2) = \bar{i}^H(2)$. Given identical demand for houses from age 3 agents in periods s and $s+1$, therefore, market clearing requires that $i^H_s(2) < \bar{i}^H(2)$, which in turn implies that $i^H_s(2) < i^H(2)$ or, eqivalently, $m^H_s > m^H$.

According to equation (18), the number of transactions in period s equals $n_s = 2 - m^H + 2\,(S^H - 1 + m^H)\,m^H_s$, while that in the old steady state is $n = 2 - m^H + 2\,(S^H - 1 + m^H)\,m^H$. A simple calculation shows that the inequality $m^H_s > m^H$ implies $n_s > n$. More precisely, we have three effects: an increase in the number of agents acquiring a house at age 2 by $i^H(2) - i^H_s(2) = m^H_s - m^H$; an increase in the number of agents who purchase a flat at age 3 by $[1 - i^H(2)]\,(m^H_s - m^H)$; and a change in the number of agents who buy a house at age 3 by $-i^H(2)\,(m^H_s - m^H)$. As $i^H(2) > \frac{1}{2}$, the second and third effect together yield a fall in the number of transactions, that is, there are fewer repeat purchases by age 3 agents. The driving force behind the overall change in the number of transactions is therefore the number of repeat purchases by age 2 agents. ■

[31] Adapting definition (15) to take account of the fact that we are solving for steady state prices, we see that the steady state inverse demand function at age 3 is $Q^{H,3}(m) = q^F + u(m)/r$ in the old steady state and $\overline{Q}^{H,3}(m) = \overline{q}^F + u(m)/r$ in the new one. So the difference is $\overline{q}^F - q^F$.

C.2 Non-Proportional Income Shocks

In contrast to Section 3, where we restricted ourselves to proportional income shocks, we now wish to consider more general shocks. Instead of supposing proportional shocks, we make the following assumptions:

Assumption C.1 *The income difference across steady states, $\overline{w}(i,j) - w(i,j)$, is increasing in the income index i at ages $j = 1, 2$.*

Assumption C.2 *Median age 1 and 2 income differences across steady states satisfy*

$$\overline{w}(\tfrac{1}{2},2) - w(\tfrac{1}{2},2) \geq \frac{r\,(1-\gamma)}{\gamma}\left[\overline{w}(\tfrac{1}{2},1) - w(\tfrac{1}{2},1)\right].$$

Thus, richer agents are assumed to gain at least as much (in absolute terms) from the income shock as poorer agents. In addition, the median age 2 income difference across steady states is assumed to be at least $r\,(1-\gamma)/\gamma$ times as large as the median age 1 income difference. Realistic parameter values will imply $r\,(1-\gamma)/\gamma \leq 1$. In other words, Assumption C.2 effectively means that the shock raises median age 2 earnings by at least as much as median age 1 earnings. These are very mild assumptions.

Proposition C.1 (Comparison of steady states) *Under Assumptions C.1 and C.2 the steady state with higher incomes is characterized by higher property prices, a higher difference between the price of houses and the price a flats, a higher number of age 2 house buyers, and a lower number of age 3 house buyers.*

Proof of Proposition C.1: As in the proof of Proposition 3.1 it suffices to show that the wealth effect on the age 2 cohort is stronger than the substitution effect on the age 3 cohort. More precisely, it is enough to prove that $\overline{W}(i^H(2)) - rq^F - [\overline{W}(i^H(2)) - r\overline{q}^F]$ exceeds $\overline{q}^F - q^F$.

As $i^H(2) \geq \frac{1}{2} > i^H(1)$, Assumptions C.1 and C.2 imply

$$\begin{aligned}
\overline{W}(i^H(2)) - W(i^H(2)) &\geq \overline{W}(\tfrac{1}{2}) - W(\tfrac{1}{2}) \\
&= \overline{w}(\tfrac{1}{2},2) - w(\tfrac{1}{2},2) + (1+r)\left[\overline{w}(\tfrac{1}{2},1) - w(\tfrac{1}{2},1)\right] \\
&\geq \left(\frac{r\,(1-\gamma)}{\gamma} + 1 + r\right)\left[\overline{w}(\tfrac{1}{2},1) - w(\tfrac{1}{2},1)\right] \\
&= \left(\frac{r}{\gamma} + 1\right)\left[\overline{w}(\tfrac{1}{2},1) - w(\tfrac{1}{2},1)\right]
\end{aligned}$$

and

$$\overline{q}^F - q^F = \frac{1}{\gamma}\left[\overline{w}(i^F(1),1) - w(i^F(1),1)\right] < \frac{1}{\gamma}\left[\overline{w}(\tfrac{1}{2},1) - w(\tfrac{1}{2},1)\right].$$

A simple calculation now leads to the desired inequality. ■

Proposition C.2 (Overshooting of the house price) *The house price in the period of the shock is higher than in the old steady state. The house price overshoots its new steady state if and only if*

$$\overline{w}(\overline{i}^H(2),1) - w(\overline{i}^H(2),1) < \overline{q}^F - q^F. \tag{C.1}$$

Proof of Proposition C.2: As in the proof of Proposition 3.2, we consider the inverse demand functions (14) and (15) for $t = s-1, s, s+1$. By exactly the same arguments as in that earlier proof, $Q_s^{H,2}(i) > Q_{s-1}^{H,2}(i)$ for all $i > i^H(1)$ and $Q_s^{H,3}(m) > Q_{s-1}^{H,3}(m)$ for all m. So $q_s^H > q_{s-1}^H = q^H$.

Now there is overshooting in the house price ($q_s^H > q_{s+1}^H = \overline{q}^H$) if and only if at the new steady state cutoff $\overline{i}^H(2)$ the $Q_s^{H,2}$ curve is above $\overline{q}^H$ as shown in Figure 4, i.e., if and only if

$$Q_s^{H,2}(\overline{i}^H(2)) - Q_{s+1}^{H,2}(\overline{i}^H(2)) = \frac{1+r}{\gamma}\left(\overline{q}^F - q^F - \left[\overline{w}(\overline{i}^H(2),1) - w(\overline{i}^H(2),1)\right]\right) > 0. \tag{C.2}$$

■

We can use the characterization of the flat price in equation (10) to translate condition (19) into the inequality

$$\overline{w}(\overline{i}^H(2), 1) - w(\overline{i}^H(2), 1) < \frac{\overline{w}(3 - S^F - S^H, 1) - w(3 - S^F - S^H, 1)}{\gamma}, \tag{C.3}$$

a sufficient condition for which is

$$\overline{w}(1, 1) - w(1, 1) < \frac{\overline{w}(3 - S^F - S^H, 1) - w(3 - S^F - S^H, 1)}{\gamma} \tag{C.4}$$

by the monontonicity of income differences. Note that this will always hold when the income shock results in a parallel shift of age 1 earnings. Thus, we will have overshooting in the house price as long as the shock does not spread age 1 incomes too drastically.

The following results now follow exactly as in the case of a proportional shock.

Proposition C.3 (Transactions in the period of the shock) *Under the overshooting condition* (19)*, the number of transactions in the period of the shock is higher than in the old steady state. This higher number of transactions is due to an increase in the number of repeat purchases by buyers of age 2.*

Proposition C.4 (Co-movement of prices and number of transactions) *Under the overshooting condition* (19)*, property prices and the number of transactions all rise in the period of the income shock.*

Note that overshooting of the house price is sufficient, but not necessary for this result. In fact, Figure 4 shows that the inequality $i_s^H(2) < i^H(2)$ will hold as long as the $Q_s^{H,2}$ curve lies sufficiently far above the $Q_{s-1}^{H,2}$ curve at the old steady state cutoff index $i^H(2)$. It can be shown that the precise condition is $Q_s^{H,2}(i^H(2)) > \overline{q}^H - r\,[\overline{q}^H - q^H - (\overline{q}^F - q^F)]$. By contrast, the overshooting condition states that $Q_s^{H,2}(\overline{i}^H(2)) > \overline{q}^H$, which is a stricter requirement as $\overline{i}^H(2) < i^H(2)$ and $\overline{q}^H - q^H > \overline{q}^F - q^F$. We argued above that the overshooting condition is very plausible. The case for the co-movement of prices and volume is thus even stronger.

D A Markov Model for Incomes: Proofs

Proof of Proposition 4.1: A straightforward adaptation of the proof of Proposition C.2. ■

Proof of Proposition 4.2: The difference between the prices agent (i, m) of age 3 is willing to pay for a house after histories $\mu\lambda$ and $\mu\kappa$ is

$$Q_\lambda^{H,3}(m) - Q_\kappa^{H,3}(m) = q_\lambda^F - q_\kappa^F + \frac{\sum_{\nu\in\mathcal{S}} \rho_{\lambda\nu}[q_{\lambda\nu}^H - q_\nu^F] - \sum_{\nu\in\mathcal{S}} \rho_{\kappa\nu}[q_{\kappa\nu}^H - q_\nu^F]}{1+r}. \tag{D.1}$$

As each of the expectations in the numerator lies between $u(\frac{1}{2})/r$ and $u(1 - S^H)/r$, we have

$$q_\lambda^F - q_\kappa^F - \frac{u(\frac{1}{2}) - u(1 - S^H)}{(1+r)\,r} \le Q_\lambda^{H,3}(m) - Q_\kappa^{H,3}(m) \le q_\lambda^F - q_\kappa^F + \frac{u(\frac{1}{2}) - u(1 - S^H)}{(1+r)\,r}. \tag{D.2}$$

History ending in t	$q^F_t - q^F_{t-1}$	$q^H_t - q^H_{t-1}$	$n_t - n_{t-1}$
1111	0	0	0
1112	+	+	+
1121	−	−	−
1122	0	−	−
1211	0	+	+
1212	+	+	+
1221	−	−	?
1222	0	0	+
2111	0	0	−
2112	+	+	?
2121	−	−	−
2122	0	−	−
2211	0	+	+
2212	+	+	+
2221	−	−	−
2222	0	0	0

Table 1: Co-movements of prices and volume in a two-state Markov model

Under condition (26), age 3 agents are willing to pay more for a house in state λ than in state κ. Condition (27) ensures that in the comparison of history $\mu\kappa$ with $\mu\lambda$, any possible upward shift in age 3 agents' willingness to pay for houses is smaller than the upward shift in age 2 agents' ability to pay. The claimed results now follow as outlined in the main text. ■

Proof of Proposition 4.3: As we saw earlier, condition (25) implies overreaction of the house price (hence $q^H_{12} > q^H_{22}$ and $q^H_{11} > q^H_{21}$), while (26) implies monotonicity of the house price in the current income state, in particular if the state was the same in the previous period (hence $q^H_{22} > q^H_{11}$). We also saw that under these conditions, the number of repeat purchases by members of the age 2 cohort is ranked the same way as the house price across all possible income histories of length two. Since a higher number of house buyers at age 2 means a lower number of house owners at age 3, i.e., a higher preference cutoff at age 3, this preference cutoff is also ranked the same way as the house price.

Regarding the simultaneous changes in prices and volume as the income process evolves, Table 1 lists all sixteen histories of length four as well as the sign of the change in the flat price, house price and number of transactions from the previous to the current period. A comparison of the second and third columns shows immediately that the property price index always moves in the same direction as the price of houses. The fourth column then shows that in twelve out of sixteen cases, the price of houses and the number of transactions move in the same direction.

The entries in the fourth column are determined according to the equation

$$n_t - n_{t-1} = m^H_{t-2} - m^H_{t-1} + 2\,(S^H - 1 + m^H_{t-1})\, m^H_t - 2\,(S^H - 1 + m^H_{t-2})\, m^H_{t-1}, \tag{D.3}$$

which is a direct consequence of (18). It is straightforward to confirm the signs in Table 1 for the eight histories starting with state 1; the cases starting with state 2 are just mirror images of these. ■

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Figure 6a: Response to permanent income shock
(Benchmark preference gradient)

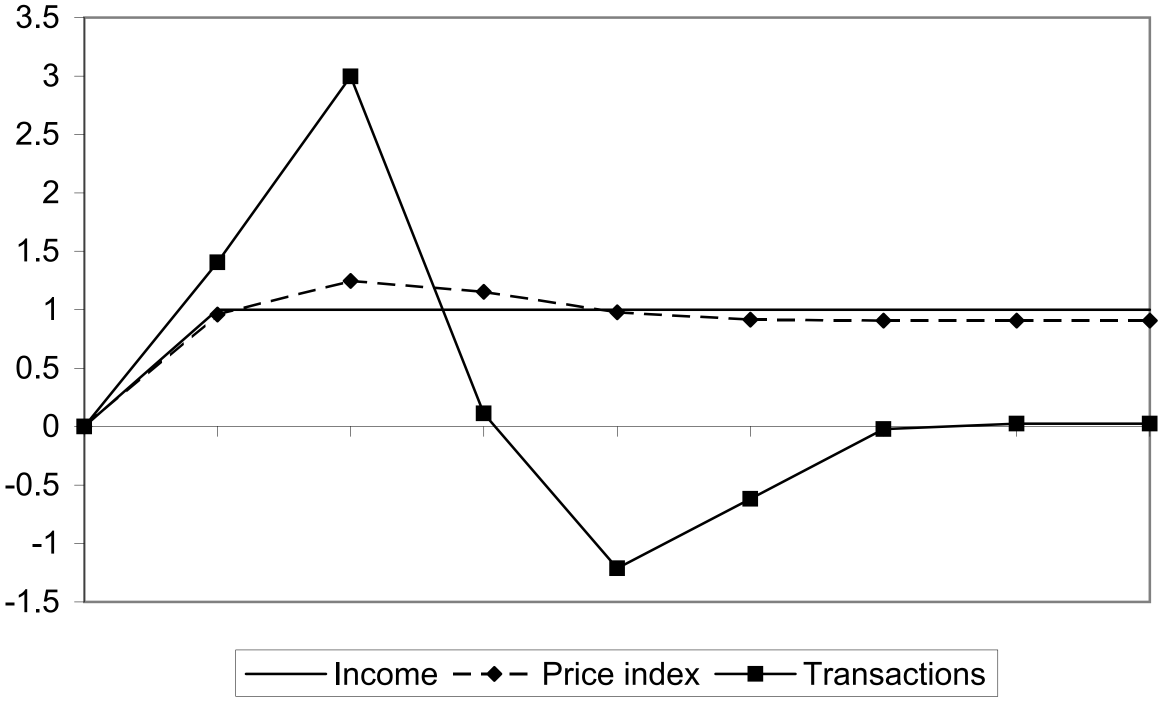

Figure 6b: Response to permanent income shock
(Steeper preference gradient)

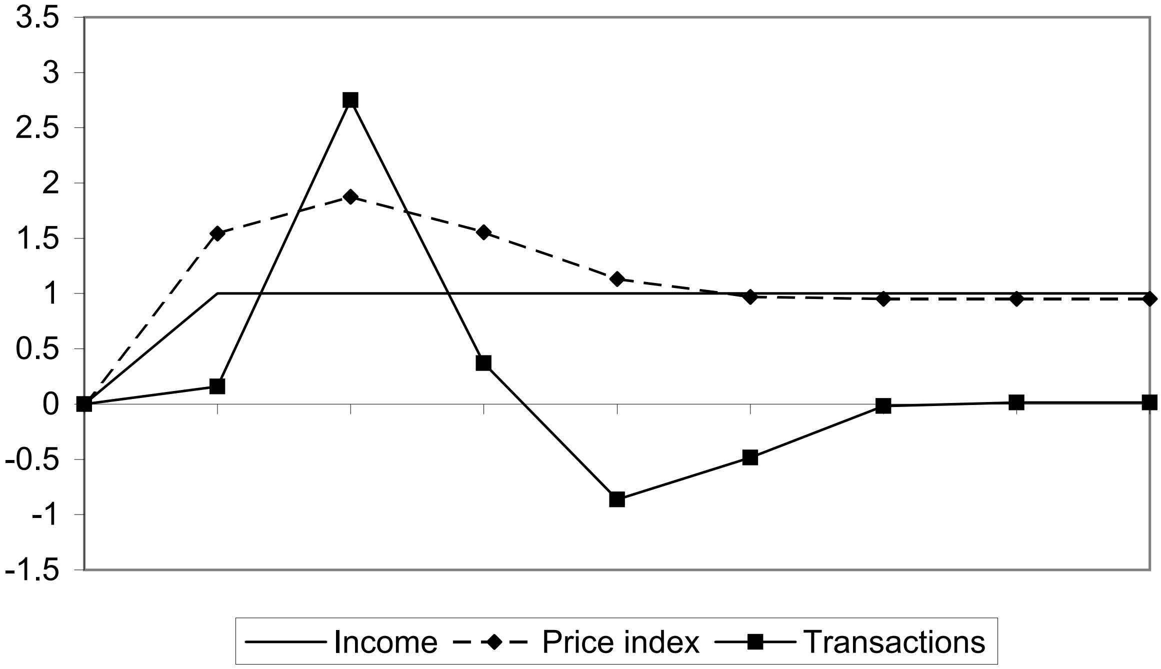

Figure 7a: Markov income process
(Benchmark preference gradient)

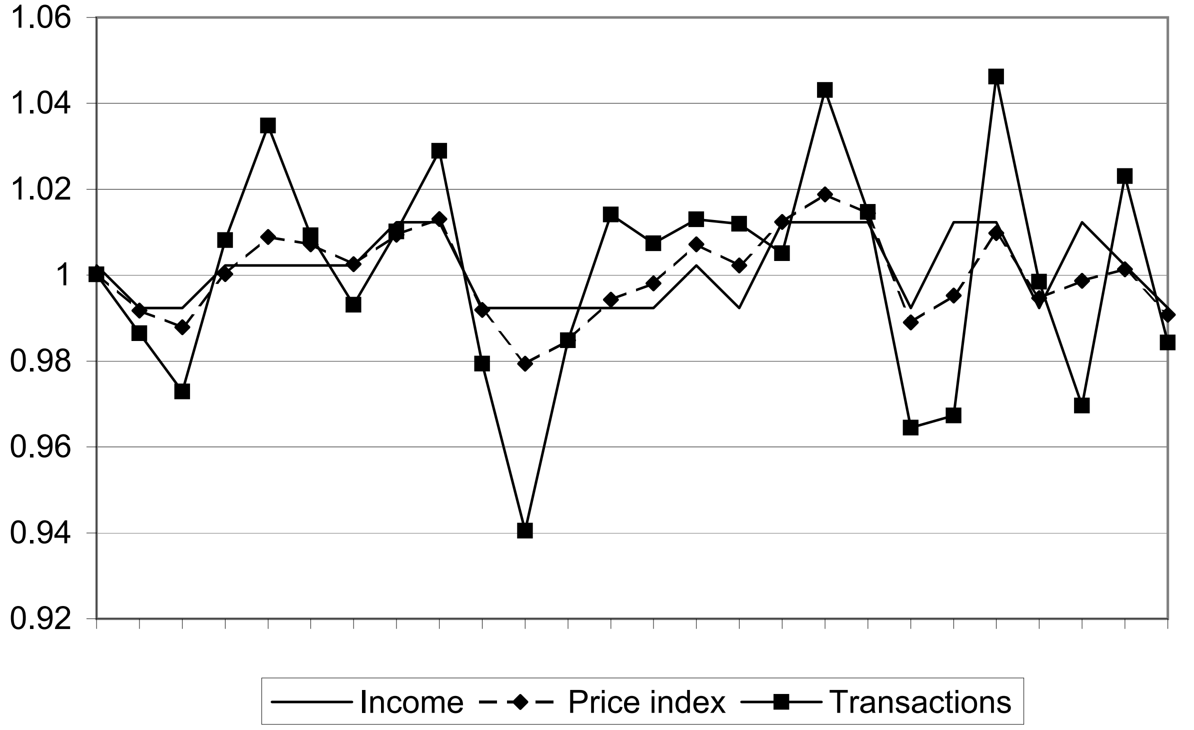

Figure 7b: Markov income process
(Steeper preference gradient)

Figure 8: US income and housing prices

(Scale adjusted and log transformed)