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TAX COMPETITION, THE DISTRIBUTION OF MNE'S OWNERSHIP AND THE WAGE FORMATION PROCESS

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Abstract

This paper shows how the distribution of the ownership of multinational companies and the labour market conditions, especially the wage formation process, influence the outcome of interjurisdictional tax competition and coordination. In particular, it sets forth that equilibrium corporate tax rate can be negative, being a subsidy to the mobile factor, financed through a tax on the immobile one, and that foreign ownership of companies enables a jurisdiction which behaves non-cooperatively to export its tax burden through a too large tax rate on profits on its territory.

Keywords: tax competition, tax coordination, multinational firm, foreign ownership, labour market, wage formation.

JEL Classification: H32, H73, H87.

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1 Introduction

For the last twenty years, a huge amount of research has been conducted on interjurisdictional tax competition in general and international tax competition for capital in particular. Let us mention the pioneering work of Gordon (1983, 1986, 1992) or Razin and Sadka (1991) as well as the fundamental discussion proposed by Edwards and Keen (1996) and the state of the art, including the distributive aspects, of Cremer et al. (1996), Oates (1999), Wellisch (2000), Wildasin (1998) or Wilson (1999), all mentioned by Wildasin (2000).

The present paper, which extends previous work, a.o. with M. Hadhri - see Gérard and Hadhri (1993, 1994), Gérard (2001) -, focuses on the effect on the outcome of tax competition and coordination, of the labour market conditions, especially the wage formation process, on the one hand, and of the distribution of the ownership of a multijurisdictional firm on the other hand. By the former it has something in common with recent contributions of Fuest and Huber (1999, 2001), by the latter with Huizinga and Nielsen (1997) and Eijfinger and Wagner (2001). The results obtained in the paper are consistent with the standard lessons that the corporate income tax rate levied at source should vanish under tax competition and that such a framework leads to tax rates on the mobile factor income which are too small as compared with those which would appear at social optimum. However those standard lessons appear to be only specific cases generated by a model which can also produce less standard results like a negative tax rate on the income of the mobile factor, and too large tax rates on that factor at non-cooperative equilibrium. The argument beyond that is developed thereafter but is basically the following one : taking into account another source of tax revenue, deemed to be immobile and inelastically supplied, allows the jurisdiction to tax that captive tax base in order to finance public goods and to subsidise the mobile factor ; otherwise an increase in the foreign ownership of the tax base is an opportunity for the jurisdiction to export the burden of the tax on the mobile factor income, and thus to increase the corresponding tax rate. The first argument in particular is consistent with the empirical observation that effective tax rates - see King and Fullerton (1984), Gérard (1993) - can be negative and that many public authorities use to subsidise investment on their territory in order to increase the welfare of the residents.

Focusing on labour market conditions points out that the process of wage formation matters. Indeed, as shown in the paper, moving from rigid wage rates determined, say, by a bargaining process outside the model, to flexible market determined wages, implies that a given tax cut will have a smaller effect in terms of both the value of the firm and the level of tax revenue ; indeed, when wage rates are variable, the tax cut can generate extra pressure on the labour market, and then higher wage rates and labour costs, which in turn push the profit and the tax base downward.

Let us add that the degree of domestic ownership of the multijurisdictional firm can be interpreted either as the degree actually observed in the jurisdiction, which is the point of view adopted in this paper, or, alternatively, as the concern of the tax designer of the jurisdiction for the holders of capital assets, a viewpoint set forth in Gérard (2001).

As shown in the paper the interaction between labour market conditions and the distribution of ownership of the multijurisdictional firms appears to a key issue here.

Finally, the corporate tax system at work in this model is a pure source one, by which is meant that the corporate tax base consists of the profit produced on the territory by the joint use of capital and labour. This approach is consistent with both the most frequently tax system applied, at least in the European Union, to profits of a foreign permanent branch or of a foreign subsidiary - see article 7 of the OECD model tax convention and the Parent-Subsidiary EU Directive of July 23, 1990 -. It is also consistent with the consolidated tax base system recently set forth by the EU Commission - see EU Commission (2001) - provided that the apportionment criterion be the profit on the territory ; that criterion does not make a problem in this paper since there in no intracompany trade.

Section 2 thereafter presents the model, successively reviewing the behaviour of the multijurisdictional firm, the condition of the labour market, the capital income of the residents and the programme of the governments. Then we turn first to the interjurisdictional social equilibrium and, second, to the non-cooperative equilibrium. Some conclusions and avenues are suggested in section 5.

The model used here is kept as simple as possible in order to help focus on the mechanisms we want to set forth. In particular private and public consumptions are regarded as perfect substitutes. In line with that assumption, the precise determination of the tax rate on labour income is left aside ; however it plays a key role, its base determining the lower limit of the corporate tax rate, and thus the highest level of the subsidy.

2 The model

This section presents the model, dealing successively with the behaviour of the multijurisdictional firm, the conditions of the labour market, the capital income of the residents and the programme of the government.

2.1 Behaviour of the multijurisdictional firm

Let us consider a multijurisdictional firm which has to allocate one unit of investment between two jurisdictions h and f in such a way that its value is maximised. The firm uses two factors, capital, deemed to be mobile - not necessarily perfectly mobile - across jurisdictions, and labour assumed to be immobile and inelastically supplied, to produce goods which are sold everywhere in the world at a price standardized to be unity. Formally its problem consists in

$$\max_{\alpha,l,\bar{l}} V = \frac{(1-\tau_h)\, T_f(\alpha_h, l_h) + (1-\tau_f)\, T_f(\alpha_f, l_f)}{\rho} \tag{1}$$

assuming that the horizon of time is infinite and the various parameters invariant over time. In that equation α_h is the fraction of the capital invested in jurisdiction h - the home jurisdiction - and $\alpha_f = 1 - \alpha_h$ its counterpart in jurisdiction f, the foreign jurisdiction. Moreover τ_h and τ_f denote the corporate tax rates and T_h and T_f the corresponding tax bases and thus the before tax profits obtained in each jurisdiction ; l_h and l_f are the levels of employment provided by the firm, and ρ stands for the discounting rate ; however, without loss of generality we can assume $\rho = 1$.

The tax bases functions are of the form,

$$T_j(\alpha_j, l_f) = f(\alpha_j, l_j) - w_j l_j \quad , \quad j = h, f \tag{2}$$

and are assumed to be quasi concave since production functions $f(\alpha_j, l_j)$ are such; w_j is the wage rate. Maximisation of (1) implies the following first order conditions,

$$(1-\tau_h)\frac{\partial T_h}{\partial \alpha_h}(\alpha_h, l_h) - (1-\tau_f)\frac{\partial T_f}{\partial \alpha_f}(\alpha_f, l_f) = 0 \tag{3}$$

and

$$\frac{\partial T_j}{\partial l_j} = \frac{\partial f(\alpha_j, l_j)}{\partial l_j} - w_j = 0 \tag{4}$$

which in turn generate the demand functions,

$$\alpha_j = \alpha_j \left(\underset{-}{\tau_j}, \underset{+}{\tau_{j'}}, \underset{-}{w_j}, \underset{+}{w_{j'}} \right) \ , j' \neq j \tag{5}$$

and

$$l_j = l_j \left(\underset{-}{\tau_j}, \underset{+}{\tau_{j'}}, \underset{-}{w_j}, \underset{+}{w_{j'}} \right) \ , j' \neq j \tag{6}$$

The second order conditions are proved in Appendix A while the signs of the partial derivatives of the demand functions come from the standard comparative statics exercise presented in Appendix 2.

Let us add that, should the mobility of capital be perfect, which is not required here,

$$\frac{\partial \alpha_j}{\partial \tau_j} \to -\infty$$

2.2 Domestic labour income and wage formation

We assume that labour is not mobile across jurisdictions and is inelastically supplied in each jurisdiction. Then two polar situations are supposed to be possible in each jurisdiction : either the wage rate w_j clears a labour market always characterized by full employment or it does not and unemployment is present. In that latter case one can imagine that the wage rate obeys a game between the employers and the employees or their representatives, which is conducted outside the model. In that latter case too the investment of the multijurisdictional firm creates additional jobs in the jurisdiction, reducing unemployment, without making any upward pressure on the wage rate. Unlike that, in the former situation, the arrival of the multijurisdictional firm - and more precisely of multijurisdictional firms since the one we consider is a representative of a series - crowds out existing jobs and pushes the wage rate up ; that latter phenomenon is not anticipated by the multijurisdictional firm since, individually, it is price taker.

Thus gross labour income $y_j = w_j L_j$ is increased by the arrival of the multijurisdictional firm either since L_j increases or since w_j goes up. However, in that latter case the operating cost of the multijurisdictional firm goes up too.

More formally,

$$\frac{\partial y_j}{\partial \alpha_j} = k_j w_j \frac{\partial l_j}{\partial \alpha_j} + (1 - k_j) L_j \frac{\partial w_j}{\partial \alpha_j} > 0 \tag{7}$$

with $0 \leq k_j \leq 1$.

Finally, net labour income is $(1 - t_j)\, y_j$.

2.3 Capital income of the residents

The residents of the jurisdiction are deemed to hold a fraction θ_j of the shares of the multijurisdictional firm. Their gross capital income is thus $\theta_j V$.

Let us add that the fact that the multijurisdictional firm has to allocate its investment between jurisdictions h and f does not preclude that its ownership is more disseminated in the world ; therefore we don't require that the sum of θ_h and θ_f equals unity but we simply assume that if θ_h goes up, then θ_f goes down.

2.4 The governments

The government of jurisdiction j is interested in maximising the welfare of the residents of that jurisdiction identified with the sum of their private and public consumptions, with private and public consumption respectively defined by,

$$c_j = (1 - t_j)\, y_j + \theta_j V \tag{8}$$

and

$$g_j = t_j y_j + \tau_j T_j \tag{9}$$

so that the welfare of the jurisdiction is

$$W_j = y_j + \theta_j V + \tau_j T_j \tag{10}$$

to be maximised subject to the budget constraint

$$t_j y_j + \tau_j T_j \geq g_j \tag{11}$$

At this stage it is worth noticing that in this model, one euro used to finance private consumption is equally valued as one euro financing public expenditures. Otherwise the allocation of the residents' income between public and private uses is left aside in the model. Accordingly the tax levied on immobile income can be used both to finance public expenditures and to allow the corporate tax rate to be changed into a subsidy to the mobile factor, which in turn increases both gross labour income and capital income of the residents.

Therefore the maximisation process can be conducted as if there is only one tax instrument.

3 The interjurisdictional game

Prior to considering the interjurisdictional non-cooperative game, we examine the outcome of the model should a benevolent social planner be at work at interjurisdictional level. That approach provides us with a cooperative solution, or tax coordination outcome, which can be used as a benchmark for comparing non-cooperative solutions.

3.1 An interjurisdictional social equilibrium

The social planner maximises $W_h + W_f$ with respect to the available instruments, actually w.r.t. τ_f and τ_h,

$$\max_{\tau_h, \tau_f} (W_h + W_f) \tag{12}$$

The first order conditions of that programme are

$$\begin{aligned} \frac{d}{d\tau_h} &= \frac{\partial y_f}{\partial \tau_h} + \frac{\partial y_h}{\partial \tau_h} \\ &\quad + (\theta_f + \theta_h) \left[-T_h + (1 - \tau_f) \frac{\partial T_f}{\partial \tau_h} + (1 - \tau_h) \frac{\partial T_h}{\partial \tau_h} \right] \\ &\quad + T_h + \tau_h \frac{\partial T_h}{\partial \tau_h} + \tau_f \frac{\partial T_f}{\partial \tau_h} \\ &= 0 \end{aligned} \tag{13}$$

and

$$\begin{aligned}\frac{d}{d\tau_f} &= \frac{\partial y_f}{\partial \tau_f} + \frac{\partial y_h}{\partial \tau_f} \\ &\quad + (\theta_f + \theta_h)\left[-T_f + (1-\tau_f)\frac{\partial T_f}{\partial \tau_f} + (1-\tau_h)\frac{\partial T_h}{\partial \tau_f}\right] \\ &\quad + T_f + \tau_f \frac{\partial T_f}{\partial \tau_f} + \tau_h \frac{\partial T_h}{\partial \tau_f} \\ &= 0 \end{aligned} \tag{14}$$

However, by the first order conditions of the value maximization of the multijurisdictional firm and the envelop theorem,

$$(1-\tau_h)\frac{\partial T_h}{\partial \tau_h} + (1-\tau_f)\frac{\partial T_f}{\partial \tau_h}$$

reduces to

$$-(1-\tau_h)(1-k_h)\,l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} - (1-\tau_f)(1-k_f)\,l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}} \tag{15}$$

and similarly for its counterpart w.r.t. τ_f, while, for the same reason,

$$\tau_h \frac{\partial T_h}{\partial \tau_h} + \tau_f \frac{\partial T_f}{\partial \tau_h}$$

turns out to be,

$$\tau_h \left[\frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} - (1-k_h)\,l_h \underset{+}{\frac{\partial w_h}{\partial \tau_h}}\right] + \tau_f \left[-\frac{1-\tau_h}{1-\tau_f}\frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_f}{\partial \tau_h}} - (1-k_f)\,l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}}\right] \tag{16}$$

and similarly again for the condition w.r.t. τ_f.

Then the first order condition for the social planner, w.r.t. τ_h, can be usefully rewritten as

$$D_h(\tau_h, \tau_f) + X_h(\tau_h, \tau_f) = 0 \tag{17}$$

with

$$\begin{aligned} D_h(\tau_h,\tau_f) &= \underset{-}{\frac{\partial y_h}{\partial \tau_h}} + (1-\theta_h)\,T_h \\ &\quad +\theta_h\left[-(1-\tau_h)(1-k_h)\,l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} - (1-\tau_f)(1-k_f)\,l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}}\right] \\ &\quad +\tau_h\left[\frac{\partial f_h}{\partial \alpha_h}\underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} - (1-k_h)\,l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}}\right] \end{aligned} \tag{18}$$

the elements of the f.o.c. which correspond to the best interest of jurisdiction h ignoring the other one, and

$$\begin{aligned} X_h(\tau_h,\tau_f) &= \underset{+}{\frac{\partial y_f}{\partial \tau_h}} - \theta_f T_h \\ &\quad +\theta_f\left[-(1-\tau_h)(1-k_h)\,l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} - (1-\tau_f)(1-k_f)\,l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}}\right] \\ &\quad +\tau_f\left[-\frac{1-\tau_h}{1-\tau_f}\frac{\partial f_h}{\partial \alpha_h}\underset{-}{\frac{\partial \alpha_f}{\partial \tau_h}} - (1-k_f)\,l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}}\right] \end{aligned} \tag{19}$$

which corresponds to the internalisation of the effect on the other jurisdiction. In that latter expression it is a key issue to point out the term $-\theta_f T_h$. Indeed that term, which illustrates the fraction of the tax base on the territory of jurisdiction h which is attributed, or "exported", to residents of jurisdiction f, is negative while its absolute value is decreasing in τ_h. It turns out that there is no guarantee that $X_h(\tau_h,\tau_f) > 0$. Unlike that it may be, at least for relatively high values of θ_f that $X_h(\tau_h,\tau_f) < 0$ so that internalising the effect of τ_h on the welfare of the residents of jurisdiction f can lead to a smaller equilibrium value of that tax parameter.

For an immediate insight, consider the peculiar case where there is only one factor, capital ; then

$$X_h(\tau_h,\tau_f) = -\theta_f T_h - \tau_f \frac{1-\tau_h}{1-\tau_f}\frac{\partial f_h}{\partial \alpha_h}\underset{-}{\frac{\partial \alpha_f}{\partial \tau_h}}$$

Let us add that the outcome of that maximization is subject to the condition that

$$\underline{\tau_j} \leq \tau_j \leq 1 \tag{20}$$

with

$$\underline{\tau_j} = -\frac{\overline{t_j} y_j}{T_j} = -\frac{y_j}{T_j} \quad , \quad \overline{t_j} = 1 \tag{21}$$

where $\overline{t_j}$ stands for the upper limit for t_j. That last observation shows the importance of the size of a jurisdiction, a point put forward by Haufler and Wooton (1999). Moreover, notice that in this equilibrium there is no interjurisdictional government transfer from one jurisdiction to another.

3.2 Non-cooperative game and Nash equilibrium

Let us now examine the outcome of the interjurisdictional game if governments decide to play non-cooperatively, thus engaging in a process of tax competition.

Consider the behavior of government h. Maximization of W_h implies τ_h such that,

$$D_h(\tau_h, \tau_f) = 0 \tag{22}$$

Decomposition of the left hand side of that equation is instructive

$$\begin{aligned} D_h(\tau_h, \tau_f) &= \underset{-}{\frac{\partial y_h}{\partial \tau_h}} \\ &\quad -\theta_h \left[T_h + (1-\tau_h)(1-k_h) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} + (1-\tau_f)(1-k_f) l_f \underset{+}{\frac{\partial w_f}{\partial \tau_h}} \right] \\ &\quad + T_h + \tau_h \left[\frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} - (1-k_h) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} \right] \end{aligned} \tag{23}$$

labour income effect The first line of the right hand side of the equation gives the effect of a change in the corporate tax rate on labour income, either through a change in the amount of employed people or in the level of the wage rate, thus

$$\frac{\partial y_h}{\partial \tau_h} = k_h w_h \underset{-}{\frac{\partial l_h}{\partial \tau_h}} + (1-k_h) L_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} \tag{24}$$

capital income effect The second line provides us with the effect of a change in the domestic corporate tax rate on the income or the wealth of the domestic shareholders of the multijurisdictional firm. That expression depends in a key manner on the size of the domestic ownership of the firm measured by parameter θ_h, and on the conditions of the labour market at home and abroad. In particular, if the wage rate is flexible in both jurisdictions, a tax cut in jurisdiction h (i) will reduce the tax liabilities given the value of tax base T_h in the jurisdiction, (ii) but also will reduce the gross profit in that jurisdiction - or will limit its increase - since the tax cut, by making location in that jurisdiction more attractive, induces pressures on the local labour market which in turn push the wage cost up and, finally, (iii) will increase the gross profit in the other jurisdiction - or will limit its decline - since the considered tax cut, by pushing away investment from the other jurisdiction, will relax pressures on that jurisdiction's labour market, pushing down the labour cost, in that jurisdiction.

government budget effect Finally, the third line shows the effect of the tax change on the level of government revenue. It consists in a direct effect T_h - a tax cut decreases tax revenue given the tax base - and a strategic or tax base effect - on the one hand, by making the jurisdiction more attractive, a tax cut expands the tax base on the territory, but, on the other hand, resulting increased unit labour cost, if salaries are flexible, pushes down that tax base since wage cost is deductible against the corporate tax base -.

The second order condition of the maximisation w.r.t. τ_h requires that

$$D_{hh}(\tau_h, \tau_f) < 0 \tag{25}$$

Assuming, in order to keep the model as tractable as possible, that y, w, l and α are linear in the tax rates - or have been linearized for modelling purposes -, then,

$$\begin{aligned} D_{hh}\left(\tau_h,\tau_f\right) &= -\theta_h \frac{\partial}{\partial \tau_h} T_h + \theta_h \left(1-k_h\right) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} + \frac{\partial}{\partial \tau_h} T_h \\ &\quad + \left[\frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} - \left(1-k_h\right) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} \right] \\ &= \left(2-\theta_h\right) \left[\frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} - \left(1-k_h\right) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} \right] \\ &\quad + \theta_h \left(1-k_h\right) l_h \underset{-}{\frac{\partial w_h}{\partial \tau_h}} \end{aligned} \qquad (26)$$

which is negative. Notice that we can justify that the expression between brackets is negative by using an incentive compatibility argument.

Now let us inspect some particular cases, before showing the results from a comparative statics exercise and turning to the derivation of the reaction functions and the discussion of the Nash equilibrium.

3.2.1 Some particular cases

Suppose first that there is only one factor, capital, and that the residents of the jurisdiction own no fraction of the multijurisdictional firm, or, equivalently, that the government of the jurisdiction has no concern for capital income. Then we will introduce labour, first assuming that wages are fixed, then allowing them to vary in line with supply and demand on the labour market.

A single factor, capital Leaving labour aside, the first order condition (22) turns out to be

$$\left(1-\theta_h\right) T_h + \tau_h \frac{\partial f_h}{\partial \alpha_h} \underset{-}{\frac{\partial \alpha_h}{\partial \tau_h}} = 0 \qquad (27)$$

and

$$\tau_h = -\frac{(1-\theta_h)\,T_h}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} \tag{28}$$

Especially, if the residents of the jurisdiction own no fraction of the multijurisdictional firm, or, if the government of the jurisdiction has no concern for capital income,

$$\tau_h = -\frac{T_h}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} = \frac{\alpha_h}{\eta_h \left|\dfrac{\partial \alpha_h}{\partial \tau_h}\right|} \tag{29}$$

the interpretation of which is in line with Ramsey (1927) rule, η_h being the elasticity of the tax base w.r.t. α_h. That equation also shows that the equilibrium value of the tax rate is positive except if

$$\left|\frac{\partial \alpha_f}{\partial \tau_f}\right| \to \infty \tag{30}$$

which characterises perfect capital mobility, then $\tau_f = 0$.

If there is some ownership of the multijurisdictional firm by resident taxpayers or if the government has concern for that ownership, then the equilibrium value of the corporate tax rate is smaller and,

$$\tau_h = -\frac{(1-\theta_h)\,T_h}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} = \frac{(1-\theta_h)\,\alpha_h}{\eta_h \left|\dfrac{\partial \alpha_h}{\partial \tau_h}\right|} \tag{31}$$

If $\theta_f = 1 - \theta_h$, which is possible but not requested,

$$\tau_h = -\frac{\theta_f T_h}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} = \frac{\theta_f \alpha_h}{\eta_h \left|\dfrac{\partial \alpha_h}{\partial \tau_h}\right|}$$

and we immediately see that the tax rate on income of the mobile factor levied at source increases with the fraction of that rate which is supported by non resident taxpayers.

two factors, labour and capital Introducing labour, and first assuming that wages are fixed in both jurisdictions - $k_j = 1$ -, we now have,

$$\tau_h = -\frac{(1-\theta_h)\,T_h}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} - \frac{\dfrac{\partial y_h}{\partial \tau_h}}{\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h}} = \frac{(1-\theta_h)\,\alpha_h}{\eta_h\left|\dfrac{\partial \alpha_h}{\partial \tau_h}\right|} - \frac{\dfrac{\partial y_h}{\partial \tau_h}}{\eta_h\dfrac{\partial \alpha_h}{\partial \tau_h}}\frac{\alpha_h}{T_h} \tag{32}$$

so that the equilibrium tax rate on corporate income is still lower, in order to take into account the positive impact of a tax cut on labour income. Moreover we can no longer exclude that it becomes negative ; therefore we actually have that τ_f is equal to the maximum of the value provided by equation (32) and the one given by equation (21).

Consider especially the case of perfect mobility of capital, then the first element of the right hand side of equation (32) vanishes while, using the comparative statics results of Appendix 2, the second element turns out to be negative and,

$$\tau_h = -\frac{\dfrac{\partial^2 f_h}{\partial l_h \partial \alpha_h}}{\eta_h\dfrac{\partial^2 f_h}{\partial l_h^2}}\frac{\alpha_h}{T_h}$$

If we further assume that wage rates are flexible in both jurisdictions - $k_j = 0$ -,

$$\tau_f = \frac{(1-\theta_h)\,T_h}{-\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h} + (1-\theta_h)\,l_h\dfrac{\partial w_h}{\partial \tau_h}} + \frac{\dfrac{\partial y_h}{\partial \tau_h} - \theta_h\left[l_h\dfrac{\partial w_h}{\partial \tau_h} + (1-\tau_f)\,l_f\dfrac{\partial w_f}{\partial \tau_h}\right]}{-\dfrac{\partial f_h}{\partial \alpha_h}\dfrac{\partial \alpha_h}{\partial \tau_h} + (1-\theta_h)\,l_h\dfrac{\partial w_h}{\partial \tau_h}} \tag{33}$$

so that the numerator of the fraction is larger while the denominator is smaller. As a consequence, the tax rate is larger. The economic argument behind that is that under wage flexibility, a tax cut pushes the wage cost upward reducing the return of the tax cut both for the firm, in terms of profit, and for the government, in terms of tax base ; therefore the government is restrained from reducing the tax rate. However now the labour conditions in the other jurisdiction also matter since, if the wage rate is flexible in the other jurisdiction, the tax cut in the home jurisdiction will push the wage rate downward in the foreign jurisdiction as a response of that latter labour

θ	no labour	fixed wages
$\theta = 0$	$\frac{\alpha_h}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert} \equiv \tau_h^*$	$\tau_h^* - \frac{\frac{\partial y_h}{\partial \tau_h}}{\eta_h \frac{\partial \alpha_h}{\partial \tau_h}} \frac{\alpha_h}{T_h}$
$\theta > 0$	$\tau_h^* - \frac{\theta_h \alpha_h}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert}$	$\tau_h^* - \frac{\theta_h \alpha_h}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert} - \frac{\frac{\partial y_h}{\partial \tau_h}}{\eta_h \frac{\partial \alpha_h}{\partial \tau_h}} \frac{\alpha_h}{T_h}$
	flexible wages	
$\theta = 0$	$\tau_h^* \frac{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert + \frac{\alpha_h}{T_h} l_h \frac{\partial w_h}{\partial \tau_h}} - \frac{\left\lvert \frac{\partial y_f}{\partial \tau_f} \right\rvert \frac{\alpha_f}{T_f}}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert + \frac{\alpha_h}{T_h} l_h \frac{\partial w_h}{\partial \tau_h}}$	
$\theta > 0$	$\tau_h^* \frac{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert - (1-\theta_h) \frac{\alpha_h}{T_h} l_h \left\lvert \frac{\partial w_h}{\partial \tau_h} \right\rvert} - \frac{\left\lvert \frac{\partial y_h}{\partial \tau_h} \right\rvert \frac{\alpha_h}{T_h} + \frac{\alpha_h}{T_h} \theta_h \left[l_h \frac{\partial w_h}{\partial \tau_h} + \left(1-\tau_f\right) l_f \frac{\partial w_f}{\partial \tau_h} \right]}{\eta_h \left\lvert \frac{\partial \alpha_h}{\partial \tau_h} \right\rvert - (1-\theta_h) \frac{\alpha_h}{T_h} l_h \left\lvert \frac{\partial w_h}{\partial \tau_h} \right\rvert}$	

Table 1: equilibrium values of the corporate tax rate

market to the shift in investment ; that phenomenon will push the home jurisdiction to reduce its corporate tax rate, and that incentive will increase with the domestic ownership of the multijurisdictional firm.

To have some additional insight consider the case $\theta_h = 0$. Then, the effect of the flexibility of wages is to push the corporate tax rate upward since the denominator of the fraction is now smaller,

$$\tau_h = \frac{T_h + \frac{\partial y_h}{\partial \tau_h}}{-\frac{\partial f_h}{\partial \alpha_h} \frac{\partial \alpha_h}{\partial \tau_h} + l_h \frac{\partial w_h}{\partial \tau_h}} \tag{34}$$

Table 1 summarises that discussion,

3.2.2 Comparative statics

A comparative statics exercise can be conducted using the property that

$$\frac{d\tau_h}{dx} = -\frac{\frac{\partial D_h}{\partial x}}{D_{hh}} \propto \frac{\partial D_h}{\partial x} \tag{35}$$

since $D_{hh} < 0$ by the second order conditions of the maximization of the residents' social welfare by the government. Therefore we observe the following results.

Variation of the domestic ownership of the multijurisdictional firm An increase in the domestic ownership of the multijurisdictional firm pushes the equilibrium value of the tax rate downward. Indeed,

$$\frac{d\tau_h}{d\theta_h} \propto -\left[T_h + (1-\tau_h)(1-k_h)\, l_h \frac{\partial w_h}{\partial \tau_h} + (1-\tau_f)(1-k_f)\, l_f \frac{\partial w_f}{\partial \tau_h}\right] < 0 \tag{36}$$

In other words, when the fraction of the corporate tax supported by the residents increases, the desired level of that tax decreases.

Variation of the domestic labour market conditions The effect of increased rigidity regarding the formation of the wage rate on the domestic labour market depends on the distribution of the ownership of the multijurisdictional firm.

Inspection of the equation

$$\frac{d\tau_h}{dk_h} \propto \left[w_h \frac{\partial l_h}{\partial \tau_h} - L_h \frac{\partial w_h}{\partial \tau_h}\right] + \theta_h \left[(1-\tau_h)\, l_h \frac{\partial w_h}{\partial \tau_h}\right] + \tau_h l_h \frac{\partial w_h}{\partial \tau_h} \tag{37}$$

first shows that if no fraction of the multijurisdictional firm is owned by resident taxpayers - $\theta_h = 0$ -, then the sign of the effect of increased rigidity will depend on the balance between the gain in terms of jobs and the losses in terms of wages, on the one hand, and the gain in terms of tax revenue from less tax base reduction on the other hand. Suppose first that the level of employment in the multijurisdictional sector is large relative to the one in the purely domestic sector, then the first term in the first bracket of the right hand side of the equation dominates the second one and the sign is negative : increased rigidity is an incentive for the government to cut corporate tax in order to get extra jobs. Unlike that, if the multijurisdictional sector is small, e.g. in a large jurisdiction with an important domestic sector, the second term of the first bracket dominates the first one and the sign is positive : an increased rigidity in the wage formation process is an incentive for the government to refrain from pushing the tax rate downward ; indeed increased rigidity prevents the labour community as a whole benefiting from the increased wage rate generated by a multijurisdictional firms' attractive corporate tax cut.

Now, when a positive fraction of the multijurisdictional firm is owned by resident taxpayers, increased rigidity is also regarded by resident shareholders as a situation which makes increased labour cost less likely and thus increases the profitability of the firm ; then the jurisdiction is more likely to call for a further tax cut when rigidity increases.

Thus the sign of the effect is more likely to be negative when domestic ownership of the multijurisdictional firm is large and when the multijurisdictional sector is also large as compared to the purely domestic one.

Variation of the foreign labour market conditions Finally, increased rigidity in the wage formation process in the other jurisdiction reduces the gain for the shareholders of the multijurisdictional firm, from a tax cut in the domestic jurisdiction ; indeed that tax cut is then less likely to generate a downward movement in the wage rate abroad. Being less profitable for the multijurisdictional firm domestic holders, the tax cut will be less important as long as the interest of those multijurisdictional firm domestic holders is a concern for the jurisdiction. This is shown by the equation below,

$$\frac{d\tau_h}{dk_f} \propto \theta_h \left[(1 - \tau_f) \, l_f \frac{\partial w_f}{\partial \tau_h} \right] \geq 0 \tag{38}$$

3.2.3 The reaction functions and the Nash equilibrium

Using equation (35), the response of jurisdiction h in terms of its corporate tax rate to a decision of jurisdiction f to change its own rate is given by,

$$\frac{d\tau_h}{d\tau_f} = -\frac{1}{D_{hh}} \frac{\partial D_h}{\partial \tau_f} > 0 \tag{39}$$

so that the tax rates are strategic complements. Indeed,

$$\frac{\partial D_h}{\partial \tau_f} = \theta_h \left[(1 - k_f) \, l_f \frac{\partial w_f}{\partial \tau_h} \right] + (1 - \theta_h) \left[\frac{\partial f_h}{\partial \alpha_h} \frac{\partial \alpha_h}{\partial \tau_f} - (1 - k_h) \, l_h \frac{\partial w_h}{\partial \tau_f} \right] > 0 \tag{40}$$

Symmetric result appears for the other jurisdiction.

The Nash equilibrium which comes out is stable if

$$|E| = \begin{vmatrix} 1 & -\frac{1}{D_{hh}}\frac{\partial D_h}{\partial \tau_f} \\ -\frac{1}{D_{ff}}\frac{\partial D_f}{\partial \tau_h} & 1 \end{vmatrix} > 0 \tag{41}$$

which, under the assumption of linearisation already issued, and the further one that the effects of the tax parameters are symmetric, is satisfied if

$$\frac{\partial f_h}{\partial \alpha_h}\left|\frac{\partial \alpha_h}{\partial \tau_h}\right| > (1-\theta_h)(1-k_h)\, l_h \left|\frac{\partial w_h}{\partial \tau_h}\right| + \theta_h (1-k_f)\, l_f \left|\frac{\partial w_h}{\partial \tau_h}\right| \tag{42}$$

which is likely to hold. Examples of sufficient conditions aret $k_f = 1$, $\theta_h = 0$ or $(1-k_h)\, l_h = (1-k_f)\, l_f$.

Let us consider still another case for illustrative purposes, the one where $k_h = k_f = 0$. Then,

$$\frac{d\tau_h}{d\tau_f} = \frac{1-\theta_h}{2-\theta_h} > 0 \tag{43}$$

Moreover, from equation (32), we can write now that

$$\tau_h = \frac{(1-\theta_h)\, a\tau_f - b}{(2-\theta_h)\, a} \tag{44}$$

where $a = -\frac{\partial f_h}{\partial \alpha_h}\frac{\partial \alpha_h}{\partial \tau_h} = \frac{\partial f_h}{\partial \alpha_h}\frac{\partial \alpha_h}{\partial \tau_f} > 0$ and $b = \frac{\partial y_h}{\partial \tau_h} > 0$, and similarly that

$$\tau_f = \frac{(1-\theta_f)\, a\tau_h - b}{(2-\theta_f)\, a}$$

Then, at Nash equilibrium,

$$\tau_h = \tau_f = -\frac{b}{a} \tag{45}$$

a negative value independent of the distribution of the ownership ; that equilibrium is stable since $a > 0$.

We can finally conduct a new exercise of comparative statics. We now have that,

$$\frac{d\tau_h^N}{dx} = \frac{1}{|E|}\left[\frac{\partial \tau_h}{\partial x} + \frac{\partial \tau_h}{\partial \tau_f}\frac{\partial \tau_f}{\partial x}\right] \tag{46}$$

so that the effect of a change in, say θ_h, can now be decomposed into a direct effect and a strategic one. Since both $|E|$ and $\frac{\partial \tau_h}{\partial \tau_f}$ are positive - assuming the equilibrium is stable - the strategic effect reinforces the direct one ; the direct one itself however is larger than in a closed economy - the case investigated previously - since $0 < |E| < 1$.

Moreover we can complete the comparative statics conducted before, adding that, using a superscript N to indicate that we are at Nash equilibrium

$$\frac{d\tau_h^N}{d\theta_f} = \frac{1}{|E|} \frac{\partial \tau_h}{\partial \tau_f} \frac{\partial \tau_f}{\partial \theta_f} \propto \frac{\partial \tau_f}{\partial \theta_f} \tag{47}$$

with,

$$\frac{\partial \tau_f}{\partial \theta_f} = -\left[T_f + (1 - \tau_f)(1 - k_f) l_f \frac{\partial w_f}{\partial \tau_f} + (1 - \tau_h)(1 - k_h) l_f \frac{\partial w_h}{\partial \tau_f} \right] < 0$$

Taken separately, an increase in the ownership of the multijurisdictional firm by residents of the competing jurisdiction pushes the corporate tax rate downward in the home jurisdiction : residents of the foreign jurisdiction indeed ask for a reduced tax rate in their own jurisdiction and residents of the home jurisdiction react by asking a further reduction of the tax rate on their territory.

However if we combine a reduction in the domestic ownership with an increase in the foreign ownership,

$$\frac{d\tau_h^N}{d\theta_h} + \frac{d\tau_h^N}{d\theta_f} \propto \frac{\partial \tau_h}{\partial \theta_h} + \frac{\partial \tau_h}{\partial \tau_f} \frac{\partial \tau_f}{\partial \theta_h} + \frac{\partial \tau_h}{\partial \tau_f} \frac{\partial \tau_f}{\partial \theta_f} \tag{48}$$

The decrease in domestic ownership pushes the tax rate upward directly and indirectly since the strategic complementarity reinforces the effect, while the corresponding increase in the foreign ownership pushes the foreign tax rate downward and the domestic rate too so that, assuming that the two strategic effects cancel, the outcome of the shift in the ownership to the other jurisdiction is an increase in the domestic corporate tax rate.

4 Conclusion

This paper has shown that the outcome of tax competition, and then of tax coordination, depends in a key manner on both labour market conditions, especially the wage formation process, and the distribution of firm ownership.

The paper is consistent with traditional lessons of tax competition models that the tax rate on mobile factor income tends to vanish and that tax competition turns out to produce too low tax rate on that income ; however it shows that those are particular cases of a model which can also produce less standard results, especially that equilibrium value of the corporate income tax rate can be negative, thus turned out to be a subsidy, and that non-cooperative mobile factor income tax rate can be too large.

In particular, the following observations have been set forth.

First, providing extra job opportunities, or higher pay, to domestic workers is *per se* an incentive to reduce the corporate tax rate which can even become negative. In that latter case, multijurisdictional firm investment is subsidised by means of money levied on wage income through a labour income tax. The revenue of that latter levy and then the level of the subsidy is obviously determined by the size of the labour income tax base, and thus by the size of the jurisdiction.

Second, entering into tax competition when domestic underemployment is high can lead to a larger cut in the corporate tax rate than when full employment is at work, especially if the best interest of the domestic holders of the multijurisdictional firm capital is taken into account. Indeed, when underemployment prevails, there is no risk that the investment leads the multijurisdictional firm to pay a higher wage rate in order to be able to hire workers.

Third, for symmetric reason engaging into tax competition with a foreign jurisdiction experimenting underemployment refrains a jurisdiction from reducing its own corporate tax rate, again when the best interest of the domestic holders of the multijurisdictional firm capital is taken into account : indeed those stockholders are less interested in moving capital away from abroad since there is no gain in terms of reduced wage cost in that latter jurisdiction.

Those latter two observations show that the effect of the labour market conditions, at home and abroad, depends in a key manner on the distribution of the multijurisdictional firm ownership and thus on the concern of the governement for capital holders.

Fourth, entering into tax competition with a jurisdiction of large multijurisdictional firm ownership leads to a higher domestic corporate tax rate, part on the tax burden being exported.

Finally it appears that substituting fiscal cooperation, or coordination, for tax competition, can lead to a larger or a smaller equilibrium tax rate depending on both the labour market conditions and the the importance of multijurisdictional firm ownership in the partner jurisdiction : if the partner jurisdiction has a high ownership, we cannot rule out that the cooperatively decided domestic corporate tax rate will be smaller.

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Appendix 1. Second order conditions of the maximization of the Value of the firm

Second order conditions first are,

$$\frac{\partial^2 V}{\partial \alpha_h^2} < 0$$

and

$$\frac{\partial^2 V}{\partial l_j^2} < 0$$

which requires

$$(1-\tau_h)\frac{\partial^2 f_h}{\partial \alpha_h^2}(\alpha_h, l_h) + (1-\tau_f)\frac{\partial^2 f_f}{\partial \alpha_f^2}(\alpha_f, l_f) < 0$$

and

$$\frac{\partial^2 f_j}{\partial l_j^2}(\alpha_h, l_h) < 0$$

satisfied by the concavity of the production functions.

Second order conditions are also that

$$\begin{vmatrix} \frac{\partial^2 V}{\partial \alpha_h^2} & \frac{\partial^2 V}{\partial \alpha_h \partial l_j} \\ \frac{\partial^2 V}{\partial l_j \partial \alpha_h} & \frac{\partial^2 V}{\partial l_j^2} \end{vmatrix} > 0$$

and

$$\begin{vmatrix} \frac{\partial^2 V}{\partial l_h^2} & \frac{\partial^2 V}{\partial l_h \partial l_f} \\ \frac{\partial^2 V}{\partial l_f \partial l_h} & \frac{\partial^2 V}{\partial l_f^2} \end{vmatrix} > 0$$

In the first matrix, the main diagonal elements are negative so that their product is positive, while the terms on the other diagonal are both positive since

$$\frac{\partial^2 f_h}{\partial \alpha_h \partial l_h} > 0 \ , \ \frac{\partial^2 f_f}{\partial \alpha_f \partial l_f} > 0$$

so that the determinant is positive if the production functions have the good shape. In the second matrix, the main diagonal elements are also both negative so that their product is positive, while now the off diagonal elements are zero since

$$\frac{\partial^2 f_j}{\partial l_j \partial l_{j'}} = 0 \ , \ j \neq j'$$

and again the determinant is positive.

Seconder order conditions are finally that

$$\begin{vmatrix} \frac{\partial^2 V}{\partial \alpha_h^2} & \frac{\partial^2 V}{\partial \alpha_h \partial l_j} & \frac{\partial^2 V}{\partial \alpha_h \partial l_h} \\ \frac{\partial^2 V}{\partial l_h \partial \alpha_h} & \frac{\partial^2 V}{\partial l_h^2} & \frac{\partial^2 V}{\partial l_h \partial l_f} \\ \frac{\partial^2 V}{\partial l_f \partial \alpha_h} & \frac{\partial^2 V}{\partial l_f \partial l_h} & \frac{\partial^2 V}{\partial l_f^2} \end{vmatrix} < 0$$

We know from the inspection of the previous conditions that

$$\begin{vmatrix} \frac{\partial^2 V}{\partial \alpha_h^2} & \frac{\partial^2 V}{\partial \alpha_h \partial l_j} & \frac{\partial^2 V}{\partial \alpha_h \partial l_h} \\ \frac{\partial^2 V}{\partial l_h \partial \alpha_h} & \frac{\partial^2 V}{\partial l_h^2} & 0 \\ \frac{\partial^2 V}{\partial l_f \partial \alpha_h} & 0 & \frac{\partial^2 V}{\partial l_f^2} \end{vmatrix}$$

so that the value of the determinant is

$$\frac{\partial^2 V}{\partial \alpha_h^2} \frac{\partial^2 V}{\partial l_h^2} \frac{\partial^2 V}{\partial l_f^2} - \frac{\partial^2 V}{\partial l_f \partial \alpha_h} \frac{\partial^2 V}{\partial l_h^2} \frac{\partial^2 V}{\partial \alpha_h \partial l_h} - \frac{\partial^2 V}{\partial l_h \partial \alpha_h} \frac{\partial^2 V}{\partial \alpha_h \partial l_j} \frac{\partial^2 V}{\partial l_f^2} < 0 \qquad (49)$$

The first term of that expression is clearly negative ; in the second term the first two elements are negative and the last one is positive so that the term itself is positive ; in the last term the first element is positive and the other two are negative so that the term is positive. As a consequence the determinant is negative.

Appendix 2. Comparative statics at firm level

Let us compute the total derivatives of those equations. We obtain

$$\begin{aligned} & \left[(1-\tau_h) \frac{\partial f_h^2}{\partial \alpha_h^2} + (1-\tau_f) \frac{\partial f_f^2}{\partial \alpha_f^2} \right] d\alpha_h + (1-\tau_h) \frac{\partial f_h^2}{\partial \alpha_h \partial l_h} dl_h \\ & - (1-\tau_f) \frac{\partial f_f^2}{\partial \alpha_f \partial l_f} dl_f - d\tau_h \frac{\partial f_h}{\partial \alpha_h} + d\tau_f \frac{\partial f_f}{\partial \alpha_f} \\ = & \ 0 \end{aligned}$$

$$\frac{\partial^2 f_h}{\partial l_h \partial \alpha_h} d\alpha_h + \frac{\partial^2 f_h}{\partial l_h^2} dl_h - dw_h = 0$$

and

$$-\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f} d\alpha_h + \frac{\partial^2 f_f}{\partial l_f^2} dl_f - dw_f = 0$$

or, in a matrix form,

$$D \begin{bmatrix} d\alpha_h \\ dl_h \\ dl_f \end{bmatrix} = \begin{bmatrix} d\tau_h \frac{\partial f_h}{\partial \alpha_h} - d\tau_f \frac{\partial f_f}{\partial \alpha_f} \\ dw_h \\ dw_f \end{bmatrix}$$

with

$$D = \begin{bmatrix} (1-\tau_h) \frac{\partial f_h^2}{\partial \alpha_h^2} + (1-\tau_f) \frac{\partial f_f^2}{\partial \alpha_f^2} & (1-\tau_h) \frac{\partial f_h^2}{\partial \alpha_h \partial l_h} & -(1-\tau_f) \frac{\partial f_f^2}{\partial \alpha_f \partial l_f} \\ \frac{\partial^2 f_h}{\partial l_h \partial \alpha_h} & \frac{\partial^2 f_h}{\partial l_h^2} dl_h & 0 \\ -\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f} & 0 & \frac{\partial^2 f_f}{\partial l_f^2} \end{bmatrix}$$

Then,

$$\begin{bmatrix} d\alpha_h \\ dl_h \\ dl_f \end{bmatrix} = D^{-1} \begin{bmatrix} d\tau_h \frac{\partial f_h}{\partial \alpha_h} - d\tau_f \frac{\partial f_f}{\partial \alpha_f} \\ dw_h \\ dw_f \end{bmatrix}$$

As a consequence,

$$\begin{aligned} d\alpha_h &= \frac{1}{|D|} \frac{\partial^2 f_h}{\partial l_h^2} \frac{\partial^2 f_f}{\partial l_f^2} \left[d\tau_h \frac{\partial f_h}{\partial \alpha_h} - d\tau_f \frac{\partial f_f}{\partial \alpha_f} \right] \\ &\quad - \frac{1}{|D|} (1-\tau_h) \frac{\partial f_h^2}{\partial \alpha_h \partial l_h} \frac{\partial^2 f_f}{\partial l_f^2} dw_h \\ &\quad + \frac{1}{|D|} (1-\tau_f) \frac{\partial f_f^2}{\partial \alpha_f \partial l_f} \frac{\partial^2 f_h}{\partial l_h^2} dw_f \end{aligned}$$

	$d\tau_h$	$d\tau_f$	dw_h	dw_f
$d\alpha_h$	$-$	$+$	$-$	$+$
$d\alpha_f$	$+$	$-$	$+$	$-$
dl_h	$-$	$+$	$-$	$+$
dl_f	$+$	$-$	$+$	$-$

Table 2: Comparative statics at firm level

$$\begin{aligned} dl_h &= -\frac{1}{|D|}\frac{\partial^2 f_h}{\partial l_h \partial \alpha_h}\frac{\partial^2 f_f}{\partial l_f^2}\left[d\tau_h \frac{\partial f_h}{\partial \alpha_h} - d\tau_f \frac{\partial f_f}{\partial \alpha_f}\right] \\ &+\frac{1}{|D|}\left[(1-\tau_h)\frac{\partial f_h^2}{\partial \alpha_h^2} + (1-\tau_f)\frac{\partial f_f^2}{\partial \alpha_f^2}\right]\frac{\partial^2 f_f}{\partial l_f^2} dw_h \\ &-\frac{1}{|D|}(1-\tau_f)\frac{\partial f_f^2}{\partial \alpha_f \partial l_f}\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f} dw_h \\ &-\frac{1}{|D|}(1-\tau_f)\frac{\partial f_f^2}{\partial \alpha_f \partial l_f}\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f} dw_f \end{aligned}$$

and

$$\begin{aligned} dl_f &= \frac{1}{|D|}\frac{\partial^2 f_h}{\partial l_h^2}\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f}\left[d\tau_h \frac{\partial f_h}{\partial \alpha_h} - d\tau_f \frac{\partial f_f}{\partial \alpha_f}\right] \\ &-\frac{1}{|D|}(1-\tau_h)\frac{\partial f_h^2}{\partial \alpha_h \partial l_h}\frac{\partial^2 f_f}{\partial l_f \partial \alpha_f} dw_h \\ &+\frac{1}{|D|}\left[(1-\tau_h)\frac{\partial f_h^2}{\partial \alpha_h^2} + (1-\tau_f)\frac{\partial f_f^2}{\partial \alpha_f^2}\right]\frac{\partial^2 f_h}{\partial l_h^2} dw_f \\ &-\frac{1}{|D|}(1-\tau_h)\frac{\partial f_h^2}{\partial \alpha_h \partial l_h}\frac{\partial^2 f_h}{\partial l_h \partial \alpha_h} dw_f \end{aligned}$$

with $|D| < 0$ by equation (49).

It turns out that the signs of the comparative statics are those given by Table 2.