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TAX POLICY IN A MODEL OF SEARCH WITH TRAINING

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Tax policy in a model of search with training

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Abstract

This paper develops a model of search on the labour market with training. The model reveals how the tax system can restore the social optimum if the Hosios condition is not satisfied in the private equilibrium. Furthermore, the effects are explored of a second-best reform from average to marginal taxes when a given amount of public revenue has to be raised. We find that (i) a marginal wage tax is less distortionary to raise revenue than is an average tax per job, provided that training is not distorted initially; (ii) this conclusion may reverse in the presence of training distortions; (iii) marginal wage taxes are less distortionary in economies characterized by commitment in wage bargaining such as the European labour market. Hence, tax reforms that reduce the average tax per job and raise the marginal wage tax, such as an EITC or a negative income tax, are more attractive in Europe than in the US.

1 Introduction

Many European countries suffer from structural labour-market problems, such as high unemployment and low participation. To tackle these problems, various tax proposals have been put forward, including cuts in payroll taxes on employers, earned income tax credits and negative income taxes (see Snower and De la Dehesa (1996), Haveman (1996), Srensen (1997), Bovenberg et al. (1999)). These proposals aim to reduce unemployment and stimulate participation, without seriously damaging the incomes of transfer recipients or cutting government spending. However, these measures typically make the tax system more progressive. This is the case if the lower average tax burden on work is financed by a higher marginal tax burden on higher incomes. Or, as in the case

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of an EITC, if a tax reduction at low incomes is phased out to reduce the budgetary costs of the tax reduction and to get back to the original tax schedule at high incomes. Accordingly, the government typically faces a trade-oœ between the positive eœects of the lower average tax per job on participation and the potential adverse incentive eœects of higher marginal taxes.

This paper illustrates this trade-oœ in a model of search on the labor market with training. On the one hand, our model is a special case of the framework developed by Pissarides (1990) and Mortensen and Pissarides (1999). In particular, we use a static (or one shot) version of their models, where agents incur search costs to ønd a vacancy and ørms post vacancies at a øxed cost. After being matched, a worker and a ørm Nash bargain about the wage. It is well-known that in this type of models taxes inÆuence the division of the surplus between worker and ørm (see for instance Mortensen and Pissarides (1998) and Pissarides (1999)). Hence, taxes inÆuence search intensity, the number of vacancies and unemployment. On the other hand, we extend the Mortensen-Pissarides framework by introducing training decisions. As stressed by the training literature, high marginal tax rates typically reduce the inc1entive to acquire skills (see e.g. Trostel (1993), Dupor et al. (1996), Srensen and Nielsen (1997), Bovenberg and van Ewijk (1997), Heckman et al. (1999)). With our model, we aim to illustrate the trade-oœ between the possible beneøcial eœect of an EITC on unemployment and the possible detrimental eœect on training.

The advantage of focusing on a static version of a search and matching model is that we are able to derive analytical solutions for the optimal taxes. Pissarides (1999) and Mortensen and Pissarides (1998) evaluate the eœects of tax reform in dynamic search models numerically using simulation techniques. In particular, Pissarides (1999) considers the eœects of wage tax reform on unemployment and wages in four diœerent models. In the case of constant real unemployment beneøts, he ønds that a higher marginal tax rate reduces unemployment. This is comparable to the results we ønd below by assuming that training costs are completely tax deductible. If training costs are only partly tax deductible, however, increasing the marginal tax rate will reduce labour market tightness and employment. Mortensen and Pissarides (1998) consider a far broader array of taxes and subsidies, including hiring subsidies and øring taxes. However, they do not derive analytical results on the optimal use of these instruments.

In our model, the tax system serves a threefold task. First, the tax system should correct for distortions in training. Subsidies on training are able to alleviate these distortions. Second, taxes need to restore ineŒciencies in labour-market tightness. The latter distortion arises from the mismatch between the marginal productivity of search (vacancies) in the matching process, and the corresponding bargaining power of the worker (ørm) in wage negotiations, which determine the private marginal beneøt from job matching.[1] We show that a combination of average taxes per job and marginal wage taxes can always be used to restore the Hosios condition by redistributing the surplus from a

[1] Equality between the marginal productivity and marignal private beneøts is known as the Hosios condition.

match between the worker and the ørm. Finally, the tax system aims to raise public revenue with least cost to the private sector. We ønd that, if labour-market tightness and training are not distorted initially, the marginal wage tax is always less distortionary than the average tax per job as an instrument to raise public revenue. The reason is that, although both taxes distort search through adversely aœecting the expected surplus for the worker, the average tax distorts labour-market tightness as well. We relate this result to the Diamond and Mirrlees (1971) intuition of production eŒciency. The result has important implications for the welfare eœects of an EITC. Indeed, if the government would adopt average taxes initially to ønance its spending (in addition to using them to correct for labour-market tightness), a small reduction in the average tax per job ønanced by a higher marginal wage tax ˘ which can be interpreted as an EITC ˘ is welfare improving.

This result may change if training is taxed on a net basis, e.g. because the government does not have access to training subsidies or because the costs of training are not fully tax deductible. One might guess that adding training distortions to the model would always call for a lower marginal tax rate. However, we ønd that the opposite may be true as well. In particular, initial training distortions introduce two additional eœects of tax reforms on welfare. On the one hand, higher marginal taxes exacerbate the initial training distortion by further reducing training eœort. On the other hand, the marginal tax reduces the surplus of a match for the ørm, thereby distorting labour-market tightness. Whereas the ørst eœect raises the distortionary impact of marginal wage taxes, the second eœect can work in both directions, depending on whether labor-market tightness is too high or too low. Accordingly, training distortions can make the introduction of an EITC either more or less attractive. This result originates in the second-best character of the model. In particular, marginal taxes not only exacerbate initial training distortions, but may also alleviate distortions associated with labour-market tightness.

The paper explores two alternative assumptions regarding training. First, bargaining parties may commit to the wage proøle before training takes place. In that case, workers and ørms share the costs and beneøts of training. In a second model, wage bargaining occurs after the training. In this latter framework, workers bear the entire cost of training, while the beneøts are shared across the worker and the ørm. It turns out that the wage proøle is Æatter in the no-commitment case than in the commitment case. This causes two additional distortions in the no-commitment case. First, training is too low since the social beneøts of training exceed the private beneøts for the worker. This distortion can be alleviated by setting training subsidies above the marginal wage tax. Second, since training costs do not reduce the surplus from a match to the ørm, they distort labour-market tightness. This calls for a higher average tax and a lower marginal tax, compared to the commitment case.

The model with commitment may be more relevant for European labour markets while the labour market in the US may be characterized by less commitment (Teulings and Hartog, 1998). A positive conclusion from our model is that it explains why the marginal tax rate in the US is lower than in Europe.

Indeed, a high marginal tax rate in the US economy is more distortive because training is more distorted initially due to the lack of commitment. As a normative conclusion, our analysis reveals that tax reforms which reduce the average tax per job and raise the marginal wage tax ˘ such as an EITC or a negative income tax ˘ tend to be more attractive in Europe than in the US.

The rest of this paper is organized as follows. The next section elaborates on the search model with training. In that model, bargaining parties commit to the wage proøle before the training decision is made. Section 2 also illustrates how taxes in the private outcome may restore the social optimum and explores a reform from average to marginal taxes if the government has a positive revenue requirement. Section 3 presents how the model changes if there is no commitment with respect to the wage proøle and reveals how the tax system can restore the social optimum in this case. Section 4 concludes. The appendix contains the proofs of all results in the paper.

2 Model with commitment

This section develops a model of search on the labour market. The model describes the matching process between vacancies posted by ørms, and workers that search for a job. A job match yields a surplus that is divided across the worker and the ørm through a bargaining process. Workers can also engage in training in order to raise their skill level and thus to receive a higher wage. Firms may inÆuence the training decision by changing the wage proøle.

In contrast to most search models, our framework is of a one-shot nature. This is a considerable simpliøcation of the dynamic models in Mortensen and Pissarides (1999). However, the static model captures a number of features of the dynamic models. Most notably, it captures the Hosios condition discussed below. The simpliøcation allows us to introduce a training decision in the model and still derive analytical solutions for the optimal tax rates. This focuses the analysis on the three distortions in the model: the hold up problem in search and vacancy creation, the distortion in training and the positive revenue requirement of the government. Moreover, the results can be interpreted in the light of the Diamond and Mirrlees (1971) production eŒciency result, which appears to be new in the search and matching literature.

In the model, sequencing in the decision process is important. In particular, the commitment model has the following timing structure.

time t	agents	ørms
0	search intensity s_i	vacancies v
1	matches $m(v,s)$	
2	bargaining V_e	
3		wage proøle (w_0, w_1)
4	training eœort σ	
5	output (y_0, y_1)	

At $t = 0$, ørms decide whether or not to open up a vacancy at a øxed cost k. At the same time, workers choose their search intensity s_i at cost $\gamma(s_i)$, where $\gamma(0) = 0, \gamma'(0) = 0, \gamma''(s_i) > 0$ and $\lim_{s_i \to 1} \gamma'(s_i) = +\infty$. The set of workers is modelled as the unit interval, hence $i \in [0,1]$.

At $t = 1$, ørms and workers are matched, where the total number of matches equals $m(v,s)$ with $s = \int_0^1 s_i di$. The matchingsfunction is homogenous of degree one in v and s and is increasing and concave in each (separate) argument. We denote the matching elasticity of search by η and the elasticity of vacancies by $1-\eta$. As argued by, among others, Blanchard and Diamond (1989) and Broersma and Van Ours (1999), a Cobb-Douglas matchingsfunction $m(v,s) = m_0 v^{1-\eta} s^{\eta}$ is a reasonable approximation in reality. We assume that the matchingsfunction is of this form and hence η is a constant. Firms and workers that are not matched have value 0.

At $t = 2$, ørms and workers bargain about the value of being employed for a worker V_e.

At $t = 3$, the ørm determines the wage proøle (w_0, w_1) where w_0 is the net (of taxes) wage for an untrained worker and w_1 the net wage of a trained worker. The proøle has to satisfy the property that the expected value of being employed equals (at least) the bargained value V_e.

At $t = 4$, the worker chooses his training eœort σ at a cost $c\sigma$. A training eœort σ brings the worker in the trained state with probability $p(\sigma)$ and leaves the worker in the untrained state with probability $1 - p(\sigma)$, where $p'(\sigma) > 0$ and $p''(\sigma) < 0$. This training technology assumption simpliøes the analysis in two dimensions. First, it captures a dynamic decision in a static framework. The literature on training generally models the incentive to train as the gain in future wage income compared to the current disutility of training, often modelled as the income forgone due to the time spend on training (see e.g. Heckman et al. (1999) and the references therein). Our formalization captures these incentives, although in a somewhat diœerent manner. In particular, our static framework does not include the time lag between the moment of training and the appearance of the beneøts in the form of higher wages. Furthermore, the disutility of training is captured by eœort costs, rather than time. The important element in the training literature, however, is the degree of tax deductibility of training costs (Trostel, 1993). We capture tax deductibility in our analysis by means of a training subsidy.[2] Second, our training technology has only two states, trained and untrained, while the training decision σ is a continuous variable. Alternatively, one could have modelled training as a deterministic process where a worker's productivity is a continuous function of training effort σ. Since workers are ex ante identical and risk neutral this is equivalent to the model above.[3] However, in this case one would have had to model how the wage depends on productivity for an interval of productivities instead of just two levels of productivity.

At $t = 5$, output is produced and wages are paid, where a trained worker produces y_1 and receives net wage w_1 and an untrained worker produces y_0 (with $y_1 > y_0$) and receives w_0.

This sequencing in the decision making process implies that bargaining parties commit to the wage profile before the training decision is made. Hence, the model does not give rise to the hold-up problem in training investments. Section 4 will elaborate on the alternative case in which bargaining parties can renegotiate the wage, after the training decision is made. However, the model in this section contains another hold-up problem, because firms and agents cannot commit to the return to search or vacancies. Hence each of the parties invests to create a joint surplus, while the division of the surplus is bargained over after these investments are sunk. Correcting this hold up problem turns out to be an important function of the tax system.

2.1 The private outcome

The process of job matching

The probablity for a worker with search intensity s_i of being matched with a firm equals

$$\frac{s_i}{s} m(v, s) = s_i m(\frac{v}{s}, 1) = s_i m(\theta)$$

where the first equality follows from the assumption that $m(s, v)$ is homogenous of degree 1 in v and s, $\theta \equiv \frac{v}{s}$ denotes the labour market tightness and $m(\theta) \equiv m(\theta, 1)$. A worker chooses s_i to maximise the expected surplus from search

$$\max_{s_i} \{s_i m(\theta) V_e - \gamma(s_i)\}$$

where the value of not being matched is assumed to be equal to 0. Looking at a symmetric equilibrium where each agent chooses the same $s_i = s$ we find that s solves

$$\gamma'(s) = m(\theta) V_e \tag{1}$$

[2] We turn to the possible limitations of our approach in the concluding remarks.

[3] This can be seen as follows. Let $y(\sigma)$ denote a worker's productivity as a function of his training effort σ. Further, let y_1 (y_0) denote a worker's productivity in trained (untrained) state. Then defining $y(\sigma) \equiv p(\sigma) y_1 + (1 - p(\sigma)) y_0$ shows the equivalence.

Hence, marginal search costs equal the expected marginal gain from search. Since we assume a strictly convex cost function for search (i.e. $\gamma''(s) > 0$), the net expected surplus for the worker, $sm(\theta)V_e - \gamma(s) = s\gamma'(s) - \gamma(s)$, is positive.

The probability that a ørm is matched with a worker equals $\frac{m(v,s)}{v} = \frac{m(\theta)}{\theta}$. Assuming free entry into the vacancy posting business, the value for a ørm of employing a worker J_e satisøes

$$\frac{m(\theta)}{\theta} J_e - k = 0 \tag{2}$$

where we use the assumption that the value of not being matched equals 0.

Hence, ørms earn zero proøts in equilibrium.[4]

The value of a job match

Given a certain training eœort σ and wage proøle (w_0, w_1), to be determined below, the value of being employed for a worker depends on the expected wage minus training costs

$$V_e = p(\sigma)w_1 + (1 - p(\sigma))w_0 - [1 - \frac{s_\sigma}{1 + s_\sigma}]c\sigma \tag{3}$$

where $\frac{s_\sigma}{1+s_\sigma}$ stands for a subsidy on training. The value of employing a worker for a ørm equals

$$\begin{aligned} J_e &= p(\sigma)(y_1 - (1+\tau)w_1) + (1 - p(\sigma))(y_0 - (1+\tau)w_0) - \tau_a \\ &= p(\sigma)y_1 + (1 - p(\sigma))y_0 - \tau_a - (1+\tau)V_e - (1+\psi)c\sigma \end{aligned} \tag{4}$$

where $(1 + \tau)w + \tau_a$ equals the total wage cost for the ørm of paying a net wage w to the worker Hence, τ denotes the marginal tax rate on wages while τ_a stands for the average tax per job. Furthermore, $1 + \psi \equiv \frac{1+\tau}{1+s_\sigma}$ so that ψ measures the net tax burden on training (see below).

The worker and the ørm bargain about V_e. This is modelled with a Nash bargaining function with the threat points for both worker and ørm equal to 0.[5] Hence V_e solves

$$\max_{V_e} V_e^\beta (p(\sigma)y_1 + (1 - p(\sigma))y_0 - \tau_a - (1+\tau)V_e - (1+\psi)c\sigma)^{1-\beta}$$

It is routine to verify that this yields the following solution

$$V_e = \frac{\beta}{1+\tau}(\bar{y} - (1+\psi)c\sigma - \tau_a) \tag{5}$$

$$J_e = (1-\beta)(\bar{y} - (1+\psi)c\sigma - \tau_a) \tag{6}$$

[4] If the cost of opening a vacancy would be a function of v (i.e. $k(v)$, where $k(.)$ is a strictly convex function), there would be a positive surplus for the ørm. The eœect on optimal wage taxes of such a generalization is analyzed by Boone and Bovenberg (1999).

[5] This follows from the one shot nature of the game. If the ørm and worker cannot agree on the value V_e, the match is dissolved. Then the worker and ørm receive the same pay oœ as the ørms and workers that were not matched at time 1. In equilibrium such disagreement never happens.

where $\bar{y} \equiv p(\sigma)y_1 + (1 - p(\sigma))y_0$ denotes the expected output of a worker. The bargaining outcome reveals that the total surplus from a match, $\bar{y} - (1 + \psi)c\sigma - \tau_a$, is divided between the employee and the employer on the basis of their respective bargaining powers, β and $1 - \beta$, respectively. The marginal wage tax, τ, reduces the value of a job for the worker. This is a well known result, see for instance Pissarides (1999: 142, 143). The reason is that a high marginal tax rate makes workers less aggressive in bargaining since a rise in the gross wage translates in a small change in the net wage. Similarly, a high τ makes employers more aggressive in bargaining, since a rise in w translates into a big rise in wage costs. In the Nash bargaining solution this effect of τ on the wage exactly cancels the higher τ for employers.

The surplus from a match, $\bar{y} - (1 + \psi)c\sigma - \tau_a$, is sum of three terms. First, there is expected output $\bar{y}$. Second, training costs are subtracted because bargaining parties commit to the bargaining solution before the training decision is made. Hence, the incidence of the training costs is divided between the worker and the firm. Apart from the gross cost of training, there is a net tax burden on training, denoted by ψ. On the one hand, training costs are subsidized through s_σ. On the other hand, the compensation to employees for the costs of training occurs through wage payments that are subject to the marginal wage tax. If $\psi > 0$, the marginal wage tax exceeds the training subsidy so that the compensation for training costs is taxed on a net basis.[6] This reduces the value of a match. Accordingly, the net tax burden on training is shared across the worker and the firm. The final term is the average tax, τ_a. By reducing the value of a match, this tax also bears on both the employer and the employee.

Using the free entry condition (2) together with (??) and (??), we can rewrite V_e as

$$V_e = \frac{\beta}{1-\beta}\frac{1}{1+\tau}\frac{k\theta}{m(\theta)} \tag{7}$$

Substituting this into the first order condition for s, equation (1), yields

$$\gamma'(s) = \frac{\beta}{1-\beta}\frac{k\theta}{1+\tau} \tag{8}$$

Training

Given the (pre-commited) value V_e, the firm will set the wage profile (w_0, w_1) in such a way that σ maximizes J_e in equation (3). Hence σ solves

$$p'(\sigma) = \frac{c(1+\psi)}{y_1 - y_0} \tag{9}$$

[6] An alternative interpretation of the training subsidy is tax deductibility. In particular, if $s_\sigma = \tau$, training costs would be fully tax deductibility. This would be the case if training costs consist of foregone working time (compare Boskin (1976)). If part of the training costs is not tax deductible, such as tuition or leisure time, we find that $s_\sigma < \tau$ or equivalently $\psi > 0$ (see e.g. Trostel (1993)).

The way that the ørm implements this value for σ is to choose the diœerence $w_1 - w_0$ such that the worker, choosing σ to maximize V_e, selects this value. Hence, equation (5) determines $w_1 - w_0$ and (??) determines V_e. It is routine to verify that this implies the following wage proøle

$$w_0 = \frac{\beta}{1+\tau}(y_0 - \tau_a) + \frac{1-\beta}{1+\tau}\left((1+\psi)c\sigma - p(\sigma)(y_1 - y_0)\right) \quad (10)$$

$$w_1 = \frac{\beta}{1+\tau}(y_1 - \tau_a) + \frac{1-\beta}{1+\tau}\left((1+\psi)c\sigma + (1-p(\sigma))(y_1 - y_0)\right) \quad (11)$$

Using equation (5) to rewrite $(1+\psi)c\sigma = p'(\sigma)\sigma(y_1 - y_0)$ in equations (6) and (7), one veriøes that the concavity of $p(.)$ implies that w_0 is below the renegotiation wage $\frac{\beta}{1+\tau}(y_0 - \tau_a)$ and w_1 above the renegotiation wage $\frac{\beta}{1+\tau}(y_1 - \tau_a)$. This is discussed in section 4.

At the level of σ, determined in (5), the free entry condition yields

$$\frac{k\theta}{m(\theta)} = (1-\beta)\left(\bar{y} - (1+\psi)c\sigma - \tau_a\right) \quad (12)$$

The three equations (4), (5) and (8) determine the three endogenous variables s, σ and θ. From these, we can derive the total number of matches $sm(\theta)$ and total unemployment $u = 1 - sm(\theta)$. Total output equals $sm(\theta)(p(\sigma)y_1 + (1-p(\sigma))y_0)$.

2.2 The social optimum

As a measure of welfare, we use total expected output minus the total costs of training, search and vacancies. Since workers and ørms are assumed to be risk neutral, this is not an unreasonable criterion. Also note that the risk neutrality assumption implies that there is no reason to redistribute income among trained, untrained and unemployed workers. When below we consider the case with positive government expenditure, $g > 0$, this welfare criterion implies that government expenditure has the same social value as private income and consumption.

$$sm(\theta)\left[p(\sigma)y_1 + (1-p(\sigma))y_0 - c\sigma\right] - \gamma(s) - ks\theta \quad (13)$$

Assume that the social planner chooses s, θ and σ to maximize welfare (9). Then the following result is a slight generalization of the Hosios (1990) condition to the case with training. For the case without training it is well known (see for instance Pissarides (1990)) that $\beta = \eta$ and $\tau = \tau_a = s_\sigma = 0$ causes the private outcome to coincide with the social optimum.

Lemma 1 If $\beta = \eta$ then

(i) $\tau = \tau_a = s_\sigma = 0$ implies that the private outcome coincides with the social optimal outcome;

(ii) $\tau > 0, \tau_a \geq 0$ and $\psi \geq 0$ implies that agents underinvest in search;

(iii) $\psi > 0$ ($\psi < 0$) implies that workers underinvest (overinvest) in training and if further $\tau_a \geq 0$ then ørms underinvest (overinvest) in vacancies;

(iv) $\tau_a > 0$ ($\tau_a < 0$) and $\tau = s_\sigma = 0$ implies that agents underinvest (overinvest) in search and ørms underinvest (overinvest) in vacancies.

If $\tau = \tau_a = s_\sigma = 0$ then

(v) $\beta < \eta$ ($\beta > \eta$) implies that tightness θ is too high (low) in the private outcome;

(vi) $\beta \neq \eta$ implies that agents underinvest in search.

The intuition for these results is as follows.

(i) If there are no distortionary taxes and the Hosios condition, $\beta = \eta$, is satisøed (see for instance Hosios (1990) or Mortensen and Pissarides (1999)) then the private returns to training, search and creating vacancies coincide with the social returns. The intuition for the Hosios condition is that an agent's (ørm's) bargaining power should coincide with the marginal contribution of search (vacancies) in establishing a match. In particular, the bargaining solution determines the marginal private beneøts of a match for the ørm and the worker. If the marginal private beneøts equal the marginal social beneøts, the hold up problem is solved as each party gets the social return on its sunk investments in search, respectively, vacancies.

(ii) If there are positive wage tax distortions ($\tau > 0$), the private return of search falls short of the social return. This follows from (4) because the marginal tax τ does yield social surplus but is not captured by the worker.

(iii) If the training decision is not distorted ($\tau = s_\sigma$ so that $\psi = 0$), the marginal tax rate has no eœect on ørms' incentives to create vacancies. This follows immediately from (??). As mentioned above, the intuition is that a high marginal tax rate makes workers less aggressive in bargaining. If the wage tax exceeds the training subsidy (i.e. $\tau > s_\sigma$ so that $\psi > 0$), the marginal tax rate reduces the ørm's incentive to create vacancies. This is because the net tax burden on training raises the øxed compensation that employers provide to workers to compensate them for the costs of training. In other words, τ creates an average tax burden comparable to τ_a. Accordingly, the total surplus available declines so that a rise in τ reduces the number of vacancies and hence labour-market tightness, θ. This is the main diœerence with a model without training where τ does not aœect θ.

(iv) The tax τ_a aœects the value of the match and hence the agents' search activity and ørms' incentives to create vacancies. According to (9) the tax τ_a does create social value, but it reduces the private return to ørms and workers. Hence it causes underinvestment in vacancies and search. Since the training decision is taken after the match, it is only aœected by the marginal tax rate τ and the training subsidy, s_σ, not by τ_a.

(v) If the Hosios condition is not satisøed, labor market tightness is too high compared to the social optimum if the workers' bargaining power β is too low compared to their productivity in establishing matches η. Tightness is too low if β exceeds η, because ørms do not receive a big enough share of the surplus and hence create too few vacancies.

(vi) Finally, agents always underinvest in search irrespective of whether their bargaining power is too big or too small. The idea is that too much bargaining power for the worker reduces the number of vacancies created and hence reduces the return to searching. Too little bargaining power reduces a worker's share of the surplus and hence reduces his search intensity.

The next lemma reveals that, if the Hosios condition is not satisøed, taxes τ, s_σ and τ_a can be used to restore the social optimal outcome, in the case with zero government expenditure, $g = 0$.

Lemma 2 If $g = 0$ and $\beta \neq \eta$ then there are taxes τ, τ_a and s_σ that restore the social optimum in the private outcome. Moreover, these taxes balance the government budget. If training subsidies are not available and $\beta \neq \eta$ then the tax instruments τ and τ_a are insuŒcient to restore the social optimum.
In particular,

$$1 + \tau = \frac{\beta}{1-\beta}\frac{1-\eta}{\eta} \tag{14}$$

$$\tau_a = \frac{\eta - \beta}{1-\beta}(\bar{y} - c\sigma) \tag{15}$$

$$s_\sigma = \tau \tag{16}$$

make the private outcome in (4), (5) and (8) equivalent to the social outcome.

The intuition is as follows. The training subsidy $s_\sigma = \tau$ ensures that training decisions are not distorted. The tax instruments τ and τ_a can be used to redistribute the surplus of a match in such a way that ørms and workers get their correct share determined by η. To illustrate, if the workers' bargaining power β exceeds their productivity η, the government uses a positive marginal tax rate τ to reduce the workers' bargaining power and subsidises the ørm by setting $\tau_a < 0$. The reason that this can be done with a balanced budget is that there are no external eœects in the model. The total private value created equals the total social value created. The value just needs to be (re)distributed in the right way.[7]

If the training subsidy would not be available ˘ e.g. because the government cannot observe training eœorts and hence they are not tax deductible ˘ it would not be possible in general to arrive at the social optimum. Indeed, the government would have access to just two instruments, τ, τ_a to correct three variables, s, σ and θ. This is not possible in general.

In the next section, we explore the eœects of a tax reform, starting from an equilibrium that is not necessarily optimal. Furthermore, we analyze the case of a positive revenue requirement for the government.

[7]Note that the balanced budget result depends on our assumption of constant returns to scale in the matchingsfunction. In particular, with decreasing returns to scale in the matchingsfunction the taxes needed to induce the social optimum in the private case yield a budget surplus for the government. With increasing returns to scale the taxes yield a deøcit.

2.3 Tax reform

In this section, public expenditures, g, are positive in the initial equilibrium. Hence, we start from a sub-optimal equilibrium since the social optimum is always characterized by zero public revenues (see Lemma 2). We assume that the level of g remains øxed, e.g. because political barriers preclude budgetary cuts. The government does not have access to a non-distortionary lump-sum tax to ønance its spending. This brings us into a second-best world. Indeed, the government has to rely on three distortionary tax instruments, τ, τ_a and s_σ, to raise public revenue. This section analyzes the optimal mix between these taxes. Furthermore, we explore the welfare eœects of a balanced budget tax reform from the average tax, τ_a, towards the marginal tax, τ, if we start from a sub-optimal initial equilibrium. This reform is interpreted as an earned income tax credit (EITC).

The government budget constraint is of the form

$$g = sm(\theta)[p(\sigma)(\tau w_1 + \tau_a) + (1 - p(\sigma))(\tau w_0 + \tau_a) - \frac{s_\sigma}{1 + s_\sigma} c\sigma]$$

The government spends g and has to ønance this with a wage tax $\tau w_1 + \tau_a$ ($\tau w_0 + \tau_a$) per matched and trained (untrained) worker. Further, from equation (??), the government pays a training subsidy $\frac{s_\sigma}{1+s_\sigma} c\sigma$ per matched worker. Using equation (??), the government budget constraint can be written as

$$g = sm(\theta)[\tau V_e + \tau_a + \psi c\sigma] \tag{17}$$

Now consider the folowing two optimization programs.

$$\max_{\tau, \tau_a, s_\sigma} sm(\theta)\left[p(\sigma)y_1 + (1 - p(\sigma))y_0 - c\sigma\right] - \gamma(s) - ks\theta \qquad (P_{C1})$$

subject to equations (4),(5),(8) and (??)

and

$$\max_{\tau, \tau_a, s_\sigma} s \qquad (P_{C2})$$

subject to equations (4),(5),(8) and (??)

The following proposition substantially simpliøes our welfare analysis.

Proposition 3 Optimization programs (P_{C1}) and (P_{C2}) are equivalent.

Hence, instead of maximizing the expression for welfare in (P_{C1}) we just need to maximize workers' search intensity s subject to the same equations determining s, σ, θ and government budget balance. The intuition why s is a suŒcient statistic for welfare (9) is as follows. Total surplus is divided between workers, ørms and the government. Since ørms make zero expected proøts by the free entry condition and because we maximize welfare for given government expenditure g, only workers' surplus creates welfare. As mentioned above, substituting (1) into workers' expected surplus yields

$$sm(\theta)V_e - \gamma(s) = s\gamma'(s) - \gamma(s)$$

which is positive and increasing in s due to the assumption that $\gamma(.)$ is convex. Hence s is a suŒcient statistic for total welfare.

Eœects on search

To arrive at the reduced form for search, we log-linearize the model around an initial equilibrium and solve the linearized model analytically. In particular, we solve the model for two exogenous variables τ and s_σ. The government budget constraint is used to adjust τ_a endogenously. Intuitively, the revenues from a higher marginal tax (or a lower training subsidy) are used to reduce the average tax per job, such that the government budget remains balanced.

Lemma 4 The reduced form for search can be written as

$$\varepsilon_s \Delta \tilde{s} = [(\beta - \eta)(\bar{y} - c\sigma) + (1 - \beta)(\tau_a + \psi c\sigma)]\tilde{\tau} - \frac{\psi c\sigma}{\varepsilon_\sigma}(\tilde{\tau} - \tilde{s}_\sigma) \qquad (18)$$

where

$$\Delta \equiv \eta(\bar{y} - c\sigma) - \frac{g}{m(\theta)s}(1 + \frac{1}{\varepsilon_s}) > 0$$

and $\varepsilon_s \equiv \frac{\gamma''(s)s}{\gamma'(s)} > 0$, $\varepsilon_\sigma \equiv -\frac{p''(\sigma)\sigma}{p'(\sigma)} > 0, \tilde{s} = \frac{ds}{s}, \tilde{\tau} = \frac{d\tau}{1+\tau}$ and $\tilde{s}_\sigma = \frac{ds_\sigma}{1+s_\sigma}$.

The determinant, Δ, should be strictly positive for the model to be stable. This means that we should be on the left side of the Laœer curve. It requires that the size of the public sector, g, is not too large. Since, at the optimum, the marginal change in search is zero, the reduced form shows some direct implications for the optimal second-best tax structure in case of a positive revenue requirement for the government, $g > 0$.

Proposition 5 Consider the case where $g > 0$.
(i) If $\beta = \eta$ then it is optimal for the government to set $\tau_a = 0$ and $\psi = 0$, irrespective of the level of g;
(ii) if $\beta \neq \eta$ then it is optimal for the government to set $\tau_a = \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma)$ and $\psi = 0$, irrespective of the level of g;
(iii) at the optimal taxes it is the case that $V_e = \eta(\bar{y} - c\sigma) - \frac{g}{sm(\theta)}$.

Hence, in a second-best framework with a positive revenue requirement, the government should always set the average tax per job and the training subsidy at their ørst-best levels derived in equations (??) and (??). Public revenues should thus be raised by the marginal wage tax alone. The intuition for this result is the following. As ørms earn zero proøts, they cannot bear the burden of taxation. Indeed, the incidence of all taxes is ultimately borne by workers in the form of a lower net surplus from search. The government can tax away this surplus through either τ or τ_a(we ignore the training subsidy for the moment). It turns out that taxing the surplus from search directly through the wage tax τ is more eŒcient than taxing it indirectly through the average tax per job. Intuitively, whereas both taxes distort the search intensity of the worker, the average tax distorts also labour-market tightness by reducing vacancies. Accordingly, the average tax per job is a relatively ineŒcient instrument to tax away the surplus from search of the worker.

This result resembles that of Diamond and Mirrlees (1971). Their model assumes perfect competition, which ensures zero proøts for the ørm, and implies that there is no such distortion as a deviation from the Hosios condition, because each production factor receives its marginal product. If the government relies on distortionary taxes to raise public revenue, Diamond and Mirrlees show that it should always maintain production eŒciency, i.e. the government should not impose taxes that distort the input mix in production. In our framework, we ønd that the government should not impose taxes that distort the ratio between vacancies and search, i.e. labour-market tightness. Hence, only deviations from the Hosios condition call for an average tax rate, τ_a, not the need to ønance $g > 0$.

In terms of the optimal incidence, the expression for V_e under (iii) summarizes this discussion. The worker gets his eŒcient (Hosios) share η of the surplus $(\bar{y} - c\sigma)$ and bears the burden of g per match completely.

Armed with the optimal structure, the reduced form reveals under which conditions an EITC improves welfare if we start from a sub-optimal equilibrium. The following result follows immediately from equation (??).

Proposition 6 Introducing an EITC, here interpreted as a rise in τ where the government budget is balanced by a reduction in τ_a, has the following implications for welfare:
(i) in case $\psi = 0$, then it raises welfare if and only if $\tau_a > \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma)$;
(ii) in case $\tau_a = \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma)$ and $\psi > 0$, then it raises welfare if and only if $1 - \beta > \frac{1}{\varepsilon_\sigma}$;
(iii) in case $\tau_a = \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma)$ and $\psi > 0$, it always raises welfare if $\tilde{\tau} = \tilde{s}_\sigma$.

The intuition for the result is the following.

(i) If training subsidies are set at their ørst-best level (i.e. $\psi = 0$), the introduction of an EITC would raise welfare if and only if the initial τ_a exceeds the optimal second-best level in proposition 5 (ii) (see also ??).

(ii) If τ_a is set at its optimal level in (??), a marginal increase in τ yields an ambiguous eœect on welfare if s_σ is øxed at too low a level, that is $\psi > 0$. This case with $\tau > s_\sigma$ seems relevant in practice. In particular, not all costs of training are tax deductible. Examples include books, materials, training hours during leisure time and pure eœort costs.

With $\tau > s_\sigma$, training is taxed on a net basis. This net tax burden on training causes two distortions. First, it distorts the distribution of the surplus from a match across the worker and the firm. In particular, the tax burden on training reduces the surplus for the firm so that a lower level of τ_a would suffice to maintain the Hosios condition. If one starts from the optimal second-best level of τ_a, a marginal reduction thus yields a first-order welfare improvement. Second, the net tax burden on training implies that training is too low. A marginal increase in τ exacerbates this distortion if the training subsidy is not adjusted accordingly. Hence, the overall welfare effect of an EITC is ambiguous and depends on the magnitude of a reform on the two distortions, i.e. the distortion in labour-market tightness and the distortion in training. The effect on the distortion in labour-market tightness is measured by $1-\beta$, which denotes the extend to which the tax burden on training reduces the surplus for the firm. If firms have relatively much bargaining power (i.e. $1-\beta$ is large), they also bear a large part of the net tax burden on training since these reduce the surplus from a match. The effect on the training distortion depends on ϵ_σ, which measures the speed with which the rate of return to training declines with the level of training. In particular, if rate of return to training declines rapidly with the level of training (i.e. ε_σ is large), the effect on training will be small. A net tax burden on training thus makes an EITC more attractive if the first effect dominates the second effect. Otherwise, an EITC becomes less attractive.

(iii) If the government would raise the training subsidy in line with τ, or equivalently all training costs would be tax deductible, it would offset the effect of τ on the training distortion. Hence, an EITC (i.e. a marginal increase in τ and s_σ and a corresponding reduction in τ_a) would unambiguously raise welfare since it alleviates the distortion in labour-market tightness.

3 Model without commitment

In the previous model, agents choose training after bargaining parties committed to the wage profile. In this section, we explore an alternative sequence of decision making. In particular, we consider the following timing structure.

time	agents	firms
0	search intensity s_i	posting vacancies v
1	matches $m(v,s)$	
2	training effort σ	
3	bargaining (w_0, w_1)	
4	output (y_0, y_1)	

Hence, households decide about their training effort before negotiating about the wage profile.

3.1 The private outcome

If ørms cannot commit to a wage proøle (w_0, w_1), the worker and ørm (re)negotiate the wage in each state, trained and untrained. Again assuming zero threat points, w_i is determined by the Nash bargaining function in state $i = 0, 1$. That is, w_i solves

$$\max_{w_i} w_i^{\beta}(y_i - (1+\tau)w_i - \tau_a)^{1-\beta}$$

Consequently

$$w_i = \frac{\beta}{1+\tau}(y_i - \tau_a) \tag{19}$$

$i = 0, 1$.

Hence, if bargaining parties negotiate the wage outcome after the training decision, the wage proøle diœers from the model with commitment above. Indeed, comparing the wages in (6) and (7) reveals that the wage proøle is less steep in the model without commitment, because in the commitment case w_0 is below $\frac{\beta}{1+\tau}(y_0 - \tau_a)$ and w_1 above $\frac{\beta}{1+\tau}(y_1 - \tau_a)$. The intuition is the following. Although the division of the surplus may not be optimal in the commitment case above (if Hosios is not satisøed), the ørm has still an incentive to set the wage diœerence $w_1 - w_0$ in such a way as to maximize the total surplus. In the no commitment case here, the training costs are already sunk when the ørm and worker bargain about the surplus. Hence the relevant surplus for the ørm is $y_i - \tau_a$, not the total surplus $\bar{y} - \tau_a - (1+\psi)c\sigma$. In other words, the ørm has neither the means nor the incentive to maximize the total surplus.

Note that the commitment case assumes commitment from both the ørm and the worker not to renegotiate wages. In particular, in the untrained state the worker has an incentive to renegotiate because the wage w_0 in (6) is below the renegotiation wage $\frac{\beta}{1+\tau}(y_0 - \tau_a)$. Similarly, in the trained state the ørm has an incentive to renegotiate because w_1 in (7) is above the renegotiation wage $\frac{\beta}{1+\tau}(y_1 - \tau_a)$. In other words, the commitment case requires an institutional setting where both parties have an incentive to stick to their agreement. Reasons may be that individual workers and ørms try to build a reputation for being able to commit. This is most likely if relationships are expected to last long. Alternatively, unions and representatives of ørms bargain at a centralized level and force their constituencies to stick to the agreement because the aggregate surplus is bigger with commitment. Either way, in our interpretation, the commitment case is more likely to come about on the European continent than in the Anglosaxon countries (Teulings and Hartog, 1998). We return to this below.

It follows that

$$V_e = p(\sigma)w_1 + (1 - p(\sigma))w_0 - \frac{1}{1+s_\sigma}c\sigma \tag{20}$$

Hence workers choose σ to solve

$$\max_{\sigma}\{p(\sigma)\frac{\beta}{1+\tau}(y_i - y_0) - \frac{1}{1+s_\sigma}c\sigma\}$$

Thus

$$p'(\sigma) = \frac{c(1+\psi)}{\beta(y_1 - y_0)} \tag{21}$$

Comparing this expression to (5), for given ψ, workers invest less in training in the no commitment case than in the commitment case. The intuition is that in the no commitment case, workers pay all training costs while they share the returns of training with ørms. Further,

$$\begin{aligned} J_e &= p(\sigma)(y_1 - (1+\tau)w_1) + (1-p(\sigma))(y_0 - (1+\tau)w_0) - \tau_a \\ &= \bar{y} - c\sigma - V_e - \frac{g}{sm(\theta)} \end{aligned} \tag{22}$$

where the last equality follows from the government budget constraint (??). The interpretation is that the total surplus after a match, $\bar{y} - c\sigma$, is distributed over employers, J_e, employees, V_e, and the government, $\frac{g}{sm(\theta)}$. Combining this equation with the free entry condition (2) yields

$$\frac{k\theta}{m(\theta)} = \bar{y} - c\sigma - V_e - \frac{g}{sm(\theta)} \tag{23}$$

The ørst order condition (1) for search intensity s remains unchanged

$$\gamma'(s) = m(\theta)V_e \tag{24}$$

3.2 Restoring the social optimum

To ønd the solution for the optimal taxes, we again simplify the analysis by proving the equivalence of two optimization programs. Consider the following optimization programs

$$\max_{\tau,\tau_a,s_\sigma} sm(\theta)\left[p(\sigma)y_1 + (1-p(\sigma))y_0 - c\sigma\right] - \gamma(s) - ks\theta \qquad (P_{NC1})$$

subject to (10), (??), (??), (??) and (??)

and

First, solve the following two maximization problems (P_{NC2})

$\max_{V_e}\{s\}$ subject to (??) and (??)

$\max_{\psi}\{p(\sigma)y_1 + (1-p(\sigma))y_0 - c\sigma\}$ subject to (10)

and let V_e^* (ψ^*) denote the value of V_e (ψ) at which the maximum is achieved, then choose τ, τ_a and s_σ that solve

$$\begin{aligned} V_e^* &= \frac{\beta}{1+\tau}\left(p(\sigma)y_1 + (1-p(\sigma))y_0 - \tau_a - \frac{1+\psi^*}{\beta}c\sigma\right) \\ 1+s_\sigma &= \frac{1+\tau}{1+\psi^*} \\ g &= sm(\theta)\left(\tau V_e^* + \tau_a + \psi^* c\sigma\right) \end{aligned}$$

Proposition 7 The optimization programs (P_{NC1}) and (P_{NC2}) are equivalent.

The gain of working with (P_{NC2}) instead of (P_{NC1}) is that we do not need to consider three ørst order conditions, but only two. Both optimization problems under (P_{NC2}) are in fact rather simple.[8]

The intuition of why this works is as follows. The reason why s is a suŒcient statistic for welfare is the same as in proposition 3. In fact, the proof of proposition 3 does not depend on the commitment assumption. Second, we use the fact that τ and τ_a inÆuence the division of the surplus between worker and ørm. Instead of determining the optimal division by diœerentiating with respect to τ and τ_a we determine the optimal incidence directly by diœerentiating with respect to V_e. This yields one ørst order condition instead of two. Second, given that the optimal division is known, we choose the training subsidy in such a way that the total surplus of the match is maximized.[9]

Proposition 8 With optimal taxes, the commitment and no commitment cases yield the same values for s, θ, σ and V_e. Comparing the optimal taxes in each case yields

$$\begin{aligned} \tau^{NC} &= \tau^C - \frac{\beta c \sigma}{V_e} \\ \tau_a^{NC} &= \tau_a^C + c\sigma \\ \psi^{NC} &= -(1-\beta) \\ \psi^C &= 0 \end{aligned}$$

where the solution to (P_{C1}) is denoted by $(\tau^C, \tau_a^C, s_\sigma^C)$ and $1+\psi^C = \frac{1+\tau^C}{1+s_\sigma^C}$ and the solution to (P_{NC1}) is denoted by $(\tau^{NC}, \tau_a^{NC}, s_\sigma^{NC})$ and $1+\psi^{NC} = \frac{1+\tau^{NC}}{1+s_\sigma^{NC}}$.

The three tax instruments τ, τ_a and s_σ can solve the problem of lack of commitment, in the sense that the values for s, θ, σ and V_e are the same with optimal taxes in the commitment and no commitment case. This follows already from proposition 7. (P_{NC2}) shows that ψ is chosen so as to maximize the total surplus of a match $p(\sigma)y_1 + (1-p(\sigma))y_0 - c\sigma$. Substituting the optimal value $\psi^C = 0$ in proposition 5 into equation (5) shows that in the commitment case, too, σ is chosen to maximize $\bar{y} - c\sigma$. Given this value of $\bar{y} - c\sigma$, the question is how this surplus net of government expenditure, $\bar{y} - c\sigma - \frac{g}{sm(\theta)}$, is distributed over workers, V_e, and ørms, J_e. This distribution then aœects s and θ. The instruments τ and τ_a are used to achieve this distribution. (P_{NC2}) makes clear that in the no commitment case V_e is chosen to maximize search s or equivalently welfare (9). The way to do this is to give workers their Hosios share η of surplus $\bar{y} - c\sigma$ minus the government expenditure requirement per match $\frac{g}{sm(\theta)}$. Thus $V_e = \eta(\bar{y} - c\sigma) - \frac{g}{sm(\theta)}$. The reason why workers bear the

[8]We could have used a similar proposition in the commitment case. But there it is not really necessary because solving P_{C2} is already straightforward.

[9]Here we use the fact that we have three instruments τ, τ_a and s_σ. Proposition 7 is no longer valid if we only consider two instruments τ and τ_a.

incidence of g completely is that ørms earn zero (expected) proøts and hence cannot bear any incidence at all. The value of V_e in the no commitment case corresponds to the value in proposition 5 (iii) for the commitment case. Hence V_e, s and θ are the same in the commitment and no commitment case. However, the values of the tax instruments diœer in the two cases.

The intuition for the tax results is the following. The model without commitment contains two additional distortions compared to the no-commitment case. First, training is too low in the no-commitment case due to the hold-up problem. Indeed, the social beneøts of training exceed the private beneøts for the worker so that the private outcome yields too little incentives for training. In the commitment case, training is distorted only if it is taxed on a net basis. Proposition 8 reveals that the hold-up problem can be alleviated in the no commitment case by setting the training subsidy above the marginal wage tax, that is training is subsidized on a net basis ($\psi < 0$). In particular, the net subsidy on training should raise the return to training for the worker so that the marginal costs and beneøts coincide. This calls for a net subsidy equal to $1 - \beta$, i.e. the share of the beneøts of training that Æow to the ørm instead of the worker. The second distortion due to the lack of commitment in wage bargaining appears because training costs do not reduce the surplus from a match to the ørm. Accordingly, it distorts labour-market tightness. Compared to the commitment case, this calls for a higher average tax and a lower marginal tax in order to redistribute surplus from the ørm to the worker. Intuitively, redistributing the surplus creates the right incentives for search and vacancy creation by equating the marginal private beneøts with the corresponding marginal productivities in the matching process.

If we start from an arbitrary tax system, the introduction of an EITC is thus more likely to improve welfare in an economy characterized by commitment on the labour market than in an economy without commitment. Interpreting the US economy as being characterized by less commitment than the economies on the European continent, this suggests the following policy conclusion. If the EITC is considered a success in the US since the welfare eœects of increased participation outweigh the adverse incentive eœects of high marginal taxes (see, for instance, Eissa and Liebmann (1995)), then worries about the detrimental eœects of a high marginal tax rate should not stop the EITC from being introduced in Europe.

4 Concluding remarks

This paper formalizes the trade-oœ between labour-market participation and training in a model of search on the labour market. In particular, it illustrates the optimal tax structure in the presence of three distortions, namely a training distortion, a public revenue requirement that has to be ønanced by distortionary taxes, and a hold-up problem that arises because ørms and workers cannot precommit to the return to search and vacancies. We are able to derive analytical results because we use a simpliøed, static representation of the search model in the tradition of Pissarides (1990) and Morensen and Pissarides (1999). Although our approach captures some important features of these dynamic models, it may also have some limitations. In particular, since we ignore dynamics, our results should be interpreted as steady-state solutions. Especially for human capital formation, this may be too simplistic, although we believe that our model captures the main incentive eœects of training. An important assumption in our model is that tax policies have no implications for the fall back position of workers and ørms, which always have a value equal to zero. If these values were positive, which is the case in a dynamic model of search, taxes may have a diœerent impact on labour-market decisions, including the training decision. This type analysis may be left for future research, but it seems unlikely that one would be able to derive analytical solutions from such a model.

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Appendix

This appendix contains the proofs of the results in this paper.

Proof of Lemma 1

The ørst order conditions for s, θ and σ in maximizing welfare (9) are respectively

$$m(\theta)(\bar{y} - c\sigma) - \gamma'(s) - k\theta = 0 \tag{25}$$
$$m'(\theta)\,[\bar{y} - c\sigma] - k = 0 \tag{26}$$
$$p'(\sigma)(y_1 - y_0) - c = 0 \tag{27}$$

where, as above, $\bar{y} = p(\sigma)y_1 + (1 - p(\sigma))y_0$. Rewriting equation (??) as

$$(1-\eta)(\bar{y} - c\sigma) = \frac{k\theta}{m(\theta)} \tag{28}$$

Substituting this into (??) yields

$$\gamma'(s) = \frac{\eta}{1-\eta}k\theta \tag{29}$$

(i) Comparing (??), (??) and (??) to the private outcome in equations (4), (5) and (8) shows that the private and social outcomes coincide if $\tau = s_\sigma = \tau_a = 0$ and $\beta = \eta$.

(ii) Comparing (??) with (4) shows that for given θ, $\tau > 0$ implies that the private s is lower than the socially optimal s. Further, (8) together with $\psi \geq 0, \tau_a \geq 0$ implies that private θ does not exceed socially optimal θ. This implies that s in (4) does not exceed s in (??).

(iii) The eœect on training follows immediately from (5) and (??). Since the value of σ in (??) maximizes $\bar{y} - c\sigma$, it follows from (??) and (8) (with the assumption that $\tau_a \geq 0$) that private θ is below socially optimal θ.

(iv) $\tau = s_\sigma = 0$ implies that $\psi = 0$ and hence both private and socially optimal σ maximize $\bar{y} - c\sigma$. Now comparing equations (??) and (8) shows that $\tau_a < 0$ implies that θ is higher in private outcome than in social optimum. Comparing (??) and (4) shows that the higher private θ implies that private search s is higher than socially optimal search.

(v) The eœects of $\beta \neq \eta$ are as follows. First, note that σ is unaœected by β in the private outcome (5). Second, the eœect of β on θ can be derived from (8) as

$$\frac{d\theta}{d\beta} = \frac{-1}{\eta}\frac{\theta}{1-\beta} < 0$$

(vi) Using this in equation (4) to ønd the eœect of β on s yields

$$\frac{ds}{d\beta} = \frac{k\theta}{\gamma''(s)}\frac{1}{(1-\beta)^2}\left(1 - \frac{\beta}{\eta}\right)\begin{cases} > 0 & \text{if} \quad \beta < \eta \\ < 0 & \text{if} \quad \beta > \eta \end{cases}$$

Hence s is always too low if $\beta \neq \eta$. QED

Proof of Lemma 2

It is routine to verify that τ, s_σ and τ_a satisfying

$$\begin{aligned} 1+\tau &= \frac{\beta}{1-\beta}\frac{1-\eta}{\eta} \\ \tau_a &= \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma) \\ s_\sigma &= \tau \end{aligned}$$

make the private outcome in (4), (8) and (5) equivalent to the social outcome in (??), (??) and (??). To check budget balance we show that $\tau(p(\sigma)w_1 + (1 - p(\sigma))w_0) + \tau_a - \frac{s_\sigma}{1+s_\sigma}c\sigma = 0$ as follows

$$\begin{aligned} \tau(p(\sigma)w_1 + (1-p(\sigma))w_0) + \tau_a - \frac{s_\sigma c\sigma}{1+s_\sigma} &= \tau V_e + \tau_a + \psi c\sigma \\ &= \frac{\tau}{1+\tau}\beta(\bar{y} - c\sigma - \tau_a) + \tau_a \\ &= (\bar{y} - c\sigma)\left\{\frac{\beta(1-\eta) - (1-\beta)\eta}{1-\eta}\left(1 - \frac{\eta-\beta}{1-\beta}\right) + \frac{\eta-\beta}{1-\beta}\right\} \\ &= 0 \end{aligned}$$

where the ørst equality follows from (??), the second equality from $\psi = 0$ and (??) and the third equality follows from the expressions for τ and τ_a above. QED

Proof of proposition 3

Using the expressions for the value of a match for the worker and the ørm (??) and (??), we write

$$(1+\tau)V_e + J_e = [\bar{y} - \tau_a - (1+\psi)c\sigma]$$

Using this expression to eliminate $\bar{y} - c\sigma$ from (9), we write welfare as

$$sm(\theta)[\tau V_e + \tau_a + \psi c\sigma] + s[m(\theta)V_e - \frac{\gamma(s)}{s}] + sm(\theta)[J_e - \frac{k\theta}{m(\theta)}]$$

Using (1) and (2) and the government budget constraint (??), we can write welfare as follows

$$g + s[\gamma'(s) - \frac{\gamma(s)}{s}]$$

Hence, welfare is determined by public expenditures, the net surplus from search and the net surplus from vacancies (which equals zero). Since we explore only balanced budget reforms ($dg = 0$), the change in welfare can be written as

$$s\gamma''(s)ds$$

Because $\gamma''(s) > 0$ for each $s \geq 0$, welfare maximization is equivalent to maximization of search. Note that this holds true for both the case with and without commitment (see below). QED

Proof of lemma 4

Log linearizing equations (4), (5) and (8) yields

$$\begin{pmatrix} \varepsilon_s & -1 & 0 \\ 0 & \eta[\bar{y} - \tau_a - (1+\psi)c\sigma] & 0 \\ 0 & 0 & \varepsilon_\sigma \end{pmatrix} \begin{pmatrix} \tilde{s} \\ \tilde{\theta} \\ \tilde{\sigma} \end{pmatrix} = \begin{pmatrix} -\tilde{\tau} \\ -\tau_a\tilde{\tau}_a - (1+\psi)\sigma c(\tilde{\tau} - \tilde{s}_\sigma) \\ -(\tilde{\tau} - \tilde{s}_\sigma) \end{pmatrix} \quad (30)$$

where $\varepsilon_s \equiv \frac{\gamma''(s)s}{\gamma'(s)}, \varepsilon_\sigma \equiv -\frac{p''(\sigma)\sigma}{p'(\sigma)}$, $\tilde{\tau} \equiv \frac{d\tau}{1+\tau}$ and $\tilde{s}_\sigma \equiv \frac{ds_\sigma}{1+s_\sigma}$.

Using equation (??), government budget constraint (??) can be written as

$$\begin{aligned} g &= sm(\theta)\left(\tau V_e + \psi c\sigma + \tau_a\right) \\ &= s\frac{\tau}{1+\tau}\frac{\beta}{1-\beta}k\theta + sm(\theta)\psi c\sigma + sm(\theta)\tau_a \end{aligned}$$

Hence

$$\begin{aligned} dg \;=\;& m(\theta)s\left(\frac{\tau}{1+\tau}\frac{\beta}{1-\beta}\frac{k\theta}{m(\theta)} + (1-\eta)(\psi c\sigma + \tau_a)\right)\tilde{\theta} + g\tilde{s} + sm(\theta)\psi c\sigma\tilde{\sigma} + \\ & sm(\theta)\left(\frac{1}{1+\tau}\frac{\beta}{1-\beta}\frac{k\theta}{m(\theta)} + \sigma c(1+\psi)\right)\tilde{\tau} + sm(\theta)\tau_a\tilde{\tau}_a - sm(\theta)c\sigma(1+\psi)\tilde{s}_\sigma \end{aligned}$$

Using $dg = 0$ we get

$$\begin{aligned}\tau_a \tilde{\tau}_a + \sigma c(1+\psi)(\tilde{\tau} - \tilde{s}_\sigma) &= -\frac{1}{1+\tau}\frac{\beta}{1-\beta}\frac{k\theta}{m(\theta)}\tilde{\tau} + \\ &+ \left(\frac{\tau}{1+\tau}\frac{\beta}{1-\beta}\frac{k\theta}{m(\theta)} + (1-\eta)(\psi c\sigma + \tau_a)\right)(-\tilde{\theta}) + \\ &+ \frac{g}{m(\theta)s}\frac{1}{\varepsilon_s}(-\tilde{\theta} + \tilde{\tau}) + \psi c\sigma \frac{(\tilde{\tau} - \tilde{s}_\sigma)}{\varepsilon_\sigma} \qquad (31)\end{aligned}$$

or equivalently, using equations (??) and (??) to gather the term Δ,

$$\Delta\tilde{\theta} = \left(\frac{1}{1+\tau}\frac{\beta}{1-\beta}\frac{k\theta}{m(\theta)} - \frac{g}{m(\theta)s}\frac{1}{\varepsilon_s}\right)\tilde{\tau} - \frac{\psi c\sigma}{\varepsilon_\sigma}(\tilde{\tau} - \tilde{s}_\sigma)$$

where

$$\Delta \equiv \eta(\bar{y} - c\sigma) - \frac{g}{m(\theta)s}(1 + \frac{1}{\varepsilon_s})$$

Using the ørst row of (??), we ønd the following reduced form for search

$$\varepsilon_s \Delta \tilde{s} = ([\beta - \eta](\bar{y} - c\sigma) + (1-\beta)(\tau_a + \psi c\sigma))\,\tilde{\tau} - \frac{\psi c\sigma}{\varepsilon_\sigma}(\tilde{\tau} - \tilde{s}_\sigma)$$

QED

Proof of proposition 5

Proposition 3 says that maximizing welfare subject to (4),(5),(8) and (??) is equivalent to maximizing search subject to these four equations. The reduced form in (??) takes these four equations already into account. To ønd the socially optimal taxes, we need (only) to solve $\frac{ds}{d\tau} = 0$ and $\frac{ds}{ds_\sigma} = 0$. From (??), it follows immediately that $\frac{ds}{ds_\sigma} = 0$ if and only if $\frac{\psi c\sigma}{\varepsilon_\sigma} = 0$, that is $\psi = 0$, or equivalently $\tau = s_\sigma$. Using this in the ørst order condition $\frac{ds}{d\tau} = 0$ yields

$$(\beta - \eta)(\bar{y} - c\sigma) + (1-\beta)\tau_a = 0$$

or equivalently

$$\tau_a = \frac{\eta - \beta}{1-\beta}(\bar{y} - c\sigma)$$

Note that these expressions for ψ and τ_a are independent of the level of g. In other words, only τ is aœected by the level of g.

(i) and (ii) follow immediately from $\psi = 0$ and $\tau_a = \frac{\eta-\beta}{1-\beta}(\bar{y} - c\sigma)$.

(iii) Noting that the total surplus $\bar{y} - c\sigma$ is divided among employed workers, ørms and the government, we can write

$$V_e + J_e + \frac{g}{sm(\theta)} = \bar{y} - c\sigma$$

or equivalently

$$\begin{aligned} V_e &= \bar{y} - c\sigma - J_e - \frac{g}{sm(\theta)} \\ &= \bar{y} - c\sigma - (1-\beta)\left(\bar{y} - c\sigma - \frac{\eta-\beta}{1-\beta}(\bar{y}-c\sigma)\right) - \frac{g}{sm(\theta)} \\ &= \eta(\bar{y}-c\sigma) - \frac{g}{sm(\theta)} \end{aligned}$$

where the second equality follows from equation (??) and the expressions for optimal taxes $\psi = 0$ and $\tau_a = \frac{\eta-\beta}{1-\beta}(\bar{y}-c\sigma)$. QED

Proof of proposition 6

The reduced form (??) is derived under the assumption that changes in τ (and s_σ) are oœset by changes in τ_a to keep a balanced budget for the government. In particular, this follows from the $dg = 0$ assumption in equation (??) above. The results (i), (ii) and (iii) follow immediately from equation (??). QED

Proof of proposition 7

It is straightforward to verify that the proof of proposition 3 also applies in the no commitment case. Hence (P_{NC1}) is equivalent to

$$\max_{\tau,\tau_a,s_\sigma} s$$
$$\text{subject to (10),(??),(??),(??) and (??)}$$

Because $1+\psi = \frac{1+\tau}{1+s_\sigma}$ we can write this as

$$\begin{aligned} &\max_{\tau,\tau_a,\psi} s \\ \text{subject to } \gamma'(s) &= m(\theta)V_e \\ \frac{k\theta}{m(\theta)} &= \bar{y} - c\sigma - V_e - \frac{g}{sm(\theta)} \\ V_e &= \frac{\beta}{1+\tau}\left(\bar{y} - \tau_a - \frac{1+\psi}{\beta}c\sigma\right) \\ p'(\sigma) &= \frac{c(1+\psi)}{\beta(y_1-y_0)} \end{aligned}$$

Since all functions used are well behaved, we can ørst solve for the optimal τ and τ_a for given ψ. Then we derive the optimal value for ψ, given that τ and τ_a are functions of ψ. Note that, for given ψ, the tax parameters τ and τ_a divide the surplus between the ørms and workers. Hence we can ørst derive the optimal incidence J_e and V_e of taxes τ and τ_a. That is, we solve

$$\max_{V_e} s$$

$$\text{subject to } \gamma'(s) = m(\theta)V_e \tag{32}$$

$$\frac{k\theta}{m(\theta)} = \bar{y}_\psi - c\sigma_\psi - V_e - \frac{g}{sm(\theta)} \tag{33}$$

for given value of ψ and thus for given values of σ_ψ and $\bar{y}_\psi$. Let $V_{e\psi}, s_\psi$ and θ_ψ denote the values that follow from this optimization problem. Then the corresponding optimal taxes, denoted τ_ψ and $\tau_{a\psi}$, follow from

$$\begin{aligned} V_{e\psi} &= \frac{\beta}{1+\tau_\psi}\left(\bar{y}_\psi - \tau_{a\psi} - \frac{1+\psi}{\beta}c\sigma_\psi\right) \\ g &= s_\psi m(\theta_\psi)\left(\tau V_{e\psi} + \tau_{a\psi} + \psi c\sigma_\psi\right) \end{aligned}$$

To ønd $V_{e\psi}, s_\psi$ and θ_ψ, we log-linearize equations (??) and (??) with respect to V_e, s and θ. This yields

$$\begin{pmatrix} \varepsilon_s & -(1-\eta) \\ \frac{-g}{sm(\theta)} & \eta(\bar{y} - c\sigma - V_e) - \frac{g}{sm(\theta)} \end{pmatrix}\begin{pmatrix} \tilde{s} \\ \tilde{\theta} \end{pmatrix} = \begin{pmatrix} \tilde{V}_e \\ -V_e\tilde{V}_e \end{pmatrix}$$

where $\tilde{s} = \frac{ds}{s}, \tilde{\theta} = \frac{d\theta}{\theta}$ and $\tilde{V}_e = \frac{dV_e}{V_e}$. Inverting the matrix on the left hand side yields

$$\begin{aligned} \begin{pmatrix} \tilde{s} \\ \tilde{\theta} \end{pmatrix} &= \frac{1}{\varepsilon_s\left[\eta(\bar{y} - c\sigma - V_e) - \frac{g}{sm(\theta)}\right] - (1-\eta)\frac{g}{sm(\theta)}} \times \\ &\quad \begin{pmatrix} \eta(\bar{y} - c\sigma - V_e) - \frac{g}{sm(\theta)} & (1-\eta) \\ \frac{g}{sm(\theta)} & \varepsilon_s \end{pmatrix}\begin{pmatrix} \tilde{V}_e \\ -V_e\tilde{V}_e \end{pmatrix} \end{aligned}$$

Hence $\frac{ds}{dV_e} = 0$ implies

$$\eta(\bar{y} - c\sigma - V_e) - \frac{g}{sm(\theta)} - (1-\eta)V_e = 0$$

or equivalently

$$V_{e\psi} = \eta(\bar{y}_\psi - c\sigma_\psi) - \frac{g}{s_\psi m(\theta_\psi)} \tag{34}$$

where s_ψ and θ_ψ follow from (??) and (??).

Now we go on to determine the optimal value of ψ. Hence we solve

$$\max_\psi s_\psi$$

$$\begin{aligned} \text{subject to } p'(\sigma_\psi) &= \frac{c(1+\psi)}{\beta(y_1 - y_0)} && (35) \\ \gamma'(s_\psi) &= m(\theta_\psi)\eta[p(\sigma_\psi) + (1-p(\sigma_\psi))y_0 - c\sigma_\psi] - \frac{g}{s_\psi} && (36) \\ \frac{k\theta_\psi}{m(\theta_\psi)} &= (1-\eta)[p(\sigma_\psi) + (1-p(\sigma_\psi))y_0 - c\sigma_\psi] && (37) \end{aligned}$$

where equations (??) and (??) follow from substituting (??) into (??) and (??).

Note that equation (??) implies that θ_ψ is increasing in $[p(\sigma_\psi) + (1 - p(\sigma_\psi))y_0 - c\sigma_\psi]$. Equation (??) implies that s_ψ is increasing in θ_ψ and increasing in $[p(\sigma_\psi) + (1-p(\sigma_\psi))y_0 - c\sigma_\psi]$. Hence to maximize s_ψ we need to chose ψ such that the value of σ_ψ implied (??) maximizes $[p(\sigma_\psi) + (1-p(\sigma_\psi))y_0 - c\sigma_\psi]$.

We have shown that optimization program (P_{NC1}) is equivalent to choosing tax instruments τ, τ_a and ψ such that

$$\begin{array}{l} V_e \text{ maximizes } s \\ \text{subject to (??) and (??)} \end{array}$$

and

$$\begin{array}{l} \psi \text{ maximizes } [p(\sigma) + (1 - p(\sigma))y_0 - c\sigma] \\ \text{subject to (10)} \end{array}$$

QED

Proof of proposition 8

Proposition 5 implies that $\psi^C = 0$. Hence it follows from (5) that the private σ^C in the commitment case solves

$$\max_{\sigma}[p(\sigma) + (1 - p(\sigma))y_0 - c\sigma] \tag{38}$$

It follows from (P_{NC2}) that

$$(p'(\sigma)(y_1 - y_0) - c)\,\frac{d\sigma}{d\psi} = 0$$

where $\frac{d\sigma}{d\psi}$ follows from (10). In other words, σ^{NC} solves (??) and thus $\sigma^{NC} = \sigma^C$. Let σ denote this value $\sigma \equiv \sigma^{NC} = \sigma^C$. Whereas $\psi^C = 0$ achieves this value of σ in the commitment case, equation (10) implies that $\psi^{NC} = -(1 - \beta)$.

In the commitment case, equations (1), (2) and proposition 5 (iii) imply

$$\begin{aligned} \gamma'(s^C) &= m(\theta^C)V_e^C \\ \frac{k\theta^C}{m(\theta^C)} &= \bar{y} - c\sigma - V_e^C - \frac{g}{s^C m(\theta^C)} \\ V_e^C &= \eta(\bar{y} - c\sigma) - \frac{g}{s^C m(\theta^C)} \end{aligned}$$

where $\bar{y}^C = \bar{y}^{NC} = \bar{y} \equiv p(\sigma)y_1 + (1 - p(\sigma))y_0$.

For the no commitment case we ønd from (??), (??) and (??) in the proof of proposition 7:

$$\begin{aligned} \gamma'(s^{NC}) &= m(\theta^{NC})V_e^{NC} \\ \frac{k\theta^{NC}}{m(\theta^{NC})} &= \bar{y} - c\sigma - V_e^{NC} - \frac{g}{s^{NC} m(\theta^{NC})} \\ V_e^{NC} &= \eta(\bar{y} - c\sigma) - \frac{g}{s^{NC} m(\theta^{NC})} \end{aligned}$$

Comparing the equations for s^C, θ^C and V_e^C with these for s^{NC}, θ^{NC} and V_e^{NC} shows immediately that $s^C = s^{NC} = s$, $\theta^C = \theta^{NC} = \theta$ and $V_e^C = V_e^{NC} = V_e$. However, the values of the tax instruments τ, τ_a, ψ to achieve these values diœer in the commitment and no commitment case.

¿From the government budget constraint (??) and the expression for $\psi^{NC} = -(1-\beta)$ we ønd

$$\frac{g}{sm(\theta)} = \tau^{NC} V_e + \tau_a^{NC} - (1-\beta)c\sigma$$

Using $V_e = \eta(\bar{y} - c\sigma) - \frac{g}{sm(\theta)}$ this can be written as

$$(1+\tau^{NC})\frac{g}{sm(\theta)} = \tau^{NC}\eta(\bar{y} - c\sigma) + \tau_a^{NC} - (1-\beta)c\sigma$$

Combining this with the expression $(1+\tau^{NC})V_e = \beta(\bar{y} - c\sigma - \tau_a^{NC})$ yields

$$\begin{aligned} \tau_a^{NC} &= \frac{\eta - \beta}{1-\beta}(\bar{y} - c\sigma) + c\sigma \\ &= \tau_a^C + c\sigma \end{aligned}$$

where the expression for τ_a^C follows from proposition 5 (ii).

Finally, the expression for τ^{NC} is derived from the government budget constraint (??) written as

$$\begin{aligned} \frac{g}{sm(\theta)} &= \tau^{NC} V_e + \frac{\eta - \beta}{1-\beta}(\bar{y} - c\sigma) + c\sigma - (1-\beta)c\sigma \\ \frac{g}{sm(\theta)} &= \tau^C V_e + \frac{\eta - \beta}{1-\beta}(\bar{y} - c\sigma) \end{aligned}$$

Substracting these two equations yields

$$0 = (\tau^{NC} - \tau^C)V_e + \beta c\sigma$$

or equivalently

$$\tau^C = \tau^{NC} + \frac{\beta c\sigma}{V_e}$$

QED