THE DESIGN OF PENSION PAY OUT OPTIONS WHEN THE HEALTH STATUS DURING RETIREMENT IS UNCERTAIN

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CESIFO WORKING PAPER NO. 2211
CATEGORY 3: SOCIAL PROTECTION
FEBRUARY 2008

PRESENTED AT CESIFO VENICE SUMMER INSTITUTE, WORKSHOP ON
'LONGEVITY AND ANNUITIZATION, JULY 2007

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Abstract

This paper examines the optimal design of pension plans when the health status during retirement is uncertain. Assuming that the health status affects both life expectancy and the marginal utility of consumption, choice between a lump-sum payment and an annuity can be welfare-enhancing if the health status is not observable by pension plans. This result holds if the marginal utility of consumption and life expectancy are negatively correlated. On equity grounds, a lump-sum option can be justified even if the marginal utility of consumption is independent of life expectancy.

JEL Code: G23, H55, D82.

Keywords: pensions, lump-sum withdrawal, annuities, longevity.

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January 2008
I thank Friedrich Breyer, Jean-Marie Lozachmeur, Wolfram Richter, Matthias Wrede and participants of the 8th European Health Economics Workshop at the University of Magdeburg, the research seminar at the University of Dortmund and the CESifo Venice Summer Institute Workshop on "Longevity and Annuitization" for helpful comments and suggestions. This research is part of the project "Lebensarbeitszeit, Humankapitalbildung und Transfersystem" funded by the Deutsche Forschungsgemeinschaft under grant BR 740/15-1.

1 Introduction

In a seminal paper, Yaari (1965) showed that individuals who maximize expected utility should annuitize all of their savings. Recently, Davidoff, Brown, and Diamond (2005) extended his analysis and showed that this result also holds under weaker conditions. Nevertheless, full annuitization remains the exception rather than the rule. The literature has therefore tried to explain why individuals only partially annuitize their wealth or choose not to annuitize at all.[1]

This paper deals with a particular deviation from Yaari's result. Frequently, individuals have a choice between a lump-sum payment and an annuity upon entering retirement. For example, this is the case in many private pension plans. Also publicly regulated programmes such as Chile's funded pension system, the Swiss occupational pension scheme or state-subsidized supplementary private pensions in Germany ('Riester pensions') allow such a choice.

An important question is whether the possibility to select a lump sum can be desirable. In a standard model, this option can only reduce welfare since individuals with a low life expectancy will opt for the lump sum. This reduces the redistribution from short-living to long-living individuals which is optimal ex ante when life expectancy is still uniform (Brugiavini, 1993, and Sheshinski, 2004).

The standard model, however, assumes that the utility function is independent of life expectancy. This is questionable as life expectancy is closely related to the health status which is likely to have an impact on the utility function. In this paper, we show that considering the links between life expectancy, health and utility can make a lump-sum option valuable. Specifically, we find that rational individuals might prefer a choice between a lump-sum payment and an annuity if the health status during retirement is uncertain and unobservable. This is the case if the marginal utility of consumption and life expectancy are negatively correlated.[2]

[1] Possible explanations include inferior returns to annuities due to administrative costs and selection effects (Friedman and Warshawsky, 1988, 1990, Mitchell, Poterba, Warshawsky, and Brown, 1999), bequest motives (Kotlikoff and Summers, 1981, Hurd, 1989, Bernheim, 1991), incomplete markets (Yagi and Nishigaki, 1993, Davidoff, Brown, and Diamond, 2005), within family-risk sharing (Kotlikoff and Spivak, 1981, Brown and Poterba, 2000), and pre-existing annuities from public pensions (Bernheim, 1991).

[2] A similar result has been obtained by Diamond (2003) in an optimal income tax framework. He finds a lump-sum option to be optimal if life expectancy and productivity are positively correlated.

A related result is obtained by Direr (2007) who extends the standard model by considering uninsurable expenses during old age. As in the present paper, individuals discover their survival probabilities after buying an annuity contract. Individuals with high life expectancy face an uninsurable expenditure risk early in old age. Direr shows that a flexible annuity plan is optimal which allows a withdrawal. It smoothes consumption for individuals who experience the expenditure shock. The existence of short-lived individuals, however, puts a limit on the withdrawal.[3]

The paper is structured as follows. In Section 2, we present the basic model and derive conditions under which choice in a pension plan is optimal. Section 3 extends the basic model in various directions. In Section 4, we discuss the implications for public pensions. Here we show that a lump-sum option can also be justified on equity grounds. For a strictly concave social welfare function, this is the case even if the marginal utility of consumption is independent of life expectancy. Section 5 concludes and points out directions for further research.

2 The basic model

2.1 Health status, marginal utility and life expectancy

Individuals are initially identical. They invest wealth Ω in a pension plan before entering retirement. Retirement is reached with probability $\delta < 1$. Upon retirement, the health status $h = g, b$ of individuals is revealed. With probability π the 'good' state g arises, with probability $1 - \pi$ the health status is 'bad'. The health status has implications both for life expectancy and the marginal utility of consumption:

[3] Zhang and Tang (2007) examine the optimal choice with an uninsurable expenditure risk in absence of a withdrawal option. They find that individuals may then prefer not to fully annuitize their wealth.

- *Health status and life expectancy*

  Individuals with health status b will only live one period after retirement (period 1). With status g, one can live up to two periods. The survival probability for period 2 is $0 < \rho < 1$. Individuals possess information on their life expectancy.[4]

- *Health status and utility*

  Utility is state-dependent. In state g, utility in each period is $u(c_t)$ where c_t is consumption in period $t = 1,2$. In stage b, utility is $\alpha u(c_t)$, with $u'(c_t) > 0$, $u''(c_t) < 0, \lim_{c_t \to 0} u'(c_t) = \infty$ and $\alpha > 0$. We leave open whether $\alpha \gtreqless 1$, i.e. whether marginal utility of consumption is higher in state b or g for a given level of consumption.[5] In particular, we do not find $\alpha > 1$ implausible.[6] Knowledge of nearby early death may make consumption more valuable. For example, individuals may want to spend money on an expensive trip they always dreamed of.[7]

For simplicity, consumption before period 1 is not modeled since it does not affect the structure of the optimal pension plan. Only the amount invested in the annuity may vary. Furthermore, we assume that the interest rate is zero. Individuals do not discount the future and have no bequest motive. Finally, we abstract from further financial risks, e.g. medical expenditure, by implicitly assuming that these are fully insured. Expected utility is thus given by

$$EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right). \tag{1}$$

[4] For evidence on this hypothesis, see Hurd and McGarry (1995).

[5] In the following, we frequently drop the qualification “for a given level of consumption” and simply speak of “marginal utility of consumption being higher in state b (g)” when it is clear that we refer to $\alpha > 1$ (< 1).

[6] Viscusi and Evans (1990) find evidence for a lower marginal utility when the health status declines. However, this study is based on chemical workers and not on elderly. In a further study using survey data on adults approaching middle age, Evans and Viscusi (1991) could not identify an effect of health on the marginal utility of consumption.

[7] In our set-up, $\alpha > 1$ implies that individuals are actually better off in state b. However, we can also write utility in the bad state as $\alpha u(c) - \kappa$ which is compatible with higher marginal utility of consumption but lower total utility. Since only marginal utility of consumption is important in the following, we stick to our simpler version.

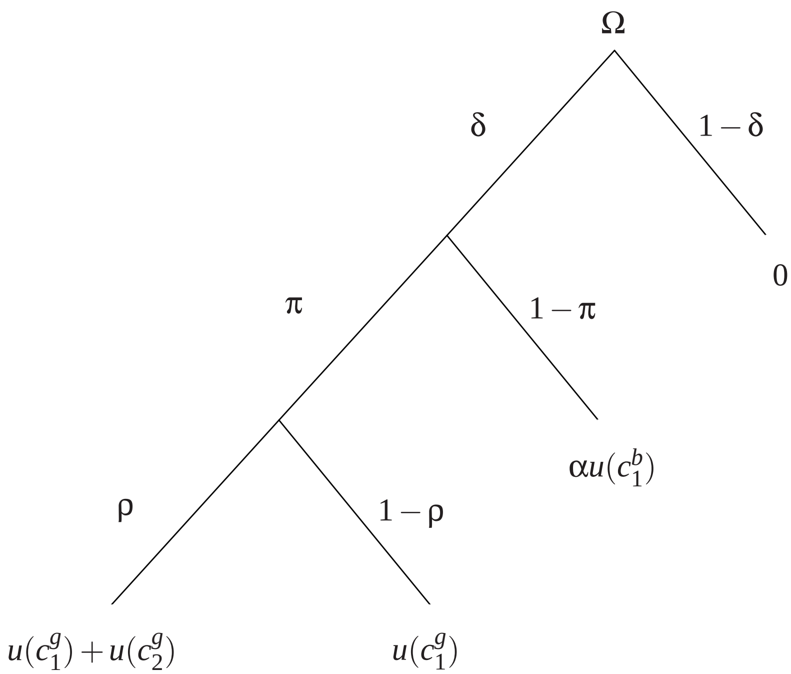

Figure 1: States of nature and utilities

Figure 1 shows the different states of nature and the corresponding utilities. Three risks which individuals would like to insure against through a pension plan can be identified:

(i) the risk to reach retirement,

(ii) the risk that marginal utility differs between the health states,

(iii) the longevity risk in state g.

We assume that pension plans are actuarially fair and maximize expected utility of individuals. This can be interpreted as the outcome of competition on the market for pension plans. Alternatively, this assumption can be justified by a public pension scheme set up to meet this objective (see Section 4). Furthermore, we rule out that individuals can draw loans on future pension payments and guarantee repayments through life insurance. Therefore, it is not possible to borrow against future income.

2.2 Observable health status

If health status is observable, pension plans can make their payments dependent on the health status and the age of the individual. We therefore solve the following problem

$$\begin{aligned}\max_{c_1^g, c_2^g, c_1^b} EU &= \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right) \\ \text{s.t.} \quad \Omega &= \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right). \end{aligned} \tag{2}$$

From the first-order conditions, we obtain that marginal utility of consumption must be the same in all states and periods, i.e.

$$u'(c_1^{g*}) = u'(c_2^{g*}) = \alpha u'(c_1^{b*}). \tag{3}$$

This implies

$$c_1^{g*} = c_2^{g*} = c^{g*}, \quad c_1^{b*} \gtrless c^{g*} \Leftrightarrow \alpha \gtrless 1.$$

Thus, a constant annuity is optimal in the good health state. The payment in the bad health state is larger if marginal utility of consumption is higher for a given level of consumption.

Proposition 2.1. *If the health status is observable, then it is optimal to pay out c_1^{b*} in the bad health state and to provide an annuity c^{g*} in the good health state. The payment in the bad health state is larger than the annuity if and only if marginal utility of consumption is higher in that state.*

2.3 Unobservable health status

In the following, we assume that the health status is not observable. Furthermore, pension plans do not possess any information on the consumption of individuals which would allow them to identify the type. Then the optimal pension plan must be incentive-compatible, i.e. no type should have an advantage by claiming to be the other type. For b-types, the incentive constraint is

$$\alpha u(c_1^b) \geq \alpha u(c_1^g) \quad \Leftrightarrow \quad c_1^b \geq c_1^g. \tag{ICB}$$

Clearly, the one-period payments for b-types cannot be smaller than the first-period payments for g-types. Note that the first best violates (ICB) if $c_1^g > c_1^b$ which corresponds to $\alpha < 1$, i.e. lower marginal utility of consumption in the bad state of health.

Incentive compatibility for g-types could be ensured if pension plans were able to punish g-types in period 2 if they claimed a one-period payment since only g-types can be alive in this period. However, it is doubtful whether courts would enforce it. We therefore do not consider this possibility.[8]

??Except for Section 3.5, we also rule out that pension plans can observe the levels of consumption. Types can therefore not be identified by their first-period consumption.

If g-types pretend to be b-types, they exchange a one-period payment for a payment stream over two periods. This raises the question how they finance their consumption in period 2. Their preferred method is to annuitize the one-period payment. In this section, we assume that they are able to do so, e.g. because pensions plan are not able to monitor further annuity purchases.[9] The price g-types must pay for an annuity will be ρ per unit consumption in period 2 since only g-types will demand annuities. g-types will therefore buy annuities up to the point where $u'(\hat{c}_1^g) = u'(\hat{c}_2^g)$. If g-types claim to be b-types and receive c_1^b in period 1, their consumption is therefore given by $\hat{c}_1^g = \hat{c}_2^g = c_1^b/(1+\rho)$ yielding expected utility in period 1

$$EU^g(t=1) = u(\hat{c}_1^g) + \rho u(\hat{c}_2^g) = (1+\rho)u(c_1^b/(1+\rho)).$$

Therefore the incentive constraint for g-types is

$$u(c_1^g) + \rho u(c_2^g) \geq (1+\rho)u(c_1^b/(1+\rho)). \qquad \text{(ICG)}$$

The first-best solution violates (ICG) if $c_1^{b*} > (1+\rho)c^{g*}$, i.e. if the payment for b-types is larger than the present value of the annuity for g-types. This is the case if α exceeds a critical value $\tilde{\alpha} > 1$. For example, if $u(c) = \ln(c)$, we have $c_1^{b*} = \alpha c^{g*}$ in the first best which yields a critical value $\tilde{\alpha} = 1+\rho$.

[8] See also Section 3.5 where we consider that b-types may live up to period 2.

[9] In Section 3.1, we allow pensions plans to prohibit the purchase of further annuities.

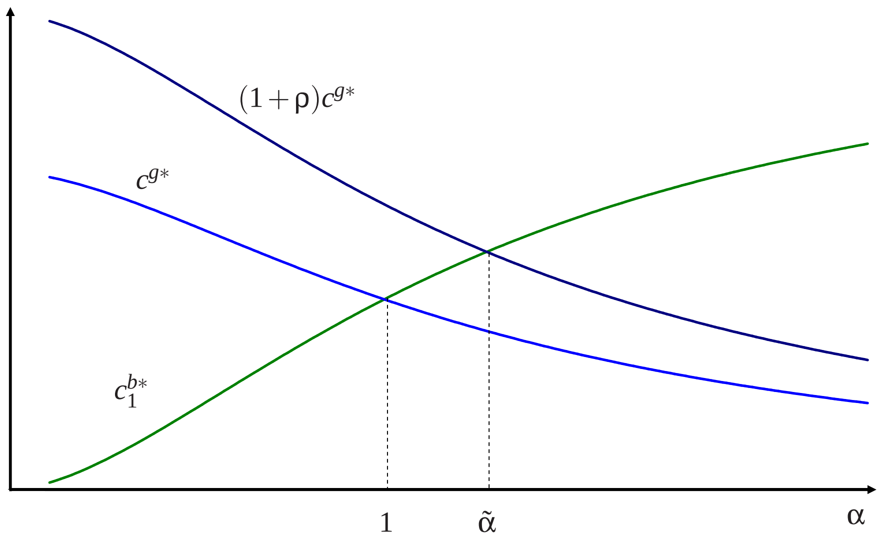

Figure 2: First-best consumption and incentive compatiblity

Figure 2 illustrates the conflict between the first best and incentive compatibility. It shows first-best consumption c^{g*} and c_1^{b*} and the present value $(1+\rho)c^{g*}$ as functions of α. If $\alpha < 1$, then $c_1^{b*} < c^{g*}$ and the incentive-constraint for b-types is violated. $\alpha > \tilde{\alpha}$ implies $c_1^{b*} > (1+\rho)c^{g*}$ and g-types have the incentive to claim c_1^{b*} and convert this into an annuity. We therefore find that the first best is incentive-compatible only if $\alpha \in [1;\tilde{\alpha}]$. In this case, the first best can be implemented by giving individuals a *choice* between a lump-sum payment c_1^{b*} and an annuity c_t^{g*}. Individuals will self-select since $c_1^{b*} \geq c^{g*}$ and $(1+\rho)c^{g*} \geq c_1^{b*}$. However, if $\alpha < 1$ or $\alpha > \tilde{\alpha}$, only a second-best solution is possible. We consider both cases in the following.

Second-best solution for $\alpha < 1$

If $\alpha < 1$, then marginal utility of consumption and life expectancy are positively correlated. The first best is not compatible with the incentive constraint for b-types (ICB). In the second-best solution, this constraint will therefore be binding. To determine the second best, we solve the problem

$$\max_{c_1^b, c_1^g, c_2^g} EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)$$

s.t.

$$\begin{aligned} \Omega &= \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) \\ \hat{c}_1^b &= \hat{c}_1^g = c_1 \end{aligned} \tag{4}$$

where (4) is the incentive constraint (ICB) with equality sign. Substituting (4) yields the Lagrangian

$$\mathcal{L} = \delta\left(\pi\left(u(c_1) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1)\right) + \lambda\left\{\Omega - \delta\left(c_1 + \pi\rho c_2^g\right)\right\}$$

with the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_1} = \delta(\pi + (1-\pi)\alpha)u'(c_1) - \lambda\delta = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g) - \lambda\delta\pi\rho = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \Omega - \delta\left(c_1 + \pi\rho c_2^g\right) = 0. \tag{7}$$

We obtain

$$(\pi + (1-\pi)\alpha)u'(c_1) = u'(c_2^g). \tag{8}$$

Since $(\pi + (1-\pi)\alpha) < 1$, this implies $c_1 < c_2^g$, i.e. the annuity rises over time.[10] Thus, the incentive for b-types to claim to be g-types leads to a distorted annuity for g-types. Ex ante, of course, all individuals are worse off. Compared to first best, it can be shown that $c_1^{b*} < c_1 < c^{g*}$.[11] A priori, it is not clear whether $c_2^g \gtreqless c^{g*}$.[12]

Second-best solution for $\alpha > \tilde{\alpha}$

In this case, marginal utility of consumption and life expectancy are strongly negatively correlated. The first best violates the incentive constraint for g-types (ICG). The second best can be found by solving the problem

$$\max_{c_1^b, c_1^g, c_2^g} EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)$$

[10] Note that we ruled out borrowing against future payments.

[11] Equation (8) implies $\alpha u'(c_1) < u'(c_2^g)$. Taking into account the budget constraint, the first-best condition (3) and the incentive constraint (4), this is only possible if $c_1 > c_1^{b*}$. Furthermore, $c_1 \geq c^{g*}$ is not compatible with the budget constraint since $c_2^g > c_1$.

[12] For constant relative risk aversion ψ, it can be shown that $c_2^g > c^{g*}$ if and only if $\psi < 1$.

s.t.

$$\Omega = \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) \tag{9}$$

$$u(c_1^g) + \rho u(c_2^g) = (1+\rho)u(c_1^b/(1+\rho)) \tag{10}$$

where (10) is the incentive constraint (ICG) with equality sign. The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \delta\left(\pi\left(u(c_1^g)+\rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)\\ &\quad +\lambda\left\{\Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right)\right\}\\ &\quad +\mu\left\{u(c_1^g)+\rho u(c_2^g) - (1+\rho)u(c_1^b/(1+\rho))\right\}\end{aligned}$$

with the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g) - \lambda\delta\pi\rho + \mu\rho u'(c_2^g) = 0 \tag{12}$$

$$\frac{\partial\mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) - \mu u'(c_1^b/(1+\rho)) = 0 \tag{13}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = \Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 \tag{14}$$

$$\frac{\partial\mathcal{L}}{\partial\mu} = u(c_1^g) + \rho u(c_2^g) - (1+\rho)u(c_1^b/(1+\rho)) = 0. \tag{15}$$

From (11) und (12) we obtain

$$u'(c_1^g) = u'(c_2^g) \quad\Rightarrow\quad c_1^g = c_2^g = c^g \tag{16}$$

and a standard annuity is optimal for g-types. Substituting into (10) yields

$$(1+\rho)u(c^g) = (1+\rho)u(c_1^b/(1+\rho)) \quad\Rightarrow\quad c_1^b = (1+\rho)c^g > c^g, \tag{17}$$

i.e. the lump-sum payment for b-types and the present value of the annuity for g-types are the same. In the first best, in contrast, $\alpha > \tilde{\alpha}$ implies that the present value of the annuity is smaller than the payment for b-types (see Figure 2). Thus, the annuity level for people in good health must be larger in the second best and the payment for b-types must be smaller. Substituting (16) and (17) into the budget constraint (9), we obtain

$$c_1^b = \frac{\Omega}{\delta} \quad\text{and}\quad c_1^g = c_2^g = \frac{\Omega}{\delta(1+\rho)} \tag{18}$$

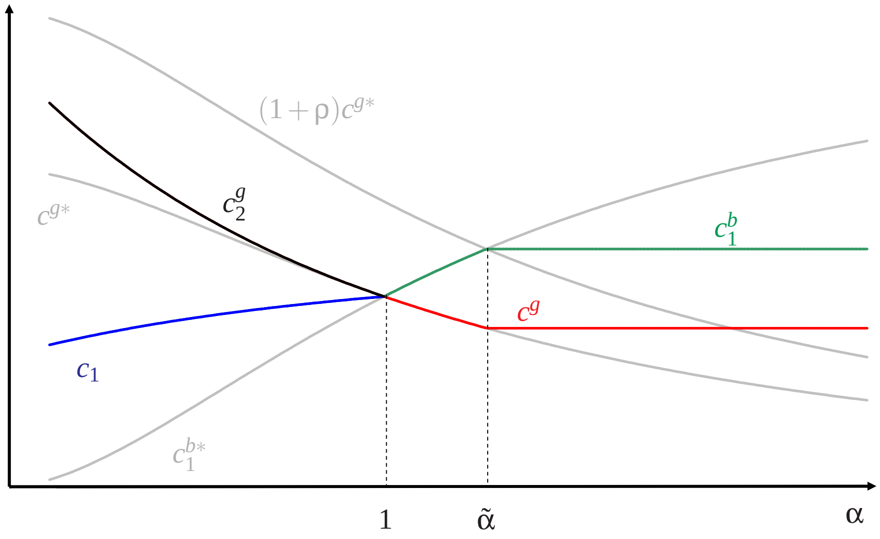

Figure 3: Second-best consumption

as the optimal solution. It can be implemented by giving individuals a choice between a lump-sum payment and an annuity.[13]

Proposition 2.2. *If the health status is unobservable, then the first best can only be implemented if life expectancy and marginal utility of consumption are weakly negatively correlated* $(1 \leq \alpha \leq \tilde{\alpha})$. *Otherwise, a second-best solution prevails. It is optimal to give individuals a choice between a lump-sum payment and an annuity if life expectancy and marginal utility of consumption are negatively correlated* $(\alpha \geq 1)$. *Otherwise, an annuity which increases over time is preferable.*

Figure 3 illustrates the second-best solution. For $\alpha < 1$, we have $c_1^{b*} < c_1 < c^{g*}$ and $c_2^g > c^{g*}$, the latter being due to a utility function with constant relative risk aversion less than one. For $\alpha > \tilde{\alpha}$, c_1^b and c^g are given by (18).

[13] It is also possible to implement the second best by paying out a lump sum c_1^b which is then annuitized by g-types. In Section 3.1 where we allow pension plans to prohibit the purchase of further annuities, however, this solution is inferior to a choice between a lump-sum payment and an annuity (see footnote 14).

3 Extensions

In this section, we extend the basic model in various directions to check whether choice between a lump-sum payment and an annuity at retirement can still remain optimal. We consider the following extensions:

1. In Subsection 3.1, we allow pension plans to monitor the purchase of further annuities.
2. Imperfect correlation between marginal utility of consumption and life expectancy is considered in Subsection 3.2.
3. Subsection 3.3 assumes that both types have the same identical close-to-death utility $\alpha u(c_t)$.
4. The possibility of moral hazard due to a state-guaranteed minimum income is examined in Subsection 3.4
5. Subsection 3.5 allows for a positive survival probability to period 2 for individuals in bad health.

3.1 Monitoring of annuity purchases possible

If annuity purchases can be monitored, pension plans can make it more difficult for g-types to pretend to be b-types by prohibiting annuitization of the lump-sum payment. Then the only way for g-types to transfer income to period 2 is to save. If they chose the lump-sum payment, they will therefore solve the problem

$$\max EU^g = u(c_1^g) + \rho u(c_2^g) \quad \text{s.t.} \quad c_1^g + c_2^g = c_1^b. \tag{19}$$

At the optimum

$$u'(\hat{c}_1^g(c_1^b)) = \rho u'(\hat{c}_2^g(c_1^b)) \tag{20}$$

and therefore $\hat{c}_1^g(c_1^b) > \hat{c}_2^g(c_1^b)$ as $\rho < 1$. With probability $1-\rho$, individuals will leave unintended bequests $c_2^g(c_1^b)$.

The new incentive constraint for g-types is

$$u(c_1^g) + \rho u(c_2^g) \geq u(\hat{c}_1^g(c_1^b)) + \rho u(\hat{c}_2^g(c_1^b)). \tag{21}$$

Clearly, the RHS of (21) will be smaller than the RHS of (ICG) for a given value of c_1^b. Thus, the corresponding critical value of $\tilde{\alpha}_{mon}$ will be higher than the critical value $\tilde{\alpha}$ without monitoring and the first best can be implemented for a larger range of α. For example, if $u(c) = \ln(c)$, we obtain

$$(1+\rho)\ln(c^g) \geq \ln\left(\frac{c_1^b}{1+\rho}\right) + \rho\ln\left(\frac{\rho c_1^b}{1+\rho}\right).$$

With the first-best condition $c^b = \alpha c^g$, this yields a critical value

$$\tilde{\alpha}_{mon} = (1+\rho)\rho^{-\frac{\rho}{1+\rho}} > 1+\rho = \tilde{\alpha}.$$

If $\alpha > \tilde{\alpha}_{mon}$, then we obtain a similar result as in section 2.3. The second best can be implemented by choice between a lump-sum payment and a constant annuity.[14] However, the present value of the annuity for g-types is lower than lump-sum payment for b-types.

Proposition 3.1. *If pensions plans can monitor further annuity purchases, then the first best can be implemented for higher levels of the marginal utility of consumption in the bad health state.*

3.2 Heterogenous marginal utility

Now we relax the assumption that marginal utility is unique given life expectancy. In each health state, marginal utility can take different values. In state b, utility is $\alpha u(c_1^b)$ with $\alpha \in [\alpha_1, \alpha_2]$. In state g, utility is $\beta u(c_1^g) + \rho\beta u(c_2^g)$ with $\beta \in [\beta_1, \beta_2]$. Expected utility is given by

$$EU = \delta\left(\pi\bar{\beta}\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\bar{\alpha}u(c_1^b)\right) \tag{22}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the average values of α and β.

In the first best, we obtain that the marginal utility of consumption should be equalized across all states, i.e.

$$\forall\alpha,\beta \quad \alpha u'(c_1^b(\alpha)) = \beta u'(c_t^g(\beta)), \quad t = 1,2.$$

[14] As opposed to the basic model, only paying out a lump-sum payment c_1^b cannot implement the second best since g-types are not allowed to annuitize.

The optimal pay outs are therefore increasing in α and β. This implies that the first best requires knowledge of these parameters which is highly unlikely. Thus, even if the health status is observable, only a second-best solution can be implemented. Normalizing $\bar{\beta} = 1$, we obtain the following condition

$$u'(c_1^g) = u'(c_2^g) = \bar{\alpha}u'(c_1^b)$$

which states that marginal utility of consumption should be equalized on average across health states.

When the health status is not observable, the incentive constraints are

$$\forall\alpha \quad \alpha u(c_1^b) \geq \alpha u(c_1^g)$$

and

$$\forall\beta \quad \beta u(c_1^g) + \rho\beta u(c_2^g) \geq (1+\rho)\beta u(c_1^b/(1+\rho)).$$

Note that α and β have no impact on the incentive constraints. Thus, they are identical to (ICB) and (ICG) and we can use the results from above by interpreting α as the average $\bar{\alpha}$. Thus, although the first best cannot be implemented, giving individuals a choice between a lump-sum payment and an annuity is still optimal if $\bar{\alpha} \geq 1$. For $1 \leq \bar{\alpha} \leq \tilde{\alpha}$, we obtain a second-best solution, otherwise a third best arises.

Proposition 3.2. *If marginal utility of consumption and life expectancy are only imperfectly correlated and neither marginal utility of consumption nor the health status are observable, then it is optimal to give individuals a choice between a lump-sum payment and an annuity if life expectancy and marginal utility are negatively correlated. Otherwise, an annuity which increases over time is preferable.*

3.3 Identical close-to-death utility

One argument for state-dependent utility is closeness to certain death. If this reasoning applies, we must also assume that the utility function is $\alpha u(c)$ for g-types in period 2. Expected utility is then given by

$$EU = \delta\left(\pi\left(u(c_1^g) + \rho\alpha u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right). \tag{23}$$

Maximizing expected utility subject to the wealth constraint (2) leads to

$$u'(c_1^{g*}) = \alpha u'(c_2^{g*}) = \alpha u'(c_1^{b*})$$

which implies for the first best

$$c_1^{b*} = c_2^{g*} \gtreqless c_1^{g*} \;\Leftrightarrow\; \alpha \gtreqless 1.$$

Thus, consumption in the last period of life is identical for both types. Second-period consumption in good health is higher if marginal utility is larger.

If the health status cannot be observed, the incentive constraint (ICB) for b-types remains unaffected and the first best cannot be implemented for $\alpha < 1$. In this case, consumption must be the same for both types in period 1. The Lagrangian is

$$\mathcal{L} \;=\; \delta\left(\pi\left(u(c_1)+\rho\alpha u(c_2^g)\right)+(1-\pi)\alpha u(c_1)\right)+\lambda\left\{\Omega-\delta\left(c_1+\pi\rho c_2^g\right)\right\}.$$

with the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1} \;=\; \delta(\pi+(1-\pi)\alpha)u'(c_1)-\lambda\delta=0 \tag{24}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} \;=\; \delta\pi\rho\alpha u'(c_2^g)-\lambda\delta\pi\rho=0 \tag{25}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} \;=\; \Omega-\delta\left(c_1+\pi\rho c_2^g\right)=0. \tag{26}$$

We obtain

$$(\pi+(1-\pi)\alpha)u'(c_1)=\alpha u'(c_2^g). \tag{27}$$

Since $\pi+(1-\pi)\alpha>\alpha$, we have $c_1>c_2^g$, i.e. the annuity falls over time. This result also holds in the first best. However, g-types' intertemporal consumption is distorted since $\pi+(1-\pi)\alpha<1$ which implies $u'(c_1)>\alpha u'(c_2^g)$. As in the basic model, g-types consume too much in the second period.

Turning to the case $\alpha>1$, we need to consider the incentive constraint for g-types if the health status is not observable. It is different from the basic model since g-types will not buy a constant annuity if they select the payment for b-types. Taking into account their optimal choice of consumption $\hat{c}_t^g(c_1^b)$ in period t if g-types choose the lump-sum payment, the incentive constraint is

$$u(c_1^g)+\rho\alpha u(c_2^g)\geq u(\hat{c}_1^g(c_1^b))+\rho\alpha u(\hat{c}_2^g(c_1^b)). \tag{28}$$

Optimality requires

$$u'(\hat{c}_1^g(c_1^b)) = \alpha u'(\hat{c}_2^g(c_1^b)) \tag{29}$$

which implies $\hat{c}_2^g(c_1^b) > \hat{c}_1^g(c_1^b)$. Again, we can determine a critical value $\tilde{\alpha}_{\text{ctd}}$ up to which the first best can be implemented. Under the assumption that g-types can buy further annuities, their budget constraint with the first-best lump-sum payment is

$$c_1^g + \rho c_2^g = c_1^{b*}.$$

This compares to a present value of consumption $c_1^{g*} + \rho c_2^{g*}$ if the annuity is chosen. Thus, the lump-sum option is inferior if $c_1^{b*} < c_1^{g*} + \rho c_2^{g*}$. Using $c_1^{b*} = c_2^{g*}$ yields the equivalent condition

$$c_1^{b*} < \frac{c_1^{g*}}{1-\rho}$$

which will be met as long as α is sufficiently small. For example, if the utility function is $u(c_t) = \ln(c_t)$, then $c_2^{g*} = \alpha c_1^{g*}$. In this case, the condition is

$$\alpha < \frac{1}{1-\rho} \equiv \tilde{\alpha}_{\text{ctd}}.$$

In the basic model, $\tilde{\alpha} = 1+\rho$. Since $\frac{1}{1-\rho} > 1+\rho$, the critical value of α is therefore larger with identical close-to-death utility. The intuition for this result is that g-types have less incentives to pretend to be b-types if their annuity is higher when old.

As long as $1 \leq \alpha \leq \tilde{\alpha}_{\text{ctd}}$, the first best can be implemented by giving individual a choice between the lump-sum payment c_1^{b*} and the increasing annuity c_1^{g*}, c_2^{g*}. If $\alpha > \tilde{\alpha}_{\text{ctd}}$, the incentive constraint (28) needs to be considered. The Lagrangian is

$$\begin{aligned}
\mathcal{L} \quad = \quad & \delta\left(\pi\left(u(c_1^g) + \rho\alpha u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right) \\
& +\lambda\left\{\Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right)\right\} \\
& +\mu\left\{u(c_1^g) + \rho\alpha u(c_2^g) - u(\hat{c}_1^g(c_1^b)) - \rho\alpha u(\hat{c}_2^g(c_1^b))\right\}
\end{aligned}$$

with the first-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1^g} &= \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 &(30)\\
\frac{\partial \mathcal{L}}{\partial c_2^g} &= \delta\pi\rho\alpha u'(c_2^g) - \lambda\delta\pi\rho + \mu\rho\alpha u'(c_2^g) = 0 &(31)\\
\frac{\partial \mathcal{L}}{\partial c_1^b} &= \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi)\\
&\quad -\mu u'(\tilde{c}_1^g(c_1^b))\frac{\mathrm{d}\tilde{c}_1^g}{\mathrm{d}c_1^b} - \mu\rho\alpha u'(\tilde{c}_2^g(c_1^b))\frac{\mathrm{d}\tilde{c}_2^g}{\mathrm{d}c_1^b} = 0 &(32)\\
\frac{\partial \mathcal{L}}{\partial \lambda} &= \Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 &(33)\\
\frac{\partial \mathcal{L}}{\partial \mu} &= u(c_1^g) + \rho\alpha u(c_2^g) - u(\tilde{c}_1^g(c_1^b)) - \rho\alpha u(\tilde{c}_2^g(c_1^b)) = 0. &(34)
\end{aligned}$$

From (30) und (31) we obtain

$$u'(c_1^g) = \alpha u'(c_2^g) \quad \Rightarrow \quad c_1^g < c_2^g \tag{35}$$

and an increasing annuity is optimal for g-types. Since $u'(\tilde{c}_1^g(c_1^b)) = \alpha u'(\tilde{c}_2^g(c_1^b))$ by (29), the binding incentive constraint (28) implies

$$c_1^b = c_1^g + \rho c_2^g, \tag{36}$$

i.e. the lump-sum payment for b-types and the present value of the annuity for g-types are the same. In the first best, we also have an increasing annuity annuity of g-types. However, the first-best lump-sum payment is larger than the present value of the annuity if $\alpha > \tilde{\alpha}_{\text{ctd}}$. Thus, the lump-sum payment must be smaller in the second best. The same result holds in the basic model. Again, the optimal solution can be implemented by giving individuals a choice between a lump-sum payment and an annuity.

Proposition 3.3. *When the utility function is the same for both types in the period prior to death and marginal utility is larger in this period, choice between a lump-sum payment and an annuity is optimal if the health status is unobservable. The only difference to the basic model is that the annuity must be increasing rather than constant.*

3.4 Minimum income and moral hazard

If society grants a minimum income to its citizens, an important concern with respect to lump-sum withdrawals is moral hazard. Individuals may then have the incentive to take the lump sum, spend it regardless of their life expectancy on immediate consumption and rely on public transfers if they live longer. To rule this out, the government may therefore require that individuals only buy pension plans which guarantee a payment that is at least as high as the guaranteed minimum income in each period.

In the following, we examine the consequences of this policy for the second-best pension plan. We denote minimum income by m and assume that in the first best, individuals do not qualify for public assistance, i.e. $c^{g*}, c_1^{b*} > m$. If the health status is not observable, the incentive constraint (ICB) for b-types and therefore the results for $\alpha < 1$ remain unchanged. However, the incentive constraint for g-types needs to modified. Since the contract for b-types cannot be lump sum, it consists of a payment c_1^b in period 1 and m in period 2 (which is never paid in the first best). If g-types pretend to be b-types, they therefore can also claim a payment m in period 2. The present value of the payment for b-types is therefore $(c_1^b + \rho m)/(1+\rho)$ and the incentive constraint is

$$u(c_1^g) + \rho u(c_2^g) \geq (1+\rho)u((c_1^b + \rho m)/(1+\rho)). \tag{37}$$

For a given value of c_1^b, it is therefore more attractive for g-types to claim to be b-types. This lowers the critical value $\tilde{\alpha}$. For logarithmic utility, we obtain

$$\tilde{\alpha}_m = 1 + \rho - \frac{\rho m}{c^{g*}} < 1 + \rho = \tilde{\alpha}.^{15}$$

As above, the first best is incentive-compatible if $1 \leq \alpha \leq \tilde{\alpha}_m$. In this case, individuals can be given a choice between the payment stream c_1^{b*}, m and a constant annuity c^{g*}. Since $\alpha > 1$ implies $c_1^{b*} > c^{g*}$, we can interpret $c_1^{b*} - c^{g*}$ as a partial lump-sum withdrawal which results in a reduction $c^{g*} - m$ in the second-period payment.

To determine the optimal pension plan for $\alpha > \tilde{\alpha}_m$, we solve the problem

$$\max EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)$$

[15] Note that $\tilde{\alpha}_m > 1$ since we assumed $c^{g*} > m$.

s.t.

$$\Omega = \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right)$$

$$u(c_1^g) + \rho u(c_2^g) = (1+\rho)u((c_1^b + \rho m)/(1+\rho)). \tag{38}$$

Setting up the Lagrangian

$$\begin{aligned}\mathcal{L} &= \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right) \\ &\quad + \lambda\left\{\Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right)\right\} \\ &\quad + \mu\left\{u(c_1^g) + \rho u(c_2^g) - (1+\rho)u((c_1^b + \rho m)/(1+\rho))\right\}\end{aligned}$$

yields the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g) - \lambda\delta\pi\rho + \mu\rho u'(c_2^g) = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) - \mu u'((c_1^b + \rho m)/(1+\rho)) = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \Omega - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = u(c_1^g) + \rho u(c_2^g) - (1+\rho)u(c_1^b/(1+\rho)) = 0. \tag{43}$$

From (39) und (40) we obtain as in the basic model

$$u'(c_1^g) = u'(c_2^g) \quad \Rightarrow \quad c_1^g = c_2^g = c^g.$$

Inserting into condition (38) yields

$$(1+\rho)u(c^g) = (1+\rho)u((c_1^b + \rho m)/(1+\rho)) \;\Rightarrow\; c_1^b = (1+\rho)c^g - \rho m. \tag{44}$$

Thus, the present value of the payment for b-types is smaller than the annuity for g-types. Furthermore, we must have $c_1^b > c^g > m$.[16] Therefore, it is optimal to give individuals a choice between a partial lump-sum withdrawal $c_1^b - c^g$ with a reduction $c^g - m$ in the second-period payment and a constant annuity paying c^g.

[16] By (44), $c_1^b \le c^g$ implies $m \ge c^g$. Since we assumed for the first best $c^{g*} > m$, this implies $c^g < c^{g*}$ and, by the budget constraint, $c_1^b > c_1^{b*} > m$. Thus, $c_1^b \le c^g$ is only possible if $c^g > m$ which is incompatible with (44). Therefore, we must have $c_1^b > c^g$ and, by (44), $c^g > m$.

Proposition 3.4. *If the government requires that individuals only buy pensions plans which guarantee a payment that is at least as high as minimum income in each period and the health status is not observable, then the first best can be implemented for a smaller range of* α*. If marginal utility of consumption is higher in the bad health state, it is optimal to give individuals a choice between a partial lump-sum withdrawal and a constant annuity.*

3.5 Positive survival probability for individuals in bad health

So far, we maintained the assumption that individuals in state b will not live more than one period. Now we allow both types to survive to the second period. The respective survival probabilities are ρ^b and ρ^g with $0 < \rho^b < \rho^g < 1$, i.e. b-types have a lower life expectancy. Expected utility is

$$EU = \delta\left(\pi\left(u(c_1^g) + \rho^g u(c_2^g)\right) + (1-\pi)\left(\alpha u(c_1^b) + \rho^b \alpha u(c_2^b)\right)\right). \qquad (45)$$

It is straightforward to show that the first-best solution requires annuities c^g and c^b with

$$c^b \gtrless c^g \quad \Leftrightarrow \quad \alpha \gtrless 1.$$

If the health status is not observable, then it is impossible to implement the first best unless $\alpha = 1$. If $\alpha < 1$, then b-types pretend to be g-types and vice versa. Consequently, one incentive constraint will be binding for $\alpha < 1$, the other for $\alpha > 1$.

In the following, we assume that pension plans can monitor and thus prohibit further annuity purchases. For simplicity, we also allow pension plans to monitor and rule out savings (see footnote 19).

Second-best solution for $\alpha < 1$

In this case, the incentive-constraint for b-types

$$u(c_1^b) + \rho^b u(c_2^b) \geq u(c_1^g) + \rho^b u(c_2^g) \qquad (46)$$

will be binding. Setting up the Lagrangian

$$\begin{aligned}\mathcal{L} &= \delta\left(\pi\left(u(c_1^g)+\rho^g u(c_2^g)\right)+(1-\pi)\left(\alpha u(c_1^b)+\rho^b\alpha u(c_2^b)\right)\right)\\ &\quad +\lambda\left\{\Omega-\delta\left(\pi c_1^g+\pi\rho^g c_2^g+(1-\pi)c_1^b+(1-\pi)\rho^b c_2^b\right)\right\}\\ &\quad +\mu\left\{u(c_1^b)+\rho^b u(c_2^b)-u(c_1^g)-\rho^b u(c_2^g)\right\}\end{aligned}$$

yields the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g)-\lambda\delta\pi-\mu u'(c_1^g)=0 \tag{47}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} = \delta\pi\rho^g u'(c_2^g)-\lambda\delta\pi\rho^g-\mu\rho^b u'(c_2^g)=0 \tag{48}$$

$$\frac{\partial\mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b)-\lambda\delta(1-\pi)+\mu u'(c_1^b)=0 \tag{49}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^b} = \delta(1-\pi)\rho^b\alpha u'(c_2^b)-\lambda\delta(1-\pi)\rho^b+\mu\rho^b u'(c_2^b)=0 \tag{50}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = \Omega-\delta\left(\pi c_1^g+\pi\rho^g c_2^g+(1-\pi)c_1^b+(1-\pi)\rho^b c_2^b\right)=0 \tag{51}$$

$$\frac{\partial\mathcal{L}}{\partial\mu} = u(c_1^b)+\rho^b u(c_2^b)-u(c_1^g)-\rho^b u(c_2^g)=0. \tag{52}$$

We obtain $c_1^b = c_2^b$ from (49) and (50). Conditions (47) and (48) yield

$$u'(c_1^g)=\frac{\lambda\pi}{\delta\pi-\mu} \quad \text{and} \quad u'(c_2^g)=\frac{\lambda\pi}{\delta\pi-\mu\frac{\rho^b}{\rho^g}}.^{17}$$

Since $\rho^g > \rho^b$, this implies

$$u'(c_2^g) < u'(c_1^g) \quad \Rightarrow \quad c_2^g > c_1^g.$$

Thus, the incentive for b-types to pretend to be g-types is countered by an increasing annuity which is less attractive for b-types. From the binding incentive constraint (46), we obtain the solution

$$c_2^g > c_1^b = c_2^b > c_1^g.$$

Thus, it is optimal to give individuals a choice between a constant and an increasing annuity in period 1.

[17] The assumption $\lim_{c_t\to 0} u'(c_t)=\infty$ ensures positive marginal utilities.

Second-best solution for $\alpha > 1$

In this case, the incentive-constraint for g-types

$$u(c_1^g) + \rho^g u(c_2^g) \geq u(c_1^b) + \rho^g u(c_2^b) \tag{53}$$

will be binding. The Lagrange function is

$$\begin{aligned}
\mathcal{L} &= \delta\left(\pi\left(u(c_1^g) + \rho^g u(c_2^g)\right) + (1-\pi)\left(\alpha u(c_1^b) + \rho^b \alpha u(c_2^b)\right)\right) \\
&\quad + \lambda\left\{\Omega - \delta\left(\pi c_1^g + \pi\rho^g c_2^g + (1-\pi)c_1^b + (1-\pi)\rho^b c_2^b\right)\right\} \\
&\quad + \mu\left\{u(c_1^g) + \rho^g u(c_2^g) - u(c_1^b) - \rho^g u(c_2^b)\right\}.
\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^g} = \delta\pi\rho^g u'(c_2^g) - \lambda\delta\pi\rho^g + \mu\rho^g u'(c_2^g) = 0 \tag{55}$$

$$\frac{\partial \mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) - \mu u'(c_1^b) = 0 \tag{56}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^b} = \delta(1-\pi)\rho^b \alpha u'(c_2^b) - \lambda\delta(1-\pi)\rho^b - \mu\rho^g u'(c_2^b) = 0 \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \Omega - \delta\left(\pi c_1^g + \pi\rho^g c_2^g + (1-\pi)c_1^b + (1-\pi)\rho^b c_2^b\right) = 0 \tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = u(c_1^g) + \rho^g u(c_2^g) - u(c_1^b) - \rho^g u(c_2^b) = 0. \tag{59}$$

We obtain $c_1^g = c_2^g$ from (54) and (55). Conditions (56) and (57) lead to

$$u'(c_1^b) = \frac{\lambda(1-\pi)}{\delta(1-\pi)\alpha - \mu} \quad \text{and} \quad u'(c_2^b) = \frac{\lambda(1-\pi)}{\delta(1-\pi)\alpha - \mu\frac{\rho^g}{\rho^b}}.^{18}$$

Since $\rho^g > \rho^b$, we obtain

$$u'(c_1^b) < u'(c_2^b) \quad \Leftrightarrow \quad c_1^b > c_2^b.^{19}$$

[18] The assumption $\lim_{c_t \to 0} u'(c_t) = \infty$ ensures positive marginal utilities.

[19] We assumed that pension plans can prohibit savings. This is necessary if the optimal solution implies $u'(c_1^b) < \rho^h u'(c_2^b)$, $h = g, b$. If savings cannot be monitored then the additional constraint $u'(c_1^b) \geq \rho^g u'(c_2^b)$ needs to be imposed which implies $u'(c_1^b) \geq \rho^b u'(c_2^b)$.

Finally, using the binding incentive constraint (53) yields

$$c_1^b > c_1^g = c_2^g > c_2^b.$$

This solution can be implemented by allowing individuals a choice between a partial lump-sum withdrawal $c_1^b - c^g$ with a reduction $c^g - c_2^b$ in the second-period payment and a constant annuity c^g.

Proposition 3.5. *If individuals in bad health have a positive probability to survive to period 2 and a lower life expectancy, the first best cannot be implemented if the health status is not observable. Assuming that pension plans can rule out further annuity purchases and savings, it is optimal to give individuals a choice between a partial lump-sum withdrawal and a constant annuity if life expectancy and marginal utility are negatively correlated. Otherwise, choice between a constant and an increasing annuity is preferable.*

4 Implications for public pensions

So far, we have left open whether the pension plan is private or public. Since we assumed that individuals are ex ante identical, however, there seems to be little justification for public intervention. This changes if we consider that individuals differ already ex ante in their type.[20] Assume that a share π lives for certain for one period and with probability ρ for two periods, and a share $1-\pi$ lives only for one period. The types are unobservable and the utility of consumption differs as in the analysis above. For an utilitarian social planner the social welfare function is then equivalent to the expected utility function (1). Thus, it is optimal for the social planner to use a lump-sum option if marginal utility of consumption and life expectancy are negatively correlated.

Furthermore, one can make an argument in favor of a lump-sum option even if the utility function in each period is not state-dependent, i.e. the per period utility

[20] In Diamond (2003, Chapter 7) individuals differ ex ante in productivity which is positively correlated with life expectancy. In an optimal income tax framework, he shows that a lump-sum option can increase social welfare since it will be chosen by the individuals with low life expectancy. As an additional screening device, it benefits individuals with low productivity.

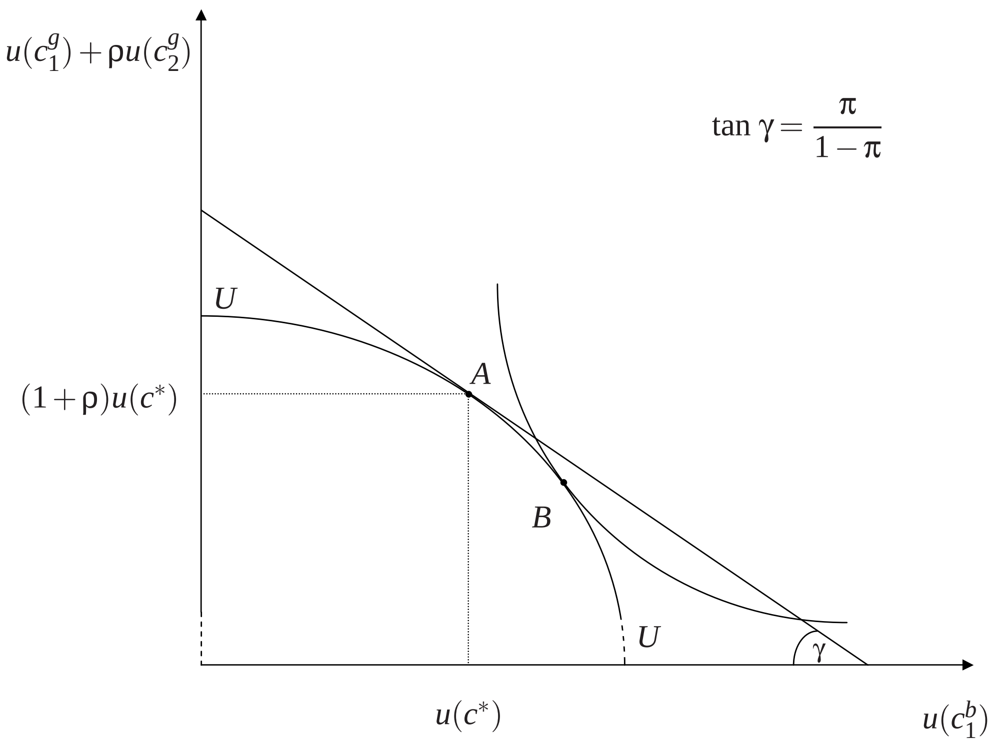

Figure 4: Optimal pension plans and social welfare

function is always $u(c_t)$.[21] Consider the social welfare function

$$\begin{aligned} \mathcal{W} &= (1-\pi)W\left(EU^g\right)+\pi W\left(EU^b\right) \\ &= (1-\pi)W\left(u(c_1^g)+\rho u(c_2^g)\right)+\pi W\left(u(c_1^b)\right) \end{aligned} \tag{60}$$

with $W'>0, W''\leq 0$. In the utilitarian case $W''=0$, the objective function is equivalent to the expected utility function (1) for $\alpha=1$. A constant annuity annuity c^* is therefore optimal. This situation is illustrated by allocation A in Figure 4. UU is the utility possibility frontier which must be strictly concave due to $u''<0$. At point A, the marginal rate of substitution between the expected utility of both types is

$$-\left.\frac{\mathrm{d}EU^g}{\mathrm{d}EU^b}\right|_{\mathrm{d}\mathcal{W}=0} = \frac{\pi}{1-\pi}.$$

In A, we have $EU^g=(1+\rho)u(c^*)>u(c^*)=EU^b$. If the social welfare function is

[21] I thank Jean-Marie Lozachmeur for making me aware of this interpretation.

strictly concave, i.e. $W'' < 0$, the social planner will therefore want to redistribute to individuals with lower total expected utility, i.e. those with low life expectancy. The optimal point will be to the right of point A, for instance in point B. Here the marginal rate of substitution is given by

$$-\left.\frac{\mathrm{d}EU^g}{\mathrm{d}EU^b}\right|_{\mathrm{d}\mathcal{W}=0} = \frac{\pi}{1-\pi}\frac{W'(EU^b)}{W'(EU^g)} > \frac{\pi}{1-\pi}.$$

Now define $\alpha \equiv W'(EU^b)/W'(EU^g) > 1$. Then the maximization of the social welfare function (60) is equivalent to maximization of expected utility (1) with $\alpha > 1$. Thus, we can reinterpret $\alpha > 1$ as the relative welfare weight given to individuals with low life expectancy at the optimum. This shows that a lump-sum withdrawal option can also be justified on pure equity grounds without the assumption of a state-dependent utility function. If α is smaller than the critical value $\tilde{\alpha}$, the first-best can be implemented; otherwise the incentive constraint for individuals with high life expectancy will be binding and only a second-best solution is possible.

Proposition 4.1. *If types differ ex ante in life expectancy and the social planner maximizes a utilitarian social welfare function, it is optimal to give individuals a choice between a lump-sum payment and an annuity if life expectancy and marginal utility of consumption are negatively correlated. If social welfare is strictly concave in the types' total expected utility, this choice is optimal even if the marginal utility of consumption is independent of life expectancy.*

5 Conclusion

This paper examined the optimal design of pension plans when the health status during retirement is uncertain. In contrast to standard models, we assumed that the health status affects both life expectancy and the marginal utility of consumption. A simple model demonstrated that choice between a lump-sum payment and an annuity can be welfare-enhancing if the health status is not observable. This result holds if the marginal utility of consumption and life expectancy are negatively correlated. This result proved robust in several extensions. For example, we allowed marginal utility of consumption to be imperfectly correlated with the health status and considered that the maximum life-span does not depend on the health status. In the latter case, the possibility of a partial lump-sum withdrawal proved to be optimal if marginal utility of consumption and life expectancy are negatively correlated. Furthermore, we showed that a lump-sum option can be justified on equity grounds. When the social welfare function is strictly concave, this result holds even if the marginal utility of consumption is independent of life expectancy.

A limitation of the analysis is that we assumed a uniform retirement age. However, health and life expectancy can be expected to have an impact on the retirement age as well. For example, McGarry (2004) finds that the less healthy are likely to retire earlier. Similarly, Hurd, Smith, and Zissimopoulos (2004) observe that those with very low subjective probabilities of survival choose a lower retirement age. An interesting question for future research is whether early retirement and lump-sum payments are substitutes if individuals value consumption higher when their health state is bad.

Finally, the paper raises the empirical question on how health affects utility and life expectancy of the elderly. In particular, it would be interesting to investigate the correlation of marginal utility of consumption and life expectancy in old age. As this paper shows this correlation is crucial for the optimal design of pension plans.

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