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SIMULATING WITH RICE COALITIONALLY STABLE BURDEN SHARING AGREEMENTS FOR THE CLIMATE CHANGE PROBLEM

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Abstract

In this paper we test empirically with the Nordhaus and Yang (1996) RICE model the core property of the transfer scheme advocated by Germain, Toint and Tulkens (1997). This scheme is designed to sustain full cooperation in a voluntary international environmental agreement by making all countries at least as well off as they would be by joining coalitions adopting emission abatement policies that maximize their coalition payoff; under the scheme no individual country, nor any subset of countries would have an interest in leaving the international environmental agreement. The simulations show that the transfer scheme yields an allocation in the core of the carbon emission abatement game associated with the RICE model. Finally, we discuss some practical implications of the transfer scheme for current climate negotiations.

Keywords: Environmental economics, climate change, burden sharing, simulations, core of cooperative games

JEL Classification: C6, C7, F2, F47

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1 Introduction and summary of results

It is well known that international environmental treaties involving substantially more emission reduction effort than the laissez-faire situation are unlikely to emerge spontaneously without international transfers. The reason is that, although there is a substantial surplus to cooperation, there might exist countries for which the abatement required by the world optimum is so large that they end up individually worse off under the cooperative solution compared to the noncooperative laissez-faire situation. Therefore, the use of international transfers is often advocated to facilitate the formation of international environmental agreements.

This paper investigates how such international transfers might look like for the climate change problem. For this purpose we consider a variant of the integrated economy-climate model RICE of Nordhaus and Yang (1996). In particular we employ the transfer scheme proposed by Chander and Tulkens (1995) and (1997) in a static context and Germain, Toint and Tulkens (1997) in a dynamic framework. The transfer rule redistributes the surplus of cooperation over noncooperation in function of the (marginal) climate change damage costs that countries will experience. In Germain, Toint and Tulkens (1997) it was shown that the transfer scheme will result in an allocation in the core of the emission abatement game provided damage costs can be written as a linear function of the stock pollutant. The core property is a necessary (but not sufficient) condition for full, voluntary cooperation among the countries involved in the transboundary pollution problem, see Tulkens (1998). If it were not satisfied, there might exist coalitions that could obtain a better outcome by coordinating their emission strategies among themselves. Such coalitions would have no incentive to join an international environmental treaty. Since for the climate change problem the linearity assumption is hard to maintain and while no analytical results are available for the nonlinear case, we turn here to an empirical nonlinear model to test the core property of the transfer mechanism proposed in Germain, Toint and Tulkens (1997). This is the main purpose of the paper.

Section 2 introduces an optimal growth model which is a version of the RICE model by Nordhaus and Yang (1996) without trade. Pareto efficient allocations are characterized in section 3. In the case of full cooperation, i.e. if the grand coalition forms, the optimality rule determining emission abatement efforts is given by a dynamic version of the Samuelson (1954) rule for the optimal provision of public goods. Marginal abatement costs are equalized across all countries and are set equal to the sum of all future discounted marginal damages from climate change. Capital accumulation is determined by a generalization of the Keynes-Ramsey rule as in, for instance, equation (6.5) in van der Ploeg and Withagen (1991).The capital accumulation rule completely internalizes all future climate damage costs in all countries of the world.

The case of full cooperation is contrasted with the absence of cooperation in section 4. In an open loop Nash equilibrium marginal abatement costs are no longer equalized across all countries. Every country reduces its emissions as to equalize its marginal abatement costs to the sum of its own future discounted marginal damages from climate change. Positive externalities from abatement policies to other countries are neglected in an open loop Nash equilibrium. Similarly, the Ramsey-Keynes rule for capital accumulation only internalizes a country's private marginal climate change damages.

In section 5, we define the concept of a *partial agreement Nash equilibrium with respect to a coalition* which is the counterpart for dynamic models of the partial agreement Nash equilibrium w.r.t. a coalition defined by Chander and Tulkens (1995) and (1997). The partial agreement Nash equilibrium concept assumes that a coalition of countries chooses emissions and production levels that maximize the coalition's joint payoff for a given emission and production strategy of the outsiders, non-members of the coalition. The outsiders on their turn maximize their individual payoff taking as given the strategies of all other players. Optimality rules driving investment and emission abatement decisions in a partial agreement Nash equilibrium w.r.t. a coalition are derived. They turn out to be a combination of the optimality rules for the Pareto efficient allocations and the standard Nash equilibrium.

Simulations with the RICE model are reported in section 6. We first construct three reference scenario's (business-as-usual, Nash equilibrium and Pareto efficient allocation without transfers) and we compare them in terms of carbon emissions, carbon concentrations, temperature change and emission abatement effort. Since we use a lower discount rate and a higher exponent of the climate change damage functions we obtain higher emission abatement figures, hence smaller temperature changes, than in the original formulation of the RICE model in Nordhaus and Yang (1996).

In section 7 the partial agreement Nash equilibrium w.r.t. a coalition concept is used to define the core of the carbon emission abatement game. The core can be interpreted as the set of allocations that sustain full cooperation and can be implemented by means of a voluntary international agreement. It is well known that without international transfers, the core property is not always met by the Pareto efficient allocations implied by the Samuelson rule. We observe this phenomenon in the simulations for *Former Soviet Union*, *China* and, to a lesser extent, for the *USA*. Without additional compensation it is unlikely that these regions would want to join an international climate treaty that implements a cost efficient climate policy.

However, this participation problem can be overcome by using the Germain, Toint and Tulkens (1997) international transfer scheme. Starting from the Pareto efficient allocation without transfers, this transfer rule give rise to an allocation in the core of the carbon emission abatement game associated with the RICE model. The compensation rule consists of a once-and-for-all transfer which distributes the surplus of cooperation over noncooperation in function of the marginal damages that countries experience. For the RICE model, we consider a dynamic extension of the transfer scheme proposed by Germain, Toint and Tulkens (1997). Since damage functions are not linear, it is not obvious that the resulting allocation is in the core of the emission abatement game. Therefore, we checked the core property by computing all partial agreement Nash equilibria w.r.t. any coalition and compared the coalition's payoff to its joint allocation of consumption under the transfer schemes. It turned out that all coalitions are better off under the transfer scheme than under the Nash equilibrium.

Finally, we make some observations on the practical implementation of the transfers. We observe a major problem in the timing of the transfers since the bulk of the surplus of cooperation only comes far in the future. In the early periods up to 2100 the surplus is in fact negative. Only after 2100 the gain in terms of avoided climate change damage becomes large enough to justify a cooperative climate policy. This implies that the transfers cannot be paid out at the beginning of the game since countries cannot borrow against future benefits. This calls for spreading out transfers over time.

2 The RICE model without trade

We consider a simple optimal growth model without international trade. Growth is driven by exogenous population growth, technological change and endogenous capital accumulation. Anthropogenic emissions of carbon dioxide are assumed proportional to output. In this respect, the model is very similar to the RICE model by Nordhaus and Yang (1996) without international trade. N denotes the set of countries/regions[1] indexed $i = 1, 2, \ldots, n$. A complete list of variables and parameters is given in appendix. We assume that there exists a world social planner who maximizes the weighted sum of the countries' discounted utilities. We use a discrete time model with a finite horizon.

$$\sum_{t=0}^{T} \sum_{i \in N} \lambda_i \frac{U_i(Z_{i,t})}{[1+\rho]^t} \; + \; w_i(K_{i,T+1}) \tag{1}$$

where λ_i denotes the welfare weight of region i and ρ stands for the discount rate. In each regions, utility is assumed to be a strictly increasing and strictly concave function of consumption. The strictly increasing and strictly concave function w_i stands for the scrap value of the terminal capital stock. The following equations describe the *economy* of a country i at time t:

$$Y_{i,t} \; = \; Z_{i,t} \; + \; I_{i,t} \; + \; C_i(\mu_{i,t}) \; + \; D_i(\Delta T_t) \tag{2}$$

$$Y_{i,t} \; = \; A_{i,t} \, F_i(K_{i,t}, L_{i,t}) \tag{3}$$

$$K_{i,t+1} \; = \; [1 \; - \; \delta_K] \, K_{i,t} \; + \; I_{i,t} \qquad ; \quad K_{i,0} \text{ given} \tag{4}$$

[1]In the sequel we will indifferently speak of regions or countries.

Equation (2) defines the claims of consumption, investment, cost of abatement and climate change damage upon production[2]. The costs of abatement and of climate change damage are assumed strictly increasing and strictly convex in abatement and temperature change respectively. (3) defines production as a strictly increasing and strictly concave function of capital and labour input. $A_{i,t}$ measures overall productivity. It is assumed that productivity increases exogenously as time goes by. Since labour supply is assumed exogenous we will omit this argument in the production function in the sequel. Labour input is subsumed in the functional form of the productivity measure $A_{i,t}$. Finally, expression (4) is a standard capital accumulation equation where δ_K stands for the rate of capital depreciation.

The *carbon emissions, the carbon cycle and climate module* are respectively modelled by the following three equations:

$$E_{i,t} = \sigma_{i,t}\,[1 - \mu_{i,t}]\,Y_{i,t} \tag{5}$$

$$M_{t+1} = [1 - \delta_M]\,M_t + \beta \sum_{i \in N} E_{i,t} \qquad \text{with } M_0 \text{ given} \tag{6}$$

$$\Delta T_t = G(M_t) \tag{7}$$

According to expression (5), carbon emissions are proportional to production. The emissions to output ratio $\sigma_{i,t}$ declines exogenously over time due to an assumed autonomous energy efficiency increase (AEEI). Emissions can be reduced at a rate $\mu_{i,t} \in [0,1]$ in every period though this is costly according to equation (2). Equation (6) describes the accumulation of carbon in the atmosphere. This process is modelled similarly to a standard capital accumulation process where δ_M denotes the natural decay rate of atmospheric carbon concentrations and β is the airborne fraction of carbon emissions. Expression (7) translates atmospheric carbon concentration levels into global mean temperature change. We assume that G is a continuous differentiable and increasing function. The function G can also stand for a more complex relationship between atmospheric carbon concentration and temperature change as is the case in the RICE model.

[2]Our formulation is slightly different from the one used by Nordhaus and Yang (1996). Translated into our notation, their formulation of the budget equation (2) would be given by:

$$\Omega_{i,t}\,Y_{i,t} \equiv \frac{1 - C_{i,t}/Y_{i,t}}{1 + D_{i,t}/Y_{i,t}}\,Y_{i,t} = Z_{i,t} + I_{i,t}$$

Basically, both formulations are identical in the sense that the costs of emission abatement and of damage from climate change reduce the amount of production that can be devoted to consumption or investment.

3 Pareto efficient allocations of consumption, production and emissions

3.1 World welfare maximization

Since the utility functions U_i are assumed strictly concave, the set of all Pareto efficient allocations can be described completely by maximizing a weighted sum of the members' utilities[3] with $(\lambda_1, \ldots, \lambda_n) = \lambda \in \Delta^{n-1}$:

$$\max_{Z_{i,t},\, I_{i,t},\, K_{i,t},\, \mu_{i,t},\, M_t} \quad \sum_{t=0}^{T} \sum_{j \in N} \frac{\lambda_j\, U_j(Z_{j,t})}{[1+\rho]^t} \;+\; w_j(K_{j,T+1}) \tag{8}$$

subject to (for all $0 \leq t \leq T$):

$$\sum_{j \in N} A_{j,t}\, F_j(K_{j,t}) \geq \sum_{j \in N} [Z_{j,t} + I_{j,t} + C_j(\mu_{j,t}) + D_j(G(M_t))] \qquad [\zeta_t]$$

$$K_{i,t+1} = [1 \;-\; \delta_K]\, K_{i,t} \;+\; I_{i,t} \qquad ; \quad K_{i,0}\text{given} \qquad [\psi_{i,t}]$$

$$M_{t+1} = [1 \;-\; \delta_M]\, M_t \;+\; \beta \sum_{j \in N} \sigma_{j,t}\, [1 \;-\; \mu_{j,t}]\, A_{j,t}\, F_j(K_{j,t}) \qquad [\phi_t]$$

The resource constraint says that overall production should be sufficient to cover overall expenses on consumption, investment, abatement costs and climate change damages. We associate Lagrange multipliers ζ_t to the world resource constraint, $\psi_{i,t}$ to the individual capital accumulation constraints and ϕ_t to the carbon accumulation process. First-order conditions for all $i \in N$ and $0 \leq t \leq T$ for an interior optimum for a given vector of welfare weights $\lambda \in \Delta^{n-1}$ are given by (the asterisk superscript refers to the values of the variables at the Pareto efficient solution):

$$\zeta_t^* \;=\; \frac{\lambda_i\, U_i'(Z_{i,t}^*)}{[1+\rho]^t} \tag{9}$$

$$\zeta_t^* \;=\; \psi_{i,t}^* \tag{10}$$

$$\begin{aligned} \psi_{i,t-1}^* \;=\; & \psi_{i,t}^* \left[A_{i,t}\, F_i'(K_{i,t}^*) \;+\; [1-\delta_K]\right] \\ & -\beta\, \sigma_{i,t}\, [1-\mu_{i,t}^*]\, A_{i,t}\, F_i'(K_{i,t}^*)\, \phi_t^* \end{aligned} \tag{11}$$

[3] $\Delta^{n-1} = \{\lambda \in \mathbb{R}_{++}^n \mid \sum_{j=1}^n \lambda_j = 1\}$ Nordhaus and Yang (1996) use so-called Negishi weights (see Negishi (1960)) induced by a competitive equilibrium in world trade. In our present model without trade, the weights λ_j need not be that specific.

$$\psi^*_{i,T} \;=\; w'_i(K^*_{i,T+1}) \tag{12}$$

$$\psi^*_{i,t}\, C'_i(\mu^*_{i,t}) \;=\; \beta\, \sigma_{i,t}\, A_{i,t}\, F_i(K^*_{i,t})\, \phi^*_t \tag{13}$$

$$\phi^*_{t-1} \;=\; G'(M^*_t) \sum_{j\in N} \psi^*_{j,t}\, D'_j(G(M^*_t)) \;+\; [1 \;-\; \delta_M]\, \phi^*_t, \quad \phi^*_T = 0 \tag{14}$$

$$\sum_{j\in N} A_{j,t}\, F_j(K^*_{j,t}) \;=\; \sum_{j\in N} \left[Z^*_{j,t} + I^*_{j,t} + C_j(\mu^*_{j,t}) + D_j(G(M^*_t))\right] \tag{15}$$

$$K^*_{i,t+1} \;=\; [1 \;-\; \delta_K]\, K^*_{i,t} \;+\; I^*_{i,t} \tag{16}$$

$$M^*_{t+1} \;=\; [1 \;-\; \delta_M]\, M^*_t \;+\; \beta \sum_{j\in N} \sigma_{j,t}\, [1 - \mu^*_{j,t}]\, A_{j,t}\, F_j(K^*_{j,t}) \tag{17}$$

Condition (9) says that for every country and in every period, the Lagrange multiplier of the resource constraint equals its discounted marginal utility of consumption times its welfare weight. The right hand side (RHS) of equation (9) is a measure for the marginal social valuation of consumption for country i in period t. Notice that these marginal social valuations are equalized across countries because the formulation of the resource constraint in (2) implies that the social planner can make use of a lump sum redistribution instrument to reallocate consumption. According to (10), the shadow cost of the resource constraint equals the shadow cost of capital. Since marginal social valuations of consumption are equalized, also the shadow cost of capital is equalized across all countries in every period. The evolution of the capital stock is described by condition (11). Last period's shadow price of capital equals the marginal valuation of the terminal capital stock according to (12). (13) determines the optimal amount of carbon emission control for country i. In a Pareto efficient allocation marginal abatement costs are a function of the shadow cost of atmospheric carbon concentrations. Expression (14) describes the evolution of the shadow price of atmospheric carbon concentration. In the last period, this shadow price is zero because there is no valuation of the terminal carbon concentration. Conditions (15), (16) and (17) repeat the resource constraint, the capital and carbon accumulation relationships.

3.2 Interpreting the first-order conditions

In this section we rewrite the first-order conditions in order to eliminate the Lagrange multipliers. We start by solving the difference equation (14) for the shadow price of carbon emissions. From the terminal condition $\phi^*_T = 0$, it follows from (14) through iterative substitution that the carbon tax at any period t is equal to the weighted sum of all future discounted marginal damages experienced by all regions in the world:

$$\phi_t^* \;=\; \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1}\, G'(M_\tau^*)\, \zeta_\tau^* \sum_{j\in N} D_j'(G(M_\tau^*)) \tag{18}$$

Notice that the optimal carbon tax takes into account the climate change damage affecting all regions in the world. Hence, the climate externality is internalized completely. Substituting for the carbon tax in (13), we can derive the rule driving the optimal amount of carbon emission control for country i in period t:

$$\frac{C_i'(\mu_{i,t}^*)}{\sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^*)} \;=\; \frac{\beta}{\zeta_t^*}\,\phi_t^* \;=\; \frac{\beta}{\zeta_t^*} \sum_{\tau=t+1}^{T} G'(M_\tau^*)\,[1-\delta_M]^{\tau-t-1}\,\zeta_\tau^* \sum_{j\in N} D_j'(G(M_\tau^*)) \tag{19}$$

This rule will be referred to in the sequel as the *Samuelson rule for the optimal emission reductions* in all countries and time periods. It is a dynamic extension of the traditional optimality rule for static public good models that was first stated by Samuelson (1954). The left hand side (LHS) of the expression stands for the marginal cost for region i of reducing its carbon emissions by an additional ton in period t. The denominator denotes gross emissions without abatement and is used to convert the units of the marginal abatement costs into US$ per ton of carbon[4]. The RHS of the expression consists of the sum of all regions' discounted future marginal damages from climate change, divided by the marginal social valuation of consumption. It is a measure of the additional climate change damage costs for all countries in the world if country i were to emit an extra ton of carbon at period t. Only the fraction of the emissions that become actually airborne is taken into account (multiplication by β). The Samuelson rule (19) thus says that all regions should reduce their emissions in such a way that their marginal abatement costs in each period t be equalized. Hence, the Samuelson rule induces both cost efficiency and allocative efficiency. Condition (19) is very similar to the optimality condition (3.8) in Kverndokk (1993).

We now derive the condition for the optimal accumulation of capital in the presence of an environmental externality. Substituting (9) into (11), we obtain:

$$[1+\rho]\,\frac{u_i'(Z_{i,t-1}^*)}{u_i'(Z_{i,t}^*)} \;=\; A_{i,t}\,F_i'(K_{i,t}^*)\left[1 \;-\; \beta\,\sigma_{i,t}\,[1-\mu_{i,t}^*]\,\frac{\phi_t^*}{\psi_{i,t}^*}\right] \;+\; 1 \;-\; \delta_K \tag{20}$$

$$\;=\; A_{i,t}\,F_i'(K_{i,t}^*)\left[1 \;-\; [1-\mu_{i,t}^*]\,\frac{C_i'(\mu_{i,t}^*)}{A_{i,t}\,F_i(K_{i,t}^*)}\right] \;+\; 1 \;-\; \delta_K \tag{21}$$

The latter condition was derived by substituting for the carbon tax from expression (19). It says that the marginal utility loss of an additional unit of investment at time $t-1$ should equal next period's marginal utility of the marginal product that can be produced with the additional investment. In the sequel, this investment rule will be referred to as the

[4]Recall that $\mu_{i,t} \in [0,1]$ has no dimension since it is the fraction of emissions that are abated.

Ramsey-Keynes optimal investment rule. Notice that the Ramsey-Keynes rule internalizes all climate change spill over effects affecting all regions in the world. Condition (20) can be interpreted as a generalization of the modified Keynes-Ramsey rule (6.5) in van der Ploeg and Withagen (1991).

4 Nash equilibrium

4.1 Domestic welfare maximization

We now describe what would happen if the countries do not succeed in signing a voluntary international environmental agreement. In order to characterize such a situation we make use of a particular form of the noncooperative Nash equilibrium in a dynamic setting, namely the open loop Nash equilibrium. An open loop Nash equilibrium (Nash equilibrium hereafter) consists of a combination of strategies, one for each player, that maximizes every country i's utility given the strategies of all other players $j \neq i$. In such an equilibrium, no individual country has an incentive to deviate given that the other countries stick to their equilibrium strategies. In the RICE model, a Nash equilibrium can be characterized by maximizing every region's utility subject to the individual resource and capital constraint and the climate module for a given emission strategy $\bar{E}_{i,t}$ of all other players $j \neq i$ and $\forall t$:

$$\max_{Z_{i,t},\, I_{i,t},\, K_{i,t},\, \mu_{i,t},\, M_t} \quad \sum_{t=0}^{T} \frac{U_i(Z_{i,t})}{[1+\rho]^t} \;+\; w_i(K_{i,T+1}) \tag{22}$$

subject to (for all $0 \leq t \leq T$):

$$A_{i,t}\, F_i(K_{i,t}) \geq Z_{i,t} \;+\; I_{i,t} \;+\; C_i(\mu_{i,t}) \;+\; D_i(G(M_t)) \qquad [\zeta_{i,t}]$$

$$K_{i,t+1} = [1 \;-\; \delta_K]\, K_{i,t} \;+\; I_{i,t} \qquad [\psi_{i,t}]$$

$$M_{t+1} = [1 \;-\; \delta_M]\, M_t \;+\; \beta\, \sigma_i\, [1 \;-\; \mu_{i,t}]\, A_{i,t}\, F_i(K_{i,t}) \;+\; \beta \sum_{j \neq i} \bar{E}_{j,t} \qquad [\phi_{i,t}]$$

We associate Lagrange multipliers $\zeta_{i,t}$ to the resource constraint, $\psi_{i,t}$ to the capital accumulation constraint and $\phi_{i,t}$ to the carbon accumulation process. First-order conditions for all $0 \leq t \leq T$ for an interior optimum are given by (the superscript "NE" refers to the equilibrium values of the variables for the Nash equilibrium):

$$\zeta_{i,t}^{NE} = \frac{U_i'(Z_{i,t}^{NE})}{[1+\rho]^t} \tag{23}$$

$$\psi_{i,t}^{NE} = \zeta_{i,t}^{NE} \tag{24}$$

$$\begin{aligned} \psi_{i,t-1}^{NE} = \; & \psi_{i,t}^{NE} \left[A_{i,t}\, F_i'(K_{i,t}^{NE}) \;+\; [1-\delta_K]\right] \\ & - \beta\, \sigma_{i,t}\, [1-\mu_{i,t}^{NE}]\, A_{i,t}\, F_i'(K_{i,t}^{NE})\, \phi_{i,t}^{NE} \end{aligned} \tag{25}$$

$$\psi_{i,T}^{NE} = w_i'(K_{i,T+1}^{NE}) \tag{26}$$

$$\psi_{i,t}^{NE}\, C_i'(\mu_{i,t}^{NE}) = \beta\, \sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^{NE})\, \phi_{i,t}^{NE} \tag{27}$$

$$\phi_{i,t-1}^{NE} = G'(M_t^{NE})\, \psi_{i,t}^{NE}\, D_i'(G(M_t^{NE})) \;+\; [1 \;-\; \delta_M]\, \phi_{i,t}^{NE} \qquad \phi_{i,T}^{NE} = 0 \tag{28}$$

$$A_{i,t}\, F_i(K_{i,t}^{NE}) = Z_{i,t}^{NE} \;+\; I_{i,t}^{NE} \;+\; C_i(\mu_{i,t}^{NE}) \;+\; D_i(G(M_t^{NE})) \tag{29}$$

$$K_{i,t+1}^{NE} = [1 \;-\; \delta_K]\, K_{i,t}^{NE} \;+\; I_{i,t}^{NE} \tag{30}$$

$$M_{t+1}^{NE} = [1 \;-\; \delta_M]\, M_t^{NE} \;+\; \beta \sum_{j\in N} \sigma_i\, [1-\mu_{i,t}^{NE}]\, A_{i,t}\, F_i(K_{i,t}^{NE}) \tag{31}$$

A Nash equilibrium is a simultaneous solution to this system of first-order conditions for all $i \in N$ and $0 \leq t \leq T$. The first two conditions (23) and (24) say that the shadow cost of capital equals the shadow cost of the resource constraint and that both are equal to the marginal, discounted utility of consumption. The evolution of the capital stock is described by conditions (25) and (26). (27) determines the optimal amount of carbon emission control for country i. Expression (28) describes the evolution of the shadow price of atmospheric carbon concentration. In the sequel, the shadow price to country i of carbon accumulation in the atmosphere will often be referred to as the *carbon tax for country i*. Condition (29) restates the budget equation, (30) and (31) repeat the capital and carbon accumulation relationships.

4.2 Interpreting the first-order conditions

We start again by solving the difference equation (28). From the terminal condition $\phi_{i,T}^{NE} = 0$ and by solving iteratively from (28), it can be shown that the carbon tax for an outsider at any period t is equal to the sum of future marginal damage caused by an additional unit of carbon emissions at time t, evaluated at the appropriate marginal utility of consumption:

$$\phi_{i,t}^{NE} \;=\; \sum_{\tau=t+1}^{T} \psi_{i,\tau}^{NE}\,[1-\delta_M]^{\tau-t-1}\,G'(M_\tau^{NE})\,D_i'(G(M_\tau^{NE})) \tag{32}$$

Notice that the carbon tax for country i in a Nash equilibrium only takes into account the climate change damage occurring within country i. In contrast to the Pareto efficient tax in (18), spill over effects to neighbouring countries are not taken into account in country i's individual decision process. Substituting for the carbon tax in (27), we can derive the rule driving the optimal amount of carbon emission control. In particular, in a Nash equilibrium, every country equalizes its marginal costs of abatement (per ton of carbon) to the marginal damage from the resulting climate change (all quantities are evaluated at the appropriate marginal utility of consumption):

$$\frac{C_i'(\mu_{i,t}^{NE})}{\sigma_{i,t}\,A_{i,t}\,F_i(K_{i,t}^{NE})} \;=\; \beta\,\frac{\phi_{i,t}^{NE}}{\psi_{i,t}^{NE}} \;=\; \frac{\beta}{\psi_{i,t}^{NE}}\sum_{\tau=t+1}^{T} \psi_{i,\tau}^{NE}\,[1-\delta_M]^{\tau-t-1}\,G'(M_\tau^{NE})\,D_i'(G(M_\tau^{NE})) \tag{33}$$

This is the traditional optimality condition for a noncooperative Nash equilibrium saying that marginal abatement costs should be equal to individual marginal damage of climate change. Notice that if preferences are quasi-linear $U_i'(Z_{i,t}^{NE}) = 1$, the latter optimality condition can be simplified to:

$$\frac{C_i'(\mu_{i,t}^{NE})}{\sigma_{i,t}\,A_{i,t}\,F_i(K_{i,t}^{NE})} \;=\; \beta\,\phi_{i,t}^{NE} \;=\; \beta\,[1+\rho]\sum_{\tau=t+1}^{T}\left[\frac{1-\delta_M}{1+\rho}\right]^{\tau-t-1} G'(M_\tau^{NE})\,D_i'(G(M_\tau^{NE}))$$

This condition is equivalent to condition (7) in Germain, Toint and Tulkens (1997) for $G' = 1$. The term $\left[\frac{1-\delta_M}{1+\rho}\right]^{\tau-t-1}$ is a deflation effect for the valuation of marginal damage in period t. Because of the discount rate and natural decay rate of carbon concentrations in the atmosphere, the effect of emitting one ton of carbon at time t gradually dies off.

We now turn to the Ramsey-Keynes condition that drives capital accumulation for country i. Substituting (23) into (25), the latter condition can be written as follows:

$$[1+\rho]\,\frac{u_i'(Z_{i,t-1}^{NE})}{u_i'(Z_{i,t}^{NE})} \;=\; A_{i,t}\,F_i'(K_{i,t}^{NE})\left[1 \;-\; [1-\mu_{i,t}^{NE}]\,\frac{C_i'(\mu_{i,t}^{NE})}{A_{i,t}\,F_i(K_{i,t}^{NE})}\right] \;+\; 1 \;-\; \delta_K \tag{34}$$

The latter condition was derived by substituting for the carbon tax from expression (32) and says that the marginal utility loss of an additional unit of investment at time $t-1$ should equal next period's marginal utility of the marginal product that can be produced with the additional investment. Compared to the case of Pareto efficient allocations, the Ramsey-Keynes rule for the Nash equilibrium does only internalize climate change damage occurring domestically. Again, negative climate change externalities to neighbouring countries are not taken into account.

5 Partial agreement Nash equilibrium w.r.t. a coalition

5.1 Definition

The previous two sections described two extreme cases of cooperation. In section 3, all countries take action to reduce their emissions of carbon dioxide and they do so by internalizing completely the external effects of their carbon emissions. In section 4, every country reduces its carbon emissions also but to a lesser extent because they only internalize the external effects of their emissions that affect their own territory. Spill over effects to neighbouring countries are not taken into account in the private decision making process. However, in reality, we often observe partial or intermediate cooperation in international environmental agreements. Hence, only some subgroup of countries affected by the problem agrees to coordinate its emission reduction policies. The 1997 Kyoto protocol is a prominent example of partial cooperation.

In order to characterize this situation of partial cooperation, we use the concept of partial agreement Nash equilibrium *w.r.t. a coalition* (PANE). This equilibrium concept was introduced by Chander and Tulkens (1995) and (1997). Suppose a coalition $S \subseteq N$ forms. In a partial agreement Nash equilibrium w.r.t. coalition S, this coalition chooses actions that are most beneficial from the group point of view while the outsiders to the coalition choose actions that maximize their individual utility. The PANE w.r.t. coalition S can be interpreted as a special type of Nash equilibrium in which a coalition S coordinates its policies taking as given the emission strategies of the outsiders who, on their turn, are playing a noncooperative Nash strategy against S. If one reinterprets the game such that the coalition of cooperating countries stands for only one player, the partial agreement Nash equilibrium w.r.t. a coalition in the original game is equivalent to an ordinary Nash equilibrium in the new game. Formally, an partial agreement Nash equilibrium w.r.t. coalition S is a combination of strategies that solves simultaneously the following maximization problems:

for all *insiders* $j \in S$ with $\lambda \in \Lambda^{s-1}$:

$$\max_{Z_{j,t},\, I_{j,t},\, K_{j,t},\, \mu_{j,t},\, M_t} \quad \sum_{t=0}^{T} \sum_{j \in S} \frac{\lambda_j\, U_j(Z_{j,t})}{[1+\rho]^t} \tag{35}$$

subject to $\sum_{j \in S} A_{j,t}\, F_j(K_{j,t}) \geq \sum_{j \in S} [Z_{j,t} + I_{j,t} + C_j(\mu_{j,t}) + D_j(G(M_t))]$

and for all *outsiders* $i \in N \setminus S$:

$$\max_{Z_{i,t},\, I_{i,t},\, K_{i,t},\, \mu_{i,t},\, M_t} \quad \sum_{t=0}^{T} \frac{U_i(Z_{i,t})}{[1+\rho]^t} \tag{36}$$

subject to $A_{i,t}\, F_i(K_{i,t}) \geq Z_{i,t} + I_{i,t} + C_i(\mu_{i,t}) + D_i(G(M_t))$. At the same time, the capital accumulation conditions (4), definition of carbon emissions (5), carbon concentration (6) and temperature change (7) equations are to be fulfilled. It is clear that this definition encompasses both the definition of Pareto efficient allocations (for $S = N$) and the definition of a Nash Equilibrium (for $S = \{i\}$) in sections 3 and 4 respectively.

5.2 First-order conditions for a partial agreement Nash equilibrium w.r.t. a coalition S

5.2.1 Insiders

First-order conditions for a partial agreement Nash equilibrium w.r.t. a coalition S can be derived in the same way as before. The insiders internalize the negative externalities from climate change among the members of the coalition only as becomes clear from the fist-order conditions for emission control rates and investment $\forall i \in S$ and $\forall 0 \leq t \leq T$:

$$\frac{C_i'(\mu_{i,t}^S)}{\sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^S)} = \frac{\beta}{\psi_t^S} \sum_{\tau=t+1}^{T} [1-\delta_M]^{\tau-t-1}\, G'(M_\tau^S)\, \psi_\tau^S \sum_{j \in S} D_j'(G(M_\tau^S)) \tag{37}$$

$$[1+\rho]\, \frac{u_i'(Z_{i,t-1}^S)}{u_i'(Z_{i,t}^S)} = A_{i,t}\, F_i'(K_{i,t}^S) \left[1 - [1-\mu_{i,t}^S]\, \frac{C_i'(\mu_{i,t}^S)}{A_{i,t}\, F_i(K_{i,t}^S)}\right] + 1 - \delta_K \tag{38}$$

5.2.2 Outsiders

Outsiders only take into account domestic climate change damages. The corresponding first-order conditions for emission control rates and investment $\forall i \in N \setminus S$ and $\forall 0 \leq t \leq T$ are given by:

$$\frac{C_i'(\mu_{i,t}^S)}{\sigma_{i,t}\, A_{i,t}\, F_i(K_{i,t}^S)} = \frac{\beta}{\psi_{i,t}^S} \sum_{\tau=t+1}^{T} \psi_{i,\tau}^S\, [1-\delta_M]^{\tau-t-1}\, G'(M_\tau^S)\, D_i'(G(M_\tau^S)) \tag{39}$$

$$[1+\rho]\, \frac{u_i'(Z_{i,t-1}^S)}{u_i'(Z_{i,t}^S)} = A_{i,t}\, F_i'(K_{i,t}^S) \left[1 - [1-\mu_{i,t}^S]\, \frac{C_i'(\mu_{i,t}^S)}{A_{i,t}\, F_i(K_{i,t}^S)}\right] + 1 - \delta_K \tag{40}$$

The outsiders only take into account domestic climate change damage. The first-order conditions characterizing a partial agreement Nash equilibrium w.r.t. a coalition $S \subseteq N$ can be interpreted as a mixture of the first-order conditions for a Pareto efficient allocation and a Nash equilibrium. For $S = N$ the insiders' conditions reduce to (19) and (21) respectively. For $S = \{i\}$ for any $i \in N$, the outsiders' conditions coincide with (33) and (34) respectively.

6 Simulations with RICE

6.1 Reinterpreting the RICE model and solving for a partial agreement Nash equilibrium w.r.t. a coalition

In this section we will use the integrated economy-climate RICE model by Nordhaus and Yang (1996) to illustrate the differences between a business-as-usual scenario, the Pareto efficient allocation without transfers and the Nash Equilibrium. The equations and parameters of the model and all basic data on GDP, population, capital stock, carbon emissions and concentration and global mean temperature were taken from the RICE model. A complete list of the equations in the simulation model is provided in the appendix. The RICE simulation model distinguishes between 6 regions in the world: *USA, Japan, European Union, China, Former Soviet Union* and *Rest Of the World.*

The difference with Nordhaus' and Yang's (1996) model is twofold. First, we do not allow for trade in consumption or carbon emissions. Second, we use different assumptions concerning the discount rate (1.5% instead of 3%), we doubled the exponent of the climate change damage function (3 instead of 1.5) and we use a linear representation of utility instead of a logarithmic. In our opinion, these assumptions are more suited to study the long term consequences of climate change.

In order to calculate partial agreement Nash equilibria w.r.t. a given coalition $S \subseteq N$, we adopted a standard numerical algorithm to calculate Nash equilibria. The coalition S is treated as one player in the emission game and in every iteration, each player chooses a strategy vector denoted by $s_i = \{(Z_{i,t}, I_{i,t}, K_{i,t}, \mu_{i,t}, M_t)_{t=0,1,2,\ldots,T}\}$ that maximizes its life time utility given the strategies of the other players. This iteration process is continued until the distance between the strategy vectors in two consecutive iterations is smaller than a given threshold value. Notice that the algorithm assumes perfect foresight on behalf of the players and hence, the resulting equilibrium is an open loop equilibrium. The algorithm is equivalent to algorithm in Yang (1998) to calculate numerically so-called "hybrid" coalition solutions. The procedure is described below:

- STEP A:
  specify some initial strategies profile $s^0 = (s_1^0, s_2^0, \ldots, s_n^0)$ and assign $s = s^0$
- STEP B:

- *step 1*: seek s'_1 that maximizes utility of agent 1 while keeping constant $s_j, \ j \neq 1$
- *step 2:* seek s'_2 that maximizes utility of agent 2 while keeping constant s'_1 and $s_j, \ j > 2$
- ...
- *step n:* seek s'_n that maximizes utility of agent n while keeping constant $s'_j, \ j < n$

- STEP C:
  If $\|s' - s\| \geq \epsilon$, set $s = s'$ and return to STEP B, otherwise stop.

The algorithm was implemented using GAMS. For the simplified RICE model without trade with 20 periods, solving for an partial agreement Nash equilibrium takes about 5 minutes on a Pentium Pro, 300mhz PC[5].

6.2 Reference solutions

6.2.1 Carbon emissions

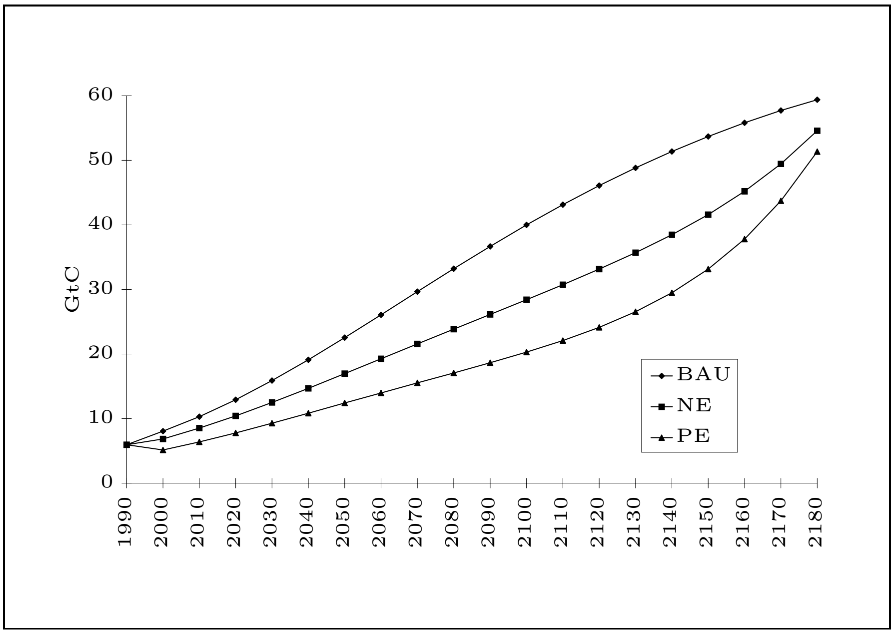
Figure 1 shows the carbon emissions in business-as-usual (BAU), in the Nash equilibrium (NE) and in the Pareto efficient (PE) allocation without transfers. We only consider carbon emissions originating from fossil fuel use. World carbon emissions in 1990 amount to approximately 6 gigatons of C. In the BAU scenario, we assume that countries do not abate their carbon emissions, i.e. $\mu_{i,t} = 0$ for all i and t. BAU emissions grow continuously to reach more than 40 gigatons of C by the year 2100. In the Nash equilibrium, emissions grow at a slower rate to reach about 28 gigatons of C by the year 2100. Pareto efficient carbon emissions in 2000 are lower than 1990 emission but start rising afterwards. By the year 2100 Pareto efficient emissions amount to some 20 gigatons of C. This is about half the BAU emission level.

6.2.2 Atmospheric carbon concentrations

Figure 2 shows the atmospheric carbon concentration in the BAU, the Nash equilibrium and the Pareto efficient scenarios. 1990 atmospheric carbon concentration amounted approximately 750 gigatons of C. Under the BAU, the atmospheric carbon concentration rises steadily and reaches about 1728 gigatons of C in 2100. Doubling of the concentration w.r.t. 1990 takes place between 2080 and 2090. In the Nash equilibrium and Pareto efficient scenario, atmospheric carbon concentrations grow at a slower rate and reach 1454 and 1238 gigatons of C respectively in 2100. Doubling of the concentration w.r.t. 1990 is postponed until after 2100. Notice that even in the Pareto efficient allocation, the carbon concentration does not level off by the year 2100.

[5] All data and programs are available from the authors upon request.

Figure 1: Carbon emissions



6.2.3 Temperature changes

Figure 3 shows the temperature increase compared to preindustrial times for the three reference scenario's. By the year 2100 temperature rises with 2.79, 2.39 and 1.99°Celsius in BAU, in the Nash equilibrium and in the Pareto efficient allocation respectively. The difference between the BAU and Pareto efficient scenario's amounts to approximately 0.80°C in 2100.

Figure 2: Atmospheric carbon concentration

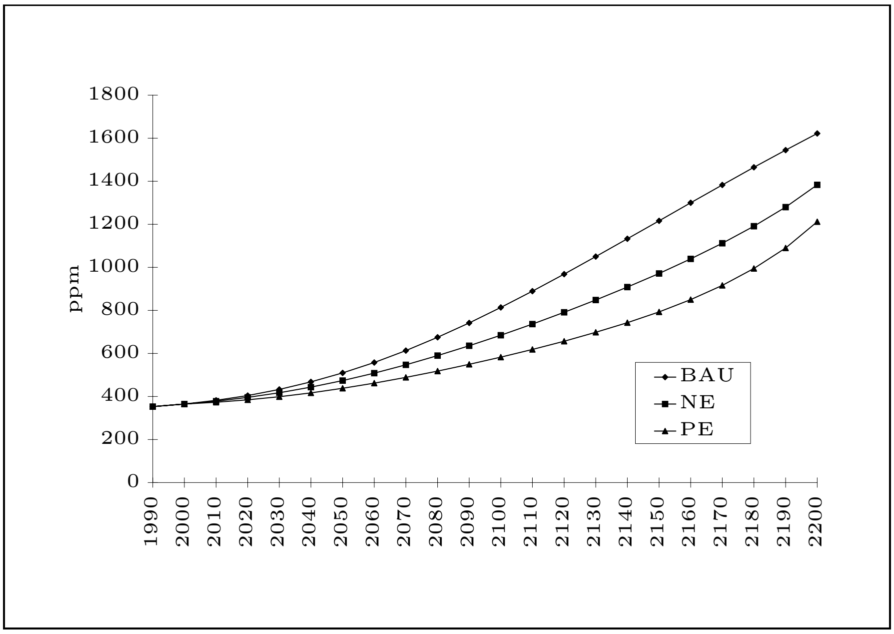
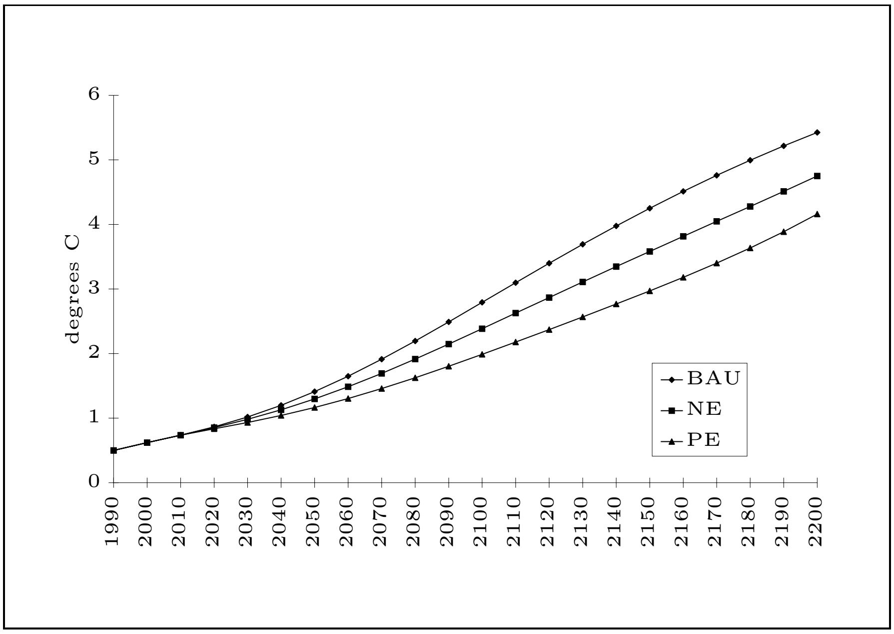


Figure 3: Temperature increases



6.2.4 Emission control rates

Figure 4 shows the time path of emission control ($\mu_{i,t}^{NE}$) for the Nash equilibrium. Overall world emission abatement w.r.t. BAU emissions rises from 15.69% in 2000 to about 28.57% in 2100 in the noncooperative scenario. The time path of emission control rate of *Rest Of the World* lies far above the control rate time paths of the other regions. This is due to two reasons. First, *ROW* has relatively cheap emission control options (see coefficient $b_{i,1}$ in Table 5). Second, it is characterized by a relatively high marginal climate change damage estimate (see coefficient $\theta_{i,1}$ in Table 5). Therefore, when equating individual marginal abatement costs to individual carbon tax, *ROW* will reduce its carbon emissions relatively more than the other regions. *Japan* and *Former Soviet Union* are situated at the other end of the spectrum. For *Japan* this is due to high emission abatement costs and for *FSU* this is due to the low climate change damage estimate.

Figure 4: Nash equilibrium emission control rates

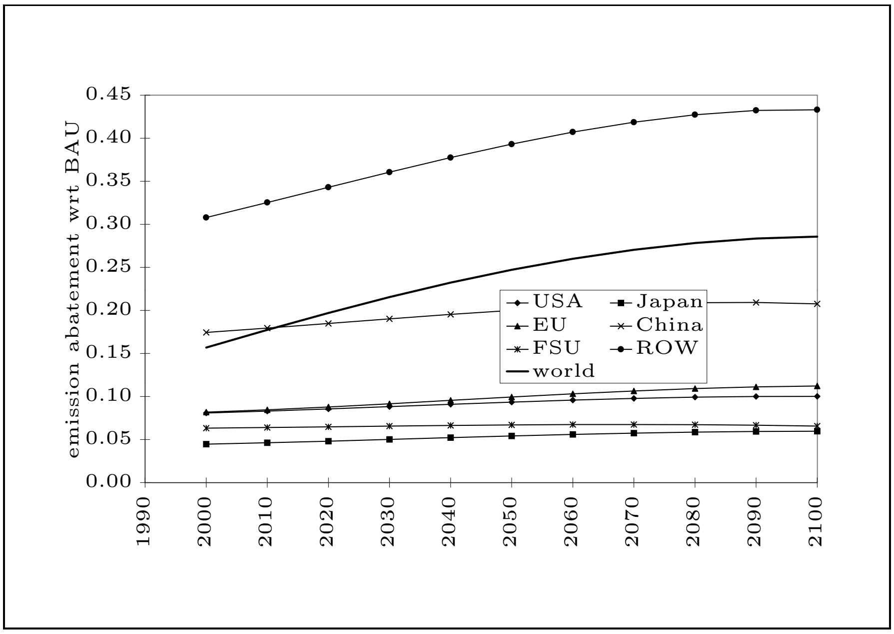


Figure 5 shows the time path of emission control rates ($\mu_{i,t}^{*}$) for the Pareto efficient allocation. Overall world emission abatement w.r.t. BAU emissions rises from 39.40% in 2000 to about 49.41% in 2100 in the Pareto efficient solution. In the world optimum case, *China* and *Former Soviet Union* should reduce their emissions relatively more than the others since they are characterized by the lowest emission abatement costs initially. The ranking of regions in terms of μ_i^* is identical to the ranking in terms of marginal emission abatement costs.

Figure 5: Pareto efficient emission control rates

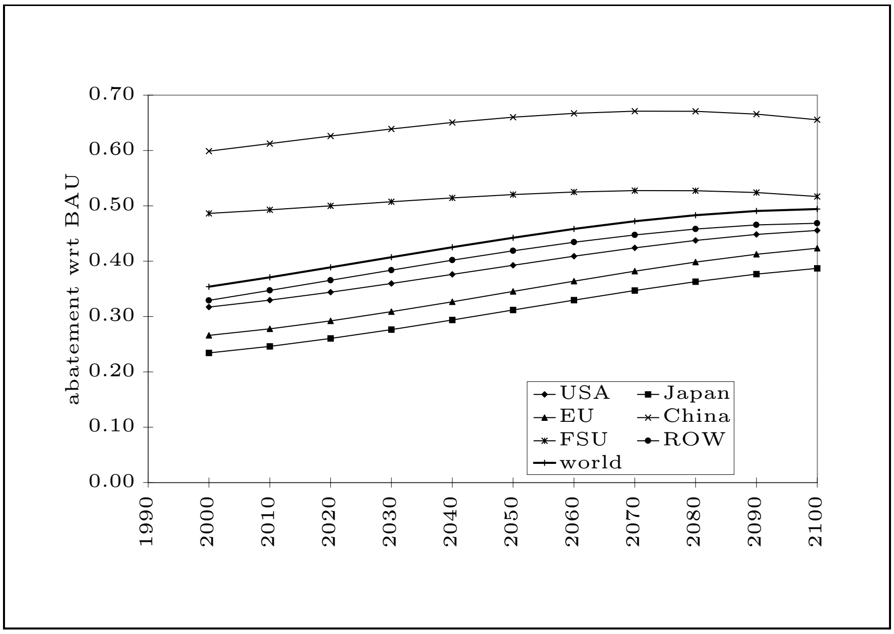
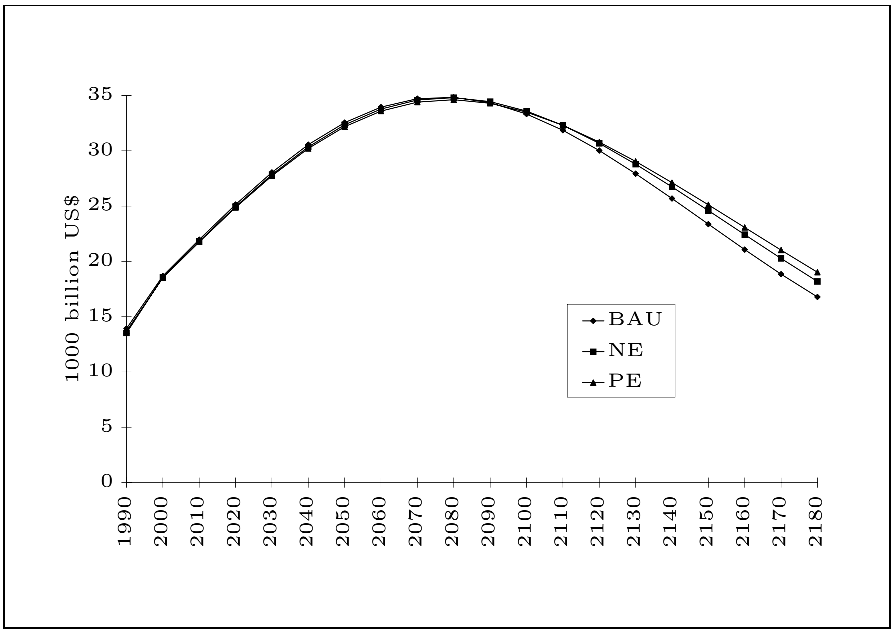


Figure 6: Discounted consumption flows



6.2.5 Consumption flows and gains from cooperation

Figure 6 shows the aggregate (world total) discounted consumption flows for the business-as-usual, the Nash equilibrium and the Pareto efficient scenarios. Generally speaking, differences between the three scenarios are small. Only towards the end of the time horizon, substantial differences are noticeable: the Pareto efficient consumption flow dominates the Nash equilibrium and business-as-usual flows.

Table 1: Aggregate discounted consumption under alternative scenarios (billion 1990US$)

scenario	BAU	NE	NE/BAU	PE	PE/BAU	PE/NE
USA	71,538	72,157	0.87	72,143	0.85	-0.02
Japan	40,130	40,444	0.78	40,558	1.07	0.28
EU	93,831	94,684	0.91	94,960	1.20	0.29
China	41,050	42,549	3.65	41,615	1.38	-2.20
FSU	20,606	20,908	1.46	20,568	-0.19	-1.62
ROW	301,539	307,914	2.11	313,010	3.80	1.66
world	568,694	578,656	1.75	582,854	2.49	0.73

The last row *world* in Table 1 reveals the overall magnitudes that are at stake. It gives the discounted 1990 value of the aggregate world consumption flows in the business-a-usual, the Nash equilibrium and Pareto efficient scenarios. Discounted consumption amounts to 568,694, 578,656 and 582,854 billion 1990US$ respectively. While the gain at the world level is substantial between the business-as-usual and Nash equilibrium (+1.75%), the additional gain obtained by moving from NE to the Pareto efficient allocation is rather small (+0.73%). From these figures it can be concluded that if our model is a correct representation of reality, the climate change problem is more one of domestic policy than of internationally coordinated policies. In other words, while there is a lot to be gained from each country adopting a rational domestic policy, there is little further to be gained by modifying these policies to take account of transboundary spillovers.

These considerations apply a fortiori to the original RICE model. Indeed, compared to Table 4, p.757 in Nordhaus and Yang (1996), our differences between the scenarios are substantially bigger. For instance, the difference in aggregate discounted consumption between the Pareto efficient outcome and the BAU scenario amounts to 14,160 billion 1990US$ in our model versus only 344 billion 1990US$ in Nordhaus and Yang (1996). This difference is explained by the fact that we use a lower discount rate and higher damage valuations.

Yet, emission abatement policies do differ substantially between the NE and the PE scenarios: while NE already implies strong emission reductions compared with BAU, PE requires even stronger abatement (remember Figures 4 and 5). Figure 3 shows similarly important reductions of temperature changes. Thus, while internationally coordinated policies appear to be importantly different from uncoordinated ones as far as environmental variables are concerned, the present model reveals that there may be some economic indifference between them.

7 Monetary transfers ensuring strategic stability

7.1 Strategic stability of an emission abatement game

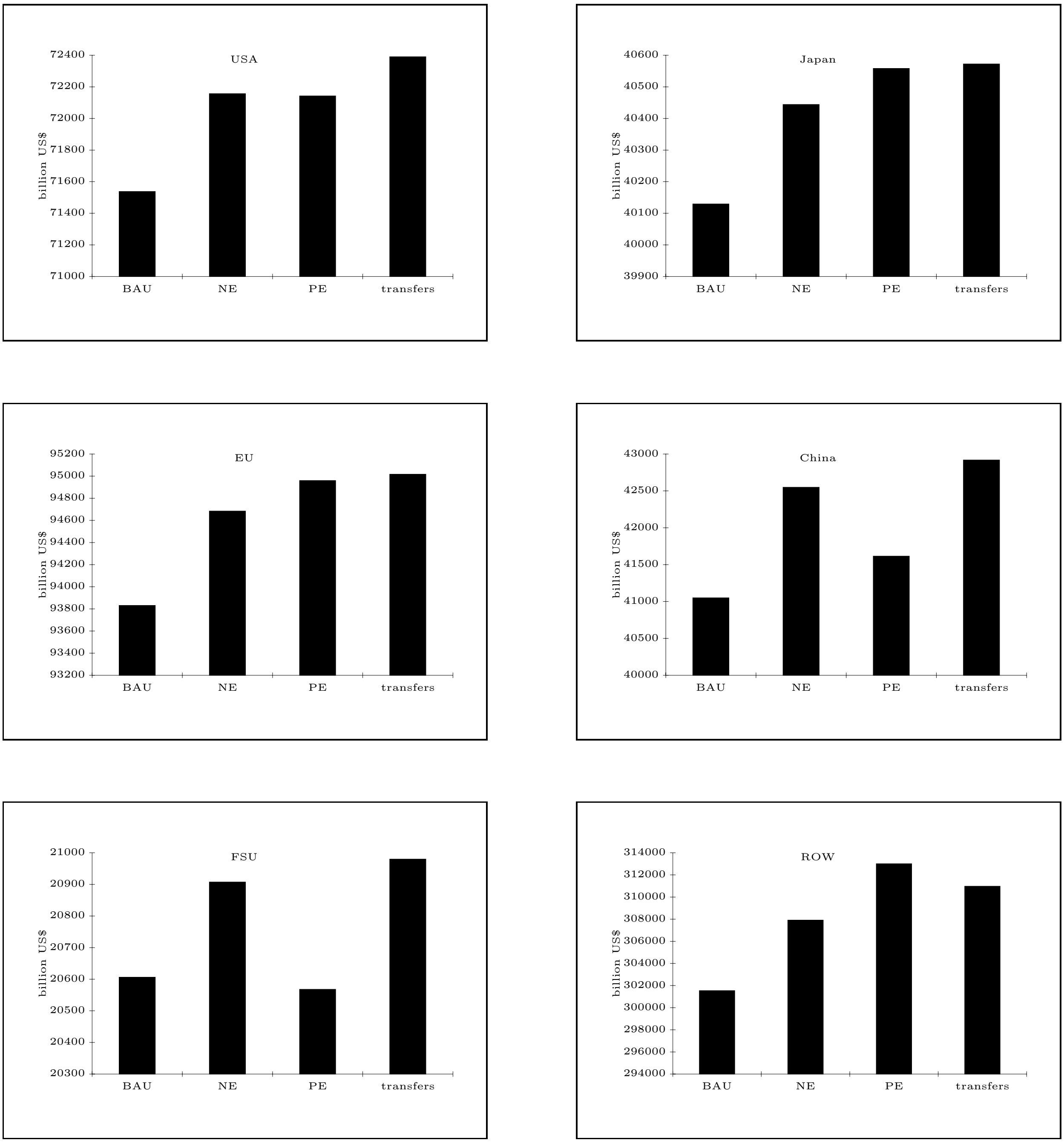
In this section we look into the question of the stability of an international environmental agreement that would implement the Pareto efficient emission policies identified in section 6. As is well known, the fact that a particular allocation constitutes a Pareto efficient allocation does not imply that all countries are better off compared to the Nash equilibrium. While many countries are net winners, some other countries can be net losers. And if a country is worse off in the Pareto efficient allocation compared to the Nash equilibrium, it will be tempted to individually deviate and leave the efficient agreement. Moreover, it might be the case that not only individual countries but also some subgroups or coalitions of countries find out that they can do better by breaking away jointly from the agreement. In both cases, any agreement to implement a Pareto efficient allocation is inherently unstable.

Figure 7 shows a comparison for every region between its discounted value of consumption in the business-as-usual scenario, the Nash equilibrium and the Pareto efficient allocation without transfers. The figures on the vertical axis are expressed in billion 1990US$. Three out of the six regions, namely *USA*, *China* and *Former Soviet Union*, are worse off under the Pareto efficient allocation compared to the Nash equilibrium. According to Table 1 above, the *USA* looses approximately 14 billion $, *China* 934 billion $ and *Former Soviet Union* 340 billion $. This phenomenon is typical for regions with relatively low marginal emission reduction costs and/or relatively low marginal willingness to pay for environmental quality. From a global point of view, it might be desirable to require a substantial contribution from low cost countries in the global emission abatement effort. However the value of the avoided climate change damage might be insufficient for these countries to compensate for such an increase in their abatement effort.

7.2 The Germain, Toint, Tulkens (1997) international transfers formula

This implies that a climate treaty implementing the Pareto efficient allocation according to the Samuelson rule (19) is unlikely to emerge as a voluntary agreement among the

Figure 7: Payoffs



major emitters of carbon dioxide. However, transfers of consumption offer a way to induce voluntary cooperation towards the optimum. In particular, we consider in this paper the transfer scheme proposed by Germain, Toint and Tulkens (1997) for stock pollution problems. In this section we present a reinterpretation of this transfer scheme for the RICE model. We start from a Pareto efficient allocation of emission abatement efforts that solves the Samuelson conditions (19) for all i and $0 \leq t \leq T$. We then modify this allocation by a transfer of the consumption good defined as follows.

Let Z_i^{NE} be the discounted consumption stream of country i under the Nash equilibrium:

$$Z_i^{NE} \;=\; \sum_{t=0}^{T} \frac{Z_{i,t}^{NE}}{[1+\rho]^t}$$

and Z_i^* as the discounted consumption stream of country i in the Pareto efficient outcome:

$$Z_i^* \;=\; \sum_{t=0}^{T} \frac{Z_{i,t}^*}{[1+\rho]^t}$$

Germain, Toint and Tulkens (1997) suggested the following transfer of consumption (with shares $\pi \in \Delta^{n-1}$):

$$\Psi_i \;=\; -[Z_i^* - Z_i^{NE}] + \frac{\pi_i}{\sum_{j \in N} \pi_j} \left[\sum_{j \in N} Z_j^* - \sum_{j \in N} Z_j^{NE} \right] \tag{41}$$

This transfer scheme yields the following consumption level for each $i \in N$:

$$\tilde{Z}_i \;=\; Z_i^* + \Psi_i \;=\; Z_i^{NE} + \frac{\pi_i}{\sum_{j \in N} \pi_j} \left[\sum_{j \in N} Z_j^* - \sum_{j \in N} Z_j^{NE} \right] \;\geq\; Z_i^{NE}$$

Notice that Z_i^{NE} and Z_i^* are time aggregates whereas π_i is not. The transfer formula taxes away the Pareto efficient consumption allocation Z_i^* and assigns the Nash equilibrium Z_i^{NE} consumption level to every country. Moreover, it divides the surplus of cooperation over noncooperation proportionally, in function of the weights π_i. Countries with a relatively high share π_i get relatively more of the surplus.

Clearly, the resulting consumption allocation is preferred over the Nash equilibrium allocation Z_i^{NE} by all $i \in N$ as long as there is a positive surplus to cooperation. Moreover, Germain, Toint and Tulkens (1997) have shown that the transfer scheme gives rise to an allocation of consumption which belongs to the *core*[6] of the cooperative emission abatement

[6]The core property ensures that under under the cooperative outcome no coalition S has an incentive to deviate by proposing an partial agreement Nash equilibrium w.r.t. coalition S such that all of its members are better off and at least one is strictly better off. This property can be interpreted as a necessary (though not sufficient) condition for a voluntary international agreement to be sustained, see Tulkens (1998).

game, provided that damage cost functions are linear, i.e. $D_i(\Delta T_t) = \pi_i \, \Delta T_t$ with $\pi_i > 0$. However, damage functions in the RICE model are nonlinear implying that is not sure that the core theorem in Germain, Toint and Tulkens (1997) carries over to the context of the RICE model. Nevertheless we can experiment with the transfer formula (41) and check by computation whether, with transfers so defined, coalitions have an interest in forming.

With nonlinear damage cost functions $D_i(\Delta T_t)$ the ratios in transfer formula (41) whereby the surplus $\left[\sum_{j \in N} Z_j^* - \sum_{j \in N} Z_j^{NE}\right]$ is shared are no longer constant over time. In order to take time into account we generalize the ratios by defining $\tilde{\pi}_i$ in the following way:

$$\tilde{\pi}_i \; = \; \sum_{0 \leq t \leq T} \psi_{i,t}^* \, D_i'(\Delta T_t^*) \tag{42}$$

with $\psi_{i,t}^* = \lambda_i U_i'(Z_{i,t}^*)/[1+\rho]^t$. $\tilde{\pi}_i$ is thus a discounted sum of country i's marginal climate change damage cost over time. The share of a country in the surplus is then $\tilde{\pi}_i / \sum_j \tilde{\pi}_j$. We will call the surplus sharing rule used in formula (41) with shares $\tilde{\pi}_i$ instead of π_i the *generalized surplus sharing rule.*

7.3 Calculating Germain, Toint, Tulkens transfers in the RICE model

The last bars in Figure 7 show the consumption allocation under the transfer scheme ("transfer"). The shares $\tilde{\pi}_i$ in the surplus are as follows: *USA*: 5.6%, *Japan*: 3.1%, *EU*: 7.9%, *China*: 8.8%, *Former Soviet Union*: 1.7% and *Rest Of the World*: 72.9%. Hence, *Rest Of the World* seizes more than 70% of the surplus. The transfer scheme (41) compensates *USA*, *China* and *Former Soviet Union* such that they are better off under the Pareto efficient allocation with transfers than if there were no cooperation at all. Hence, they have no incentive to deviate individually.

Notice that the regions are not treated identically by the transfer scheme. In particular, after compensations have taken place, *Rest of the World* and *Japan* end up in between their consumption levels in the cooperative and noncooperative cases. *European Union* is almost equally well after transfers compared to the Pareto efficient allocation without transfers.

Figure 7 also shows that especially *Rest of the World* should pay for the compensations to *China* and *Former Soviet Union.* The reason for this is that *Rest of the World* is regarded as one of the big winners of a climate agreement since they would experience most of the damages from climate change. Their gain is in fact to a large extent avoided environmental damage. From a distributional point of view, one might question this implication of the transfer rule. However, we would argue that the relevant welfare comparison is not between the last two bars of Figure 7 but instead between the Nash equilibrium (second bar) and the optimum with transfers (last bar). In this perspective, *Rest of the World* is slightly better off under the climate treaty with transfers compared to the case without climate treaty.

7.4 Checking the core property

As indicated above, Germain, Toint and Tulkens (1998) only established the core property of their transfer scheme for linear damage functions. Since in the RICE model damage costs are nonlinear functions of temperature change (which is itself a nonlinear function of the concentration and emissions of CO_2) it is by no means certain that the transfers we have computed above will yield an allocation in the core of the emission game for the functional forms we have chosen in our simulations. We therefore checked explicitly the core constraints for the allocations after transfers have taken place. In order to transform the emission abatement model into a transferable utility game, we will assume in the sequel that utility is linear in consumption and furthermore that all regions are characterized by the same welfare weights $\lambda_i = 1$. In a TU framework, it is sufficient to check the following inequalities in order to verify whether the transfer mechanism yields a core allocation:

$$\sum_{j \in S} Z_j^S \leq \sum_{j \in S} Z_j^* + \Psi_j = \sum_{j \in S} \tilde{Z}_j$$

for all coalitions of $S \subseteq N$.

The number of coalitions is given by $2^{\#N} = 64$ in the simulation model with 6 regions. A complete list of all coalition values and their payoff under the transfer rule is reported in Table 2. Payoff figures are reported in billion 1990US$. The first column contains a six digit key from which the structure of the coalition can be deducted. If a region is a member of the coalition, it obtains a 1 at the appropriate position in the key. For instance, the key "100000" refers to $S = \{USA\}$, "010000" refers to $S = \{Japan\}$, "001000" refers to $S = \{EU\}$, "000100" refers to $S = \{China\}$, "000010" refers to $S = \{FSU\}$, "000001" refers to $S = \{ROW\}$, "111000" refers to $S = \{USA, Japan, EC\}$, ''111111" refers to $S = N$ and so on. Column two ($W(S)$) contains the value of a coalition under its corresponding partial agreement Nash equilibrium and column three (Z^*) contains the value of a coalition in the Pareto efficient allocation without transfers. Columns four and five show the difference between Z^* and $W(S)$ in absolute amounts and in percentages. Column six (Ψ) shows the transfers as computed from formula (41) with shares adjusted as in (42). Column seven ($Z^* + \Psi$) contains the value of the coalition in the Pareto efficient allocation after these transfers have taken place. The last two columns show the difference between $Z^* + \Psi$ and $W(S)$ in absolute amounts and in percentages.

The entries in Table 2 have been sorted in increasing coalition size. The first six lines refer to the payoff of the individual countries in the Nash equilibrium. The next 15 lines refer to all pairs, the next 20 lines to coalitions of size three and so on. The last line refers the Pareto efficient solution $S = N$. As the transfers Ψ_i should balance, we verify that $(2) - (1) = 0$ and $(3) - (1) = 0$ for the last line.

The difference (2)-(1) measures the difference in the value of a coalition between the partial agreement Nash equilibrium w.r.t. itself when it stands alone and its value in the Pareto efficient allocation without transfers. If this difference is negative for some coalition S this means that S is worse off at that allocation compared to the value it could obtain by leaving the agreement. Hence, full cooperation cannot be sustained because the voluntary participation constraint for coalition S is not satisfied. For the particular simulation that we report, this is indeed the case for the singletons *China*, *Former Soviet Union* and for *USA*. Hence, it is unlikely that these regions would accept to join full cooperation without additional compensation.

There are also some intermediate coalitions with $\#S \geq 2$ for which the voluntary participation constraint is violated. In particular coalitions containing either *China* or *Former Soviet Union* and that do not contain *Rest Of the World* are vulnerable to this objection.

A special case is coalition "111010" (next to last line in the group of coalitions of size 4) containing *USA*, *Japan*, *EU* and *Former Soviet Union*. This coalition approximately corresponds to the group of countries that signed the Kyoto Protocol in December 1997. It turns out that this coalition loses at the Pareto efficient allocation without transfers compared to the partial agreement Nash equilibrium w.r.t. itself. Hence, without transfers from the outsiders, the Kyoto coalition has no incentive to strive for a world Pareto efficient solution.

Column (3) reports the value of the coalitions after transfers have taken place. It turns out that *Rest Of the World* is giving transfers to *China* and *Former Soviet Union* and to a lesser extent to OECD countries. The reason why *Rest Of the World* has to compensate the other regions is due to the fact that this region is gaining the most from the cooperative solution in terms of avoided climate change damage. Part of this gain is used to persuade other regions to participate in a full cooperative solution. If the difference (3)-(1) is positive, the coalition obtains a higher payoff under the transfer scheme than it would obtain under its open loop Nash equilibrium w.r.t. itself and hence, the corresponding core constraint is satisfied. As can be seen from Table 2, the core constraints are met for all possible coalitions. Hence, the allocation with transfers is a core allocation for the emission abatement game associated to the RICE model.

Table 2: Payoff intermediate coalitions (billion 1990US$)

key	$W(S)$	Z^*	$Z^*-W(S)$	(%)	Ψ	$Z^*+\Psi$	$Z^*+\Psi-W(S)$	(%)
coalitions of 1 country								
100000	72,157	72,143	-14	-0.020	248	72,391	234	0.324
010000	40,444	40,558	114	0.282	14	40,572	128	0.317
001000	94,684	94,960	276	0.291	57	95,018	333	0.352
000100	42,549	41,615	-934	-2.195	1,304	42,919	370	0.869
000010	20,908	20,568	-340	-1.624	412	20,980	72	0.346
000001	307,914	313,010	5,096	1.655	-2,035	310,975	3,061	0.994
coalitions of 2 countries								
000011	329,387	333,578	4,190	1.272	-1,623	331,955	2,567	0.779
000101	352,046	354,625	2,578	0.732	-731	353,893	1,847	0.525
000110	63,471	62,183	-1,288	-2.030	1,716	63,899	428	0.674
001001	402,894	407,970	5,077	1.260	-1,978	405,992	3,099	0.769
001010	115,602	115,528	-74	-0.064	469	115,997	396	0.342
001100	137,254	136,575	-679	-0.495	1,361	137,936	682	0.497
010001	348,466	353,568	5,102	1.464	-2,021	351,547	3,081	0.884
010010	61,353	61,126	-227	-0.370	426	61,552	199	0.324
010100	82,997	82,173	-824	-0.992	1,318	83,491	494	0.596
011000	135,130	135,518	388	0.287	72	135,590	460	0.340
100001	380,502	385,153	4,651	1.222	-1,787	383,365	2,864	0.753
100010	93,070	92,711	-359	-0.386	660	93,370	301	0.323
100100	114,722	113,758	-964	-0.840	1,552	115,309	588	0.512
101000	166,848	167,103	255	0.153	305	167,408	560	0.336
110000	112,603	112,701	98	0.087	262	112,963	360	0.320
coalitions of 3 countries								
000111	373,653	375,193	1,540	0.412	-319	374,873	1,220	0.327
001011	424,466	428,538	4,072	0.959	-1,566	426,972	2,506	0.590
001101	447,356	449,585	2,229	0.498	-674	448,911	1,555	0.348
001110	158,215	157,143	-1,072	-0.678	1,773	158,916	701	0.443
010011	369,976	374,136	4,160	1.124	-1,609	372,527	2,551	0.690
010101	392,717	395,183	2,466	0.628	-717	394,466	1,748	0.445
010110	103,933	102,741	-1,191	-1.146	1,730	104,471	539	0.518
011001	443,490	448,528	5,038	1.136	-1,964	446,565	3,075	0.693
011010	156,057	156,086	29	0.019	483	156,570	513	0.329
011100	177,725	177,133	-592	-0.333	1,375	178,509	784	0.441
100011	402,046	405,721	3,674	0.914	-1,375	404,345	2,299	0.572

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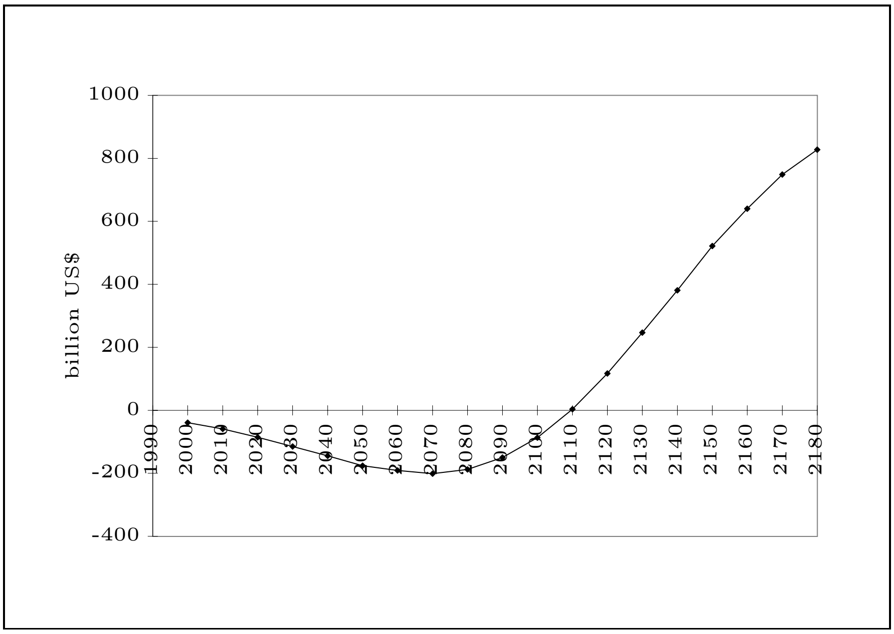
key	$W(S)$	Z^*	$Z^*-W(S)$	(%)	Ψ	$Z^*+\Psi$	$Z^*+\Psi-W(S)$	(%)
100101	424,891	426,767	1,876	0.442	-483	426,284	1,393	0.328
100110	135,672	134,326	-1,346	-0.992	1,964	136,289	617	0.455
101001	475,601	480,113	4,512	0.949	-1,730	478,383	2,782	0.585
101010	187,786	187,671	-115	-0.061	717	188,388	602	0.321
101100	209,479	208,718	-761	-0.363	1,609	210,327	848	0.405
110001	421,097	425,711	4,614	1.096	-1,773	423,938	2,840	0.675
110010	133,523	133,269	-254	-0.191	674	133,943	419	0.314
110100	155,189	154,316	-873	-0.563	1,566	155,882	693	0.446
111000	207,304	207,661	357	0.172	319	207,981	677	0.326
coalitions of 4 countries								
001111	469,052	470,153	1,101	0.235	-262	469,891	839	0.179
010111	414,358	415,751	1,394	0.336	-305	415,446	1,088	0.263
011011	465,099	469,096	3,997	0.859	-1,552	467,545	2,445	0.526
011101	488,071	490,143	2,072	0.425	-660	489,483	1,412	0.289
011110	198,703	197,701	-1,002	-0.504	1,787	199,489	785	0.395
100111	446,559	447,335	776	0.174	-72	447,264	704	0.158
101011	497,242	500,681	3,439	0.692	-1,318	499,363	2,121	0.427
101101	520,315	521,728	1,413	0.272	-426	521,302	987	0.190
101110	230,476	229,286	-1,190	-0.516	2,021	231,307	831	0.361
110011	442,678	446,279	3,601	0.814	-1,361	444,918	2,240	0.506
110101	465,605	467,326	1,721	0.370	-469	466,856	1,252	0.269
110110	176,156	174,884	-1,272	-0.722	1,978	176,862	706	0.401
111001	516,241	520,671	4,430	0.858	-1,716	518,955	2,714	0.526
111010	228,255	228,229	-26	-0.011	731	228,960	706	0.309
111100	249,973	249,276	-697	-0.279	1,623	250,899	927	0.371
coalitions of 5 countries								
011111	509,800	510,711	912	0.179	-248	510,463	664	0.130
101111	542,068	542,296	228	0.042	-14	542,282	214	0.039
110111	487,304	487,894	589	0.121	-57	487,836	532	0.109
111011	537,918	541,239	3,321	0.617	-1,304	539,935	2,017	0.375
111101	561,071	562,286	1,215	0.216	-412	561,874	803	0.143
111110	270,989	269,844	-1,144	-0.422	2,035	271,879	891	0.329
full cooperation of 6 countries								
111111	582,854	582,854	0	0.000	0	582,854	0	0.000

7.5 The transfers and time

The transfers as defined by (42) and whose numerical values are reported in column six (Ψ) in Table 2 above are single numbers representing the 1990 present value of consumption flows over 220 years. They cannot realistically be conceived of as being paid as a lump sum transfer at time $t = 0$. Can they instead be spread over time? The answer is no.

Indeed, figure 8 shows the differences in world consumption levels at each time t between the noncooperative Nash equilibrium and the Pareto efficient allocation: $\sum_{j \in N}[Z^*_{j,t} - Z^{NE}_{j,t}]\,[1+\rho]^{-t}$. It is interesting to see that up to the year 2100, the noncooperative solution dominates the Pareto efficient allocation. After 2100, the order of dominance is reversed. In total, the sum of the gains after 2100 more than compensates for the initial losses. This means that we are in a situation as in Assumption 3 in Germain, Toint and Tulkens (1997). Obviously, the countries cannot borrow against future gains in order to compensate for early losses. We should therefore design a transfer scheme in such a way that the regions most affected initially be compensated partially by the less affected regions. An attempt to design such a transfer scheme with transfers evolving over time is reported in Germain, Toint, Tulkens and De Zeeuw (1998). Computational complexity of this scheme requires however further research before it can be applied to the RICE model.

Figure 8: Difference in discounted consumption between Nash equilibrium and Pareto efficient allocation



8 Conclusion

In this paper we have tested empirically with the Nordhaus and Yang (1996) RICE model the core property of the transfer mechanism advocated by Germain, Toint and Tulkens (1997).

First-order optimality conditions have been derived for the allocations of consumption and abatement effort in Pareto efficient allocations, in open loop Nash equilibria and in partial agreement Nash equilibria w.r.t. a coalition. These optimality conditions can all be interpreted as generalizations of the Samuelson rule for the optimal provision of public goods and of the Ramsey-Keynes rule for the optimal allocation of investment across time. We then turned to a transfer rule that is designed to sustain full cooperation in a voluntary international environmental agreement by making all countries at least as well off as they would be by joining coalitions acting alone and adopting emission abatement policies that maximize their coalition payoff. Hence under the transfer scheme no individual country, nor any subset of countries has an interest in leaving the international environmental agreement. The simulations with the RICE model have shown that the transfer scheme gives rise to an allocation in the core of the carbon emission abatement game associated with the RICE model, even though damage functions are nonlinear.

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Appendix

A simplified version of the RICE model

A complete list of the equations used in our simplified version of the RICE model is given below:

$$U_i(Z_{i,t}) = Z_{i,t} \tag{43}$$

$$Y_{i,t} = Z_{i,t} + I_{i,t} + C_{i,t} + D_{i,t} \tag{44}$$

$$Y_{i,t} = A_{i,t}\, K_{i,t}^{\gamma}\, L_{i,t}^{1-\gamma} \tag{45}$$

$$C_{i,t} = Y_{i,t}\, b_{i,1}\, \mu_{i,t}^{b_{i,2}} \tag{46}$$

$$D_{i,t} = Y_{i,t}\, \theta_{i,1}\, \Delta T_t^{\theta_{i,2}} \tag{47}$$

$$K_{i,t+1} = [1 - \delta_K]\, K_{i,t} + I_{i,t} \qquad K_{i,0} \text{ given} \tag{48}$$

$$E_{i,t} = \sigma_{i,t}\,[1 - \mu_{i,t}]\,Y_{i,t} \tag{49}$$

$$M_{t+1} = [1 - \delta_M]\,M_t + \beta \sum_{i \in N} E_{i,t} \qquad M_0 \text{ given} \tag{50}$$

$$F_t = \frac{4.1\ \ln(M_t/M_0)}{\ln(2)} + F_t^x \tag{51}$$

$$T_t^o = T_{t-1}^o + \tau_3\,[T_{t-1}^a - T_{t-1}^o] \tag{52}$$

$$T_t^a = T_{t-1}^a + \tau_1[F_t - \lambda T_{t-1}^a] - \tau_2[T_{t-1}^a - T_{t-1}^o] \tag{53}$$

Table 3: List of variables

$Y_{i,t}$	production
$A_{i,t}$	productivity
$Z_{i,t}$	consumption
$z_{i,t}$	per capita consumption
$I_{i,t}$	investment
$K_{i,t}$	capital stock
$L_{i,t}$	population
$C_{i,t}$	cost of abatement
$D_{i,t}$	damage from climate change
$E_{i,t}$	carbon emissions
$\sigma_{i,t}$	emission-output rate
$\mu_{i,t}$	emission abatement
M_t	atmospheric carbon concentration
F_t	radiative forcing
T_t^a	temperature increase atmosphere
T_t^o	temperature increase deep ocean

Table 4: List of parameters

ϵ	inequality aversion	0
ρ	discount rate	0.015
δ_K	capital depreciation rate	0.10
γ	capital productivity parameter	0.25
β	airborne fraction of carbon emissions	0.64
δ_M	atmospheric carbon removal rate	0.0833
τ_1	parameter temperature relationship	0.226
τ_2	parameter temperature relationship	0.44
τ_3	parameter temperature relationship	0.02
λ	parameter temperature relationship	1.41
M_0	initial carbon concentration	590
T_0^a	initial temperature atmosphere	0.50
T_0^o	initial temperature deep ocean	0.10

Table 5: Parameter values

	$\theta_{i,1}$	$\theta_{i,2}$	$b_{i,1}$	$b_{i,2}$	λ_i
USA	0.01102	3.0	0.07	2.887	1/6
Japan	0.01174	3.0	0.05	2.887	1/6
EC	0.01174	3.0	0.05	2.887	1/6
China	0.01523	3.0	0.15	2.887	1/6
FSU	0.00857	3.0	0.15	2.887	1/6
ROW	0.02093	3.0	0.10	2.887	1/6

Table 6: Parameter values

	Y_i^0	K_i^0	L_i^0	E_i^0
USA	5,464.796	14,262.51	250.372	1.360
Japan	2,932.055	8,442.25	123.537	0.292
EC	6,828.042	18,435.71	366.497	0.872
China	370.024	1,025.79	1,133.683	0.669
FSU	855.207	2,281.90	289.324	1.066
ROW	4,628.621	9,842.22	3,102.689	1.700