ADVERSE SELECTION IN THE ANNUITY MARKET WITH SEQUENTIAL AND SIMULTANEOUS INSURANCE DEMAND

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CESIFO WORKING PAPER NO. 783
CATEGORY 3: SOCIAL PROTECTION
OCTOBER 2002

CESifo *Working Paper No. 783*

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Abstract

This paper investigates the effect of adverse selection on the private annuity market in a model with two periods of retirement. In order to introduce the existence of limited-time pension insurance, we assume that for each period of retirement separate contracts can be purchased. Demand for the two periods can be decided either sequentially or simultaneously. We show that different risk-groups prefer different types of contracts, and that only the sequential contracts, which are favourable for the long-living individuals, represent an equilibrium.

JEL Classification: D82, D91, G22.

Keywords: annuity markets, adverse selection, uncertain lifetimes, equilibrium.

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1. Introduction

Social security systems, which in many industrialised countries are organised according to the pay-as-you-go method, are threatened by the ageing of the population due to a decrease in fertility and an increase in life-expectancy. This problem is recognised by academics as well as by politicians, and several possible measures to maintain the financial stability of the system are suggested. One of these measures is a reduction of the pension payments, and it seems in fact unavoidable that it will be implemented to some degree. If this is the case, then a natural strategy for the individuals is to raise private provision for retirement, in particular by an increased purchase of life annuities. As governments want to prevent old-age poverty, they tend to encourage private pension insurance through tax incentives.[1]

However, there are concerns that the market for annuities does not offer a suitable supplement to the public pension system. One obvious argument is that it cannot incorporate redistribution, as the public system does for several reasons. Another argument concentrates on the phenomenon of adverse selection, which is a common problem that affects the efficient working of insurance markets. The present paper studies this problem in the context of specifically designed contracts for old-age insurance.

Generally, adverse selection occurs with asymmetric information, that is, when the insurer has less information than the individual as to the probability that the insured event occurs. In case of annuities, this means that companies have less information on life-expectancy of an annuitant than the individual herself. As a consequence, returns from annuities cannot reflect individual life-expectancy but only overall life-expectancy, which in turn will induce high-risk individuals (that is, the long-living) to buy more annuities than low-risk individuals. This is the standard observation, discussed in various contributions to the literature (see, e.g., Pauly 1974, Eckstein et al. 1985, Abel 1986, Mitchell et al. 1999, Walliser 2000).

However, there is a further consequence of the adverse-selection problem, namely that the time structure of the benefits matters. Individuals with low life-expectancy put less weight on pension payouts in later periods than individuals with high life-expectancy. This aspect is

[1] Tax incentives are granted in many industrialised countries, e. g. in Great Britain, U.S.A, Canada and Sweden. Moreover, the recent reform of social security in Germany aims at cutting public pensions and inducing individuals to pay contributions of four percent of income to a private old-age insurance by granting a tax release. Similarly, in Austria contributions to a private old-age insurance are subsidised by a premium since 2000.

neglected in the usual overlapping-generations model with one working period and one period of retirement. But in reality the time of retirement must not be seen as a single, homogeneous period, for which provision can be made through a one-and-for-all contract only, with a fixed and constant (in nominal or real terms) payout. Planning individuals, being aware of some estimate of their life-expectancy, will attempt to make provision in accordance with this estimate, which means that they want to use more differentiated instruments. In practice, they can buy an insurance contract with payouts increasing or decreasing over time, or they can buy a limited-time contract for the earlier phase of retirement and then use another instrument to provide for the rest of their lifetime.[2]

In order to analyse the consequences on the functioning of the annuity market of the fact that the time structure of the payouts matters, one has to extend the standard model by assuming that retirement consists of more periods and that provision can be made separately for each of them. In a previous paper (Brunner and Pech 2000) we introduced a model with one working period and two periods of retirement, where two groups of individuals with differing life-expectancy buy an annuity contract which runs for the whole time of retirement, but with payouts possibly varying over time.[3] We have shown that in this framework an equilibrium in the sense of Nash-Cournot may but need not exist.[4]

In the present contribution we consider a similar model, but with different types of contracts. We again assume that individuals live for one working period and for at most two periods of retirement, but now contracts run for one period only; for the second period, a new contract has to be bought. By this formulation we take account of the fact that in reality term-insured pension contracts exist, which provide payouts only for a limited time, given that the individual is alive. For the rest some other form of provision must be made.[5]

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[2] Poterba (1997) emphasizes the importance of the wide range of different annuity products for the growth of the U.S. annuity market. He provides a typology of individual annuities with respect to the terms under which accumulated capital is dispersed during the liquidation phase. In particular, he distinguishes between two broad classes of individual annuities, that are deferred and immediate annuities, depending on whether there is a waiting period between the premium payment and the beginning of the annuity payouts or not.
The role of annuity contracts with escalating payouts in the U.K. annuity market is studied by Finkelstein and Poterba (2002).

[3] Yagi and Nishigaki (1993) also employed a model with one working period and two periods of retirement in order to discuss optimal insurance demand of a representative individual. They showed that constant annuity payouts over time are inefficient, given that the individual rate of time preference differs from the interest rate. However, they did not consider the adverse-section problem and its impact on the existence of equilibria.

[4] With these life annuities, firms can separate individuals according to their life-expectancy by a variation of the payouts over time. In fact, only a separating equilibrium (compare Rothschild and Stiglitz 1976) can occur.

[5] Townley and Boadway (1987) studied the functioning of the annuity market when individuals save out of their payouts from the limited-time pension contract. In contrast, we consider the case that they can buy a second annuity to provide for the remaining time.

The important issue which we address is that individuals can choose between two strategies to provide for the second period of retirement: simultaneously, that is, individuals buy an additional contract already in the working period, or sequentially, that is, only those individuals who have survived to the first period of retirement, purchase an additional contract on the spot market. We show that in our model individuals in general chose only one of these alternatives, depending on the rates of return. However, in a first-best equilibrium the rates of return, which then correspond to the individual life-expectancies, assume such values which make individuals indifferent between the two alternatives, because each provides the same consumption path over lifetime.

This is no longer true if asymmetric information, where the rates of return are distorted by adverse selection, is introduced in our model. Then, under the assumption of price competition between annuity companies, the rate of return for any contract is the same for both risk-groups, and only a situation where both groups buy the same type of second-period contract is feasible. Further, it turns out that the type of contract chosen to provide for the second period of retirement affects also the rate of return in the first period. In particular, we find that the two strategies have differing consequences for the welfare of the individuals, because they allow different consumption paths over the time of retirement: long-living individuals, who put more weight on consumption in the second period, prefer the regime when all individuals make sequential provision, while short-living individuals prefer the regime with simultaneous provision. Finally, we find that only the former regime, favourable to the long-living individuals, represents a Nash-Cournot equilibrium.

In the following Section 2 we introduce the basic model and show that either simultaneous or sequential annuity contracts are chosen. We characterise demand in both cases. In Section 3 we analyse the consequences of adverse selection for the rates of return, consumption and welfare of the individuals. Moreover, the existence of an equilibrium is proved. Section 4 provides concluding comments.

2. Sequential and simultaneous demand for annuities

2.1. The basic model

Consider an economy with N individuals who live for a maximum of three periods $t = 0,1,2$. In the working period $t = 0$, each individual earns a fixed labour income w_0. At the end of period 0 she retires and lives for at most two further periods. Survival to the retirement period $t = 1$ occurs with probability π_1^i, $0 < \pi_1^i < 1$. In the same way, given that an individual is alive in period 1, survival to period 2 occurs with probability π_2^i, $0 < \pi_2^i < 1$.

Provision for old age can be made through three types of contracts, which are offered by insurance companies:

- A_1 denotes the quantity of a contract, which is bought in working period 0 and offers an immediate payout q_1A_1 in retirement period 1
- A_2 denotes the quantity of a contract, which is bought in retirement period 1 and offers an immediate payout q_2A_2 in retirement period 2.
- D_2 denotes the quantity of a contract, which is bought in working period 0 and offers a deferred payout r_2D_2 in retirement period 2.

That is, each type of contract offers payouts for one period of retirement, but they differ in the date of purchase and the waiting period for the payout to begin: provision for retirement period 1 is made through A_1, while provision for retirement period 2 can be made through A_2 (bought by those only, who survive to retirement period 1) and/or through D_2 (bought already in the working period). q_1, q_2 and r_2 are the corresponding payouts per unit of insurance. The important point of our discussion concerns the difference in the rates of return, whether the individuals use A_2- or D_2-contracts to have a guaranteed income in the second period of retirement.

We assume that the individuals have no bequest motive, which means that saving is not an attractive strategy for them to provide for old-age. This follows from the fact that the rate of return of annuities is higher than the interest rate, as annuities allow to avoid (and redistribute) unintended bequests (see Yaari 1965). Further, in order to concentrate on the design of the annuity contracts and to simplify the analysis, the assumption is made that no public pension system exists. The budget equation of an individual i for the working period 0 is

$$c_0^i = w_0 - A_1^i - D_2^i. \tag{2.1}$$

Moreover, given that individual i is alive in the retirement period 1, she can spend an amount A_2^i from her income w_1^i in order to make additional provision for consumption in the retirement period 2, and consumes an amount c_1^i. This gives us the budget equations for the two retirement periods t = 1,2:

$$c_1^i = q_1 A_1^i - A_2^i, \tag{2.2}$$

$$c_2^i = q_2 A_2^i + r_2 D_2^i. \tag{2.3}$$

Preferences over lifetime consumption of an individual i are time-separable and are represented by expected utility with a per-period utility function u depending on consumption. An individual i is confronted with the following two-stage decision problem: in the working period 0, she decides on the quantities A_1^i and D_2^i of annuities, thus on her consumption level in period 0 and on her income w_t^i in each of the two retirement periods t = 1,2. For this decision she takes into account her optimal annuity demand A_2^i and her optimal consumption levels in periods 1 and 2, which she will choose in period 1, given that then she is alive. Formally, this two-stage problem can be written as:

$$t = 0: \quad \max \quad u(c_o^i) + \pi_1^i \varphi^i(q_1 A_1^i, q_2, r_2 D_2^i), \qquad \text{s. t. (2.1)}, \tag{2.4}$$

$$t = 1: \quad \max \quad u(c_1^i) + \pi_2^i u(c_2^i), \qquad \text{s. t. (2.2) and (2.3)}, \tag{2.5}$$

where $\varphi^i(w_1^i, q_2, w_2^i) \equiv \max_{c_1^i, c_2^i, A_2^i} \{u(c_1^i) + \pi_2^i u(c_2^i) \mid c_1^i = w_1^i - A_2^i,\ c_2^i = w_2^i + q_2 A_2^i\}$.

Concerning the A_2-contract, we in fact assume that the individuals are informed about its return q_2 already in the working period 0, in other words, that the insurance companies can credibly commit to offer those contracts with return q_2 one period later. Otherwise φ^i would not be well-defined. Further, we assume $u'(c_t^i) > 0$, $u''(c_t^i) < 0$ and $\lim_{c \to 0} u'(c) = \infty$. Notice that the specification of the decision problem means that the individuals have no bequest motive and do not discount future consumption for any reason other than risk aversion.

By inserting (2.1) into (2.4) and differentiating with respect to A_1^i and D_2^i as well as inserting (2.2) and (2.3) into (2.5) and differentiating with respect to A_2^i, we obtain the Kuhn-Tucker conditions of this maximization problem:

$$-u'(w_0 - A_1^i - D_2^i) + \pi_1^i q_1 \frac{\partial \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_1^i} = 0\,, \tag{2.6}$$

$$D_2^i > 0 \quad \text{and} \quad -u'(w_0 - A_1^i - D_2^i) + \pi_1^i r_2 \frac{\partial \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_2^i} = 0 \text{ or} \tag{2.7a}$$

$$D_2^i = 0 \quad \text{and} \quad -u'(w_0 - A_1^i - D_2^i) + \pi_1^i r_2 \frac{\partial \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_2^i} \le 0\,, \tag{2.7b}$$

$$A_2^i > 0 \quad \text{and} \quad -u'(q_1 A_1^i - A_2^i) + \pi_2^i q_2 u'(r_2 D_2^i + q_2 A_2^i) = 0 \text{ or} \tag{2.8a}$$

$$A_2^i = 0 \quad \text{and} \quad -u'(q_1 A_1^i - A_2^i) + \pi_2^i q_2 u'(r_2 D_2^i + q_2 A_2^i) \le 0\,, \tag{2.8b}$$

where by application of the Envelope Theorem

$$\frac{\partial \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_1^i} = u'(c_1^i)\,, \tag{2.9}$$

$$\frac{\partial \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_2^i} = \pi_2^i u'(c_2^i)\,. \tag{2.10}$$

Obviously, an individual i always has a positive annuity demand A_1^i for the first-period contract, since this is the only possibility to provide for first-period consumption, while she can decide either to buy the immediate annuity contract (i.e. $A_2^i > 0$, $D_2^i = 0$) or the deferred contract (i.e. $A_2^i = 0$, $D_2^i > 0$) or both kind of contracts (i.e. $A_2^i > 0$, $D_2^i > 0$) in order to make provision for consumption in the second retirement period. The following Lemma shows that the latter case is in general excluded.

Lemma 1: In general, it is not optimal for an individual i to choose both $D_2^i > 0$ and $A_2^i > 0$. The inequality $q_1 q_2 < r_2$ ($q_1 q_2 > r_2$) implies $D_2^i > 0$ and $A_2^i = 0$ ($A_2^i > 0$ and $D_2^i = 0$, resp.).

Proof: By use of the equation in (2.8a), (2.9) and (2.10) the term $\pi_1^i q_1 \,\partial\varphi^i(w_1^i, q_2, w_2^i)/\partial w_1^i$ in (2.6) can be written as $\pi_1^i q_1 q_2 \,\partial\varphi^i(w_1^i, q_2, w_2^i)/\partial w_2^i$, which in general will not be equal to the term $\pi_1^i r_2 \,\partial\varphi^i(w_1^i, q_2, w_2^i)/\partial w_2^i$ in (2.7a). As we know that (2.6) must always be fulfilled, this means that the equations in (2.7a) and (2.8a) cannot hold simultaneously. By the same reasoning, one observes that if (2.8a) holds, (2.7b) can be fulfilled only if $r_2 \leq q_1q_2$, and analogous for (2.7a) and (2.8b). Q.E.D.

One unit of income invested into the first-period contract transforms into a payout of q_1, out of which an A_2-contract can be bought, which offers q_2 as the rate of return. Therefore, the decisive relation is $q_1q_2 \gtrless r_2$, whether provision for the second period of retirement should be the made through a A_2- or a D_2-contract. (Only in case of $q_1q_2 = r_2$, the individuals would be indifferent. For the analysis of the present section we rule out this specific parameter constellation.)

2.2. The influence of the rates of return on annuity demand

For the discussion of how annuity demand depends on the rates of return, we note as a preparation:

Lemma 2: $\varphi^i(w_1^i, q_2, w_2^i)$ is strictly concave with respect to w_1^i and w_2^i.

Proof: See Appendix.

In the following, we distinguish between two different situations, whether an individual expresses annuity demand sequentially or simultaneously.

Sequential annuity demand: $A_1^i > 0,\ A_2^i > 0,\ D_2^i = 0$.

Let $A_t^i(q_1,q_2)$, t =1,2, be annuity demand arising from a two-stage decision process, whose optimal solution is characterised by (2.6) and (2.8a), for given q_1 and q_2.

Lemma 3: We denote the Arrow-Pratt coefficients of relative risk aversion for the functions u and φ by $R_u^i \equiv -c_t^i u''(c_t^i)/u'(c_t^i)$, $R_\varphi^i \equiv -w_1^i(\partial^2\varphi^i(w_1^i,q_2,w_2^i)/\partial {w_1^i}^2)/(\partial\varphi^i(w_1^i,q_2,w_2^i)/\partial w_1^i)$. For any given rates of return q_1 and q_2, the effect of a marginal increase in q_t, t = 1,2, on the annuity demand $A_t^i(q_1,q_2)$ of an individual i for each contract depends on the coefficients R_φ^i and R_u^i of relative risk aversion in the following way:

$$\frac{\partial A_1^i(q_1,q_2)}{\partial q_1} \gtreqless 0, \text{ if } R_\varphi^i \lesseqgtr 1, \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial q_1} > 0, \text{ if } R_\varphi^i \le 1, \text{ otherwise undetermined,}$$

$$\frac{\partial A_1^i(q_1,q_2)}{\partial q_2} \gtreqless 0, \text{ if } R_u^i \lesseqgtr 1, \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial q_2} \gtreqless 0, \text{ if } R_u^i \lesseqgtr 1.$$

Proof: See Appendix.

To provide some intuition for this result we consider case (i), the effect of a marginal increase of q_1 on $A_1^i(q_1,q_2)$. We compute $\partial A_1^i(q_1,q_2)/\partial q_1$ by implicit differentiation of (2.6) as

$$\frac{\partial A_1^i(q_1,q_2)}{\partial q_1} = -\frac{\pi_1^i(\partial\varphi^i(w_1^i,q_2,w_2^i)/\partial w_1^i + q_1 A_1^i \partial^2\varphi^i(w_1^i,q_2,w_2^i)/\partial w_1^{i\,2})}{u''(w_0 - A_1^i) + \pi_1^i q_1^2\, \partial^2\varphi^i(w_1^i,q_2,w_2^i)/\partial w_1^{i\,2}}. \qquad (2.11)$$

Since the denominator of the RHS of (2.11) is negative due to strict concavity of $u(c_t^i)$ and of the value function $\varphi^i(w_1^i,q_2,w_2^i)$ (see Lemma 2), $\partial A_1^i(q_1,q_2)/\partial q_1$ has the same sign as the numerator of the RHS of (2.11). One observes that the numerator is just the derivative of $\pi_1^i \partial\varphi^i/\partial A_1^i$ $(= \pi_1^i q_1 \partial\varphi^i/\partial w_1^i)$ with respect to q_1, i.e. it describes the effect of an increase in q_1 on the marginal utility of A_1^i in period 1, if we take φ^i as a utility function, depending on w_1^i. It is intuitively plausible that demand for annuity A_1^i increases (which means that instantaneous consumption c_0^i decreases), if the marginal utility of future income from the annuity increases with q_1. Moreover we note that this effect on the derivative of φ^i can immediately be seen to have the same sign as $1 - R_\varphi^i$, which explains the result.

Similar considerations apply for the other cases. We only mention that, as annuity demand $A_2^i(q_1,q_2)$ is normal with respect to income in period 1, the decisive question for the cross effect in case (ii) is whether an increase of q_1 increases w_1^i. This certainly occurs if demand for $A_1^i(q_1,q_2)$ increases with q_1 (i.e., if $1 - R_\varphi^i \ge 0$). Otherwise the effect is undetermined.

Simultaneous annuity demand: $A_1^i > 0,\ A_2^i = 0,\ D_2^i > 0$.

Now we consider annuity demand $A_1^i(q_1,r_2)$ and $D_2^i(q_1,r_2)$, determined by (2.6) and the equation in (2.7a). With $A_2^i = 0$ these equations reduce to (see (2.9), (2.10))

$$-u'(w_0 - A_1^i - D_2^i) + \pi_1^i q_1 u'(q_1 A_1^i) = 0, \qquad (2.12)$$

$$-u'(w_0 - A_1^i - D_2^i) + \pi_1^i \pi_2^i r_2 u'(r_2 D_2^i) = 0. \tag{2.13}$$

Lemma 4: The effect of a marginal increase in q_1 and r_2, resp., on annuity demand depends on the coefficient R_u^i of relative risk aversion in the following way:

$$\frac{\partial A_1^i(q_1,r_2)}{\partial q_1} \gtreqless 0 \text{ if } R_u^i \lesseqgtr 1, \qquad \frac{\partial D_2^i(q_1,r_2)}{\partial q_1} \lesseqgtr 0 \text{ if } R_u^i \lesseqgtr 1,$$

$$\frac{\partial A_1^i(q_1,r_2)}{\partial r_2} \lesseqgtr 0 \text{ if } R_u^i \lesseqgtr 1, \qquad \frac{\partial D_2^i(q_1,r_2)}{\partial r_2} \gtreqless 0 \text{ if } R_u^i \lesseqgtr 1.$$

Proof: See Appendix.

The intuitive reason for these results is analogous to that offered above. The relevant issue is how an increase of q_1 or r_2 affects marginal utility of the respective annuity. $R_u \leq 1$ means that marginal utility increases, then A_1^i increases with q_1 and decreases with r_2, and vice versa for D_2^i.

2.3. The influence of the survival probabilities on annuity demand

As a next step we show how demand for annuities depends on the individuals' probabilities of survival.

Lemma 5:

(i) In case of sequential annuity demand, i.e. $A_1^i > 0$, $A_2^i > 0$, $D_2^i = 0$: For any given rates of return q_1, q_2, we have

$$\frac{\partial A_1^i(q_1,q_2)}{\partial \pi_1^i} > 0, \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial \pi_1^i} > 0,$$

$$\frac{\partial A_1^i(q_1,q_2)}{\partial \pi_2^i} > 0, \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial \pi_2^i} > 0.$$

(ii) In case of simultaneous annuity demand, i.e. $A_1^i > 0$, $A_2^i = 0$, $D_2^i > 0$: For any given rates of return q_1, r_2, we have

$$\frac{\partial A_1^i(q_1,r_2)}{\partial \pi_1^i} > 0, \qquad \frac{\partial D_2^i(q_1,r_2)}{\partial \pi_1^i} > 0\,,$$

$$\frac{\partial A_1^i(q_1,r_2)}{\partial \pi_2^i} < 0\,, \qquad \frac{\partial D_2^i(q_1,r_2)}{\partial \pi_2^i} > 0\,.$$

Proof: See Appendix.

We find that generally annuity demand reacts positively, if any probability of survival increases. However, there is an essential difference between the two cases, which concerns the cross effect of π_2^i on the first-period contract. With sequential decisions, an increase of the probability of survival to the second-period of retirement increases demand A_1^i, because this allows to buy more insurance for period 2. On the other hand, with simultaneous decisions an increase of π_2^i means that insurance for the first period of retirement is substituted by insurance for the second period of retirement. Note further that an increase in π_1^i clearly increases the probability $\pi_1^i\pi_2^i$ of survival to the second period as well, hence demand for the second-period contracts rises in both cases.

2.4. Individually fair contracts

An annuity is said to be *individually fair*, if expected payouts equal its price. This requires for the A_1-, A_2- and D_2-contract that the respective condition

$$1 - \pi_t^i q_t = 0\,,\ t = 1,2, \tag{2.14}$$

$$1 - \pi_1^i \pi_2^i r_2 = 0\,. \tag{2.15}$$

hold. Obviously, this implies that the annuity companies make zero expected profits, given that identical individuals buy these contracts.

Lemma 6: Given individually fair contracts, any individual is indifferent between choosing an A_2- or D_2-contract for the second period or retirement. She chooses the same level of consumption in every period t = 0,1,2.

Proof: The zero-profit conditions (2.14), (2.15) imply $q_1 q_2 = r_2$, which is the condition for indifference, as mentioned above. Considering (2.6) – (2.10), one observes that the zero-profit conditions also imply $u'(c_0^i) = u'(c_1^i) = u'(c_2^i)$, irrespective of the chosen contracts.

Q.E.D.

In a first-best world, where every individual can buy an annuity contract whose rate of return is precisely adjusted to her life-expectancy, it does not matter, which type of contract is chosen for provision for the second period of retirement. Anyone offers an optimal smoothing of consumption. However, in reality, asymmetric information prevents the supply of first-best contracts. This has consequences for the functioning of the annuity market in our model, as will be discussed in the next section.

3. The consequences of asymmetric information on the rates of return and on the existence of equilibria

3.1. Adverse selection with two groups of individuals

Suppose that the otherwise identical individuals are divided into two groups i = L,H, characterised by different risks of a long life, i.e. by different probabilities of survival $\pi_t^H > \pi_t^L$ for t = 1,2. Let γ_0 and $1 - \gamma_0$, resp., denote the shares of the high-risk and low-risk individuals in period 0, with $0 < \gamma_0 < 1$. The probabilities π_t^i and γ_0 are public information, known by the annuity companies. But it is the private information for each individual to know her type, i.e. her probability of survival. As a consequence, there is an adverse-selection problem in the annuity market. Moreover, we assume that there is perfect competition among the annuity companies and that they cannot monitor whether consumers buy annuities from other insurance companies, which seems to be a reasonable assumption frequently made for the annuity market (see e.g. Pauly 1974, Abel 1986, Brugiavini 1993, Walliser 2000). It follows that in equilibrium for each type of contract only one price, that is one payout q_1, q_2, r_2, resp., per unit of annuity, can exist for each period t = 1,2.

Consider the individually fair rates of return of the A_1-, A_2- and D_2-contract as defined by (2.14) and (2.15). It is obvious from our assumption on the probabilities of survival that these rates are higher for type-L individuals than for type-H individuals. Therefore, asymmetric information excludes the first-best solution, because different rates of returns cannot exist. Both types of individuals buy the same contract, which is called a pooling situation.

Moreover, the same argument implies that only a situation, where both groups use the same type of contract in order to provide for the second period of retirement, can prevail. That is, either both groups use the A_2-contract for the second period of retirement or both groups use the D_2-contract. This follows from the fact that only one rate of return q_1 for the first-period contract A_1 can exist, and that each group chooses either the A_2- or the D_2-contract, depending on whether $q_1q_2 \gtrless r_2$ (see Lemma 1). Thus we distinguish between two different regimes, where all individuals demand either sequential or simultaneous pooling contracts:

sequential regime: $A_1^i > 0$, $A_2^i > 0$, $D_2^i = 0$ for i = L, H,

simultaneous regime: $A_1^i > 0$, $A_2^i = 0$, $D_2^i > 0$ for i = L, H.

As a next step we discuss, to which extent the adverse-selection problem matters in the two regimes, that is, whether individuals with a long life-expectancy buy a larger amount of the different types of contracts.

Lemma 7:

(i) In the sequential regime, for any rates of return q_1,q_2, an individual with high survival probabilities demands larger quantities of annuities than an individuals with low survival probabilities, i. e. $A_t^H(q_1,q_2) > A_t^L(q_1,q_2)$, t = 1,2.

(ii) In the simultaneous regime, for any rates of return q_1,r_2, an individual with high survival probabilities demands a larger quantity D_2 than an individual with low survival probabilities, i.e. $D_2^H(q_1,r_2) > D_2^L(q_1,r_2)$. The ratio of demand for the A_1-contract is undetermined.

Proof: Follows immediately from Lemma 5 and $\pi_t^L < \pi_t^H$, t = 1,2. Q.E.D.

If one defines the problem of adverse selection by the extent to which the ratio of aggregate group-H demand to aggregate group-L demand exceeds $\gamma_0/(1-\gamma_0)$ (the ratio in case that demand is proportional to the population shares), we find that this problem certainly occurs for both contracts in the sequential regime; in the simultaneous regime it occurs for the second-period contract, while for the first-period contract it is mitigated by the fact that an increase of π_2^i decreases demand for the A_1-contract.

The consequence of the over-representation of high-risk individuals among aggregate annuity demand is that in equilibrium insurance companies offer a rate of return which is lower than the actuarially fair rate corresponding to the average probability of survival of the population. The respective rates are determined by the condition that, due to the assumption of perfect competition in the annuity market, the expected profits of a (pooling) contract, bought by both groups L and H, must be equal to zero. As $q_t \pi_t^i$ is the expected payout for group i, the zero-profit-conditions for the A_1- and the A_2-contract read

$$(1-\gamma_0)A_1^L(1-q_1\pi_1^L)+\gamma_0 A_1^H(1-q_1\pi_1^H)=0 \tag{3.1}$$

$$(1-\gamma_1)A_2^L(1-q_2\pi_2^L)+\gamma_1 A_2^H(1-q_2\pi_2^H)=0\,. \tag{3.2}$$

Since type-H individuals have a higher probability to survive to retirement period 1, i.e. $\pi_1^H > \pi_1^L$, their share in period 1 will rise to $\gamma_1 \equiv \gamma_0\pi_1^H/(\gamma_0\pi_1^H+(1-\gamma_0)\pi_1^L)$, while the share of type-L individuals reduces to $(1-\gamma_1)$. Thus relatively more type-H individuals will buy an A_2-contract for the retirement period 2. In the simultaneous regime, where the A_1-contract is supplemented by the D_2-contract, the expected payout from the latter is $r_2\pi_1^i\pi_2^i$, and the zero-profit condition reads

$$(1-\gamma_0)D_2^L(1-r_2\pi_1^L\pi_2^L)+\gamma_0 D_2^H(1-r_2\pi_1^H\pi_2^H)=0\,. \tag{3.3}$$

Note that the equilibrium rates of return cannot be computed explicitly from (3.1) – (3.3), because in each equation annuity demand depends on the respective rates. Nevertheless, if one takes the ratio of aggregate demand of group L to that of group H as exogenous for the moment, one observes that the respective equilibrium rate is lower, the lower this ratio. In the next section we turn to a closer analysis of this relation.

3.2. Rates of return and consumption in both regimes

Our aim is to study how the functioning of the annuity market is affected, when annuity companies can offer both kinds of second-period contracts, namely immediate as well as deferred annuities. For this purpose, from now on we assume that instantaneous utility is logarithmic, i.e.

$$u(c_t^i) = \ln(c_t^i) \text{ for } t = 0,1,2. \tag{3.4}$$

Logarithmic utility has the convenient property that annuity demand of an individual i for any contract does not depend on the payoff of this contract, which helps to keep the analysis simple.[6] First we determine annuity demand for logarithmic utility in both regimes. (In the following, a tilde refers to the sequential regime, while a bar refers to the simultaneous regime.)

Sequential regime: The conditions (2.6) and (2.8a) together with (2.9) determine annuity demand $\tilde{A}_1^i$ and $\tilde{A}_2^i$ for each single-period contract. For logarithmic utility one computes

$$\tilde{A}_1^i = \frac{\pi_1^i(1+\pi_2^i)}{1+\pi_1^i(1+\pi_2^i)} w_0, \tag{3.5}$$

$$\tilde{A}_2^i = q_1 \frac{\pi_1^i \pi_2^i}{1+\pi_1^i(1+\pi_2^i)} w_0. \tag{3.6}$$

Simultaneous regime: By use of (2.12), (2.13) and (3.4) we obtain annuity demand $\overline{A}_1^i$ for the first-period contract and annuity demand $\overline{D}_2^i$ for the second-period contract as

$$\overline{A}_1^i = \frac{\pi_1^i}{1+\pi_1^i(1+\pi_2^i)} w_0, \tag{3.7}$$

$$\overline{D}_2^i = \frac{\pi_1^i \pi_2^i}{1+\pi_1^i(1+\pi_2^i)} w_0. \tag{3.8}$$

Lemma 8: The consumption level c_0^i of an individual i in working period 0 is the same irrespective whether she chooses sequential or simultaneous pension insurance.

Proof: This result follows from the fact that $\tilde{A}_1^i = \overline{A}_1^i + \overline{D}_2^i$: substituting (3.5) and $D_2^i = 0$ into (2.1) gives the same consumption level c_0^i as substituting (3.7) and (3.8) into (2.1). Q.E.D.

Note that Lemma 8 holds for any contracts with rates of return q_1,q_2 and q_1,r_2, since the relevant annuity demand and thus consumption in period 0 does not depend on the rates of return, while they influence the level of consumption in the retirement periods 1 and 2. Thus,

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[6] Note that the coefficient of relative risk aversion occurring in Lemmas 3 and 4 is equal to one for logarithmic utility.

for a comparison of these under both regimes, we have to determine the rates of return explicitly for both regimes by inserting annuity demand into the respective zero-profit conditions.

Sequential regime: Solving (3.1), together with (3.5), for q_1 and (3.2), together with (3.6), for q_2 gives

$$\tilde{q}_1 \equiv \frac{1+\tilde{\rho}_1}{\pi_1^L + \pi_1^H \tilde{\rho}_1}, \tag{3.9}$$

$$\tilde{q}_2 \equiv \frac{1+\tilde{\rho}_2}{\pi_2^L + \pi_2^H \tilde{\rho}_2}, \tag{3.10}$$

where $\tilde{\rho}_1 \equiv \gamma_0 \tilde{A}_1^H / ((1-\gamma_0)\tilde{A}_1^L)$ and $\tilde{\rho}_2 \equiv \gamma_1 \tilde{A}_2^H / ((1-\gamma_1)\tilde{A}_2^L)$.

Simultaneous regime: Analogously, solving (3.1), together with (3.7), for q_1 and (3.3), together with (3.8), for r_2 yields

$$\bar{q}_1 \equiv \frac{1+\bar{\rho}_1}{\pi_1^L + \pi_1^H \bar{\rho}_1}, \tag{3.11}$$

$$\bar{r}_2 \equiv \frac{1+\bar{\rho}_2}{\pi_1^L \pi_2^L + \pi_1^H \pi_2^H \bar{\rho}_2}, \tag{3.12}$$

where $\bar{\rho}_1 \equiv \gamma_0 \bar{A}_1^H / ((1-\gamma_0)\bar{A}_1^L)$ and $\bar{\rho}_2 \equiv \gamma_0 \bar{D}_2^H / ((1-\gamma_0)\bar{D}_2^L)$.

Lemma 9: Comparing the two regimes, the following relations hold

(i) between the ratios of aggregate annuity demand of group H to that of group L for the different types of contracts: $\tilde{\rho}_1 > \bar{\rho}_1$, $\tilde{\rho}_2 > \bar{\rho}_2$, $\bar{\rho}_2 > \tilde{\rho}_1$,

(ii) between the rates of return from the different types of contracts: $\tilde{q}_1 < \bar{q}_1$, $\tilde{q}_2 < \bar{r}_2$, $\bar{r}_2 < \tilde{q}_1 \tilde{q}_2$.

Proof: See Appendix.

The inequalities $\tilde{\rho}_1 > \bar{\rho}_1$ and $\tilde{q}_1 < \bar{q}_1$ indicate that the adverse-selection problem for the first-period contract is more severe in the sequential regime than in the simultaneous regime (compare the discussion after Lemma 7). The intuitive reason for this result is the following:

in the simultaneous regime, annuity demand $\bar{A}_1^i$ for the first-period contract satisfies only the need for future consumption in period 1. In contrast, in the sequential regime, annuity demand $\tilde{A}_1^i$ for the first-period contract has to satisfy the need for future consumption in both retirement periods 1 and 2, since part of the returns $\tilde{q}_1\tilde{A}_1^i$ is used for $\tilde{A}_2^i$. High-risk individuals choose a higher demand $\tilde{A}_2^i$ than low risk-individuals (see Lemma 7), which in turn intensifies adverse selection for the first-period contract. Hence $\tilde{\rho}_1 > \bar{\rho}_1$, which affects the payoffs accordingly, inducing $\tilde{q}_1 < \bar{q}_1$.

The essential reason, why $\bar{\rho}_2 < \tilde{\rho}_2$ and $\bar{r}_2 > \tilde{q}_2$ hold, is that from period 0 (when the D_2-contract is bought) to period 1 (when the A_2-contract is bought) the share of the high-risk individuals in the population rises, i.e. $\gamma_0 < \gamma_1$, because of $\pi_1^L < \pi_1^H$. As the ratio of individual demand is the same for both contracts ($\tilde{A}_2^H / \tilde{A}_2^L = \bar{D}_2^H / \bar{D}_2^L$, see (3.6) and (3.8)), these shares indeed are responsible for the lower rate of return of the A_2-contract compared to that of the D_2-contract.

A further important result of Lemma 9 is the inequality $\tilde{q}_1\tilde{q}_2 > \bar{r}_2$. Remember that $\tilde{q}_1\tilde{q}_2$ is the payout resulting from one unit of income, invested in the working period 0 in order to provide for the second period of retirement in the sequential regime. It is larger than $\bar{r}_2$, the corresponding payout in the simultaneous regime. This can be explained by the fact that in the former regime provision for period 2 is made via the first-period contract A_1, which is bought by the low-risk individuals to a larger extent than the D_2-contract (note that $\tilde{\rho}_1 < \bar{\rho}_2$). In other words, in the sequential regime the high-risk individuals, when insuring for the second period, profit from being for the first period in a pool with the low-risk individuals, who put particular weight on insurance for this period, due to their short life-expectancy. In a sense, this result represents the counterpart to the above argument explaining why $\tilde{q}_1 < \bar{q}_1$.

Having determined annuity demand and the rates of return in both regimes, we are ready to calculate the consumption levels in retirement periods 1 and 2.

Lemma 10:

(i) In retirement period 1, consumption of any individual i = L,H, is lower in the sequential regime than in the simultaneous regime, i.e. $c_1^i(\tilde{q}_1, \tilde{q}_2) < c_1^i(\bar{q}_1, \bar{r}_2)$ for i = L,H.

(ii) In retirement period 2, consumption of any individual i = L,H, is higher in the sequential regime than in the simultaneous regime, i.e. $c_2^i(\tilde{q}_1, \tilde{q}_2) > c_2^i(\bar{q}_1, \bar{r}_2)$ for i = L,H.

Proof: See Appendix.

Consumption in each period of retirement depends on the amount of annuities bought for that period and on their rates of return. Lemma 10 shows that the relations between the respective rates of return ($\tilde{q}_1 < \overline{q}_1$, $\bar{r}_2 < \tilde{q}_1 \tilde{q}_2$), as found in Lemma 9 (ii), are in fact decisive for the distribution of consumption over the two periods of retirement in the two regimes. With the sequential regime, more consumption is postponed to the second period of retirement, while the simultaneous regime induces individuals to consume relatively more in the first period of retirement.

For an illustration of the results of this section, we determine the consumption possibility curves in (c_1^i, c_2^i)-space under each regime. For this, we use the convenient property that the consumption level in period 0 of an individual i is independent of the chosen regime.

Sequential regime: Substituting $\tilde{A}_2^i = c_2^i / \tilde{q}_2$ (see (2.3) for $D_2^i = 0$) into $c_1^i = \tilde{q}_1 \tilde{A}_1^i - \tilde{A}_2^i$, one obtains the consumption possibility curve, denoted by CPC_{SE}^i, as

$$c_2^i = \tilde{q}_1 \tilde{q}_2 \tilde{A}_1^i - \tilde{q}_2 c_1^i. \tag{3.13}$$

This relation, which is depicted in Figure 1, describes the feasible consumption bundles (c_1^i, c_2^i) for an individual i who invests the fixed amount $\tilde{A}_1^i = w_0 - c_0^i$ into the first-period contract. She can consume all returns $\tilde{q}_1 \tilde{A}_1^i$ in period 1 ($c_2^i = 0$ gives $c_1^i = \tilde{q}_1 \tilde{A}_1^i$) or transform part of it into second-period consumption, by buying the sequential second-period contract. If she transforms everything ($c_1^i = 0$), then $c_2^i = \tilde{q}_1 \tilde{q}_2 \tilde{A}_1^i$ results. An individual i chooses demand $\tilde{A}_2^i$ by maximizing $u(c_1^i) + \pi_2^i u(c_2)$ subject to the consumption possibility set. The payoffs $\tilde{q}_1, \tilde{q}_2$ in turn are determined such that the zero-profit conditions with actual demand of both groups are fulfilled.

Simultaneous regime: Analogously, by use of $\tilde{A}_1^i = \overline{A}_1^i + \overline{D}_2^i$ (see proof of Lemma 8) and $c_1^i = \overline{q}_1 \overline{A}_1^i$, $c_2^i = \bar{r}_2 \overline{D}_2^i$, the consumption possibility curve, denoted by CPC_{SI}^i, is derived as

$$c_2^i = \bar{r}_2 \tilde{A}_1^i - \frac{\bar{r}_2}{\overline{q}_1} c_1^i, \tag{3.14}$$

which is depicted in Figure 2. In this regime, the trade-off between consumption in period 1 and in period 2 is $-\bar{r}_2/\bar{q}_1$, which is the ratio of the returns to investing (in the working period 0!) one unit into the contract for the second and for the first retirement period, resp. Clearly, maximum consumption in each period results if everything (i.e., $\tilde{A}_1^i = w_0 - c_0^i$) is invested into the corresponding annuity: $c_2^i = 0$ gives $c_1^i = \bar{q}_1\tilde{A}_1^i$, $c_1^i = 0$ gives $c_2^i = \bar{r}_2\tilde{A}_1^i$. Again, an individual i chooses demand $\bar{A}_1^i$, $\bar{D}_2^i$ by maximizing $u(c_1^i) + \pi_2^i u(c_2^i)$ subject to the consumption possibility set, and the payoffs $\bar{q}_1, \bar{r}_2$ are those which fulfil the zero-profit condition with actual demand.

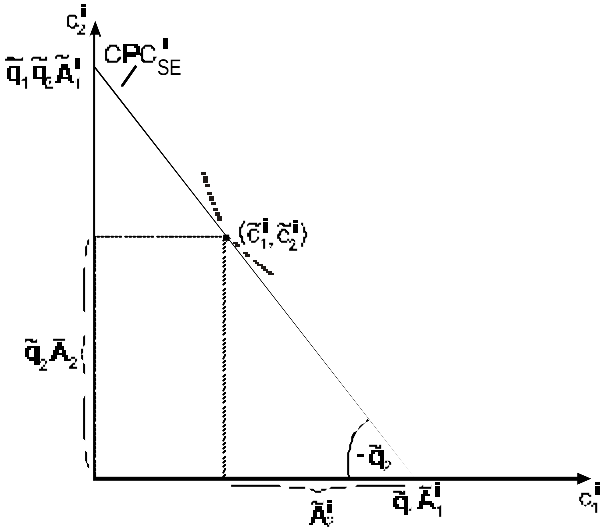

Figure 1

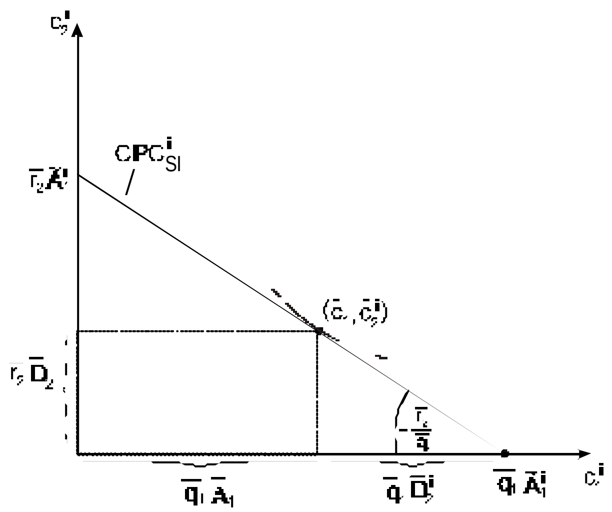

Figure 2

A comparison of the consumption possibility curves in both regimes demonstrates that the curve CPC_{SI}^i crosses the c_1^i-axis at a higher level than the curve CPC_{SE}^i, since $\tilde{q}_1 < \bar{q}_1$. The opposite holds for its crossing with the c_2^i-axis, since $\bar{r}_2 < \tilde{q}_1\tilde{q}_2$. It follows that the CPC's intersect and that CPC_{SI}^i is flatter, i.e. $-\bar{r}_2/\bar{q}_1 > -\tilde{q}_2$. This inequality is responsible for the fact that relative consumption c_2^i/c_1^i is lower in the simultaneous regime than in the sequential regime, as shown in Lemma 10.

3.3. Equilibrium

Now we turn to an analysis of whether either or both of the two regimes constitute an equilibrium. We call a set of contracts a pooling equilibrium in the sense of Nash-Cournot, if together with annuity demand of both groups i = L,H the respective zero-profit condition for each contract is fulfilled and if no other contract exists, which is preferred by at least one group $i \in \{L,H\}$ and which allows a nonnegative profit.

Proposition 1: The sequential contracts with rates of return $\tilde{q}_1, \tilde{q}_2$ represent an equilibrium.

Proof: If an additional A_1-contract with rate of return $q_1 > \tilde{q}_1$ was offered, both groups would buy that and the insurance company would make a loss. (Remember from (3.5) that demand is independent of q_1, hence $\tilde{q}_1$ is the unique payout which fulfils the zero-budget condition.)

Analogously, an insurance company offering an A_2-contract with rate of return $q_2 > \tilde{q}_2$ would make a loss (due to (3.6), demand does not depend on q_2).

Finally, if an alternative D_2-contract with a rate of return $r_2 > \tilde{q}_1\tilde{q}_2$ was offered, again both groups would buy that (see Lemma 1) and the insurance company would make loss. This follows from the fact that demand is independent of r_2 (see 3.8) and the zero-profit conditions require $\bar{r}_2 < \tilde{q}_1\tilde{q}_2$ (see Lemma 9 (iii)). Q.E.D.

Proposition 2: The simultaneous contracts with rates of return $\bar{q}_1, \bar{r}_2$ do not constitute an equilibrium.

Proof: Given the simultaneous contracts, an insurance company can additionally offer a sequential A_2-contract with rate of return $\tilde{q}_2$. For this, note that $\tilde{q}_2$ does not depend on q_1, hence firms make a nonnegative profit by this offer. We know that $\bar{q}_1 > \tilde{q}_1$ and $\tilde{q}_1\tilde{q}_2 > \bar{r}_2$ from Lemma 9. Hence $\bar{q}_1\tilde{q}_2 > \bar{r}_2$, which means that any individual will accept the offer (see Lemma 1). Q.E.D.

Remember that it is in the working period 0 when an individual opts either for the D_2- or the A_2-contract. Hence, the assumption made in section 2.1 that in period 0 insurance companies can credibly commit to offer the sequential contract with rate of return $\tilde{q}_2$ one period later, is essential for these results.

Intuitively there are two reasons why the sequential regime with rates of return $\tilde{q}_1, \tilde{q}_2$ constitutes an equilibrium: i) From the results before we know that the sequential regime allows higher consumption levels in the second retirement period. Thus, it is plausible that no better D_2-contract can be offered without making a loss. ii) No higher rate of return than $\tilde{q}_1$

can be granted for the A_1-contract, in view of the fact that individuals use part of the returns from this A_1-contract to provide for the second period via the A_2-contract.

Conversely, the simultaneous regime with rates of return $\bar{q}_1, \bar{r}_2$ is not an equilibrium, because firms can additionally offer an A_2-contract with return $\tilde{q}_2$, which combined with the existing A_1-contract with return $\bar{q}_1$ allows higher consumption levels in both retirement periods. (Obviously however, the existing A_1-contract with return $\bar{q}_1$ would make a loss in this case, because $\tilde{q}_1$ is the highest return compatible with sequential contracts.)

In the final step of our analysis, we study welfare of both types of individuals i = L,H in the two regimes, in order to find out whether the equilibrium outcome - the sequential contracts - is a favourable solution for one or both risk groups. We know from the discussion in section 3.2 that the consumption possibility curves of both regimes intersect and that the sequential regime allows higher consumption to the left of the point of intersection, but lower consumption to the right. Thus, it is unclear from the analysis of the respective consumption possibilities alone, in which regime an individual of type i is better off.

Proposition 3: An individual of type L is better off in the simultaneous regime with rates of return $\bar{q}_1, \bar{r}_2$, while an individual of type H is better off in the sequential regime with rates of return $\tilde{q}_1, \tilde{q}_2$.

Proof: See Appendix.

We give a graphical illustration of Proposition 3 in Figure 3, where the consumption possibility curves (3.13) and (3.14) of both types of individuals i = L,H are drawn, denoted by CPC_{SE}^{L} and CPC_{SI}^{L} for a low-risk type and by CPC_{SE}^{H} and CPC_{SI}^{H} for a high-risk type. Note that due to adverse selection the long-living individuals make more provision for retirement, therefore their consumption possibility curves are above those of the short-living.

Essential for the result of Proposition 3 is that at any combination (c_1^i, c_2^i) the slope $-c_2^i/(\pi_2^i c_1^i)$ of the indifference curve is steeper for a type-L individual than for a type-H individual, as $\pi_2^L < \pi_2^H$. As one can show, this property implies that, irrespective of the regime, the optimal combination for a type-L individual is to the right of the point of intersection of her consumption possibility curves CPC_{SE}^{L} and CPC_{SI}^{L}, while the optimal consumption bundle for a type-H individual lies to the left of the intersection of CPC_{SE}^{H} and

CPC^{H}_{SI}. Consequently, since the simultaneous regime allows higher consumption possibilities to the right of the point of intersection, it is preferred by a type-L individual. The opposite holds for a type-H individual.

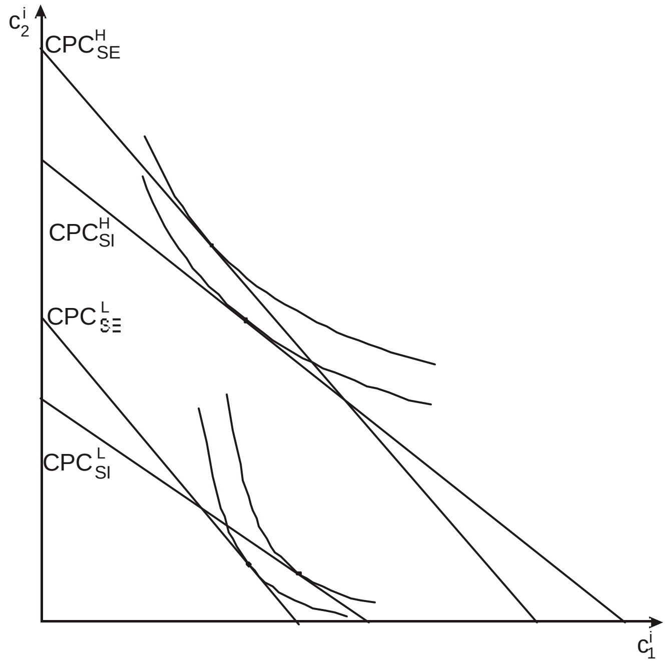

Figure 3

This result conforms with the intuition that the short-living individuals, who put more weight on consumption in period one, are indeed better off with that regime which provides more consumption in this period (Lemma 10). Conversely, the long-living individuals are better off with the sequential regime, which provides more consumption in period two.

4. Conclusion

Provision for old age can be made through a variety of annuity products, which differ in the terms concerning asset accumulation and the payout path. In the present paper we have concentrated on annuities which run over a limited time only and have to be supplemented by a second contract. This additional contract can either be bought simultaneously with the first or later, when an individual knows that she has survived some years of retirement. We have characterised demand, given these two possibilities, and we have studied the consequences of the adverse-selection phenomenon in this market. The results show that different risk-groups of individuals prefer different strategies, but only a situation, where all individuals demand sequential contracts represents an equilibrium. This is favourable for the high-risk group, while the low-risk group would be better off with the simultaneous regime. This result, though derived in a specific framework, shows some similarity to conclusions from other models with asymmetric information, where typically the low-risk groups do not receive their first-best contract.

The main conclusion from our contribution is that adverse selection has more severe consequences on the annuity market than recognised in studies using the standard overlapping-generations model. These mainly concentrate on the influence of adverse selection on a single rate of return for a uniform period of retirement. By extending this model and making the realistic assumption that provision for retirement need not be made through a once-and-for-all annuity contract, but can be made through different contracts for earlier and later phases of retirement, one finds that adverse selection also affects the choice of contracts as well as the existence and properties of equilibria.

For this analysis, a crucial issue is the range of annuity contracts or their combinations available to the individuals. Further research is needed in order to clarify the functioning of the market, if additional types of contracts, for instance packages of first- and second-period insurance, are introduced. A particularly interesting question is whether with appropriate contracts our undesirable result that the adverse-selection problem not only reduces the rates of return but also leads to an equilibrium regime, which is unfavourable for the short-living individuals, can be overcome.

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Appendix

Proof of Lemma 2:

From (2.9) we have

$$\partial^2\varphi^i / \partial {w_1^i}^2 = u''(w_1^i - A_2^i)(1 - \partial A_2^i / \partial w_1^i).$$

In case $A_2^i = 0$, $D_2^i > 0$, we have $\partial A_2^i / \partial w_1^i = 0$, while in case $A_2^i > 0$, $D_2^i = 0$, implicit differentiation of (2.8a), with $w_1^i = q_1 A_1^i$, gives us

$$\partial A_2^i / \partial w_1^i = u''(w_1^i - A_2^i) / (u''(w_1^i - A_2^i) + \pi_2^i q_2^2 u''(q_2 A_2^i)),$$

which lies between 0 and 1. Hence, $\partial^2\varphi^i / \partial {w_1^i}^2 < 0$ in both cases.

From (2.10) we find (note that $c_2^i = w_2^i + r_2 D_2^i$)

$$\partial^2\varphi^i / \partial {w_2^i}^2 = \pi_2^i u''(w_2^i + q_2 A_2^i)(1 + q_2 \partial A_2^i / \partial w_2^i).$$

In case $A_2^i = 0$, $D_2^i > 0$ we again have $\partial A_2^i / \partial w_2^i = 0$, while in case $A_2^i > 0$, $D_2^i = 0$, implicit differentiation of (2.8a), with $w_2^i = r_2 D_2^i$, gives us

$$\partial A_2^i / \partial w_1^i = -\pi_2^i q_2 u''(w_2^i + q_2 A_2^i) / (u''(w_1^i - A_2^i) + \pi_2^i q_2^2 u''(w_2^i + q_2 A_2^i)) > 0$$

Hence $\partial^2\varphi^i / \partial {w_2^i}^2 < 0$ in both cases. Q.E.D.

Proof of Lemma 3:

We denote by W^i the LHS of (2.6) and by V^i the LHS of the equation in (2.8a), where $D_2^i = 0$. Implicit differentiation gives

$$\begin{pmatrix} \frac{\partial A_1^i}{\partial q_1} & \frac{\partial A_1^i}{\partial q_2} \\ \frac{\partial A_2^i}{\partial q_1} & \frac{\partial A_2^i}{\partial q_2} \end{pmatrix} = -\begin{pmatrix} \frac{\partial W^i}{\partial A_1^i} & \frac{\partial W^i}{\partial A_2^i} \\ \frac{\partial V^i}{\partial A_1^i} & \frac{\partial V^i}{\partial A_2^i} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial W^i}{\partial q_1} & \frac{\partial W^i}{\partial q_2} \\ \frac{\partial V^i}{\partial q_1} & \frac{\partial V^i}{\partial q_2} \end{pmatrix}, \qquad \text{(A1)}$$

where (observe the strict concavity of $u(c_t^i)$ and of $\varphi^i(w_1^i, q_2, w_2^i)$, as shown in Lemma 2):

$$\frac{\partial W^i}{\partial A_1^i} = u''(w_0 - A_1^i) + \pi_1^i q_1^2 \frac{\partial^2 \varphi(q_1 A_1^i, q_1, w_2^i)}{\partial {w_1^i}^2} < 0, \quad \frac{\partial W^i}{\partial A_2^i} = 0, \tag{A2}$$

$$\frac{\partial V^i}{\partial A_1^i} = -q_1 u''(q_1 A_1^i - A_2^i) > 0, \quad \frac{\partial V^i}{\partial A_2^i} = u''(q_1 A_1^i - A_2^i) + \pi_2^i q_2^2 u''(q_2 A_2^i) < 0. \tag{A3}$$

With the Arrow-Pratt coefficients of relative risk aversion R_u^i and R_φ^i we get

$$\frac{\partial V^i}{\partial q_1} = -A_1^i u''(q_1 A_1^i - A_2^i) > 0, \tag{A4}$$

$$\begin{aligned}\frac{\partial V^i}{\partial q_2} &= \pi_2^i u'(q_2 A_2^i) + \pi_2^i q_2 A_2^i u''(q_2 A_2^i) \\ &= \pi_2^i (1 - R_u^i) u'(q_2 A_2^i),\end{aligned} \tag{A5}$$

where (A5) has the same sign as $1 - R_u^i$.

$$\begin{aligned}\frac{\partial W^i}{\partial q_1} &= \pi_1^i \frac{\partial \varphi^i(w_1^i, q_1, w_2^i)}{\partial w_1^i} + \pi_1^i q_1 A_1^i \frac{\partial^2 \varphi^i(w_1^i, q_1, w_2^i)}{\partial {w_1^i}^2} \\ &= \pi_1^i \left(1 - R_\varphi^i\right) \frac{\partial \varphi^i(w_1^i, q_1, w_2^i)}{\partial w_1^i},\end{aligned} \tag{A6}$$

where (A6) has the same sign as $1 - R_\varphi^i$.

$$\frac{\partial W^i}{\partial q_2} = \pi_1^i q_1 \frac{\partial^2 \varphi^i(w_1^i, q_1, w_2^i)}{\partial w_1^i \partial q_2}. \tag{A7}$$

The sign of (A7) is determined by the following considerations: Since $\partial \varphi^i(w_1^i, q_2, w_2^i) / \partial w_1^i =$ $= u'\left(w_1^i - A_2^i(w_1^i, q_2)\right)$ (see (2.9), here $A_2^i(w_1^i, q_2)$ denotes annuity demand A_2^i for fixed w_1^i, determined by (2.8a)), we have

$$\frac{\partial^2 \varphi^i(w_1^i, q_2, w_2^i)}{\partial w_1^i \partial q_2} = -u''(q_1 A_1^i - A_2^i) \frac{\partial A_2^i(w_1^i, q_2)}{\partial q_2}. \tag{A8}$$

Further, as $-u''(q_1 A_1^i - A_2^i) > 0$, the LHS of (A8) has the same sign as $\partial A_2^i(w_1^i, q_2) / \partial q_2$, which is determined by implicit differentiation of (2.8a) as

$$\frac{\partial A_2^i(w_1^i,q_2)}{\partial q_2}=-\frac{\pi_2^i u'(q_2A_2^i)+\pi_2^i q_2 A_2^i u''(q_2A_2^i)}{u''(w_1^i-A_2^i)+\pi_2^i q_2^2 u''(q_2A_2^i)}. \tag{A9}$$

As the denominator of the RHS of (A.9) is negative due to concavity of u, $\partial A_2^i(w_1^i,q_2)/\partial q_2$ has the same sign as the numerator of the RHS of (A.9). Substituting the relative risk aversion R_u^i into (A.9), it follows that $\partial A_2^i(w_1^i,q_2)/\partial q_2 \gtrless 0$, if $R_u^i \lesseqgtr 1$. The same holds for $\partial^2\varphi^i(w_1^i,q_2,w_2^i)/\partial w_1^i\partial q_2$ and we have for the sign of (A.7)

$$\frac{\partial W^i}{\partial q_2} \gtrless 0 \Leftrightarrow R_u^i \lesseqgtr 1.$$

Now let

$$N \equiv \frac{\partial W^i}{\partial A_1^i}\frac{\partial V^i}{\partial A_2^i} > 0, \tag{A10}$$

where the positivity follows from the inequalities above. Inverting the first matrix on the RHS of (A1) and multiplying gives:

$$\frac{\partial A_1^i(q_1,q_2)}{\partial q_1}=-\frac{1}{N}\left(\underbrace{\frac{\partial V^i}{\partial A_2^i}}_{<0}\frac{\partial W^i}{\partial q_1}-\underbrace{\frac{\partial W^i}{\partial A_2^i}\frac{\partial V^i}{\partial q_1}}_{=0}\right)\gtrless 0 \Leftrightarrow R_\varphi^i \lesseqgtr 1, \tag{A.11}$$

$$\frac{\partial A_2^i(q_1,q_2)}{\partial q_1}=-\frac{1}{N}\left(-\underbrace{\frac{\partial V^i}{\partial A_1^i}}_{<0}\frac{\partial W^i}{\partial q_1}+\underbrace{\frac{\partial W^i}{\partial A_1^i}\frac{\partial V^i}{\partial q_1}}_{<0}\right) > 0, \text{ if } R_\varphi^i \le 1, \text{ otherwise undetermined}, \tag{A.12}$$

$$\frac{\partial A_1^i(q_1,q_2)}{\partial q_2}=-\frac{1}{N}\left(\underbrace{\frac{\partial V^i}{\partial A_2^i}}_{<0}\frac{\partial W^i}{\partial q_2}-\underbrace{\frac{\partial W^i}{\partial A_2^i}\frac{\partial V^i}{\partial q_2}}_{=0}\right)\gtrless 0 \Leftrightarrow R_u^i \lesseqgtr 1, \tag{A.13}$$

$$\frac{\partial A_2^i(q_1,q_2)}{\partial q_2}=-\frac{1}{N}\left(-\underbrace{\frac{\partial V^i}{\partial A_1^i}}_{<0}\frac{\partial W^i}{\partial q_2}+\underbrace{\frac{\partial W^i}{\partial A_1^i}}_{<0}\frac{\partial V^i}{\partial q_2}\right)\gtrless 0 \Leftrightarrow R_u^i \lesseqgtr 1. \tag{A.14}$$

Q.E.D.

Proof of Lemma 4:

Let now denote W^i and V^i the LHS's of (2.12) and (2.13), resp. The formula for implicit differentiation of these equations is the same as (A1), when A^i_2 is replaced by D^i_2 and q_2 by r_2.

One derives immediately

$$\frac{\partial W^i}{\partial A^i_1} = u''(c^i_0) + \pi^i_1 q^2_1 u''(c^i_1) < 0, \qquad \frac{\partial W^i}{\partial D^i_2} = u''(c^i_0) < 0\,, \tag{B1}$$

$$\frac{\partial V^i}{\partial A^i_1} = u''(c^i_0) < 0, \qquad \frac{\partial V^i}{\partial D^i_2} = u''(c^i_0) + \pi^i_1 \pi^i_2 r^2_2 u''(c^i_2) < 0\,, \tag{B2}$$

$$\frac{\partial V^i}{\partial r_2} = \pi^i_1\left(u'(q_1 A^i_1) + q_1 A^i_1 u''(q_1 A^i_1)\right) = \pi^i_1 (1 - R_u) u'(q_1 A^i_1)\,, \qquad \frac{\partial W^i}{\partial r_2} = 0\,, \tag{B3}$$

$$\frac{\partial V^i}{\partial q_1} = 0\,, \qquad \frac{\partial V^i}{\partial r_2} = \pi^i_1 \pi^i_2\left(u'(r_2 D^i_2) + r_2 D^i_2 u''(r_2 D^i_2)\right) = \pi^i_1 \pi^i_2 (1 - R_u) u'(r_2 D^i_2)\,. \tag{B4}$$

With

$$N \equiv (u''(c^i_0) + \pi^i_1 q^2_1 u'(c^i_1)(u''(c^i_0) + \pi^i_1 \pi^i_2 r^2_2 u''(c^i_2)) - (u''(c^i_0))^2 > 0 \tag{B5}$$

and the definition of R^i_u we get, using (A11) – (A14):

$$\frac{\partial A^i_1(q_1, r_2)}{\partial q_1} = -\frac{1}{N}\frac{\partial V^i}{\partial D^i_2}\frac{\partial W^i}{\partial q_1} \gtrless 0 \Leftrightarrow R^i_u \lessgtr 1$$

$$\frac{\partial D^i_2(q_1, r_2)}{\partial q_1} = \frac{1}{N}\frac{\partial V^i}{\partial A^i_1}\frac{\partial W^i}{\partial q_1} \lessgtr 0 \Leftrightarrow R^i_u \lessgtr 1$$

$$\frac{\partial A^i_1(q_1, r_2)}{\partial r_2} = \frac{1}{N}\frac{\partial W^i}{\partial D^i_2}\frac{\partial V^i}{\partial r_2} \lessgtr 0 \Leftrightarrow R^i_u \lessgtr 1$$

$$\frac{\partial D^i_2(q_1, r_2)}{\partial r_2} = -\frac{1}{N}\frac{\partial W^i}{\partial A^i_1}\frac{\partial V^i}{\partial r_2} \gtrless 0 \Leftrightarrow R^i_u \lessgtr 1.$$

Q.E.D.

Proof of Lemma 5:

(i) We define W^i and V^i as in the proof of Lemma 3. The formula for implicit differentiation of (2.6) and the equation in (2.8a) is the same as (A1), where q_1, q_2 are replaced by π_1^i, π_2^i, resp.

We find

$$\frac{\partial W^i}{\partial \pi_1^i} = q_1 \frac{\partial \varphi}{\partial w_1^i} > 0,$$

$$\frac{\partial W^i}{\partial \pi_2^i} = \pi_1^i q_1 \frac{\partial^2 \varphi^i(q_1 A_1^i, q_2, w_2^i)}{\partial w_1^i \partial \pi_2^i} = -\pi_1^i q_1 u''(c_1^i) \frac{\partial A_2^i(w_1^i, q_2, w_2^i)}{\partial \pi_2},$$

due to (2.9) and (2.2), where $\partial A_2^i(w_1^i, q_2, w_2^i)/\partial \pi_2$ denotes the change of annuity demand in period 1, for fixed w_1^i, if π_2^i increases. It is determined by implicit differentiation of the equation in (2.8a) as

$$\frac{\partial A_2^i(w_1^i, q_2, w_2^i)}{\partial \pi_2} = -\frac{\partial V^i / \partial \pi_2}{\partial V^i / \partial A_2} = -\frac{q_2 u'(c_2^i)}{u''(c_1^i) + \pi_2 q_2^2 u''(c_2^i)} > 0\,.$$

(Compare the analogous procedure (A7) – (A9), concerning the influence of q_2). Altogether, we find $\partial W^i / \partial \pi_2^i > 0$.

Furthermore:

$$\frac{\partial V^i}{\partial \pi_1^i} = 0, \qquad \frac{\partial V^i}{\partial \pi_2^i} = q_2 u'(c_2^i) > 0.$$

Using these computations, together with (A2), (A3) and (A10), in (A11) – (A14) gives

$$\frac{\partial A_1^i(q_1,q_2)}{\partial \pi_1^i} = -\frac{1}{N}\frac{\partial V^i}{\partial A_2^i}\frac{\partial W^i}{\partial \pi_1^i} > 0 \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial \pi_1^i} = \frac{1}{N}\frac{\partial V^i}{\partial A_1^i}\frac{\partial W^i}{\partial \pi_1^i} > 0$$

$$\frac{\partial A_1^i(q_1,q_2)}{\partial \pi_2^i} = -\frac{1}{N}\frac{\partial V^i}{\partial A_2^i}\frac{\partial W^i}{\partial \pi_2^i} > 0 \qquad \frac{\partial A_2^i(q_1,q_2)}{\partial \pi_2^i} = -\frac{1}{N}\left(-\frac{\partial V^i}{\partial A_1^i}\frac{\partial W^i}{\partial \pi_2^i} + \frac{\partial W^i}{\partial A_1^i}\frac{\partial V^i}{\partial \pi_2^i}\right) > 0\,.$$

To see part (ii), we define W^i and V^i as in Lemma 4 and use the same formula (A1) for implicit differentiation of (2.12), (2.13), where A_2^i, q_1, q_2 are replaced by D_2^i, π_1^i, π_2^i resp.

We find

$$\frac{\partial W^i}{\partial \pi_1^i} = q_1 u'(c_1^i) > 0, \qquad \frac{\partial W^i}{\partial \pi_2^i} = 0,$$

$$\frac{\partial V^i}{\partial \pi_1^i} = \pi_2 r_2 u'(c_2^i) > 0, \qquad \frac{\partial V^i}{\partial \pi_2^i} = \pi_1^i r_2 u'(c_2^i) > 0.$$

Using these computations, together with (B1), (B2) and (B5), in (A11) – (A14), it follows (note that $q_1 u'(c_1^i) = \pi_2^i r_2 u'(c_2^i)$ due to (2.12) and (2.13)):

$$\begin{aligned}\frac{\partial A_1^i(q_1,r_2)}{\partial \pi_1^i} &= -\frac{1}{N}\left(\frac{\partial V^i}{\partial D_2^i}\frac{\partial W^i}{\partial \pi_1^i} - \frac{\partial W^i}{\partial D_2^i}\frac{\partial V^i}{\partial \pi_1^i}\right)\\ &= -\frac{1}{N}\Big((u''(c_0^i) + \pi_1^i \pi_2^i r_2^2 u''(c_2^i))q_1 u'(c_1^i) - u''(c_0^i)\pi_2 r_2 u'(c_2^i)\Big)\\ &= -\frac{1}{N}\pi_1^i \pi_2^i r_2^2 u''(c_2^i))q_1 u'(c_1^i) > 0,\end{aligned}$$

$$\begin{aligned}\frac{\partial D_2^i(q_1,r_2)}{\partial \pi_1^i} &= -\frac{1}{N}\left(-\frac{\partial V^i}{\partial A_1^i}\frac{\partial W^i}{\partial \pi_1^i} + \frac{\partial W^i}{\partial A_1^i}\frac{\partial V^i}{\partial \pi_1^i}\right)\\ &= -\frac{1}{N}\Big((-u''(c_0^i)q_1 u'(c_1^i) + (u''(c_0^i) + \pi_1^i q_1^2 u''(c_1^i))\pi_2 r_2 u'(c_2^i)\Big)\\ &= -\frac{1}{N}\pi_1^i q_1^2 u''(c_1^i))\pi_2^i r_2 u'(c_2^i) > 0,\end{aligned}$$

$$\frac{\partial A_1^i(q_1,r_2)}{\partial \pi_2^i} = \frac{1}{N}\frac{\partial W^i}{\partial D_2^i}\frac{\partial W^i}{\partial \pi_2^i} < 0,$$

$$\frac{\partial D_1^i(q_1,r_2)}{\partial \pi_2^i} = -\frac{1}{N}\frac{\partial W^i}{\partial A_i^i}\frac{\partial V^i}{\partial \pi_2^i} > 0.$$

Q.E.D.

Proof of Lemma 9:

(i) From (3.5) and (3.7) it follows that $\tilde{A}_1^i = (1+\pi_2^i)\overline{A}_1^i$, i = L,H. Using this relation, $\tilde{\rho}_1$ can be written as $\overline{\rho}_1 (1+\pi_2^H)\big/(1+\pi_2^L)$, which is larger than $\overline{\rho}_1$, as $\pi_2^H > \pi_2^L$.

From (3.6) and (3.8), it follows that $\tilde{A}_2^i = \tilde{q}_1\overline{D}_2^i$. Using this relation, together with $\gamma_1/(1-\gamma_1) = \gamma_0\pi_1^H/((1-\gamma_0)\pi_1^L)$, $\tilde{\rho}_2$ can be written as $\overline{\rho}_2\, \pi_1^H\big/\pi_1^L$, which is larger than $\overline{\rho}_2$, as $\pi_1^H > \pi_1^L$.

From (3.5) and (3.8) it follows that $\tilde{A}_1^i = (1+\pi_2^i)\overline{D}_2^i\big/\pi_2^i$. Using this relation, $\tilde{\rho}_1$ can be written as $\overline{\rho}_2(\pi_2^L + \pi_2^H\pi_2^L)\big/(\pi_2^H + \pi_2^L\pi_2^H)$, which is smaller than $\overline{\rho}_2$, as $\pi_2^H > \pi_2^L$.

(ii) First, we calculate the difference ($\overline{q}_1 - \tilde{q}_1$) from (3.9) and (3.11) as

$$\frac{(\pi_1^H - \pi_1^L)(\tilde{\rho}_1 - \overline{\rho}_1)}{(\pi_1^L + \pi_1^H\tilde{\rho}_1)(\pi_1^L + \pi_1^H\overline{\rho}_1)},$$

which is positive due to $\tilde{\rho}_1 > \overline{\rho}_1$ and $\pi_1^H > \pi_1^L$.

Next, we show that $\tilde{q}_2 < \overline{r}_2$: Using (3.10) and the relation $\tilde{\rho}_2 = \overline{\rho}_2\, \pi_1^H\big/\pi_1^L$ (see step (i) of this proof), $\tilde{q}_2$ can be transformed to

$$\tilde{q}_2 = \frac{\pi_1^L + \pi_1^H\overline{\rho}_2}{\pi_1^L\pi_2^L + \pi_1^H\pi_2^H\overline{\rho}_2}, \qquad \text{(C1)}$$

From (3.12) and (C1) it follows that the relative rate of return is equal to

$$\frac{\overline{r}_2}{\tilde{q}_2} = \frac{1+\overline{\rho}_2}{\pi_1^L + \overline{\rho}_2\pi_1^H}, \qquad \text{(C2)}$$

which is greater than 1 due to $\pi_1^i < 1$, for i = L,H.

Finally, we show that $\tilde{q}_1\tilde{q}_2 > \overline{r}_2$, which is equivalent to $\tilde{q}_1 - \overline{r}_2\big/\tilde{q}_2 > 0$. Substituting (3.9) and (C2), the difference $\tilde{q}_1 - \overline{r}_2\big/\tilde{q}_2$ can be written as

$$\frac{(\pi_1^H - \pi_1^L)(\overline{\rho}_2 - \tilde{\rho}_1)}{(\pi_1^L + \pi_1^H\overline{\rho}_2)(\pi_1^L + \pi_1^H\tilde{\rho}_1)},$$

which is positive, since $\pi_1^H > \pi_1^L$ and $\overline{\rho}_2 > \tilde{\rho}_1$. Q.E.D.

Proof of Lemma 10:

(i) We know that $c_1^i(\overline{q}_1,\overline{r}_2)=\overline{q}_1\overline{A}_1^i$ and that $c_1^i(\tilde{q}_1,\tilde{q}_2)=\tilde{q}_1\tilde{A}_1^i-\tilde{A}_2^i=\tilde{q}_1(\tilde{A}_1^i-\overline{D}_2^i)=\tilde{q}_1\overline{A}_1^i$ (see the proofs of Lemma 9 (i) and 8). Together with $\tilde{q}_1<\overline{q}_1$ (see Lemma 9 (ii)), it follows that $c_1^i(\tilde{q}_1,\tilde{q}_2)<c_1^i(\overline{q}_1,\overline{r}_2)$.

(ii) We know that $c_2^i(\overline{q}_1,\overline{r}_2)=\overline{r}_2\overline{D}_2^i$ and that $c_2^i(\tilde{q}_1,\tilde{q}_2)=\tilde{q}_2\tilde{A}_2^i=\tilde{q}_1\tilde{q}_2\overline{D}_2^i$. Together with $\tilde{q}_1\tilde{q}_2>\overline{r}_2$ (see Lemma 9 (ii)), it follows that $c_2^i(\tilde{q}_1,\tilde{q}_2)>c_2^i(\overline{q}_1,\overline{r}_2)$. Q.E.D.

Proof of Proposition 3:

We show that for an individual of type L, the optimal consumption bundles (c_1^i,c_2^i) in both regimes lie to the right of the point of intersection of the consumption possibility curves (3.13) and (3.14), while the opposite holds true for an individual of type H. From this we conclude that an individual of type L prefers the simultaneous regime with the rates of returns $\overline{q}_1$ and $\overline{r}_2$, while an individual of type H prefers the sequential regime with the payouts $\tilde{q}_1,\tilde{q}_2$.

We calculate the point of intersection of the possibility curves of an individual i = L,H, under both regimes by solving (3.13) and (3.14) for c_1^i and substituting (3.5), which gives

$$c_1^i(S)\equiv\frac{\pi_1^i(1+\pi_2^i)}{1+\pi_1^i(1+\pi_2^i)}\,\frac{\overline{q}_1(\overline{r}_2-\tilde{q}_1\tilde{q}_2)}{\overline{r}_2-\overline{q}_1\tilde{q}_2}\,w_0\,. \tag{D1}$$

Moreover, we have from (2.2), (3.5) and (3.6)

$$c_1^i(\tilde{q}_1,\tilde{q}_2)=\tilde{q}_1\frac{\pi_1^i}{1+\pi_1^i(1+\pi_2^i)}w_0 \tag{D2}$$

and from (2.2), (3.7) and $A_2^i=0$

$$c_1^i(\overline{q}_1,\overline{r}_2)=\overline{q}_1\frac{\pi_1^i}{1+\pi_1^i(1+\pi_2^i)}w_0\,. \tag{D3}$$

As a preparation, we show in step (i) that $c_1^H(\overline{q}_1,\overline{r}_2)<c_1^H(S)$ and in step (ii) that $c_1^L(\tilde{q}_1,\tilde{q}_2)>c_1^L(S)$.

(i) The inequality $c_1^H(\bar{q}_1,\bar{r}_2) < c_1^H(S)$ reduces to

$$1 < \frac{(1+\pi_2^H)(\tilde{q}_1\tilde{q}_2-\bar{r}_2)}{\bar{q}_1\tilde{q}_2-\bar{r}_2} \tag{D4}$$

by use of (D1) and (D3). Note first that $\bar{q}_1 > \tilde{q}_1$ (see Lemma 9), hence $\bar{q}_1\tilde{q}_2 > \bar{r}_2$, because we already know that $\tilde{q}_1\tilde{q}_2 > \bar{r}_2$. Thus (D4) is equivalent to

$$\bar{q}_1\tilde{q}_2 - \tilde{q}_1\tilde{q}_2(1+\pi_2^H) + \bar{r}_2\pi_2^H < 0\,. \tag{D5}$$

Using the relation (C2) and inserting for $\tilde{q}_1, \bar{q}_1$, (see (3.9) and (3.11)), we can transform (D5) to:

$$\frac{1+\bar{\rho}_1}{\pi_1^L+\bar{\rho}_1\pi_1^H} - \frac{1+\tilde{\rho}_1}{\pi_1^L+\tilde{\rho}_1\pi_1^H}(1+\pi_2^H) + \frac{1+\tilde{\rho}_2}{\pi_1^L+\tilde{\rho}_2\pi_1^H}\pi_2^H < 0\,. \tag{D6}$$

Substituting for $\tilde{\rho}_1$, $\tilde{\rho}_2$, $\bar{\rho}_1$ and using the abbreviation $\sigma \equiv \gamma_0(1+\pi_1^L+\pi_1^L\pi_2^L)\pi_1^H /$ $((1-\gamma_0)(1+\pi_1^H+\pi_1^H\pi_2^H)\pi_1^L)$, the LHS of (D6) can be written as

$$\frac{1+\sigma}{\pi_1^L+\sigma\pi_1^H} - \frac{1+\sigma\dfrac{1+\pi_2^H}{1+\pi_2^L}}{\pi_1^L+\sigma\pi_1^H\dfrac{1+\pi_2^H}{1+\pi_2^L}}(1+\pi_2^H) + \frac{1+\sigma\dfrac{\pi_2^H}{\pi_2^L}}{\pi_1^L+\sigma\pi_1^H\dfrac{\pi_2^H}{\pi_2^L}}\pi_2^H,$$

which reduces to

$$\frac{\sigma\pi_1^L(\pi_1^L-\pi_1^H)(\pi_2^L-\pi_2^H)^2}{(\pi_1^L+\sigma\pi_1^H)(\pi_1^L\pi_2^L+\sigma\pi_1^H\pi_2^H)(\pi_1^L(1+\pi_2^L)+\sigma\pi_1^H(1+\pi_2^H))}.$$

This expression is negative due to $\pi_1^L < \pi_1^H$.

(ii) Second, we have to show that $c_1^L(\tilde{q}_1,\tilde{q}_2) > c_1^L(S)$, which by use of (D1) and (D2) reduces to

$$\tilde{q}_1 > \frac{(1+\pi_2^L)\bar{q}_1(\tilde{q}_1\tilde{q}_2-\bar{r}_2)}{\bar{q}_1\tilde{q}_2-\bar{r}_2}, \tag{D7}$$

which is equivalent to (see the considerations following (D4))

$$\bar{q}_1\bar{r}_2(1+\pi_2^L) - \tilde{q}_1(\bar{r}_2+\bar{q}_1\tilde{q}_2\pi_2^L) > 0\,. \tag{D8}$$

Using the relation (C2) and inserting for $\tilde{q}_1, \bar{q}_1$ (see (3.9) and (3.11)) gives

$$\frac{1+\bar{\rho}_1}{\pi_1^L+\bar{\rho}_1\pi_1^H} \quad \frac{1+\tilde{\rho}_2}{\pi_1^L+\tilde{\rho}_2\pi_1^H}(1+\pi_2^L)-\frac{1+\tilde{\rho}_1}{\pi_1^L+\tilde{\rho}_1\pi_1^H}\left(\frac{1+\tilde{\rho}_2}{\pi_1^L+\tilde{\rho}_2\pi_1^H}+\frac{1+\bar{\rho}_1}{\pi_1^L+\bar{\rho}_1\pi_1^H}\pi_2^L\right)>0. \quad \text{(D9)}$$

Substituting for $\tilde{\rho}_1, \tilde{\rho}_2, \bar{\rho}_1$ and using the abbreviation σ, the LHS of (D9) can be written as

$$\frac{1+\sigma}{\pi_1^L+\sigma\pi_1^H} \quad \frac{1+\sigma\frac{\pi_2^H}{\pi_2^L}}{\pi_1^L+\sigma\pi_1^H\frac{\pi_2^H}{\pi_2^L}}(1+\pi_2^L)-\frac{1+\sigma\frac{1+\pi_2^H}{1+\pi_2^L}}{\pi_1^L+\sigma\pi_1^H\frac{1+\pi_2^H}{1+\pi_2^L}} \quad \left(\frac{1+\sigma\frac{\pi_2^H}{\pi_2^L}}{\pi_1^L+\sigma\pi_1^H\frac{\pi_2^H}{\pi_2^L}}+\frac{1+\sigma}{\pi_1^L+\sigma\pi_1^H}\pi_2^H\right),$$

which reduces to

$$\frac{\sigma^2(\pi_1^H-\pi_1^L)(\pi_2^H-\pi_2^L)^2}{(\pi_1^L+\sigma\pi_1^H)(\pi_1^L\pi_2^L+\sigma\pi_1^H\pi_2^H)(\pi_1^L(1+\pi_2^L)+\sigma\pi_1^H(1+\pi_2^H))}.$$

This expression is positive due to $\pi_1^H > \pi_1^L$.

Finally we know from Lemma 10 that $c_1^H(\tilde{q}_1,\tilde{q}_2) < c_1^H(\bar{q}_1,\bar{r}_2)$, which together with $c_1^H(\bar{q}_1,\bar{r}_2) < c_1^H(S)$ implies that for both regimes consumption lies to the left of the point of intersection $c_1^H(S)$ of the consumption possibility curves of both regimes. As to the left of $c_1^H(S)$ the consumption set is larger for the sequential regime than for the simultaneous regime, it follows that the type-H individuals are better off with the sequential regime.

Similarly, $c_1^L(\bar{q}_1,\bar{r}_2) > c_1^L(\tilde{q}_1,\tilde{q}_2)$ (see Lemma 10) and $c_1^L(\tilde{q}_1,\tilde{q}_2) > c_1^L(S)$ (see step (ii)) imply, by analogous reasoning, that the type-L individuals are better off with the simultaneous regime. Q.E.D.

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