Monetary Policy, Model Uncertainty and Exchange Rate Volatility

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Abstract

This paper proposes an explanation of the shifts in the volatility of exchange rate returns that relies on standard present value exchange rate models. Agents are uncertain about the true data generating model and deal with the model uncertainty by making inference on the models and their parameters–a mechanism I call model learning. I show how model learning may lead agents to focus excessively on a subset of fundamental variables. As a result, exchange rate volatility is mainly determined by the dynamics of this subset of fundamentals. As agents switch between models the nominal exchange rate volatility varies accordingly even though the underlying fundamentals processes remain time-invariant. I investigate the relevance of this result empirically within the Taylor-rule based exchange rate model applied to the British Pound/US Dollar exchange rate. The results suggest that the observed change in volatility was triggered by a shift between models.

JEL-Code: F31, F41, E44.

Keywords: exchange rate economics, monetary policy, model uncertainty.

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1 Introduction

One of the well known and much documented facts in international economics are shifts in the volatility of exchange rates. Floating exchange rates display low and high volatility regimes, as documented by Engel and Hamilton (1990). Numerous researchers attempted to link these volatility shifts to the dynamics of macroeconomic fundamental variables. Mussa (1986), Gerlach (1988), Baxter and Stockman (1989), and Flood and Rose (1995), for example, observed that in low inflation countries the variability of most of the aggregate variables is unaffected by the exchange rate regime. As a result, the consensus emerged that there is remarkably little evidence of a systematic relationship between the volatilities of exchange rates and macroeconomic variables. Yet, this stylized fact is inconsistent with theories that model the exchange rate as a variable reflecting underlying economic shocks.

The empirical literature suggests parameter instability as an explanation of the difficulties in finding a link between the macroeconomic variables and exchange rates. Schinasi and Swami (1989) document this fact and show that time-varying parameter exchange rate models outperform the random walk in out of sample forecasting. Cheung, Chinn and Pascual (2005), Rossi (2006) and Sarno and Valente (2008) find that the predictive power of different fundamental exchange rate models depends on the currency and the forecast horizon considered.

This paper proposes a theoretical framework which can explain a time-varying link between exchange rates and the underlying macroeconomic variables. This is demonstrated using a standard present value exchange rate model, where agents are uncertain about the true data generating model and corresponding parameters. They deal with this model uncertainty by making the inference on the models and their parameters and they face costs to estimating these models. This mechanism will be called 'model learning'. The paper demonstrates that learning about the model of the exchange rate may lead agents to focus excessively on a subset of fundamental variables at different points in time. When agents choose misspecified models it alters the weight placed on selected fundamental variables relative to others. Because agents are uncertain about which approximating model is best, and the sample size is limited, they switch between misspecified models over time. The present value exchange rate model has self-referential structure and thus the chosen model feeds back into the actual exchange rate. As a result, as agents switch between models the nominal exchange rate volatility varies accordingly, even though the underlying fundamentals processes remain time-invariant.

I provide an empirical illustration of this theoretical result. I show how the changes in the volatility of British Pound/US Dollar returns can be explained by a shift in the model and therefore macroeconomic variables. These macroeconomic variables are implied by the Taylor-rules of the two countries.

Since conventional macroeconomic models failed to explain the foreign exchange markets behavior, most of the recent literature explores numerous forms of bounded rationality. Jeanne and Rose (2002) show that the presence of noise trading can create additional exchange rate volatility and generate shifts in volatility regimes depending on the monetary policy regime.

As boundedly rational agents typically may not have complete information of the economic environment, numerous researchers consider different learning mechanisms of agents. Arifovic (1996) develops a two countries' overlapping generations model where agents update their decisions using a selection mechanism based on a genetic algorithm. She finds persistent fluctuations of the exchange rate in this model. Gourinchas and Tornell (2004) develop a nominal exchange rate determination model in which investors constantly try to determine whether interest rate shocks are transitory or persistent. They show that misperception of the agents can account for several anomalies in the exchange rate data.

This paper is closely related to studies by Lewis (1989a and 1989b) and Bacchetta and Van Vincoop (2004). Lewis (1989a and 1989b) analyzes the exchange rate dynamics generated by a change in the process of fundamental variables when the agents use Bayesian updating. Bacchetta and Van Vincoop (2004) present a theoretical framework where incomplete and heterogeneous information in the foreign exchange market can lead investors to attach excessive weight to an observed fundamental. Both approaches make a case of 'imperfect knowledge' of agents. While Lewis' (1989a and 1989b) representative agent does not know the process followed by fundamentals, Bacchetta and Van Vincoop's (2004) heterogeneous agents do not know the information sets of the other market participants. In this paper, the agents face model uncertainty, so that they do not know either the parameters or the model structure. and they need to employ model learning.

From the theoretical perspective, this paper fits into recent literature on econometric model uncertainty. Branch and Evans (2006b and 2007) study model uncertainty and its implications on the dynamics of inflation and GDP. They entail model uncertainty through dynamic predictor selection and parameters drift. They demonstrate that the econometric model uncertainty results in the dynamic paths of inflation and output which are consistent with the observed empirical regularities.

Cho and Kasa (2008) propose a model validation process as a framework for model uncertainty faced by the economic agents. More precisely, they assume that agents continue to use a model until it is statistically rejected by a general specification test and another model is randomly selected. Their results suggest that model validation process may explain the persistence of the Fed's belief in an exploitable Phillips Curve.

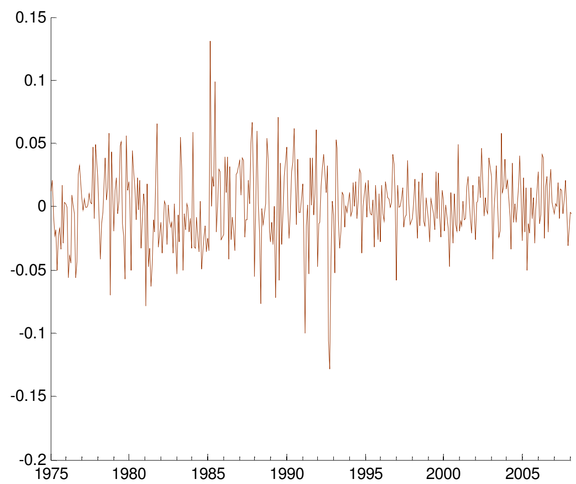
This paper makes several contributions relative to the previous work on econometric model uncertainty. Unlike Branch and Evans (2006b and 2007), who assume that agents must select restricted models, this paper does not directly assume the underparametrization. Instead it explicitly models the cost of estimating larger models by using BIC as the model selection criterion. Asymptotically, BIC will prefer the correct model, thus emphasizing that transitional learning dynamics can be empirically important. Unlike Kasa and Cho (2008), who assume that the rejection of the current model leads to the random choice of the new model, in this paper the new model is chosen on the basis of fit-parsimony trade-off. Furthermore, relative to Cho and Kasa (2008) who assume that agents apply the Kullback-Leibler Information Criterion (KLIC), the use of BIC is simple, and places econometrician and economic agent on equal ground.

The remainder of this paper is organized as follows. In the second section, I provide an illustration of the stylized fact that the shifts in the volatility of the exchange rate are unrelated to the dynamics of macroeconomic variables. For this purpose, I analyze the dynamics of the British Pound/US dollar returns. The third section presents the general asset pricing model of the exchange rate, describes the mechanism of expectation formation and derives the resulting equilibrium and its characteristics. Section 4 introduces an empirical illustration of theoretical result using the Taylor-rule model of the exchange rate. The fifth section demonstrates the results of numerous estimations and calibrations and Section 6 concludes.

2 Shifts in the volatilities of exchange rate and macroeconomic variables

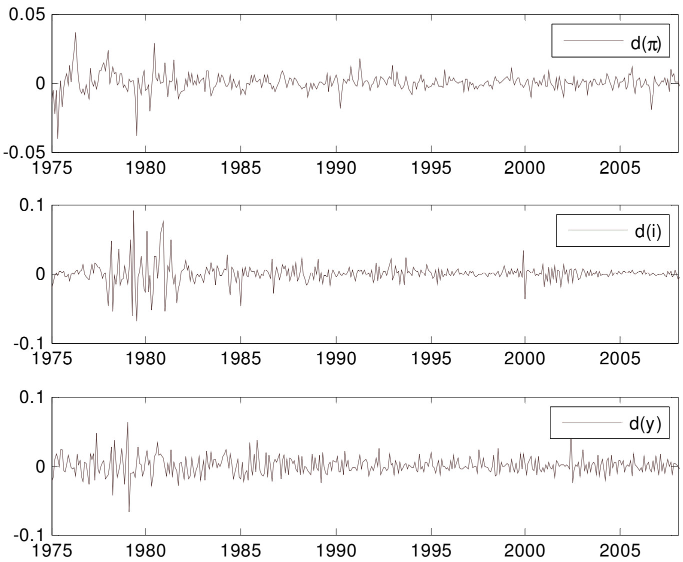
Figure 1 displays post Bretton Woods British Pound/US Dollar returns. It suggests that the volatility of the returns decreased after 1993.

Figure 1: **GBP/USD returns**



The exchange rate is usually modeled as a variable reflecting underlying macroeconomic shocks. Most of these models relate exchange rate movements to the behavior of domestic and foreign variables as interest and inflation rates but also some measures of the economic activity like output gap[1].

Figure 2: **Macroeconomic volatility**



$d(\pi)$, $d(i)$ and $d(y)$ denote the first difference of inflation rate, interest rate and output gap, respectively. All the 3 series are differentials between the UK and US variables.

Figure 2 plots the differentials between the UK and the US interest and inflation rates and output gap, during the post Bretton-Wood sample. All of the three series display high volatility figures at the end of the 1970s and the beginning of the 1980s which are mainly due to the macroeconomic instability during the Great Inflation. As the monetary tightening was brought in by Paul Volcker in the US and Margaret Thatcher in the UK in the beginning of the 1980s, inflation rate differential and especially interest rate differential, became much less volatile. Yet, there is no evidence of decrease in the volatilities around the beginning of 1993, the date when the variability of the British Pound/US Dollar exchange rate appears to have declined.

[1] Most of the macroeconomic models of the exchange rate include UIP and either relative or absolute version of PPP.

Table 1: **Breaks in the volatility of British Pound/US Dollar returns and macroeconomic series**

Exchange rate returns volatility	
Break date	1993M3***
Regime	**Estimate**
1975M1-1993M3	0.0356***
1993M4-2008M12	0.0215***

Macroeconomic Volatility		
Interest rate differential	1st Break date	1981M9***
	2nd Break date	1989M4***
Inflation rate differential	1st Break date	1980M5***
Output gap differential	1st Break date	1986M8***

CI stands for confidence intervals. *** denote significance at the 1 percent level.

To be sure that the visual inspection is accurate Table 1 reports tests for changes in the volatilities of the exchange rate and the three macroeconomic variables. For this purpose, I test for multiple structural breaks using the procedure proposed by Bai and Perron (1998 and 2003)[2]. The tests suggest that the British Pound/US Dollar returns experienced a structural break in March 1993, as Figure 1 suggested. This break cuts the exchange rate volatility into 2 regimes which are reported in the lower panel of Table 1. During the second regime, the exchange rate volatility was almost twice as high as during the first one. Note also that, during the recent financial crisis, the volatility of the British Pound/US Dollar returns did not increase as it has been the case of other asset returns.

The macroeconomic series also experience changes in volatilities and these occur in the 1980s. None of them however is located close enough to the observed shifts in the volatility of the exchange rate to be their direct cause. In this paper, I propose an explanation for this apparent disconnect between dynamics of exchange rates and fundamentals.

3 A general model of the exchange rate

In this section, I consider the general model set-up which nests several fundamental models of the exchange rate[3]. I use this framework to demonstrate analytically how the shifts in the volatility of the exchange rate can be related to the dynamics of underlying macroeconomic variables.

[2] The details of the tests for multiple structural breaks are reported in Appendix A.

[3] I can represent in this form Frenkel-Bilson (1978, 1976) model, Hooper-Morton (1982) model and Taylor rule model (Engel and West 2005).

As proposed by Mussa (1979), I model the exchange rate as an asset price that is a forward-looking and expectations-determined variable. The exchange rate, s_t, is a convex combination of the log fundamental variables $\mathbf{f}_t = (f_{1,t}, ..., f_{n,t})'$ and the expected future exchange rate

$$s_t = (1-\theta)\boldsymbol{\phi}'\mathbf{f}_t + \theta\hat{E}_t s_{t+1} \tag{1}$$

where θ is a weight on the expectations, $\boldsymbol{\phi}$ is a $(n \times 1)$ vector of fundamental variables' coefficients and $\hat{E}_t$ denotes the expectation conditional on information up to time t. The expectation is based on the available information which may be incomplete. A more precise definition of $\hat{E}_t$ will be discussed below. Assuming rational expectations $\hat{E}_t = E_t$ and solving model (1) forward yields

$$s_t = (1-\theta)\boldsymbol{\phi}' \sum_{l=0}^{T} \theta^l E_t \mathbf{f}_{t+l} + \theta^T E_t s_{t+T}. \tag{2}$$

Letting $T \to \infty$ and imposing the no-bubbles condition, such that $\lim_{T\to\infty} \theta^T E_t s_{t+T} = 0$, the present value representation is

$$s_t = (1-\theta)\boldsymbol{\phi}' \sum_{l=0}^{\infty} \theta^l E_t \mathbf{f}_{t+l} \tag{3}$$

Under the assumption that the fundamental variables in vector $\mathbf{f}_t$ follow a first-order vector autoregressive process,

$$\mathbf{f}_t = \mathbf{A}\mathbf{f}_{t-1} + \boldsymbol{\varepsilon}_t \tag{4}$$

where $\boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Sigma}_\varepsilon)$, the rational expectations solution to this model is

$$s_t^{RE} = (1-\theta)\boldsymbol{\phi}' (\mathbf{I}_n - \theta\mathbf{A})^{-1} \mathbf{f}_t \tag{5}$$

3.1 Expectations of agents

This paper presumes that agents do not have the perfect knowledge about the economic environment. Accordingly, they do not know the model of the economy and they behave as econometricians in that they choose the best model using econometric techniques. This is the assumption proposed in adaptive learning literature for example Bray (1982), Frydman (1982), Bray and Savin (1986), Sargent (1993), Marcet and Sargent (1989) and Evans and Honkapohja (2001). The impact of econometric model uncertainty on the dynamics of the macroeconomic dynamics has been recently studied by Branch and Evans (2006b and 2007) and Cho and Kasa (2008).

In this paper, agents are assumed to choose the best model according to the Bayesian Information Criterion (BIC, Schwarz 1978). BIC asymptotically chooses the true model but in finite samples it favors parsimony. In addition, the model that minimizes BIC is likely to provide the most accurate forecasts. Since models with many parameters often fit the historical data well, but forecast poorly, BIC balances goodness-of-fit with a penalty for model complexity.

The model learning mechanism is as follows. Using standard OLS techniques, agents estimate all the possible combinations of the fundamental variables of the model and choose the one that minimizes BIC.

3.1.1 Parameter learning

Suppose the model of the exchange rate in (1) includes n fundamental variables. Then there are $j = 1, \ldots, m$ combinations of n fundamental variables, where $m = 2^n - 1$.[4] $\mathbf{X}_j$ is the data matrix with regressors and $\boldsymbol{\beta}_j$ is the vector of corresponding coefficients. For each possible combination of regressors (corresponding to distinct forecasting models) $\mathbf{X}_j$ with $j = 1, \ldots, m$, the coefficients in vector $\boldsymbol{\beta}_j$ are estimated and evaluated using BIC by agents at every period.

Assume agents believe that the exchange rate is a linear function of the fundamental variables:

$$s_t = \boldsymbol{\beta}_{j,t-1}\mathbf{X}_{j,t} + \boldsymbol{\eta}_{j,t} \tag{6}$$

where $\mathbf{X}_{j,t}$ are fundamentals with corresponding coefficients $\boldsymbol{\beta}_{j,t-1}$and $\boldsymbol{\eta}_{j,t}$ is a vector of the *iid* shocks. Agents make a forecast of s_t with current fundamentals in $\mathbf{X}_{j,t}$ and past values of the coefficients, $\boldsymbol{\beta}_{j,t-1}$.

Given initial values of the model parameters in vector $\boldsymbol{\beta}_{j,0}$, the OLS procedure can be written as a recursive algorithm:

$$\begin{aligned} \boldsymbol{\beta}_{j,t} &= \boldsymbol{\beta}_{j,t-1} + t^{-1}\mathbf{R}_{j,t}\mathbf{X}_{j,t-1}\left(s_t - \boldsymbol{\beta}'_{j,t-1}\mathbf{X}_{j,t}\right) \\ \mathbf{R}_{j,t} &= \mathbf{R}_{j,t-1} + t^{-1}\left(\mathbf{X}'_{j,t}\mathbf{X}_{j,t} - \mathbf{R}_{j,t-1}\right) \end{aligned} \tag{7}$$

where $\boldsymbol{\beta}_{j,t}$ is the vector of parameters' estimates at t and $\mathbf{R}_{j,t-1} = t^{-1}\sum_{i=1}^{t-2}\mathbf{X}_{j,i}\mathbf{X}'_{j,i}$ is the fundamentals covariace matrix.

3.1.2 Model Learning

This paper assumes that agents learn about the model using BIC. Given a family of models including the true model, the probability that BIC selects the correct one approaches one as the sample size increases. Thus, asymptotically, the correct model of the exchange rate should be chosen by agents. However, Hansen and Sargent (2000) argue that historical times series are not long enough to recognize the data generating model. More precisely, when the sample size is finite, the BIC will select a misspecified model with a positive probability. In this paper, I focus on this case, where, misspecified models can govern the short term exchange rate dynamics.

Given the family of models, $\mathbf{X}_j$, which are the combinations of fundamental variables $\mathbf{f}_j$ and the vector of corresponding coefficients $\boldsymbol{\beta}_j$, agents' PLM is as follows

$$s_t = \boldsymbol{\beta}_{j,t-1}\mathbf{X}_t^{ML_{t-1}} \tag{8}$$

where ML stands for model learning and the resulting forecast of the future exchange rate is

$$\hat{E}_t s_{t+1} = \boldsymbol{\beta}_{j,t-1}\mathbf{A}\mathbf{X}_t^{ML_{t-1}} \tag{9}$$

[4] The empty set is naturally excluded from the models' set, hence -1.

where $\mathbf{A}$ is the VAR matrix and it is assumed to be known. Otherwise it can be also estimated by a regression of the fundamentals in $\mathbf{f}_t$ on $\mathbf{f}_{t-1}$. At each point in time, agents compare all the available models, m, and choose the one that satisfies the following condition

$$\mathbf{X}_t^{ML_{t-1}} = \arg\min_{X_{j,t}} \text{BIC, for } j = 1, \ldots, m, \tag{10}$$

BIC is defined for each model as

$$\text{BIC}_{j,t-1} = \log\left(\frac{\text{SSE}_{j,t-1}}{t-1}\right) + \frac{n\log t - 1}{t-1}, \text{ for } j = 1, \ldots, m \tag{11}$$

where

$$\text{SSE}_{j,t-1} = \sum_{i=0}^{t-1} \left(s_i - \mathbf{X}_{j,i}\boldsymbol{\beta}_{j,i-1}\right)' \left(s_i - \mathbf{X}_{j,i}\boldsymbol{\beta}_{j,i-1}\right). \tag{12}$$

Note that agents use BIC based on the information up to $t-1$. This avoids the simultaneity problem that would arise if the current forecast errors, $\mathbf{SSE}_{j,t}$, and therefore current exchange rate, s_t were taken into account.

The equilibrium stochastic process followed by the exchange rate (actual law of motion, ALM) is obtained by substituting the market forecast, equation (9), into the model (1).

$$s_t = (1-\theta)\,\boldsymbol{\phi}\mathbf{f}_t + \theta\boldsymbol{\beta}_{j,t-1}\mathbf{A}\mathbf{X}_t^{ML_{t-1}} \tag{13}$$

When a perceived law of motion (PLM) has the structure of the Rational Expectations Equilibrium (REE) the LS estimates asymptotically converge to the RE values[5]. Thus, when agents know that the exchange rate is a linear combination of the fundamentals in $\mathbf{f}_t$, they learn the RE solution in (5). However, if they use model learning, and the sample size is finite, underparametrization might occur. Such underparametrization means that the agents' model omits relevant variables. In this case, the REE cannot be reached. The model in (1) has a self-referential structure so that agents' forecasts feed back into the model of the economy. As a result, the exchange rate departs from the value that would prevail if they had complete information about the model.

3.2 Underparametrization and the resulting equilibrium

Since BIC tends to penalize complex models heavily, especially in small samples, the undeparametrization of the true model might occur. Econometric literature comparing different model selection criteria by Lütkepohl (1985) and Mills and Prasad (1992) shows that BIC tends to underfit the considered specifications.

In what follows, I study the characteristics of the equilibrium that would arise if underparametrization occurred. Such an equilibrium can arise only if

[5] In addition to the REE structure, the E-stability condition needs to be met. I define this concept after Evans and Honkapohja (2001) in the following section.

agents use all the available information optimally. Optimal use of information implies that agents cannot detect mistakes they are making while using an underparametrized model. If they could, they would simply change the model. By imposing orthogonality conditions between forecast errors and the underparametrized model one can insure that agents cannot detect the underparametrization. Since the agents' information set is limited relative to the RE case, the resulting solution is a Restricted Perceptions Equilibrium (RPE) as defined by Evans and Honkapohja (2001).

3.2.1 Restricted perceptions equilibrium

In order to see the results of potential underparametrization on the exchange rate process, consider a simple case. Suppose that the model of the exchange rate includes two fundamental variables, that is, $n = 2$, so that

$$s_t = (1-\theta)\boldsymbol{\phi}'\mathbf{f}_t + \theta\hat{E}_t s_{t+1} \tag{14}$$

and that the selection criterion in (11) leads the agents to choose a model with only one fundamental variable f_1: $\boldsymbol{\beta}_j\mathbf{X}_t^{ML} = \beta_1 f_{1,t}$. Thus, their PLM is

$$s_t = \beta_1 f_{1,t} \tag{15}$$

This gives the following forecast for $t+1$:

$$E_t s_{t+1} = \beta_1 \left(a_{11} f_{1,t} + a_{12} f_{2,t}\right) \tag{16}$$

where β_1 is a LS estimate of the belief parameter. Note that I also use the fact that the fundamentals follow a VAR(1) as defined in (4) and therefore the forecast for the next period $t+1$ also incorporates the coefficients a_{11} and a_{12}. The equilibrium stochastic process followed by the exchange rate (ALM) is obtained by substituting the market forecast, equation (16), into the model (14)

$$s_t = \chi_1 f_{1,t} + \chi_2 f_{2,t} \tag{17}$$

$$\chi_1 = (1-\theta)\phi_1 + \theta a_{11}\beta_1 \tag{18}$$

$$\chi_2 = (1-\theta)\phi_2 + \theta a_{12}\beta_1 \tag{19}$$

Following Branch and Evans (2006b, 2007), assume that agents' beliefs (PLM) are optimal (within their misspecification) so that they satisfy the following orthogonality condition:

$$E\left(f_{1,t}\left(s_t - \hat{\beta}_1 f_{1,t}\right)\right) = 0 \tag{20}$$

In the equilibrium, the parameter $\hat{\beta}_1$ must satisfy this orthogonality condition and be consistent with the ALM of the economy, (14). The fixed points in vector

$\boldsymbol{\chi} = (\chi_1, \chi_2)'$ of such a process describe RPE. Substituting the actual law of motion, equation (14) for s_t and solving for the belief parameter, $\hat{\beta}_1$, yields

$$\hat{\beta}_1 = \chi_1 + \chi_2 \frac{Ef_{1,t}f_{2,t}}{Ef_{1,t}^2} = \chi_1 + \chi_2 a_{11}^{-1} \chi_2 \Omega_{11}^{-1} \Omega_{12} \tag{21}$$

where $E \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} (f_1 \; f_2)' = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}$. Using the belief parameter, given by (21), in equations (18) and (19) and solving for the ALM parameters χ_1 and χ_2, I find

$$\begin{pmatrix} 1-\theta a_{11} & -\theta a_{11}\xi \\ -\theta a_{12} & 1-\theta a_{12}\xi \end{pmatrix} \boldsymbol{\chi} = (1-\theta)\boldsymbol{\phi} \tag{22}$$

$$\mathbf{B}\boldsymbol{\chi} = (1-\theta)\boldsymbol{\phi} \tag{23}$$

where the vector $\boldsymbol{\chi} = (\chi_1, \chi_2)'$ and $\xi = \Omega_{11}^{-1}\Omega_{12}$ and $\boldsymbol{\phi} = (\phi_1, \phi_2)$. Denote the matrix premultiplying $\boldsymbol{\chi}$ by $\mathbf{B}$. A RPE exists (and is unique) provided the inverse of $\mathbf{B}$ exists. The inverse of $\mathbf{B}$ exists if $\theta\left(a_{12}\xi + a_{11}\right) \neq 1$.

Proposition 1. *Provided* $\theta\left(a_{12}\xi + a_{11}\right) \neq 1$, *RPE exists.*

All the proofs are reported in Appendix B.

The resulting RPE is described by the fixed points in $\boldsymbol{\chi}$:

$$\hat{\chi}_1 = \frac{(1-\theta)\left(1-\theta\xi a_{12}\right)}{1-\theta\left(a_{12}\xi + a_{11}\right)}\phi_1 + \frac{(1-\theta)\,\theta\xi a_{11}}{1-\theta\left(a_{12}\xi + a_{11}\right)}\phi_2 \tag{24}$$

$$\hat{\chi}_2 = \frac{(1-\theta)\theta a_{12}}{1-\theta\left(a_{12}\xi + a_{11}\right)}\phi_1 + \frac{(1-\theta)\left(1-\theta a_{11}\right)}{1-\theta\left(a_{12}\xi + a_{11}\right)}\phi_2 \tag{25}$$

and the equilibrium process of the exchange rate follows

$$s_t^{RPE} = \hat{\chi}_1 f_{1,t} + \hat{\chi}_2 f_{2,t} \tag{26}$$

This equilibrium arises between optimally misspecified believes and the stochastic process of the exchange rate. These beliefs are optimal because they give the best linear forecast when agents are assumed to know only one explanatory variable. The linear projection of the exchange rate s_t on the fundamental variable f_1 is orthogonal and thus, given the information set, the forecast error is the smallest possible.

3.2.2 Stability analysis

There is a unique RPE for given parameter values of $\hat{\beta}_1$, θ and ϕ. I now examine the conditions necessary for stability of this equilibrium when agents use adaptive learning. In other words the question is whether agents using adaptive learning can find the estimate of $\hat{\beta}_1$ defined by RPE in (21).

It is important to note that stability analysis carried out here is not global but partial. In fact, it is verified whether the agents can find the estimate of $\hat{\beta}_1$ while they stick to the same model. In other words, I do not check for the conditions under which the parameter $\hat{\beta}_1$ would be learnt if agents had an opportunity to change the model of the exchange rate.

Agents base their decisions on their own estimates of the model's parameters. If estimates of parameters change, agents adjust their behavior accordingly. Moreover, agents' actions generate the data on which the estimates of parameters are calculated through the self-referential nature of the exchange rate equation, which makes learning an endogenous process. To correctly specify the model, agents would need to take the endogeneity into account, but because they do not, it is not certain that they will learn the RPE parameter value $\hat{\beta}_1$. This question is analyzed by determining E-stability (Expectational Stability) principle[6].

Calculate the T-map for $\hat{\beta}_1$ using equations (21), (18) and (19)[7].

$$\begin{aligned} T\left(\hat{\beta}_1\right) &= \chi_1 + \chi_2\xi \\ &= (1-\theta)\phi_1 + \theta\hat{\beta}_1 a_{11} + \left((1-\theta)\,\phi_2 + \theta\hat{\beta}_1 a_{12}\right)\xi \end{aligned} \tag{27}$$

$T\left(\hat{\beta}_1\right)$ is a map from the space of beliefs to outcomes. In equilibrium, they need to converge so that $\frac{d\hat{\beta}_1}{d\tau} = T\left(\hat{\beta}_1\right) - \hat{\beta}_1 = 0$.

The fixed point of the T-map is given by

$$\hat{\beta}_1 = \frac{1-\theta\,(\phi_1 + \xi\phi_2)}{1-\theta\,(a_{11} + \xi a_{12})} \tag{28}$$

Proposition 2. *Provided* $\theta\,(a_{11} + \xi a_{12}) < 1$*, the solution in* (28) *is locally E-stable.*

3.3 Model implications and numerical examples

The existence and E-stability of the RPE, is the key result as it implies that in case of underparametrization, the exchange rate dynamics are not govern by the REE. Instead, they hover around the RPE. In what follows I examine in what dimensions the dynamics generated by the REE and the RPE are different.

Under RE the equilibrium exchange rate process follows

$$s_t^{RE} = (1-\theta)\boldsymbol{\phi}'\left(\mathbf{I}_n - \theta\mathbf{A}\right)^{-1}\mathbf{f}_t \tag{29}$$

and RPE is

$$s_t^{RPE} = \begin{pmatrix} f_{1,t} & f_{2,t} \end{pmatrix} \begin{bmatrix} \hat{\chi}_1 \\ \hat{\chi}_2 \end{bmatrix} \tag{30}$$

[6] The E-stability (Expectational Stability) principle determines the stability in learning models.

[7] T-map represents regressors' parameters of the ALM. I seek to find fixed points of this map. Those are the points that the parameters estimated by agents converge to.

where χ_1 and χ_2 are defined in (24) and (25).

Assume a special case where $\phi_1 = \phi_2$, $a_{11} = a_{22}$ and $a_{12} = a_{21}$[8]. Then the weights given to both fundamental variables in the exchange rate process in (29) are equal, while in (30) the first fundamental variable f_1 receives a heavier weight. Since the exchange rate process in (30) is a linear combination of two processes f_1 and f_2, its statistical properties will obviously be described more closely by the one with the heavier weight (f_1 in this case). This is true when $a_{11} > a_{12}$. The proof is sketched in Appendix B.

In a dynamic setup, when agents are allowed to choose the best forecasting model according to the BIC, the selected variable (or the model) will receive a heavier weight than the remaining fundamentals, and dominate the statistical properties of the exchange rate process. These properties will shift if the best forecasting model changes.

If the agents use model learning, the exchange rate will potentially drift from a given RPE to another one. The resulting exchange rate dynamics, and the volatility in particular, should alter along with corresponding RPEs.

3.3.1 Decreasing gain model learning

First the model dynamics with decreasing gain are analyzed as in (7), so that the standard least squares updating takes place. In particular, I am interested in the question weather the exchange rate converges asymptotically to the REE when agents use model learning. For this purpose, the equations (13), (7) and (10) are simulated where the true model includes two fundamentals $\mathbf{f}_t = (f_{1,t}, f_{2,t})$ and the values of the parameters are chosen to ensure the E-stability, $\theta = 0.8$, $\boldsymbol{\phi} = (0.14, 0.14)$ and $\mathbf{A} = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$, $\Sigma_\epsilon = \begin{bmatrix} 1.1 & 0.0 \\ 0.0 & 0.9 \end{bmatrix}$ and initial conditions for remaining parameters are displayed in Table 2. The model has been simulated over 10000 periods.

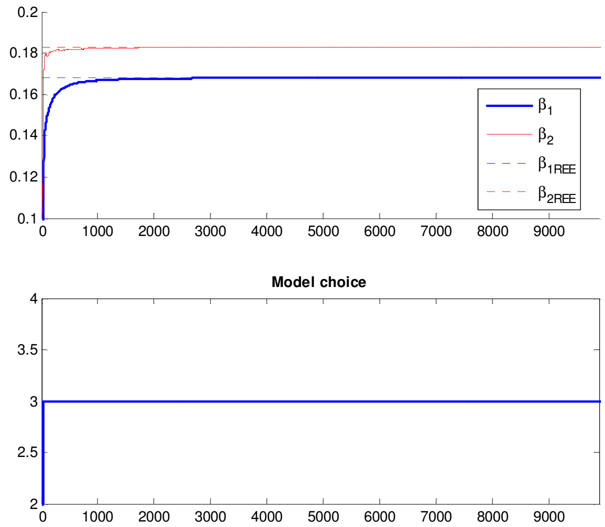
Table 2: **Initial values in decreasing and constant gain simulations**

$\boldsymbol{\beta}_{j,0}$	s_0	$\mathbf{X}_0^{ML}$	$R_{j,0}$
$\mathbf{0}$	RE	f_1, f_2	I_n

Figure 3 demonstrates the results of the simulation. The lower panel of the figure shows the model prevailing at each point in time and the upper panel displays the corresponding coefficients, β_1 and β_2. The paths followed by the updated parameters are represented by solid lines. The dotted lines correspond to $\bar{\beta}^1_{RE}$ and $\bar{\beta}^2_{RE}$. Figure 3 displays no shifts between the models and the third model, including both fundamentals prevails during the whole simulation

[8] Obviously, these are special cases which have low probability to occur in the data. They help however in understanding how this underparametrization may generate shifts in statistical regimes of the exchange rate. In the empirical part, these assumptions are relaxed and I rely on the data properties.

Figure 3: **LS coefficients of the underparametrized models and choice of the model**



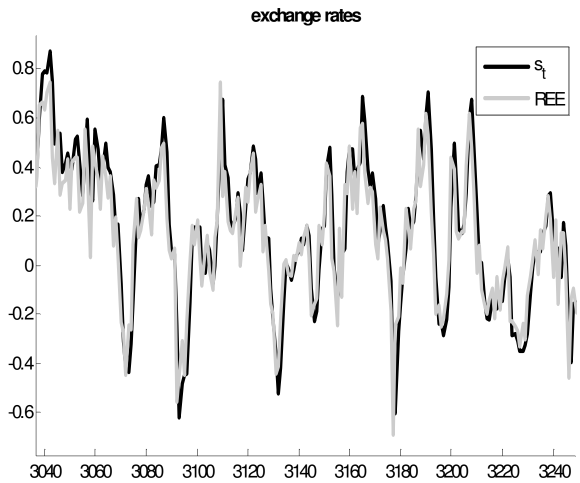
period. Accordingly, coefficients β_1 and β_2 converge to their equilibrium values $\bar{\beta}^1_{RE}$ and $\bar{\beta}^2_{RE}$ very rapidly. As a result, as illustrated in Figure 4, the exchange rate closely mimics the movements of the exchange rate implied by the REE.

These results are representative in a sense that they display the convergence to the REE. Numerous simulation exercises over a large parameter space satisfying the E-stability condition, demonstrated that under the decreasing gain model learning, the specification always asymptotically converges to the REE.

3.3.2 Constant gain model learning

When the economy is in the calm regime, it is optimal to use constant estimates and hence a decreasing gain LS algorithm. However, when there is a structural change in the economy i.e. it follows a stochastic process with parameter values that evolve over time, a constant gain learning rule or ”perpetual learning” will better track the evolution of the parameters than a decreasing gain rule. Pesaran and Pick (2008) provide an econometric evidence for this argument. They show that forecasts based on a single estimation window (decreasing gain) lead to a smaller bias either if there are no structural breaks or they are very small. When a structural break is large, the forecasts which exponentially down-weight

Figure 4: **Exchange rates under LS model learning and REE structure.**



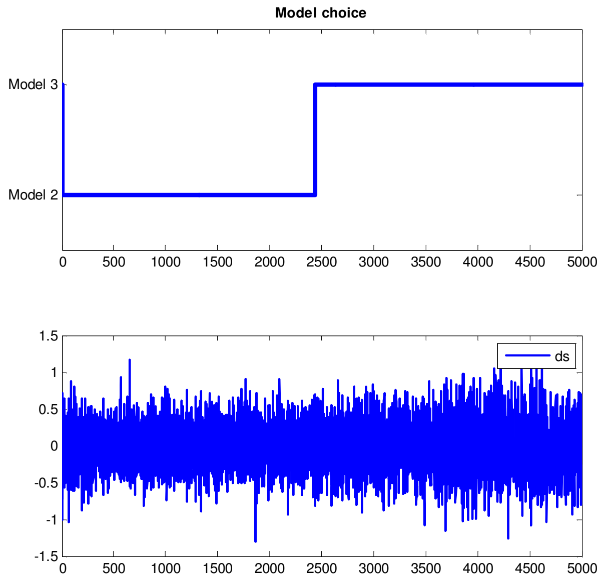
observations (constant gain) perform better.

Since the main interest of this paper lies in the data generating processes with structural breaks, it is natural to apply the constant gain LS algorithm to update the models' parameters. In particular, it might be that once the time-variation of the parameter estimates is taken into account the shifts in the model are redundant. Therefore, instead of decreasing gain t^{-1} in (7) I use a constant gain κ_1. A constant gain implies that an econometrician puts more weight on the more recent data. Accordingly, the BIC has to be also adjusted (see for instance Brailsford et al. 2002). An observation that dates i periods back in the constant gain algorithm receives the weight $(1-\kappa_1)^{i-1}$ so that BIC in (11) becomes:

$$\text{BIC}_{j,t-1} = \log\left(\frac{\text{SSE}_{j,t-1}}{t-1}\right) + \frac{n \log \sum_{i=1}^{t-1}(1-\kappa_1)^{i-1}}{\sum_{i=1}^{t-1}(1-\kappa_1)^{i-1}}, \text{ for } j = 1, \ldots, m \quad (31)$$

The model is simulated over 5000 periods with $\theta = 0.9$, $\boldsymbol{\phi} = (1.5,\, 1.5)$, $\mathbf{A} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.3 \end{bmatrix}$, and $\boldsymbol{\Sigma}_\epsilon = \begin{bmatrix} 0.8 & 0 \\ 0 & 1.0 \end{bmatrix}$. The constant gain is assumed to be a small value $\kappa_1 = 0.0054$ and the the initial values of model estimated parameters are the same as in standard LS simulation and they are summarized in Table 2.

Figure 5: **Exchange rate returns under constant gain model learning**

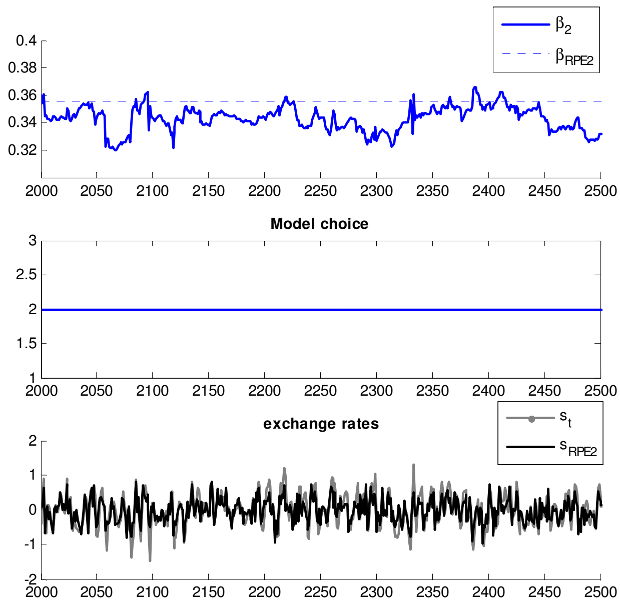


The lower panel of the figure plots the exchange rate returns and the higher panel shows the corresponding choice of the model.

Figure 5 displays the main results of the simulation. The lower panel of the figure plots the exchange rate returns and the higher panel shows the corresponding choice of the model. The figure demonstrates how the shift in the model affects the volatility dynamics. Clearly, the change in the specification from the model 2 to model 3 generates much higher volatility, as illustrated by lower panel of Figure 5. This is the case because the exchange rate series implied by the model with both fundamental variables (model 3) exhibits a much higher variability than the model with the second fundamental only (model 2).

Table 3 displays the volatility figures for all the models and all the regimes. Note that the second model displays the lowest and the third model the highest volatility. The volatility of the first model is higher then the one of the second because of assumed higher autoregressive parameter in A. Accordingly, Table 3 indicates that the volatility of the RPE_2 implied series is lower than RPE_1, both being less volatile than the REE series. Therefore, in the first regime

Figure 6: **Updated RPE coefficient and the choice of model under constant gain learning**



when the exchange rate is driven by RPE_2, the exchange rate volatility is lower than in the second regime that is generated by the REE dynamics. During both regimes the mean of the three model based exchange rate returns, $ds_t^{RPE_1}$, $ds_t^{RPE_2}$, ds_t^{REE}, and hence of the resulting returns, ds_t, remain very close to 0.

Figure 5 shows that the model converges to the REE specification. However, this depends on the parametrization of the model[9]. Below, I also present the results of a simulation with different model parametrization where the misspecified model survives.

Figure 6 demonstrates that underparametrization can persist when a small constant gain value is used. In this particular simulation, the parameters were set to the following values: $\kappa_1 = 0.01$, $\theta = 0.8$, $\boldsymbol{\phi} = (1.2, 1.2)$, $\mathbf{A} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}$,

[9]Numerous simulations showed that the exchange rate does not necessarily converge to the RE structure model. However, the conditions under which the underparametrized model persists have not been developed. This is beyond the scope of this paper being however an interesting question for future research.

Table 3: **Volatility of the exchange rate returns implied by model learning, RPE1, RPE2 and REE**

	Volatility			
Regime	ds_t	$ds_t^{RPE_1}$	$ds_t^{RPE_2}$	ds_t^{REE}
1+2	0.3251	0.3454	0.2680	0.3950
1	0.2908	0.3423	0.2648	0.3903
2	0.3547	0.3482	0.2710	0.3994

Volatility has been calculated as a sample standard deviation. It has been computed for the first difference of the exchange rate series implied by the model-learning, RPE_1, RPE_2 and REE. 1 corresponds to the first regime that ends in period 2436, when the model shifts; 2 corresponds to the second regime that starts after this date. $1+2$ stands for the whole sample period.

$\mathbf{\Sigma}_\epsilon = \begin{bmatrix} 0.6 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$. Note that in the upper panel of Figure 6, as the the constant gain updating is used, the coefficient β_1 is very volatile. As a result, the volatility of the exchange rate also rises and the convergence towards the RPE fixed point is impossible. Instead, the exchange rate converges to a distribution centered around the RPE as the lowest panel of Figure 6 illustrates. Thus, the constant gain learning can generate the excess volatility vis-à-vis fundamentals observed in the data.

4 Empirical illustration

The theoretical set up introduced in previous sections is very general and does not specify underlying fundamentals. For the empirical illustration of the theoretical result, I need to indicate the macroeconomic variables determining the exchange rate process.

4.1 The Taylor rule framework and the British Pound dynamics

A central bank may want to react to and smooth the exchange rate movements especially in a small open economies, where inflation fluctuations are likely to have a substantial international relative price component. In line with this argument, Ball (1999) and Svensson (2000) use an open economy model to demonstrate that including the exchange rate into the interest rule of the central bank leads to lower fluctuations of real GDP and inflation.

Several recent studies show that some of the central banks do, in fact, include the exchange rate in their interest rate rules (see for instance Clarida, Galí and Gertler 1998, and Lubik and Schorfheide 2007). In particular, Lubik and Schorfheide (2007) show that this is the case for the Bank of England.

In addition, as noted in Table 1, volatility of the British Pound/US Dollar returns experienced a significant decrease in the beginning of 1993 and this occurred shortly after the Bank of England adopted an inflation targeting strategy. Hence, the reaction function of the Bank of England seems to be an appropriate framework to study the link between the dynamics of the exchange rate and underlying macroeconomic variables.

By specifying Taylor rules for two countries and subtracting one from the other, an equation of the exchange rate determination can be specified. This so called Taylor-rule model of the exchange rate has recently proved to be very successful empirically. Engel and West (2006) find that the Taylor rule exchange rate model supports German data. Similarly, Mark (2006) shows that adaptive learning of the Taylor-rule fundamental variables provides a possible framework for understanding real USD/DM exchange rate dynamics. Clarida and Waldman (2007) find a positive correlation between the announcement of higher inflation and a currency appreciation in countries where the central banks have an inflation target implemented within a Taylor Rule.

The study by Molodsova and Papell (2008) shows that the Taylor-rule model outperforms the random walk in the out-of-sample predictability for 11 out of 12 currencies against the U.S. dollar over the post-Bretton Woods float. Furthermore, they show that the predictability is much stronger with Taylor rule models than with conventional interest rate, purchasing power parity, or monetary models. Motivated by these results, in what follows, I assume that the Taylor-rule-model of the exchange rate is the real model of the economy.

4.2 Taylor-rule model of the exchange rate

The Taylor-rule model of the exchange rate is a simple, empirical model that builds on the reaction functions of two countries. There is a large number of Taylor rule specifications that have been used in the literature and tested empirically (see for instance Taylor, 1999). As a consequence, the exchange rate models derived from the two Taylor rules can take different forms. In what follows, I present a general version of this model.

Assume that a home monetary authority, the Fed, sets interest rates according to a simple interest rule,

$$\tilde{\imath}_t = a_0 + a_1\pi_t + a_2 y_t + v_t \tag{32}$$

The rule implies that the current interest rate $\tilde{\imath}_t$ responds to current inflation π_t and output gap y_t. a_1 and a_2 describe how strongly the central bank is reacting to the deviations from the targeted variables and v_t is a shock to the monetary policy rule. The foreign central bank (Bank of England) follows the Taylor rule that also includes the real exchange rate,

$$\tilde{\imath}_t^* = a_0^* + a_1^*\pi_t^* + a_2^* y_t^* + a_3^* q_t + v_t^* \tag{33}$$

where the stars denote the foreign country variables and coefficients. The real exchange rate is defined as a ratio of home prices to foreign prices so that after the log transformation we have $q_t = p_t - (s_t + p_t^*)$ or $q_t = \hat{p}_t - s_t$ where $\hat{p}_t = p_t - p_t^*$. When the home prices rise relative to the foreign ones, the central bank might increase the interest rate $\tilde{\imath}_t^*$ to impede inflationary pressures so that I expect $a_3^* > 0$. The UIP condition is

$$i_t = i_t^* + \hat{E}_t s_{t+1} - s_t + u_t \tag{34}$$

where u_t is an exogenous risk premium shock. The market interest rate at home is $i_t = \tilde{\imath}_t + \tau_t$ and abroad $i_t^* = \tilde{\imath}_t^* + \tau_t^*$. Combining the two Taylor rules, one can derive a standard asset pricing exchange rate equation[10],

$$s_t = (1-\theta)\,\boldsymbol{\phi}'\mathbf{f}_t + \theta \hat{E}_t s_{t+1} + \epsilon_t \tag{35}$$

where $\theta = (1 - a_3^*)$, $\boldsymbol{\phi}' = \begin{pmatrix} \frac{\bar{a}_0}{a_3^*}, & -\frac{a_1}{a_3^*}, & -\frac{a_2}{a_3^*}, & \frac{a_1^*}{a_3^*}, & \frac{a_2^*}{a_3^*}, & 1, & 1 \end{pmatrix}$ and $\mathbf{f}_t' = \begin{pmatrix} 1, & \pi_t, & y_t, & \pi_t^*, & y_t^*, & \hat{p}_t, & \hat{\imath}_t \end{pmatrix}$, $\bar{a}_0 = a_0^* - a_0$, $\hat{p}_t = p_t - p_t^*$, and $\hat{\imath}_t = i_t - i_t^*$. The current exchange rate is a function of a set of home and foreign fundamental variables, the expected future exchange rate and shocks in ϵ_t. ϵ_t is a linear combination of shocks to monetary policy rules, to the base rates and to the UIP, $\epsilon_t = (\nu_t^* - \nu_t) + (\tau_t^* - \tau_t) - u_t$. In the empirical applications, instead of the real exchange rate, q_t, the first difference, Δq_t, is used as a stationary regressor in the foreign Taylor rule (33).

Given that the Taylor rule model implies 7 regressors (including a constant) given by $\mathbf{f}_t$, there are therefore $m = 2^7 - 1 = 127$ possible combinations of them where the empty set is ruled out.

Assuming rational expectations, $\hat{E} = E$, and solving the model forward gives the RE solution as in (5) where the number of Taylor fundamentals $n = 7$:

$$s_t^{RE} = (1-\theta)\boldsymbol{\phi}' \left(\mathbf{I}_7 - \theta \mathbf{A}\right)^{-1} \mathbf{f}_t = \mathbf{B}^{RE}\mathbf{f}_t. \tag{36}$$

In order to calculate the RE Taylor rule model of the exchange rate, I first estimate the Taylor rules as in (32) and (33) where the fundamentals are lagged one period and I calculate implied coefficients in $\boldsymbol{\phi}'$, and $\theta = (1 - a_3^*)$. Next, I estimate a VAR(1) with the fundamentals in $\mathbf{f}_t$ to obtain $\mathbf{A}$. The RE is then calibrated using (35) and monthly data.

4.3 The model under evolving monetary policy

The exchange rate in (35) is a function of two main components. The first component is directly derived from the Taylor rules of the two central banks and thus depends on the way their monetary policies evolve. The second component is the expected future exchange rate which is based on the model learning. It is assumed that agents construct their models based on the Taylor rule fundamentals. As a result, both elements of the exchange rate process in (35) depend on the same set of Taylor rule fundamentals, and therefore, the monetary policy rules affect expectations of agents[11]. If these rules change the expectations are likely to adjust too.

[10] The derivation of the Taylor rule model can be found in the Appendix C.

[11] There is also certainly the reverse causality that the expectations of agents affect the monetary policy set up. In this paper, however, I overlook this relationship as I am principally interested in how the expectations influence the exchange rate dynamics.

In order to discriminate between the effects of the evolving monetary policy and the expectations themselves on the exchange rate dynamics, I allow the parameters of reaction functions of the central banks to vary over time in response to the shifts of the monetary regimes. There is a strong empirical evidence indicating that Taylor rules' parameters are time varying and I explicitly model this[12]. I assume that the parameters of the Taylor rules in (32) and (33) are time-varying and they evolve according to SRAs. The home Taylor rule

$$\tilde{\imath}_t = a_{0,t-1} + a_{1,t-1}\pi_{t-1} + a_{2,t-1}y_{t-1} + v_t \tag{37}$$

follows the SRA of the form

$$\begin{aligned} \mathbf{b}_{1,t-1} &= \mathbf{b}_{1,t-2} + \kappa_2 \mathbf{R}_{1,t-1}^{-1}\mathbf{f}_{1,t-1}\left(i_{t-1} - \mathbf{b}'_{1,t-2}\mathbf{f}_{1,t-1}\right) \\ \mathbf{R}_{1,t-1} &= \mathbf{R}_{1,t-2} + \kappa_2\left(\mathbf{f}_{1,t-1}\mathbf{f}'_{1,t-1} - \mathbf{R}_{1,t-2}\right) \end{aligned} \tag{38}$$

where $\mathbf{f}_{1,t-1} = (1, \pi_{t-1}, y_{t-1})'$ and $\mathbf{b}_{1,t-1} = (a_{0,t-1}, a_{1,t-1}, a_{2,t-1})'$. The foreign reaction function follows

$$\tilde{\imath}^*_t = a^*_{0,t-1} + a^*_{1,t-1}\pi^*_{t-1} + a^*_{2,t-1}y^*_{t-1} + a^*_{3,t-1}q_{t-1} + v^*_t \tag{39}$$

and it evolves according to the following SRA

$$\begin{aligned} \mathbf{b}_{2,t-1} &= \mathbf{b}_{2,t-2} + \kappa_2 \mathbf{R}_{2,t-1}^{-1}\mathbf{f}_{2,t-1}\left(i^*_{t-1} - \mathbf{b}'_{2,t-2}\mathbf{f}_{2,t-1}\right) \\ \mathbf{R}_{2,t-1} &= \mathbf{R}_{2,t-2} + \kappa_2\left(\mathbf{f}_{2,t-1}\mathbf{f}'_{2,t-1} - \mathbf{R}_{2,t-2}\right) \end{aligned} \tag{40}$$

where $\mathbf{f}_{2,t-1} = \left(1, \pi^*_{t-1}, y^*_{t-1}, q_t\right)'$ and $\mathbf{b}_{2,t-1} = \left(a^*_{0,t-1}, a^*_{1,t-1}, a^*_{2,t-1}, a^*_{3,t-1}\right)'$. The exchange rate process in (35) will consequently depend on the time varying parameters $\boldsymbol{\phi}'_{\mathbf{t-1}}$

$$s_t = (1-\theta_t)\,\boldsymbol{\phi}'_{\mathbf{t-1}}\mathbf{f}_{t-1} + \theta_t \hat{E}_t s_{t+1} + \epsilon_t \tag{41}$$

where $\boldsymbol{\phi}'_{\mathbf{t-1}} = \begin{pmatrix} \frac{\bar{a}_{0,t-1}}{a^*_{3,t-1}}, & -\frac{a_{1,t-1}}{a^*_{3,t-1}}, & -\frac{a_{2,t-1}}{a^*_{3,t-1}}, & \frac{a^*_{1,t-1}}{a^*_{3,t-1}}, & \frac{a^*_{2,t-1}}{a^*_{3,t-1}}, & 1, & 1 \end{pmatrix}$ and $\theta_t = 1 - a^*_{3,t-1}$. Note also that since Taylor-rule parameters are most likely subject to structural breaks, the constant gain updating algorithm is used in (37) and (39) where κ_2 denotes the constant gain sequence. The discount factor, θ_t, is time-varying as it is implied by the weight the central bank attributes to the real exchange rate, $a^*_{3,t-1}$, which itself is changing over time. Note also that in Taylor rules (37) and (39) the timing structure differs from the one in (32) and (33). Equations (37) and (39) use the lagged regressors to avoid the potential endogeneity issues. The resulting exchange rate process in (41) is driven by the past fundamentals $\mathbf{f}_{t-1}$.

The agents' perceived law of motion (PLM) is as follows:

$$s_{t+1} = \boldsymbol{\beta}'_{j,t-1}\mathbf{A}\mathbf{X}^{ML}_{t-1} + \eta_{j,t-1} \tag{42}$$

[12] See for instance Cogley and Sargent (2005) and Orphanides and Williams (2005).

where $\dim(\mathbf{X}_{t-1}^{ML}) \leq \dim(\mathbf{f}_{t-1})$, and the forecast

$$\hat{E}_t s_{t+1} = \boldsymbol{\beta}_{j,t-1}^{'} \mathbf{A} \mathbf{X}_{t-1}^{ML} \quad (43)$$

$$\mathbf{X}_{t-1}^{ML} = \arg\min_{X_{j,t}} \text{BIC, for } j = 1, ..., m, \quad (44)$$

with BIC defined for each model as

$$\text{BIC}_{j,t-1} = \log\left(\frac{\text{SSE}_{j,t-1}}{t-1}\right) + \frac{n \log \sum_{i=1}^{t-1} (1-\kappa_1)^{i-1}}{\sum_{i=1}^{t-1} (1-\kappa_1)^{i-1}}, \text{ for } j = 1, ..., m \quad (45)$$

where

$$\text{SSE}_{j,t-1} = \sum_{i=0}^{t-1} \left(s_i - \mathbf{X}_{j,i}\boldsymbol{\beta}_{j,i}\right)' \left(s_i - \mathbf{X}_{j,i}\boldsymbol{\beta}_{j,i}\right). \quad (46)$$

4.4 Estimation procedure

The proposed exchange rate model is defined by a system of equations (41), (38), (40) where the agents forecast is defined by (43), (44), (45), and (46). Both, the model parameters $\boldsymbol{\beta}_{j,t-1}$ and the Taylor rules parameters are estimated by constant gain Least Squares and the best forecasting model is selected based on BIC. The free parameters are then the constant gain coefficients κ_1 and κ_2 and the initial conditions for the estimated parameters $\boldsymbol{\beta}_{j,0}$, $\mathbf{b}_{1,0}$, and $\mathbf{b}_{2,0}$.[13] Since the fitted values of κ_1 and κ_2 depend strongly on the initial values of those parameters they need to be estimated simultaneously. The estimation procedure used here closely follows the version of Method of Simulated Moments (MSM) proposed by Adam et. al (2008).

Define the set of moments in the exchange rate data

$$\hat{S} \equiv [\mu_{ds}, \sigma_{ds}]^{'} \quad (47)$$

where μ_{ds} is a sample mean of the exchange rate return and σ_{ds} denotes its volatility measured as sample standard deviation. Define a corresponding set of moments for the model learning implied series as

$$\hat{S}_{ML} \equiv \left[\mu_{ds_{ML}}, \sigma_{ds}^{ML}\right]^{'} \quad (48)$$

The aim is to find the initial values $\hat{\boldsymbol{\beta}}_{j,0}$, $\hat{\mathbf{b}}_{1,0}$, $\hat{\mathbf{b}}_{2,0}$, and the gain parameters $\hat{\kappa}_1$ and $\hat{\kappa}_2$ to minimize the distance between the data and model implied moments. Define $\hat{\lambda}_0 \equiv \left(\hat{\boldsymbol{\beta}}_{j,0}, \hat{\mathbf{b}}_{1,0}, \hat{\mathbf{b}}_{2,0}\right)$ and the estimated parameters

$$\left(\hat{\lambda}_0, \hat{\kappa}_1, \hat{\kappa}_2\right) \equiv \arg\min_{\lambda_0, \kappa_1, \kappa_2} \left[\hat{S} - \hat{S}_{ML}\right]^{'} W \left[\hat{S} - \hat{S}_{ML}\right] \quad (49)$$

where W is a weighting matrix which is a diagonal matrix with i-th entry defined as $1/\ \hat{\sigma}_{S_i}^2$. $\hat{\sigma}_{S_i}^2$ is an estimated variance of each of the moments included in $\hat{S}$,

[13] Note that there is no reason that I assume that the forgetting factors for the exchange rate updating, κ_1, and for the central banks reaction functions, κ_2, are equal.

and the variance of σ_{ds} is calculated as $\frac{\hat{\sigma}_{ds}^2}{2T}$. This procedure finds the model parameters that generate series matching the data as closely as possible but giving less weight to statistics with larger variance. The entries of the weighting matrix W are always computed based on the data $\hat{\sigma}_{S_i}^2$ so that the criterion of fit remains the same and facilitates the comparison between the models.

4.5 Data

The proposed exchange rate model is defined by a system of equations (41), (38), (40) where agents forecast is defined by (43), (44), (45), and (46). The inputs for the estimation are the nominal exchange rate and fundamental variables. The data is monthly and covers post Bretton-Woods sample starting in 1974M1 and ending in 2009M1. The nominal exchange rate is expressed as the number of US Dollars per British Pound. The US is a home country and the UK is the foreign economy. Output is measured as the log of seasonally adjusted industrial production, prices as the log of the CPI, inflation as the first difference of log prices, interest rate by money market rate, and the exchange rate as the log of the end of the period rate. The output gap series are constructed as deviations of actual output from the Hodrick-Prescott (1997) trend[14]. Data was mainly obtained from the International Financial Statistics (IFS) and the Federal Reserve Economic Data (FRED)[15].

5 Results

I now turn to investigate whether I can find support for the model learning applied to the Taylor-rule model of the exchange rate in the data. First I examine how the shifts of the model affect the exchange rate dynamics and the volatility changes in particular. Second, I analyze other dimensions of the data generated by model learning and compare them to other specifications.

5.1 Volatility shifts

Table 4 reports the results of the optimization defined in (49). Note first that the value of estimated gain sequence, $\hat{\kappa}_2$, is high relative to what has been found in the literature. The gain coefficient has been however usually estimated to fit the inflation and output data, as in Branch and Evans (2006a) and Orphanides and Williams (2005). In this paper, its value is selected to match the exchange rate volatility which is much higher than the one of inflation or output.

Such a high value of the constant gain can occasionally generate the explosive paths for updated coefficients $\boldsymbol{\beta}_{j,t}$. In order to avoid this issue, a "projection facility" similar to Marcet and Sargent (1989) is employed. The idea behind is that the agents ignore the observations that are not in line with their priors.

[14] Molodsova and Papell (2008) show that this measure of output gap has proved to be the best in the Taylor-rule model for the British Pound/US Dollar exchange rate.

[15] Details on the data sources and data construction can be found in Appendix 3.C.

Table 4: **Parameters estimated by MSM**

Estimated parameter				
$\hat{\boldsymbol{\beta}}_{j,0}$	$\hat{\mathbf{b}}_{1,0}$	$\hat{\mathbf{b}}_{2,0}$	$\hat{\kappa}_1$,	$\hat{\kappa}_2$
0.7	0.3	0.6	0.0582	0.1201

$\hat{\boldsymbol{\beta}}_{j,0}$, $\hat{\mathbf{b}}_{1,0}$, $\hat{\mathbf{b}}_{2,0}$ denote the estimated initial values for the parameters on the estimated fundamental-based models, home and foreign Taylor rules, respectively. $\hat{\kappa}_1$ and $\hat{\kappa}_2$ are estimated constant gain parameters in the time-varying Taylor rules and model learning.

One of the characteristics of the constant gain learning is that it produces higher volatility and, in general, richer dynamics of updated series. In order to make sure that they are not eliminated, the projection facility is put in place only if the model generates the volatility of the exchange rate returns twice as high as the one in the data. The algorithm in (7) is appended by the following rule

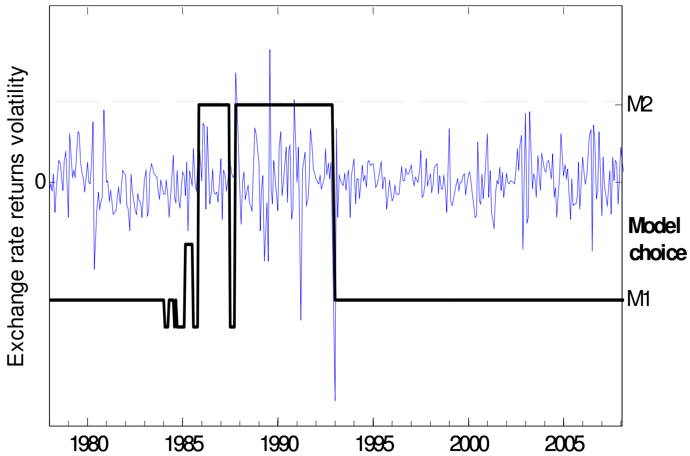
$$\begin{cases} & \text{if } \sigma_j > 1.96\sigma_s \\ \boldsymbol{\beta}_{j,t} = & r.h.s \text{ of } (7) \quad \text{if } \left|\boldsymbol{\beta}_{j,t}\right| < 2\boldsymbol{\beta}_{j,t}^{RPE} \\ & \boldsymbol{\beta}_{j,t} = \boldsymbol{\beta}_{j,t}^{RPE}, \quad \text{otherwise} \end{cases} \tag{50}$$

where σ_j is the volatility of the returns implied by the model j and σ_s is a corresponding volatility in the data. $\boldsymbol{\beta}_{j,t}^{RPE}$ is the RPE value of the coefficient $\boldsymbol{\beta}_{j,t}$. The computation of $\boldsymbol{\beta}_{j,t}^{RPE}$ for $j = 1, \ldots, n$, where $n = 7$ in case of Taylor-rule model can be found in Appendix B.

Figure 7 plots the main results of the estimation exercise where the values of the parameters satisfying (49) are presented in Table 4. The figure displays the model choice and the volatility of the British Pound/US Dollar returns during the sample period between the beginning of 1975 and the end of 2008. During the first year of the sample period (which is not presented here) the choice of the model is very volatile due to a small number of observations available. The model choice becomes stable in the beginning of 1975, the first year included in the plot. Because frequent shifts between the models may affect the resulting exchange rate volatility, the first year (12 observations) are dropped from the calculated statistics

Figure 7 reports the British Pound/US Dollar returns and the models chosen by agents at each point in time. The degree of underparametrization increases in the right y-axis. Although the agents can choose between 127 available combinations of Taylor-rule variables, only two of them prevailed during the sample period and these were reported. The model that has the RE structure i.e. it includes all the variables of the Taylor-rule equation as in (35) has never been selected as the best forecasting model. In the following section, the properties of the RE based model will be studied in more detail. The model denoted by M1 includes 3 variables and a constant, $\boldsymbol{\beta}_j^{'}\mathbf{X}_t^{ML,1}$, where $\mathbf{X}_t^{ML,1} = (1, \pi_t^*, \hat{\imath}_t)$, and $\boldsymbol{\beta}_j$ is a vector of corresponding coefficients. Thus, this model encompasses the UK inflation rate, π_t^*, and the interest rate differential, $\hat{\imath}_t$. The second

Figure 7: **Choice of the best model to predict GBP/USD exchange rate**



model, denoted as M2 includes 2 fundamentals and a constant, $\boldsymbol{\beta}_j^{'}\mathbf{X}_t^{ML,2}$, where $\mathbf{X}_t^{ML,2} = (1, \pi_t^*, y_t)$, and $\boldsymbol{\beta}_j$ is a vector of estimated coefficients. This model is based on the UK inflation rate and the US output gap.

Figure 7 shows that the model learning makes agents regularly underparametrize the Taylor-rule model of the exchange rate. The rational expectations model has never been chosen during the sample period. Put differently, the cost always exceeds the benefit of using the complete model. Therefore, in line with the results suggested in Section 5.3.2, underparametrization of the true model under constant gain model learning can persist during the whole sample period.

There are two main changes of the model. The first model, M1, dominates until January 1984 and the second model, M2, until December 1993. From the econometric point of view both models include the same number of regressors. Thus, it is impossible to claim either that agents learn larger model or that they limit the number of variables to forecast the exchange rate. The second shift between the models takes place in January 1993, shortly after the Bank of England changed its monetary policy strategy. Figure 7 also suggests that the exchange rate returns generated by the model learning exhibit a structural break at this point in time. This is verified by applying the test by Bai and Perron (1998) to the model-based returns. Results of the test are reported in Table 5.

Table 5: **Timing of breaks in model based exchange rate returns and in the data**

Volatility of the exchange rate returns		
	Model Learning	Data
Break date	1993M3***	1993M3***
Regime 1	0.0312***	0.0356***
Regime 2	0.0288***	0.0215***

*** denotes significance at the 1, 5 and 10 percent levels, respectively. The first regime corresponds to 1975M1-1993M3 and the secont to 1993M4-2008M8.

The table illustrates the two regimes in the dynamics of the volatility of the exchange rate returns. The structural break occurred, as in the data, in March 1993. Also both, in the data and model based series, this break cuts the returns into two volatility regimes, the high one before the break, and the low volatility regime after the shift. Note that the first change of the models, in February 1984, did not trigger the shift in the exchange rate volatility.

5.2 Model learning versus the RE benchmark

The previous section demonstrates that the proposed model can replicate the volatility dynamics of the British Pound/US Dollar returns. This section shows how well the model matches the data in other dimensions, mean and volatility of returns. In particular, it demonstrates how well the model fits both moments relative to the rational expectations benchmark.

Figure 8 plots the British Pound/US Dollar exchange rate, the series under RE and under model learning. It clearly shows that the exchange rate under model learning moves closely to the data while the RE series deviates considerably from it.

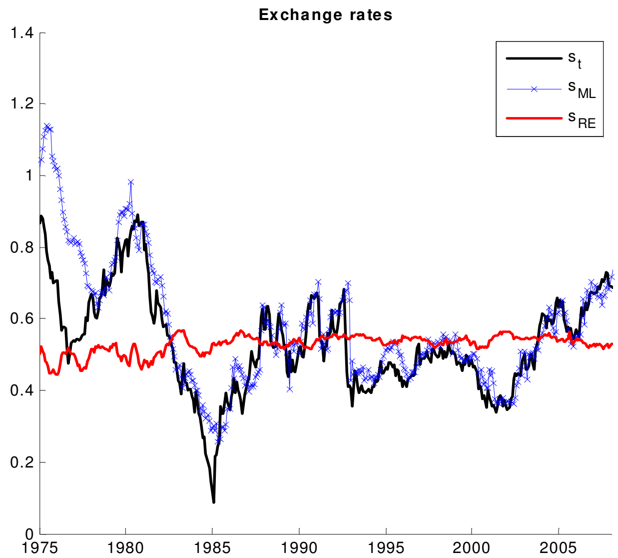
Table 6: **Exchange rate returns statistics in the data, RE and ML**

	Data	RE	ML
Moment			
μ_{ds}	-0.0010	0.0001	-0.0006
σ_{ds}	0.0303	0.0078	0.0300
μ_s	0.5288	0.5330	0.5800

μ_{ds} denotes the mean of exchange rate returns, σ_{ds} their volatility measured as sample standard deviation and μ_s is the sample mean of the loglevel exchange rate. The statistics are calculated on the sample period between 1975M1 and 2008M8.

Table 6 summarizes the moments of the exchange rate series obtained from the model learning specification, RE model and in the data. As already demonstrated in the previous section, the model learning returns display the mean

Figure 8: **Data, RE and ML based exchange rates**



and the volatility matching the figures observed in the data. Under RE, as in the data, the mean return is close to 0; however, the volatility is almost 4 times lower than the data indicates. This is so called excess volatility puzzle which states that under RE the exchange rate volatility implied by the fundamentals is much lower than the one observed in the data. This paper shows that even using the same fundamentals one can reach the high exchange rate volatility by introducing learning dynamics. This result has been also demonstrated by Kim (2009) and Lewis and Markiewicz (2009).

Visibly the RE based returns do not account for structural break in the volatility of the returns in the beginning of 1993, as suggested by the data. Table 11 in the appendix reports the result of the structural break test carried on RE series. Finally, as can be seen in the last row of Table 6, the mean of the loglevel exchange rate is well approximated by RE series although it moves rather independently from it (see Figure 8).

5.3 Comparison of model learning with RPEs

Model learning clearly outperforms the RE based model. This, however, is not a challenging task as the RE model is based on constant coefficients and therefore I analyze the dynamics generated by other RPE specification under constant gain learning. In particular, the single RPE results from a much simpler model where agents just follow a single underparametrized specification. If such a model describes better the exchange rate dynamics, there is no need for model learning. By estimating the parameters satisfying (49) I obtain the best fit for the mean and the volatility of the returns. Since there are 127 models available I select only a set of them and describe how well they match the data. First, it seems natural to me to analyze what would happen if there was no shift in the model in the beginning of 1993. Accordingly, I assess the fit of the model including 2 variables, namely, UK inflation rate, π_t^*, and US output gap, y_t (and a constant). In addition to this particular RPE, I also report the statistics for the two mostly used models of the exchange rate, namely the relative PPP and the UIP. Finally, I consider the fit of the model that has the structure of the RE equilibrium so that it includes all the Taylor-rule fundamentals. In addition to the sample statistics, I report structural break test for the series generated by the above mentioned models.

Table 7: **Statistics of the exchange rate returns under RE, RPEs, ML and in the data**

Moment	Data	ML	RE*
μ_{ds}	-0.0010	-0.0006	-0.0025
σ_{ds}	0.0303	0.0300	0.0601
μ_s	0.5288	0.5800	0.0514
		RPE	
	$1, \pi_t^*, y_t$	PPP (π_t^*, π_t)	UIP $(\hat{\imath}_t)$
Moment			
μ_{ds}	-0.0024	-0.0012	-0.0003
σ_{ds}	0.0453	0.0597	0.0643
μ_s	0.5818	0.1089	0.0498

μ_{ds} denotes the mean of exchange rate returns, σ_{ds} their volatility measured as sample standard deviation and μ_s is the sample mean of the loglevel exchange rate. ML stands for model learning, RE* for rational expectations structure, and RPE stands for restricted perception equilibrium. All the statistics are calculated on the sample period between 1975M1 and 2008M8.

Table 7 summarizes the statistics resulting from the calibration of different models with the values of the estimated parameters, reported in Table 4. In particular, the calibrations are carried out with the constant gains parameters $\hat{\kappa}_1 = 0.0582$ and $\hat{\kappa}_2 = 0.1201$. Note that RE* indicates that the model has RE equilibrium structure in the sense that it includes all the Taylor-based fundamentals. However, unlike the RE model from the previous section, this one has time varying-parameters due to the constant gain updating mechanism.

Table 8: **Estimated breaks and volatilities: test by Bai and Perron (1998)**

Volatility of the exchange rate returns			
Break date	Data	ML	RE*
	1993M3***	1993M3***	1985M1***
	0.0356***	0.0312***	0.0987***
	0.0215***	0.0288***	0.0074***
		RPE	
	$1, \pi_t^*, y_t$	PPP (π_t^*, π_t)	UIP $(\hat{\imath}_t)$
Break date	1998M10***	1985M1***	1985M1***
Regime 1	0.0307***	0.0997***	0.1164***
Regime 2	0.0687***	0.0068***	0.0079***

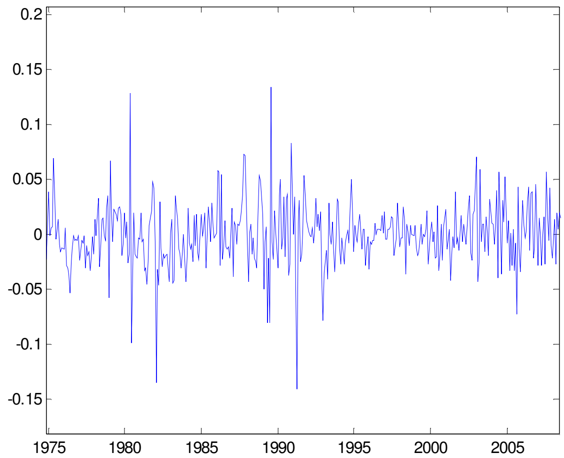
Table reports results of the test by Bai and Perron (1998) applied to the returns volatility. *** denotes significance at the 1, 5 and 10 percent levels, respectively. ML stands for model learning, and RPE stands for restricted perception equilibrium. RE* means that the model has the rational expectations equilibrium structure. All the statistics are calculated on the sample period between 1975M1 and 2008M8. The Regime 1 corresponds to the sample period starting in 1975M1 until the break date. The Regime 2 starts one month after the break and ends in 2008M8. The numbers in the table report the estimated volatilities during the two regimes.

Table 7 demonstrates that the model learning specification gives the best overall fit. All the other models display too high volatility relative to the data. In particular, it is interesting to see what pattern the returns would display if the model that was chosen before January 1993 prevailed until the end of the sample period. The result is demonstrated in Figure 9. The figure rather suggests a break around 1998. Still, I calculate the model generated volatility after January 1993 and I find that it would reach 0.0555, the number more than twice as high as in the data. This suggests that the change of the model in January 1993 was necessary to account for the decrease of the volatility. The overall volatility generated by this model is also higher than the one found in the data and the break occurs in October 1998 (see Tables 7 and 8).

None of the alternative models can generate returns with a structural shift in the volatility corresponding to the one found in the data. This is suggested by the results reported in Table 8. While all of the series exhibit volatility shifts, none of them occurs in the beginning of 1993. Therefore, the proposed model learning procedure outperforms all the other specifications.

From the empirical study, it becomes clear that the model learning provides the best fit and, at the same time, generates desired shift in the volatility. Since the break occurs shortly after the Bank of England introduced inflation targeting, the two events might be related. However, in the framework of the proposed model, it does not seem possible to assess the direct link from the new monetary regime to the lower exchange rate volatility, without speculating. Therefore, I leave this question to further research.

Figure 9: **Exchange rates returns generated by model 2**



6 Conclusion

This paper proposes an explanation for shifts in the volatility of the exchange rate returns. Its key assumption is that agents face model uncertainty and behave as econometricians to deal with it. They apply model learning which is as follows. First, given a model of the exchange rate, they estimate the coefficients of all the possible combinations of the variables within this model. Second, using a model selection criterion, they choose the best set of variables to use as a forecast of the future exchange rate.

Using a version of the asset pricing model, the paper demonstrates that learning about the model of the exchange rate may lead agents to focus excessively on a subset of fundamental variables at different points in time. When agents choose misspecified models it alters the weight placed on selected fundamental variables relative to the others. Because, the asset pricing equation has self-referential structure, the chosen model feeds back into the actual exchange rate. As a result, as agents switch between models the nominal exchange rate volatility varies accordingly even though the underlying fundamentals processes remain time-invariant.

The model learning framework was introduced into the Taylor-rule based model of the exchange rate and applied to the British Pound/US Dollar exchange rate. Numerous estimation and calibration exercises suggest that the agents shift model in January 1993, roughly the date when a significant shift in the volatility of the British Pound/US dollar returns has been found. This change was also captured by the structural break test in the series generated by the asset pricing equation under model learning. None of the alternative specifications proved to pass this test.

Finally, summary statistics show that the model learning based exchange rate displays properties similar to those of the actual exchange rate data in several dimensions. In addition to capturing the desired shift in the variability, it also matches the overall volatility and the mean in level and returns.

References

[1] Adam, Klaus, Albert, Marcet, and Juan Pablo, Nicolini, (2008) 'Stock Market Volatility and Learning,' *manuscript.*

[2] Arifovic, Jasmina, (1996) 'The Behavior of the Exchange Rate in the Genetic Algorithm and Experimental Economies,' *The Journal of Political Economy,* Vol. 104, No. 3, 510-541.

[3] Bacchetta, Philippe, and Eric, van Wincoop, (2004) 'A Scapegoat Model of Exchange Rate Fluctuations,' *American Economic Review, Papers and Proceedings*, 94, 114-118.

[4] Bai, Jushan, and Pierre, Perron, (1998) 'Estimating and testing linear models with multiple structural changes,' *Econometrica* 66, 47–78.

[5] Bai, Jushan, and Pierre, Perron, (2003) 'Computation and analysis of multiple structural change models,' *Journal of Applied Econometrics,* 18, 1-22.

[6] Ball, Laurence (1999) '*Policy Rules for Open Economies*' In Monetary Policy Rules, edited by John B. Taylor, 127-144. Chicago: University of Chicago Press.

[7] Baxter, Marianne, and Alan, C. Stockman, (1989) 'Business cycles and the exchange-rate regime,*' Journal of Monetary Economics,* 23, 377-400.

[8] Bilson, John, (1978) 'The Monetary Approach to the Exchange Rate: Some Empirical Evidence,' *IMF Staff Papers*, 25, 48-75.

[9] Brailsford, T. J., Jack H. W. Penm, and R. D. Terrell (2002) 'Selecting the forgetting factor in subset autoregressive modelling.' *Journal of Time Series Analysis* 23, 629-649.

[10] Branch, William, and George, W. Evans, (2006a) 'A Simple Recursive Forecasting Model', *Economics Letters*, 91(2), 158-166.

[11] Branch, William, and George, W. Evans, (2006b) 'Intrinsic Heterogeneity in Expectation Formation', *Journal of Economic Theory*, 127(1), 264-295.

[12] Branch, William, and George, W. Evans, (2007) 'Model uncertainty and endogenous volatility,' *Review of Economic Dynamics,* 10, 207-237.

[13] Bray, Margaret, (1982) 'Learning, Estimation, and the Stability of Rational Expectations,' *Journal of Economic Theory,* 26, 318-339.

[14] Bray, Margaret, and Eugene, Savin, (1986) 'Rational Expectations Equilibria, Learning and Model Selection,' *Econometrica,* 54, 1129-1160.

[15] Cheung, Yin-Wong, and Menzie, D. Chinn, (2001) 'Currency Traders and Exchange Rate Dynamics: A Survey of the U.S. Market,' *Journal of International Money and Finance*, 20(4), 439-471.

[16] Cheung, Yin-Wong, Menzie, D. Chinn, and Pascual, Antonio, G., (2005) 'Empirical Exchange Rate Models of the Nineties: Are Any Fit to Survive?,' *Journal of International Money and Finance,* 24, 1150-1175.

[17] Clarida, Richard, Jordi, Galí, and Mark, Gertler, (1998) 'Monetary Rules in Practice: Some International Evidence,' *European Economic Review,* 6, 1033-1067.

[18] Clarida, Richard, and Daniel, Waldman, (2007) 'Is Bad news about Inflation Good news for the Foreign Exchange Rate?,' *NBER Working Paper* 13010.

[19] Cho, In-Koo, and Kenneth, Kasa, (2008) 'Learning and Model Validation', *manuscript*, Simon Fraser University.

[20] Cogley, Timothy, and Thomas, J., Sargent, (2005) 'Drifts and Volatilities: Monetary Policies and Outcomes in the PostWWII U.S., *Review of Economic Dynamics* 8, 262–302.

[21] Engel, Charles, and James, D., Hamilton, (1990) 'Long Swings in the Dollar: Are They in the Data and Do Markets Know It?,' *American Economic Review*, 80(4), 689-713.

[22] Engel, Charles, and Kenneth, D., West, (2005) 'Exchange Rate and Fundamentals,' *Journal of Political Economy*, 113(3), 485-517.

[23] Engel, Charles, and Kenneth, D., West, (2006) 'Taylor Rules and the Deutschmark-Dollar Real Exchange Rate,' *Journal of Money, Credit, and Banking,* 38(5), 1175-1194.

[24] Evans, W. George, and Seppo, Honkapohja, (2001) *'Learning and Expectations in Macroeconomics,'* Princeton University Press, 421.

[25] Flood, Robert, and Andrew, Rose, (1995) 'Fixing exchange rates: a virtual quest for fundamentals,' *Journal of Monetary Economics,* 36, 3-38.

[26] Frenkel, Jacob, (1976) 'A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence,' *Scandinavian Journal of Economics,* 78, 200-224.

[27] Frydman, Roman, (1982) 'Towards an Understanding of Market Processes: Individual Expectations, Learning, and Convergence to Rational Expectations Equilibrium,' *American Economic Review,* 72(4), 652-668.

[28] Gerlach, Stefan, (1988) 'World business cycles under fixed and flexible exchange rates,' *Journal of Money Credit and Banking,* 20, 621-632.

[29] Gourinchas, Pierre-Olivier, and Aaron, Tornell, (2004) 'Exchange Rate Puzzles and Distorted Beliefs,' *Journal of International Economics,* 64(2), 303-333.

[30] Hansen, Lars, and Thomas, Sargent, (2001) 'Acknowledging misspecification in macroeconomic theory,' *Review of Economic Dynamics,* 4, 519-535.

[31] Hodrick, Robert, and Prescott, Edward, (1997) 'Postwar U.S. Business Cycles: An Empirical Investigation,' *Journal of Money, Credit, and Banking,* 29(1), 1-16.

[32] Hooper, Peter, and John, Morton, (1982) 'Fluctuations in the Dollar: A Model of Nominal and Real Exchange Rate Determination,' *Journal of International Money and Finance,* 1, 39-56.

[33] Jeanne, Olivier, and Rose, Andrew, (2002) 'Noise trading and exchange rate regimes,' *Quarterly Journal of Economics,* 117(2), 537- 569.

[34] Kim, Young S. (2009) 'Exchange Rates and Fundamentals Under Adaptive Learning', *Journal of Economic Dynamics and Control*, 33(4), 843-863.

[35] Lewis, Karen, (1989a) 'Changing Beliefs and Systematic Rational Forecast Errors with Evidence from Foreign Exchange,' *American Economic Review*, 79(4), 621-636.

[36] Lewis, Karen, (1989b) 'Can learning affect exchange rate behavior? The case of the dollar in the early 1980's,' *Journal of Monetary Economics,* 23, 79-100.

[37] Lewis, Vivien, and Markiewicz, Agnieszka, (2009) 'Misspecification, Learning and the Exchange Rate Disconnect Puzzle', *The B.E. Journal of Macroeconomics,* 9(1) (Topics), 13.

[38] Lubik, Thomas, and Frank, Schorfheide, (2007) 'Do central banks respond to exchange rate movements? A structural investigation,' *Journal of Monetary Economics,* 54, 1069-1087.

[39] Lütkepohl, Helmut, (1985) 'Comparison of Criteria For Estimating the Order of a Vector Autoregressive Process', *Journal of Time Series Analysis,* 6, 35-52.

[40] Marcet, Albert, and Sargent, Thomas, (1989) 'Convergence of Least Squares Learning Mechanisms in Self-Referential Linear Stochastic Models,' *Journal of Economic Theory*, 48(2), 337-368.

[41] Mark, Nelson, (2006) 'Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics,' *NBER Working Paper* 11061.

[42] Mills, Jeffrey, and Kislaya, Prasad, (1992) 'Comparison of Model Selection Criteria', *Econometric Reviews,* 11, 201-233.

[43] Molodtsova, Tanya, and David, Papell, (2008) 'Out-of-Sample Exchange Rate Predictability with Taylor Rule Fundamentals,' Journal of International Economics*, forthcoming.*

[44] Mussa, Michael, (1979) 'Empirical Regularities in the Behaviour of Exchange Rates and Theories of the Foreign Exchange Market,' *Carnegie-Rochester Series on Public Policy,* 11, 9-58.

[45] Mussa, Michael, (1986) 'Nominal exchange-rate regimes and the behavior of real exchange rates: evidence and implications,' *Carnegie-Rochester Series on Public Policy,* 25, 117-214.

[46] Orphanides, Athanasios, and Williams, John C., (2005) 'The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations,' *Journal of Economic Dynamics and Control,* 29(11), 1927-1950.

[47] Pesaran, M. Hashem, and Pick, Andreas, (2008) Forecasting Random Walks Under Drift Instability, *CESifo Working Paper* 2293.

[48] Rossi, Barbara, (2006) 'Are Exchange Rates Really Random Walks? Some Evidence Robust to Parameter Instability,' *Macroeconomic Dynamics,* 10(1), 20-38.

[49] Sargent, Thomas, (1993) *Bounded rationality in macroeconomics.* Oxford University Press, New York, NY, p 184.

[50] Sarno, Lucio, and Valente, Giorgio, (2008) 'Exchange Rates and Fundamentals: Footloose or Evolving Relationship?' *Journal of European Economic Association,* forthcoming.

[51] Schinasi, Garry, and P. A. V. B., Swami, (1989) 'The out-of-sample forecasting performance of exchange rate models when coefficients are allowed to change,' *Journal of International Money and Finance*, 8, 375-390.

[52] Svensson, Lars (2000) 'Open Economy Inflation Targeting,' *Journal of International Economics,* 50, 155-183.

[53] Schwarz, Gideon, (1978) 'Estimating the dimension of a model,' *Annals of Statistics,* 6, 461-464.

[54] Taylor, John, (1993) 'Discretion versus policy rules in practice,' *Carnegie-Rochester Conference on Public Policy,* 39, 195-214.

[55] Taylor John, (1999) *Monetary Policy Rules*, Chicago: University of Chicago Press. 127-144.

Appendix A: Structural break tests

I test for structural breaks using the procedure proposed by Bai and Perron (1998, 2003). The test is applied to the absolute value of the demeaned series $|d\widehat{x}_t - \hat{\mu}|$. The sequential procedure is as follows. First, I estimate up to 5 breaks in the series. Second, I apply the test, which is designed to detect the presence of $(j + 1)$ breaks conditional on having found j breaks $(j = 0, 1, \ldots, 5)$. The statistical rule is to reject j in favour of a model with $(j+1)$ breaks if the overall minimal value of the sum of squared residuals (over all the subsamples where an additional break is included) is sufficiently smaller than the sum of squared residuals from the model with j breaks. The dates of the selected breaks are the ones associated with this overall minimum. I identify the breaks if the test statistic allows rejection of the null hypothesis at at least a 10 per cent level of significance.

I use the code accompanying the paper by Bai and Perron (2003) which can be found on the website:
http://people.bu.edu/perron/code.html

Consider the case of the unconditional volatility of the British Pound/US Dollar returns, where I test for a structural break in the mean of the absolute value of the demeaned series, $y_t = |ds_t - \hat{\mu}|$. The results of the structural break test for this series are reported in the second column of Table 8. I find that there is a break in the volatility of exchange rate returns in March 1993.

Table 9: **Timing of breaks in British Pound/US Dollar returns**

Exchange rate returns volatility	
Break date	1993M3***
90% CI	(1992M6-1996M7)
Regime	**Estimate**
1975M1-1993M3	0.0356***
1993M4-2008M12	0.0215***

CI stands for confidence intervals. ***, ** and * denote significance at the 1, 5 and 10 percent levels, respectively.

I also test for unconditional volatility shifts in macroeconomic series, namely interest rate and inflation rate differentials. I use again the test proposed by Bai and Perron (1998, 2003) which is applied to the absolute value of the demeaned series $|d\widehat{x}_t - \hat{\mu}|$, where $d\widehat{x}_t$ is an inflation rate differential, the first difference of interest rate differential and output gap differential. Table 10 reports the results.

Table 10: **Timing of breaks in macroeconomic series**

Interest rate differential		
1st Break date	1981M9***	
90% CI	(1981M6, 1989M11)	
2nd Break date	1989M4***	
90% CI	(1987M6, 1992M8)	
Regime	Estimate	St. Error
1973M3-1981M9	0.13288	0.0213
1981M10-1989M4	0.06905	0.0073
1989M5-2006M6	0.03777	0.0039

Inflation differential		
Break date	1980M5***	
90% CI	(1980M2, 1983M9)	
Regime	Estimate	St. Error
1973M3-1980M5	0.00696	0.0009
1980M6-2006M6	0.00314	0.0002

Output gap differential		
Break date	1986M8***	
90% CI	(1985M11,1990M1)	
Regime	Estimate	St. Error
1973M3-1986M8	0.01393	0.0011
1986M9-2006M6	0.00754	0.0005

CI stands for confidence intervals. ***, ** and * denote significance at the 1, 5 and 10 percent levels, respectively.

Table 11: **Breaks in the volatility of the exchange rate under RE**

RE returns volatility	
Break date	1985M2***
Regime	**Estimate**
1975M1-1985M2	0.00845***
1985M3-2008M8	0.00449***

CI stands for confidence intervals. *** denote significance at the 1 percent level.

Appendix B: Proofs

Proof of Proposition 2.

Given the T-map for β_1

$$\begin{aligned} T(\beta_1) &= (1-\theta)\phi_1 + \theta\beta_1 a_{11} + ((1-\theta)\phi_2 + \theta\beta_1 a_{12})\xi \\ DT(\beta_1) &= \frac{(1-\theta)(\phi_1+\phi_2\xi)}{1-\theta(a_{11}+a_{12}\xi)} + ke^{(\theta(a_{11}+a_{12}\xi)-1)t} \end{aligned}$$

The E-stability of the solution $\bar{\beta}_1 = \frac{(1-\theta)(\phi_1+\phi_2\xi)}{1-\theta(a_{11}+a_{12}\xi)}$ requires that $\theta(a_{11}+a_{12}\xi) < 1$. This condition is an equivalent to transversality condition in the RE context. By imposing it is ensured that the fundamental equilibria are stable and therefore that bubbles cannot occur.

$\hat{\chi}_{1,RPE} > \hat{\chi}_{2,RPE}$

The REE and RPE are:

$$s_t^{RE} = \boldsymbol{\chi}_{\mathbf{RE}}^{-1}\mathbf{f}_t \quad (51)$$

$$s_t^{RPE} = \hat{\boldsymbol{\chi}}'_{\mathbf{RPE}}\mathbf{f}_t \quad (52)$$

where

$$\begin{aligned} \chi_{1,RE} &= \frac{(1-\theta)((1-\theta a_{22})\phi_1 + \theta a_{12}\phi_2)}{D_1} \\ \chi_{2,RE} &= \frac{(1-\theta)(\theta a_{21}\phi_1 + (1-\theta a_{11})\phi_2)}{D_1} \end{aligned}$$

with $D_1 = (1-\theta a_{11})(1-\theta a_{22}) - \theta^2 a_{12}a_{21}$

and

$$\hat{\chi}_{1,RPE} = \frac{(1-\theta)((1-\theta\xi a_{12})\phi_1 + \theta\xi a_{11}\phi_2)}{D_2} \quad (53)$$

$$\hat{\chi}_{2,RPE} = \frac{(1-\theta)(\theta a_{12}\phi_1 + (1-\theta a_{11})\phi_2)}{D_2} \quad (54)$$

where $D_2 = 1 - \theta\left(a_{12}\xi + a_{11}\right)$ and $\xi = \frac{Ef_1f_2}{Ef_1^2}$. Under the assumption that $\phi_1 = \phi_2$, $a_{11} = a_{22}$ and $a_{12} = a_{21}$, $\left(1 - \theta a_{22}\right)\phi_1 + \theta a_{12}\phi_2 = \theta a_{21}\phi_1 + \left(1 - \theta a_{11}\right)\phi_2$ and hence $\chi_{1,RE} = \chi_{2,RE}$. In this special case, the RPE coefficients imply $\left(1 - \theta\xi a_{12}\right)\phi_1 + \theta\xi a_{11}\phi_2 > \theta a_{12}\phi_1 + \left(1 - \theta a_{11}\right)\phi_2$ if $a_{11} > a_{12}$.

RPE for a general case

Note that the RPE presented here is a case where one variable, $f_{i,t}$, at the time is chosen and the true model includes all the other variables $f_{l,t}$ where $i, l = 1, \ldots, n$ and $i \neq l$. As a reminder the exchange rate equation is as follows

$$s_t = (1-\theta)\boldsymbol{\phi}'\mathbf{f}_t + \theta\hat{E}_t s_{t+1} \tag{55}$$

where there are $i = 1, \ldots, n$ fundamentals in $\mathbf{f}_t$. The exchange rate forecast based on the $i-$th predictor is

$$\hat{E}_t s_{t+1} = \beta_i\left(a_{i,i}f_{i,t} + \sum_{\substack{l=1\\ l\neq i}}^{n} a_{i,l}f_{l,t}\right)$$

Hence

$$s_t = \lambda_1 f_{i,t} + \lambda_2\sum_{\substack{l=1\\ l\neq i}}^{n}\left(\phi_j + a_{i,l}\right)f_{l,t} \tag{56}$$

where $\lambda_1 = (1-\theta)\phi_i + \theta a_{i,i}\beta_i$ and $\lambda_2 = (1-\theta) + \theta\beta_i$.

Apply orthogonality conditions between the forecast erors and agents' forecasts

$$E\left(f_{i,t}\left(s_t - \beta_i f_{l,t}\right)\right) = 0$$

Substitute for s_t equation (56) and solve for β_i.

$$\beta_i = \lambda_1 + \lambda_2\sum_{\substack{l=1\\ l\neq i}}^{n}\left(\phi_l + a_{i,l}\right)\xi$$

where $\xi = \frac{Ef_if_j}{Ef_i^2}$. Plugg back in the definitions of λ_1 and λ_2 to obtain

$$\beta_i^{RPE} = \frac{(1-\theta)\left(\phi_i + \sum_{\substack{l=1\\ l\neq i}}^{n}\left(\phi_l + a_{i,jl}\right)\xi\right)}{1-\theta\left(a_{i,i} + \sum_{\substack{l=1\\ l\neq i}}^{n}\left(\phi_l + a_{i,l}\right)\xi\right)}$$

The parameters θ and ϕ are time-varying due to the evolving monetary rules introduced in Section 4.3. As a result, β_i^{RPE} will also be time-varying. For the projection facility I use $\bar{\beta}_i^{RPE}$ which is defined as a sample average.

Appendix C: Derivation of the Taylor rule model of the exchange rate

The home central bank follows the Taylor rule rule

$$\tilde{\imath}_t = a_0 + a_1\pi_t + a_2 y_t + v_t \tag{57}$$

The foreign central bank follows the rule that also includes the real exchange rate

$$\tilde{\imath}^*_t = a^*_0 + a^*_1\pi^*_t + a^*_2 y^*_t + a^*_3 q_t + v^*_t \tag{58}$$

where $q_t = \hat{p}_t - s_t$ and $\hat{p}_t = p_t - p^*_t$. The UIP condition is

$$i_t = i^*_t + \hat{E}_t s_{t+1} - s_t + u_t \tag{59}$$

where u_t is an exogenous risk premium shock. The market interest rate at home is $i_t = \tilde{\imath}_t + \tau_t$ and abroad $i^*_t = \tilde{\imath}^*_t + \tau^*_t$. Substruct the foreign Taylor rule (58) from the home Taylor rule (57):

$$i_t - i^*_t = a_0 + a_1\pi_t + a_2 y_t - a^*_0 - a^*_1\pi^*_t - a^*_2 y^*_t - a^*_3 q_t + v_t - v^*_t + \tau_t - \tau^*_t$$

Use UIP in (59) to obtain the following

$$\hat{E}_t s_{t+1} - s_t = a_0 + a_1\pi_t + a_2 y_t - a^*_0 - a^*_1\pi^*_t - a^*_2 y^*_t - a^*_3 p_t + a^*_3\left(\hat{E}_t s_{t+1} - (i_t - i^*_t)\right) + v_t - v^*_t + \tau_t - \tau^*_t$$

One can write this specification in the following asset pricing equation form

$$s_t = (1-\theta)\,\boldsymbol{\phi}'\mathbf{f}_t + \theta\hat{E}_t s_{t+1} + \epsilon_t \tag{60}$$

where $\theta = (1-a^*_3)$, $\boldsymbol{\phi}' = \begin{pmatrix} \frac{\bar{a}_0}{a^*_3}, & -\frac{a_1}{a^*_3}, & -\frac{a_2}{a^*_3}, & \frac{a^*_1}{a^*_3}, & \frac{a^*_2}{a^*_3}, & 1, & 1 \end{pmatrix}$ and $\mathbf{f}'_t = \begin{pmatrix} 1, & \pi_t, & y_t, & \pi^*_{t,} & y^*_t, & \hat{p}_t, & \hat{\imath}_t \end{pmatrix}$, $\bar{a}_0 = a^*_0 - a_0$, $\hat{p}_t = p_t - p^*_t$, $\hat{\imath}_t = i_t - i^*_t$ and $\epsilon_t = (\nu^*_t - \nu_t) + (\tau^*_t - \tau_t) - u_t$.

Appendix D: Data

Table 12: **Description of the data sources and the construction of variables**

Variable	Country	Measure	Source
Exchange rate	USD/UKP	Market rate- end of the month	IFS
Interest rate	US	1-Month Certificate of Deposit: Secondary Market Rate	FRED
Interest rate	UK	UK Treasury Bill Discount Rate, 3 Month	BoE
Output	US	Total Indutrial Production, s.a., 2002=100	Fed
Output	UK	Total production industries, s.a, 2005=100	ONS
Output gap	US	$y_t^{g*}= y_t^{*} - y_t^{HP*(a)}$	Fed+$^{(b)}$
Output gap	UK	$y_t^{g}= y_t - y_t^{HP(a)}$	ONS+$^{(b)}$
Prices	US	Consumer Price Index (CPI), all items	IFS
Prices	UK	Consumer Price Index (CPI)	IFS
Inflation	US	First difference of log of CPI	ONS
Inflation	UK	First difference of log CPI	BLS

The sample for all the series goes from 1973M3 to 2006M5. FRED denotes Federal Reserve Economic Data,BoE Bank of England, Fed Federal Reserve, ONS Office for National Statistics and US Bureau for Labor Statistics.$^{(a)}$Output gap is calculated as deviations of actual output from the H-P trend. $^{(b)}$Author's calculations.

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