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JOB SEARCH AND SAVINGS: WEALTH EFFECTS AND DURATION DEPENDENCE

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Abstract

In this paper we consider a risk averse worker who is moving back and forth between employment and unemployment; layoffs are random and beyond the worker's influence, while the re-employment chance is directly affected by search effort. We characterize the worker's optimal savings and job-search behavior as well as the resulting consumption paths and wealth formation. In general, all decisions will depend on the current level of wealth: First, the choice of search effort increases as wealth decreases, a finding which is in accordance with our empirical duration analysis using micro data on unemployment spells. Second, consumption increases with wealth both when the worker is employed and unemployed. Third, savings provide insurane against income fluctuations but this insurance is not perfect; precautionary savings are built up during employment spells and run down during unemployment spells but the consumption path is never going to be completely smooth over states. Finally, our results suggest that the worker's search intensity and hence the probability of leaving unemployment will exhibit positive duration dependence over unemployment spells via its inverse relationship with the worker's wealth.

JEL Classification: D1, J4, J6.

Keywords: Search, consumption smooting, duration dependence.

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1 Introduction

Job search is, like the savings decision, an important intertemporal allocation problem facing a worker. An unemployed worker decides how much effort to put into job search by essentially weighing the shortening of the jobless period and thereby increasing future expected income against loss of leisure today. As future expected income bounds the consumption possibilities an intertemporal allocation decision is thus made involving both leisure and consumption. A worker who undertakes considerations of this kind, is also expected to carefully allocate consumption over time. In the face of fluctuating income, for instance due to the worker alternating between employment and unemployment, this amounts to another intertemporal allocation problem - the savings decision. Hence, search and savings are two sides of the same coin, both are undertaken at present in order to increase future consumption possibilities; yet until recently they have rarely been analyzed as interrelated problems. In the labor literature, the savings decision is typically mute, because of risk neutrality of the worker or because wealth cannot be stored or cannot be transferred between individuals. Risk neutrality is somewhat unsatisfactory when studying an ordinary bread winner in a labor market with unemployment. Furthermore, insurance and capital markets are rarely completely imperfect in terms of allowing wealth to be stored or transferred.[1] In the macro literature workers might be risk averse but then the search decision is typically mute and unemployed workers become re-employed at an exogenous rate.

The literature on job search is, of course, aware of having traded richness in the description of the searchers' consumption and savings decision for further depth in the description and understanding of the search behavior. This makes sense as long as the savings decision has little or no effect on the search behavior being studied and certain aspects of the search model are probably not affected much by the savings decision.

Caution is, however, recommended here as a number of recent studies suggest that savings influence search behavior. In this paper, we consider a risk averse and hence consumption smoothing worker who chooses search effort optimally when unemployed. Over time the

[1] Even when capital markets are imperfect, wealth can be stored to some extent; there are other ways of going about smoothing consumption over time, for instance via the timing of purchases of durable goods (see Browning and Crossley (1998)).

worker is moving back and forth between employment and unemployment. Layoffs are random and beyond the worker's influence, while the re-employment chance can be affected by search effort. We derive a number of propositions from this model, some of which we are able to test empirically. First, we characterize the worker's optimal savings and search behavior and the resulting consumption paths and wealth formation: workers do smooth consumption but the consumption path is not going to be completely smooth over time; search effort increases as wealth decreases; and wealth decreases during spells of unemployment. Second, the latter two of these features imply that the probability of leaving unemployment increases with the duration of the unemployment spell.

Finally, we test the main predictions of the model using Danish micro data. The data set does not have information on the time or effort expended on search but it does have information on the duration of individuals unemployment spells and on their wealth. First we test whether wealth influences the probability of leaving unemployment (the unemployment hazard rate) and we find the expected negative relationship predicted by our theory. We cannot directly test the positive duration dependence prediction, as we do not have information on individual's current wealth during unemployment spells. However, our model predicts that the wealth levels of different individuals will eventually converge as an unemployment spell persists. And as the wealth levels converge, so will the search behavior. Thus, one would expect that the effect on the unemployment hazard rate from differences in initial wealth levels should fade as the duration of a spell increases. Indeed, we find such a relationship. Essentially, the effect of marginal increases in initial wealth on the hazard rate is all but gone after six months. Hence, the empirical analysis supports the basic results of our model.

Our work is related to the large literature on job search following McCall (1970) and Mortensen (1977); it is related, in particular, to Burdett and Mortensen (1978) and Danforth (1979) and some more recent literature like Acemoglu and Shimer (1999), Wang and Williamson (1999), and Gomes, Greenwood and Rebelo (2001) all of which have moved beyond the expected income maximization or risk neutrality assumption. Danforth (1979) considers the reservation wage decision of a risk averse unemployed worker when consumption smoothing is explicitly modelled. While Burdett and Mortensen (1978) combine search and an intertemporal income-leisure allocation problem with risk averse workers in an analysis of the labor supply decision. In both studies employment is an absorbing state, that is, once a job is found, the worker never leaves this job. The more recent literature (see e.g. Acemoglu and Shimer (1999) and Wang and Williamson (1999)) uses general equilibrium search models in the tradition of Diamond (1982), Mortensen (1982), and Pissarides (1990) to study the functioning and optimality of unemployment insurance schemes when workers are risk averse and save. The analytical results of the dynamic model in Acemoglu and Shimer (1999) are still based on employment being an absorbing state. Wang and Williamson (1999) allow for job separation in a model where individuals both search and save. By ways of numerical methods they then characterize optimal unemployment insurance schemes. In a related article, Gomes et al. (2001) use a job search model that includes savings and job separations to study labor market and business cycle regularities. In their model workers choose a reservation wage given a constant search effort whereas in our model workers choose a level of search effort given a fixed wage opportunity. Still we arrive at compatible necessary conditions for a monotone relationship between wealth and the search effort/reservation wage decision. However, Gomes et al. (2001) do not show that these necessary conditions are satisfied in their model. In the calibrated version of their model these conditions do hold, though. This is the case for both Wang and Williamson (1999) and Gomes et al. (2001).

In this paper we provide both analytical and empirical results using a model where employment is not an absorbing state; workers do alternate between spells of employment and unemployment; during employment (unemployment) they save (dissave) because an uncertain future is bound to change their fortune (misfortune).

The duration dependence result is also related to a growing empirical literature. It is a common empirical observation that the rate at which an unemployed worker leaves the unemployment pool (i.e., the unemployment hazard rate) is a function of the length of the unemployment spell. The controversy is over the sign of the effect, that is, whether the hazard rate depend negatively or positively on duration. There are quite a few explanations for both signs in the literature. Berkovitch (1990) suggests there is a stigma associated with long unemployment spells so that the hazard rate would show negative duration dependence. Others have associated negative duration dependence with loss of absolute or relative skills due to inactivity or separation from innovations. In Mortensen (1986) a simple liquidity constraint is built into a basic search model which generates a decreasing reservation wage as the unemployed worker moves closer and closer to the constraint. This would point to positive duration dependence. Danforth (1979) states a similar result in a somewhat more general setting. Workers are risk averse and it is established that the reservation wage is lower, the lower the level of the worker's wealth.

Another common explanation of the duration dependence of the hazard rate is that there is unobserved heterogeneity in the work force. If some workers tend to leave the unemployment pool faster than others, then the pool of long time unemployed will have a relatively high rate of workers with low transition probabilities. This will lead to the observation that the hazard rate exhibits negative duration dependence. While in fact, the hazard rate is constant for each worker. Van den Berg and Van Ours (1996) estimate duration dependence while controlling for unobserved heterogeneity using US data, and find significant effects of unobserved heterogeneity. They also find negative duration dependence for some groups and positive duration dependence for others, and conclude that this may be due to differences in stigmatization effects.[2]

The arguments above are all augmentations of the basic search model. The basic search model, with the assumption of expected income maximizing agents, does not in itself display duration dependence. The augmentations all work to make certain facets of the model time dependent, such as assuming that the offer arrival rate decreases as a spell progresses, or that unemployment benefits are time dependent, etc. The duration dependence is thus a result of the assumption of time dependence of what is essentially exogenous parameters. In this paper, we show that duration dependence can occur without appealing to the influence of outside factors. If one assumes that agents are risk averse, that they maximize expected utility, and that they have access to capital markets, duration dependence results. It is not that the transition probability is changing simply because the clock is ticking. Rather, the hazard rate becomes a function of wealth and as an unemployment spell extends, the unemployed worker chooses to reduce wealth and subsequently chooses to search harder. Therefore, the worker's hazard rate is increasing.

[2] Specifically, they find that the hazard rate for white males exhibits negative duration dependence, not much duration dependence for white females and positive duration dependence for black males and females. This paper suggests that one should try to control for wealth effects in explaining these differences.

The paper proceeds by presenting the model in section 2. Then comparative statics and duration dependence are discussed in sections 3 and 4. Section 5 presents the empirical analysis and Section 6 concludes the paper.

2 The Model

Consider a simple search model in which a worker moves back and forth between unemployment and employment according to a two-state Markov process. It is assumed that the worker is risk averse and therefore want to smooth consumption over states. This smoothing is accomplished by use of capital markets where the worker's savings can be placed.

An employed worker has no control over the probability of being separated from a job,[3] whereas an unemployed worker is assumed to be able to influence the probability of moving back into employment via the choice of search intensity. To simplify matters, it is assumed that utility is additive separable over time as well as over consumption c and search s.[4] Specifically, we assume that in any given period t, the worker's utility from consumption and search is $v(c_t, s_t) = u(c_t) - e(s_t)$, where $c_t \geq 0$ and $s_t \in [0, 1]$. The function $u(\cdot)$ is assumed to be strictly increasing and strictly concave, while the function $e(\cdot)$ is assumed to be strictly increasing and strictly convex with $e(0) = 0$. For convenience, both functions are assumed to be bounded and differentiable. Finally, the worker discounts future utility by the rate $\rho > 0$.

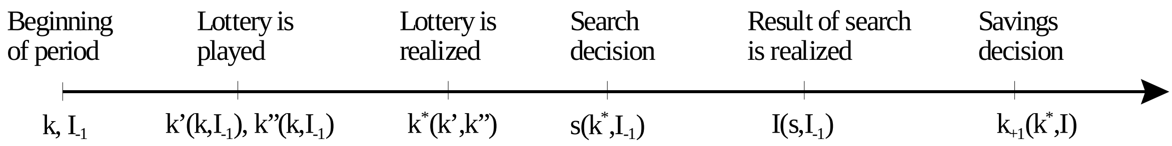
All jobs pay the same wage rate w. A worker who fails to obtain employment gets compensation b. This will be referred to as unemployment benefits but could also include income from a secondary labor market, utility from more leisure time, surplus from home production, etc.

The worker's wealth at the beginning of period t, k_t, is assumed to be bounded both above and below, that is $k_t \in \left[\underline{k}, \bar{k}\right]$. Finally, when we write "for all k_t" below we mean for all $k_t \in \left[\underline{k}, \bar{k}\right]$. A lower bound *per se* can be justified as a borrowing limit imposed by the capital market. Aiyagari (1994) points out that a lower bound on wealth can also be motivated by requiring asymptotic present value budget balance (i.e. $\lim_{t \to \infty} k_t/(1+r)^t \geq 0$)

[3] Thus, we are ignoring issues such as job retention effort and on the job search.

[4] This rules out issues such as habit persistence and the like.

Figure 1: The Timing of a Worker's Decisions within a Period.



combined with non-negative consumption. The upper bound is imposed in order to bound the problem and ensure existence of a solution. A general equilibrium argument could justify that in equilibrium, the interest rate must be such that people do not have infinite wealth (and thereby an infinite supply of capital). So, in general equilibrium and if set high enough, the upper limit will not be binding.

Let $I_t \in \{b, w\}$ be the income in period t. Thus, state variables at the beginning of period t are k_t and I_{t-1}, where I_{t-1} tracks the worker's employment status in the previous period. The timing of events in each period is as follows (see Figure 1): Given (k_t, I_{t-1}) the worker first enters the gambling stage, and has a choice of participating in a lottery with immediate realization denoted k_t^* and with expected value k_t. Note that the lottery holds no risk premium and will therefore only be entered into if the worker is risk loving. Second, given k_t^* the search and separation stage follows: If $I_{t-1} = b$, that is, the worker was unemployed in the previous period, the worker decides on how much effort, s_t, to put into the job search. Immediately after this, the result of the job search is realized and the worker either continues to be unemployed in period t or moves into employment. If $I_{t-1} = w$, the worker is separated from her job with probability η and becomes unemployed in period t; with probability $(1 - \eta)$ the worker continues to be employed in period t. In either case, the income in period t, I_t, is being determined. Finally, based on k_t^* and I_t, the worker decides on how much to leave in savings for the next period, or in other words, how much to consume in the present period, which then determines k_{t+1}.

A lottery is introduced because one cannot, in general, rule out local convexities in the worker's value functions and therefore a worker might want to participate in risky lotteries even when there is no risk premium to the lottery. The introduction of the lottery will eliminate these convexities in vital places in the model.[5] As such, the assumption of the existence of lotteries can be viewed as a sufficient condition for our results but in fact, simulations of our model suggest that it is not a necessary condition since all value functions turn out to be perfectly concave for a very wide range of model parameters even without the introduction of lotteries. We justify the assumption of lotteries by appealing to the existence of a lottery industry; and the assumption of a fixed unemployment insurance scheme with benefit b, on the basis that it is practically impossible to buy comprehensive insurance against unemployment. Either unemployment insurance is compulsory as is the case in most countries or there is a take-it-or-leave-it insurance package as in Denmark and Sweden.

The lottery is modelled as yielding return $k_t' \leq k_t$ with probability α and return $k_t'' \geq k_t$ with probability $1 - \alpha$. The parameter α is set so that the expected return of the lottery equals k_t, that is $\alpha = (k_t'' - k_t) / (k_t'' - k_t')$. The worker has access to a full menu of these lotteries in the sense of being able to choose k_t' and k_t'' freely as long as the returns lie within the wealth bounds. Non-participation in the lottery is given by $k_t' = k_t'' = k_t$.

Hence, the worker's problem regarding search and consumption, and thus savings, is then to solve the following problem,

$$\begin{aligned} \max_{\{s_t, c_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} (1+\rho)^{-t} v(c_t, s_t) \\ s.t. \quad : \quad & k_{t+1} = (1+r) k_t^* + I_t - c_t, \end{aligned}$$

where $k_t^* = k_t$, k_t', or k_t'' according to whether the worker chooses to participate in the period t lottery or not and what comes out it. The income $I_t \equiv w n_t + (1 - n_t) b$ where $n_t \in \{0, 1\}$; $n_t = 0$ implying that the worker is unemployed and $n_t = 1$ that the worker is employed. The state variable n_t follows a Markov process with transition function $P(n_{t-1}, n_t)$; $P(0,1) = s_t$, $P(0,0) = 1 - s_t$, $P(1,0) = \eta$, and $P(1,1) = 1 - \eta$.

In order to fully specify the details and orders of moves, the model is expressed in terms of Bellman equations. Let $V_g(k)$ and $U_g(k)$ be the value functions for the gambling stage of

[5] The issue of possible problems with convexities of value functions or concavities of cost functions in these types of models is noted in papers such as Gomes et al. (2001), Hopenhayn and Nicolini (1997) and Phelan and Townsend (1991).

period t, given wealth k_t for $I_{t-1} = w$ and $I_{t-1} = b$, respectively. Thus,

$$\begin{aligned} V_g(k_t) &= \max_{k'_t, k''_t} \left[\frac{k''_t - k_t}{k''_t - k'_t} V(k'_t) + \frac{k_t - k'_t}{k''_t - k'_t} V(k''_t) \right] \\ U_g(k_t) &= \max_{k'_t, k''_t} \left[\frac{k''_t - k_t}{k''_t - k'_t} U(k'_t) + \frac{k_t - k'_t}{k''_t - k'_t} U(k''_t) \right], \end{aligned}$$

where V and U are the value functions at the search and separation stage given the employment status of the previous period:

$$\begin{aligned} V(k_t) &= (1-\eta) V_c(k_t) + \eta U_c(k_t) \\ U(k_t) &= \max_{s \in [0,1]} \left[-e(s) + s V_c(k_t) + (1-s) U_c(k_t) \right], \end{aligned}$$

where, finally, V_c and U_c are the value functions at the consumption and savings stage, which depend on the employment status of the current period, period t:

$$\begin{aligned} V_c(k_t) &= \max_{k_{t+1} \in \left[\underline{k}, \min\left(k_t(1+r)+w, \bar{k}\right)\right]} \left[u\left((1+r) k_t + w - k_{t+1}\right) + \frac{V_g(k_{t+1})}{1+\rho} \right] \\ U_c(k_t) &= \max_{k_{t+1} \in \left[\underline{k}, \min\left(k_t(1+r)+b, \bar{k}\right)\right]} \left[u\left((1+r) k_t + b - k_{t+1}\right) + \frac{U_g(k_{t+1})}{1+\rho} \right]. \end{aligned}$$

These six value functions represent the model and form the basis for the results we arrive at below.

3 Comparative Statics

In this section we charecterise the comparative statics of the model and given the way we have defined the value functions, we need only keep track of two periods at a time, and we suppress the index for the current period. For instance, k and k_{+1} will be the workers wealth in two successive periods, the current and the next period.

To proceed, a little more notation is needed. Let $c(k;e)$ and $c(k;u)$ be the optimal choices of consumption given wealth k under employment and unemployment, respectively; let $s(k)$ be the optimal choice of search given wealth k; and let the optimal choices of next period's wealth be defined by $k_{+1}(k;e) \equiv k+w-c(k;e)$ and $k_{+1}(k;u) \equiv k+b-c(k;u)$ again according to whether the worker is employed or unemployed in this period. It is assumed that the only constraint that may be binding is the lower bound on wealth. The first order conditions associated with the optimal choices of consumption and search effort give us the following characterizations:

$$u'(c(k;e)) = \frac{V_g'(k_{+1}(k;e))}{1+\rho} + \lambda_V(k) \tag{1}$$

$$u'(c(k;u)) = \frac{U_g'(k_{+1}(k;u))}{1+\rho} + \lambda_U(k) \tag{2}$$

$$e'(s(k)) = V_c(k) - U_c(k), \tag{3}$$

where λ_V and λ_U are the Lagrange multipliers associated with the lower bound on wealth. By the envelope theorem it follows that:

$$V_g'(k) = \frac{V(k'') - V(k')}{k'' - k'} \text{ and } U_g'(k) = \frac{U(k'') - U(k')}{k'' - k'}, \ \forall\ k'' > k'. \tag{4}$$

If the worker chooses not to participate in the lottery, that is $k = k' = k''$, then (4) becomes:

$$V_g'(k) = V'(k) \ \text{ and } U_g'(k) = U'(k), \ \forall\ k'' = k' = k.$$

Furthermore,

$$V'(k) = (1-\eta) V_c'(k) + \eta U_c'(k), \ U'(k) = s(k) V_c'(k) + (1 - s(k)) U_c'(k), \tag{5}$$

$$V_c'(k) = u'(c(k;e))(1+r), \text{ and } U_c'(k) = u'(c(k;u))(1+r). \tag{6}$$

The main focus of this paper is to determine how the choice of search intensity is influenced by wealth. To this end, (3) is differentiated with respect to k. This yields:

$$s'(k) = \frac{\partial s(k)}{\partial k} = \frac{V_c'(k) - U_c'(k)}{e''(s(k))} \gtreqless 0 \text{ for } V_c'(k) - U_c'(k) \gtreqless 0. \tag{7}$$

From (6) and (7) it follows that $s'(k) \leq 0$ globally, if $c(k;e) \geq c(k;u)$ for all k, which is to say that search intensity increases when k falls. Furthermore, if k is decreasing over the unemployment spell, $k_{+1}(k;u) \leq k$, this will yield positive duration dependence of the worker's search intensity. In other words, the worker will search harder the longer is the period of unemployment. Notice that the search intensity of a risk neutral worker is constant over the duration of an unemployment spell.

4 Wealth Effects, Insurance and Duration Dependence

In this section we characterize how wealth affects search behavior and consumption, and derive the implied duration dependence. We shall see that search effort is inversely related to wealth and that workers dissave when they are unemployed. Also, that the consumption path is not going to be completely smooth over time. To establish these results we need to add a technical assumption to our description of the model above, namely that the period length is sufficiently small so that $s(k)$ and η (the transition probabilities between states) will be small relative to unity; specifically we need that $s(k) + \eta \leq 1$. Furthermore, we will be needing three lemmas. Lemma 1 establishes concavity of all value functions except $U(k)$, and Lemma 2 associates first derivatives across value functions. Both these lemmas are proven in the appendix.

Lemma 1 *The value functions $V_g(k)$ and $U_g(k)$ are concave, and $V(k)$, $U_c(k)$, and $V_c(k)$ are strictly concave.*

Lemma 2 $V_g'(k) = V'(k)$ *and* $U_g'(k) = U'(k'(k)) = U'(k''(k))$ *for all* k, *where* $(k'(k), k''(k))$ *is an interior solution to the optimal lottery for a worker who was unemployed in the previous period.*

The key to all of our results and the main difficulty with providing characterizations when individuals alternate between employment and unemployment is to establish that $V_c'(k) - U_c'(k) < 0$ for all k, which we do in the next lemma.

Lemma 3 $V_c'(k) - U_c'(k) < 0$ *for all* k.

Proof. First, consider the case where $k_{+1}(k; e) \leq k_{+1}(k; u)$. This implies that $c(k; e) > c_u(k; u)$. By (6) it then immediately follows that $V_c'(k) - U_c'(k) < 0$.

Now, consider the case where $k_{+1}(k; e) > k_{+1}(k; u)$. By concavity of $V(k)$ it follows that $V_g(k) = V(k)$. Thus it must be that:

$$\begin{aligned} V_c(k) &= \max_{k_{+1} \in \Gamma_w(k)} \left[u((1+r)k + w - k_{+1}) + \frac{V(k_{+1})}{1+\rho} \right] \\ &= \max_{k_{+1} \in \Gamma_w(k)} \left[u((1+r)k + w - k_{+1}) + \frac{(1-\eta)V_c(k_{+1}) + \eta U_c(k_{+1})}{1+\rho} \right], \end{aligned} \tag{8}$$

where $\Gamma_i(k) = \left\{\tilde{k} \in \mathbb{R} \mid \underline{k} \leq \tilde{k} \leq \min\left[(1+r)k + i, \bar{k}\right]\right\}$, $i \in \{w, b\}$. Similarly, $U_c(k)$ can be written in terms of V_c and U_c. But in this case, one cannot disregard the lottery, so the expression is somewhat more complicated:

$$\begin{aligned} U_c(k) &= \max_{k_{+1} \in \Gamma_b(k)} \Big\{ u((1+r)k + b - k_{+1}) + \\ & \frac{1}{1+\rho} \max_{k',k''} \left[\frac{k'' - k_{+1}}{k'' - k'} \max_{s \in [0,1]} \left[-e(s) + sV_c(k') + (1-s)U_c(k')\right] + \right. \\ & \left. \left. \frac{k_{+1} - k'}{k'' - k'} \max_{s \in [0,1]} \left[-e(s) + sV_c(k'') + (1-s)U_c(k'')\right] \right] \right\}. \end{aligned} \tag{9}$$

Let S be the set of all bounded, continuous functions. Then, (8) and (9) together define the mapping $T : S \times S \rightarrow S \times S$ or written explicitly, $(V_c, U_c)(k) = T(V_c, U_c)(k)$. Denote by $T_V(V_c, U_c)(k) = V_c(k)$ the first dimension of this mapping and similarly, $T_U(V_c, U_c)(k) = U_c(k)$ the second dimension. It is readily seen that T maps $S \times S$ into itself since the right hand sides of (8) and (9) are maximizations of bounded, continous functions over compact sets. Thus, the solution to these maximization problems must exist and be continous and bounded. The mapping T is furthermore easily verified as being a contraction mapping (see the Appendix). This immediately implies existence of a unique fix point (V_c^*, U_c^*). Also, the contraction mapping property of T implies that for some closed set $S' \subseteq S$, if $T(S') \subseteq S'' \subseteq S'$, then $(V_c^*, U_c^*) \in S''$.

In the following, it will be shown that T maps the closed set of functions S' defined by:

$$S' = \{(V_c, U_c) \in S \times S \mid V_c'(k) - U_c'(k) \leq 0 \; \forall k\},$$

into the set S'' defined by:

$$S'' = \{(V_c, U_c) \in S \times S \mid V_c'(k) - U_c'(k) < 0 \; \forall k\}.$$

Thus, by the argument above it must be that the fix point of the mapping is characterized by $V_c' - U_c' < 0$. Define $k_u' \equiv k'(k_{+1}(k;u)) \leq k_{+1}(k;u) < k_{+1}(k;e)$. The derivatives of the mapping are:

$$
\begin{aligned}
T_V'(V_c, U_c)(k) &= u'(c(k;e))(1+r) \\
&= \frac{1+r}{1+\rho}\left[(1-\eta) V_c'(k_{+1}(k;e)) + \eta U_c'(k_{+1}(k;e))\right] + (1+r)\lambda_V(k) \quad (10) \\
T_U'(V_c, U_c)(k) &= u'(c(k;u))(1+r) \\
&= \frac{1+r}{1+\rho} U_g'(k_{+1}(k;u)) + (1+r)\lambda_U(k) \\
&= \frac{1+r}{1+\rho}\left[s(k_u') V_c'(k_u') + (1-s(k_u')) U_c'(k_u')\right] + (1+r)\lambda_U(k). \quad (11)
\end{aligned}
$$

The lower bound on wealth represents a limitation on the agent's ability to insure against the low income state. The Lagrange multipliers state the value of a marginal relaxation of this lower bound. Thus, it must be that for a given k, a relaxation of the lower bound is more valuable when in the low income state (when unemployed) than in the high income state, that is $\lambda_U(k) \geq \lambda_V(k)$. If for a given k and a given state of income i, the lower bound is not binding, then $\lambda_i(k) = 0$, $i \in \{V, U\}$. Now, assume that $V_c'(k) - U_c'(k) \leq 0$ for all k and subtract (11) from (10):

$$
\begin{aligned}
T_V'(V_c, U_c)(k) - T_U'(V_c, U_c)(k) &= T_V'(V_c, U_c)(k) - \\
&\quad \frac{1+r}{1+\rho}\left[s(k_u') V_c'(k_u') + (1-s(k_u')) U_c'(k_u')\right] - \\
&\quad (1+r)\lambda_U(k) \\
&< T_V'(V_c, U_c)(k) - \\
&\quad \frac{1+r}{1+\rho}\left[s(k_u') V_c'(k_{+1}(k;e)) + (1-s(k_u')) U_c'(k_{+1}(k;e))\right] \\
&\quad - (1+r)\lambda_U(k) \\
&= \frac{1+r}{1+\rho}\left[1 - s(k_u') - \eta\right]\left[V_c'(k_{+1}(k;e)) - U_c'(k_{+1}(k;e))\right] \\
&\quad + (1+r)(\lambda_V(k) - \lambda_U(k)) \\
&\leq 0,
\end{aligned}
$$

where the strict inequality follows from strict concavity of $V_c(\cdot)$ and $U_c(\cdot)$ and that $k_u' < k_{+1}(k;e)$. The weak inequality follows from the assumption that the period length is sufficiently small so that $1 - s(k_u') - \eta \geq 0$, the assumption that $V_c'(k) - U_c'(k) \leq 0$ for all k and finally that $\lambda_V(k) \leq \lambda_U(k)$ for all k. Thus, it has been shown that for a sufficiently small period length, $T(S') \subseteq S''$ and therefore the fix point of T must be characterized by $V_c'(k) - U_c'(k) < 0$. ■

With this result, we can then characterize how search effort is affected by wealth, simply by applying (7) and remembering that the effort function $e(s)$ is assumed to be strictly convex:

Proposition 1 *Search effort increases as wealth decreases; $s'(k) < 0$ for all k.*

Thus, the search effort decision has been characterized. Turning to the consumption decisions, it is seen by the following proposition that savings indeed provide insurance against income fluctuations but that the insurance is imperfect:

Proposition 2 *For all k, consumption increases as wealth increases, $c'(k;e) > 0$ and $c'(k;u) > 0$. Also, for any given k consumption when employed is strictly greater than consumption when unemployed, $c(k;e) > c(k;u)$. Furthermore, for all $k > \underline{k}$ it must be that:*
i) if $r \leq \rho$ then $b + rk < c(k;u) < c(k;e)$,
ii) if $r = \rho$ then $b + rk < c(k;u) < c(k;e) < w + rk$.

Proof. Lemma 1 and the envelope conditions (6) immediately yield that consumption must be increasing in wealth both when employed and unemployed. The conclusion that $c(k;u) < c(k;e)$ follows directly from lemma 3 and the envelope conditions (6).

The result that $b + rk < c(k;u)$ follows from the fact that for $r \leq \rho$ it must be that $k_{+1}(k;u) < k$ for all $k > \underline{k}$. This is seen from the following argument: In Lemma 2 it was established that for an interior solution to the lottery, $U_g'(k_{+1}(k;u)) = U'(k''(k_{+1}(k;u)))$. Now let $k_u'' \equiv k''(k_{+1}(k;u))$. By the first order and envelope conditions, it then follows that:

$$U_c'(k) = \frac{1+r}{1+\rho}\left[s(k_u'')V_c'(k_u'') + (1 - s(k_u''))U_c'(k_u'')\right] + (1+r)\lambda_U(k). \tag{12}$$

By the period length being sufficiently small, it is given in Lemma 3 that $V_c'(k) - U_c'(k) < 0$ for all k. Re-writing (12), it follows that

$$U_c'(k) - \frac{1+r}{1+\rho}U_c'(k_u'') - (1+r)\lambda_U(k) = \frac{1+r}{1+\rho}s(k_u'')\left[V_c'(k_u'') - U_c'(k_u'')\right]. \tag{13}$$

Thus, it must be that the right hand side of (13) is strictly negative. Now suppose, contrary to the claim, that $k_{+1}(k;u) \geq k$. This first of all implies that $\lambda_U(k) = 0$ since the lower bound must not have been binding. Furthermore, it implies that $k''_u \geq k$. But since $U'_c(\cdot)$ is strictly concave and $r \leq \rho$ this must mean that the left hand side of (13) is positive, yielding a contradiction of the inequality. This establishes that $b + rk < c(k; u) < c(k; e)$ when $r \leq \rho$ and $k > \underline{k}$. If $k = \underline{k}$, an unemployed worker is no longer able to insure against the low income state and will simply consume the income $b + rk$.

The result that $c(k; e) < w + rk$ when $r = \rho$ follows from the fact that for $r \geq \rho$ it must be that $k_{+1}(k; e) > k$ for all $k > \underline{k}$, which can be shown by an argument similar to the one just given (for the fact that for $r \leq \rho$ it must be that $k_{+1}(k; u) < k$ for all $k > \underline{k}$). Thus, combining these two results for the case of $r = \rho$, we get that $b + rk < c(k; u) < c(k; e) < w + rk$. ■

Hence, unemployed workers always dissave as long as their wealth is above the minimum level, $\underline{k}$, while employed workers save when $r = \rho$ but might dissave if $r < \rho$. In the case where $r = \rho$, the only savings motive in play is the precautionary one. There is no speculative motive since the return on savings is exactly offset by the rate of time preference. Thus, this case allows us to study savings as an insurance mechanism in isolation. For a given k, the income in the unemployed state is $b + rk$ and similarly when employed, the income is $w + rk$. Hence, proposition 2 shows that savings do allow for some insurance against income fluctuations but that this insurance is imperfect since the consumption path is not perfectly smooth over states. This is not surprising since any transition between income states is never fully anticipated. Allowing r to differ from ρ implies the introduction of a distinct cost to the use of savings as insurance against income fluctuations. For instance, if $r < \rho$ there is a negative real return on savings. In order to insure against income fluctuations, the worker is supposed to build up savings in times of employment. But due to the negative real return on wealth there now is a capital loss associated with this strategy. Thus, one will find that in the case of $r < \rho$, wealthy workers may in fact dissave during times of employment because the speculative motive for saving is dominating the need for additional insurance at these high wealth levels.

All in all, the imperfectness of the use of savings as insurance against unemployment suggests that there may be ample room for welfare improving government unemployment insurance schemes. An obvious path of future research is to characterize the optimal unemployment benefit profiles for this case. Hopenhayn and Nicolini (1997) and Shavell and

Weiss (1979) show that in the case where workers are not allowed to save, the optimal benefit profiles are generally downward sloping. However, once unemployment insurance is complementing the workers' own savings, this conclusion is no longer obvious. In fact, Wang and Williamson (1999) suggest, via numerical examples, that in the case where savings are included in the problem, the optimal benefit profiles may have upward sloping parts.

As a direct extension of the arguments made in the proof of proposition 2, it can be shown that for the case where $r \leq \rho$, the search intensity of an unemployed worker will exhibit positive duration dependence throughout the unemployment spell. This result follows from the basic relationship between the choice of search intensity and the worker's wealth as established in proposition 1.

Proposition 3 *The worker's search intensity exhibits positive duration dependence if* $r \leq \rho$.

Proof. The result follows directly from two observations. 1) $k_{+1}(k; u) < k$ whenever $r \leq \rho$, that is, an unemployed worker will monotonically decrease wealth. This was shown in the proof of Proposition 2. 2) In Proposition 1, it was established that the search intensity is decreasing in the worker's wealth. Thus, it must be that the search intensity increases as the unemployment spell progresses to the point where the worker's wealth reaches the lower bound. After this point, the search intensity remains constant. ■

For a sufficiently small interest rate relative to workers time preferences, we have shown that the worker will monotonically decrease wealth and as such, the search intensity will strictly increase during spells of unemployment, up til the point where the lower bound on wealth has been reached. This result is similar to the results that Danforth (1979) derives under the assumption that employment is an absorbing state, which is to say that once the worker finds employment, she will remain employed in that job forever. This turns out to be a very crucial assumption. If we make the same assumption in our model (i.e. $\eta = 0$), all of our results are obtained immediately, even without allowing for lotteries. Also, our results are obtained under a broader range of utility specifications than in Danforth (1979) where it is assumed that utility is in the decreasing absolute risk aversion category. If one is interested in the savings decision from an insurance point of view, the assumption that the worker need not worry about insurance once employment is found is unfortunate. Thus, this paper can be seen to generalize the results for the dependence of the search decision on wealth, to models that are better suited for insurance analysis purposes. Two such examples are the recent analyses of Wang and Williamson (1999) and Acemoglu and Shimer (1999), where optimal unemployment insurance schemes are derived in models where the workers are insured against unemployment via both their own savings as well as unemployment benefits. A similar problem is analyzed in Pissarides (2000) where instead of unemployment insurance, forms of employment protection such as severance pay and advance notice of job termination are the insurance mechanisms to supplement workers' own savings. However, due to the analytical difficulties with this setup, these papers only achieve analytical results by assuming away most of the wealth effects on the search decisions. The present paper should facilitate further analytical work on search models with job separation and where wealth is allowed to affect search behavior.

5 Empirical Analysis of Wealth Effects

The main theoretical results of this paper suggest that empirical unemployment duration analyses should include wealth as an explanatory variable whenever possible. In particular, all else being equal, a worker with more wealth should have a lower unemployment hazard rate due to the fact that her search intensity will be lower. This section will test this hypothesis using Danish micro data from the period 1980-1994.[6]

The data consists of 13,151 unemployment spells ranging from 4 weeks to 52 weeks. The unemployment spells are observed on a weekly basis. Spells of longer duration were censored at 52 weeks. Spells of less that 4 weeks were dropped from the dataset. These brief spells consist to a large extend of vacations. They most likely also represent short jobless periods due to mismatch between the end of one job and the start of another where the new job was found while still working in the old job. Such spells do not represent the kind of search that this model envisions. Including the less-than-4-week spells in the analysis did not change the point estimates significantly but lowered the statistical significance of the estimates. The data only considers workers between the ages of 18 and 66. The unemployment spell data is then merged with administrative databases which provide demographic and financial

[6] The data set was generously made available by The Centre of Labor Market Studies in Århus, Denmark.

Table 1: Summary Statistics

Variable	Minimum	Maximum	Mean	Standard Deviation
Years of education	9.00	18.00	11.96	2.63
1=Female	0.00	1.00	0.47	0.50
Spouse's income	-159,457.80	886,542.30	0.00	136,779.00
# of children	0.00	12.00	1.65	1.89
Years of work experience	0.26	50.84	17.48	8.94
UI compensation rate	0.10	0.90	0.62	0.17
1=Upper management	0.00	1.00	0.14	0.35
1=Lower management	0.00	1.00	0.20	0.40
1=Salaried worker	0.00	1.00	0.29	0.46
1=Skilled worker	0.00	1.00	0.12	0.32
Wage	55.00	872.00	138.40	56.35
1=Owner of real estate	0.00	1.00	0.73	0.45
Net wealth	-1,046,000.00	1,046,000.00	96,510.00	253,081.30

N=13,151

All income and wealth amounts are in 1994 DKK.

information about the workers associated with the unemployment spells. This information is observed on a yearly basis. Thus, changes in a worker's wealth is not observed during an unemployment spell. One can attempt to compensate for this by inferring how wealth could be changing during the spell and thus how the search intensity changes with wealth. This could be achieved by using a structural estimation based on the model set out above. However, rather than doing this, the analysis in the following will be a reduced form analysis. The wealth of a worker is set to be the wealth level at the last day of the year prior to the year of the unemployment spell. While imperfect, this should capture the worker's wealth level at the beginning of the spell. Along with a range of other co-variates, the study will then determine the impact of these co-variates on the hazard rate of the individual worker. The data are summarized in table 1. The residual category related to the job category dummies is unskilled workers. The spouse's income is defined as the difference from the mean spousal income.

The analysis assumes a basic proportional hazard model similar to Meyer (1990). Thus, the individual hazard rate is assumed to take the following form,

$$\lambda_i(t) = \lambda_0(t)\exp(z_i'\beta), \tag{14}$$

where $\lambda_0(t)$ is the baseline hazard common to all individuals. Expression (14) is assumed to be continuous in time. Also, note that the individual co-variates z_i are constant over the unemployment spell. This is dictated by the data. Since workers are observed to leave unemployment only on a weekly basis rather than continously, it is not known at exactly which point in time a worker leaves the unemployment pool. Denote by w_i the week in which worker i leaves the unemployment pool. Meyer (1990) derives the log-likelihood expression for this case, which is given by:

$$L(\gamma,\beta) = \sum_{i=1}^{N}\left[\delta_i \log\left\{1-\exp\left[-\exp\left(\gamma(w_i)+z_i'\beta\right)\right]\right\} - \sum\nolimits_{t=1}^{w_i-1}\exp\left[\gamma(t)+z_i'\beta\right]\right], \tag{15}$$

where $\delta_i = 0$ denotes that spell i is right censored and $\delta_i = 1$ that the spell ending was observed. Furthermore $\gamma(t)$ is defined by:

$$\gamma(t) \equiv \log\left(\int_t^{t+1}\lambda_0(s)\,ds\right),\; t = 1,\ldots,T-1. \tag{16}$$

One can interpret this model as a discrete model, in which case $\gamma(t)$ is the estimated baseline hazard at time t. Alternatively, the model can be interpreted as a continous time hazard model, where the data do not allow for identification of the continous baseline hazard. There will be an infinite number of baseline hazard functions $\lambda_0(\cdot)$ which will satisfy (16). Estimations based on (15) will be referred to as specification 1.

In the model set out above, it is fairly straight forward to establish that $k_{+1}(k;u)$ is a contraction mapping. This implies that there is a unique fix point $k^* = k_{+1}(k^*;u)$ of the mapping. Hence, all initial wealth levels will eventually converge to this fix point (which happens to be the lower bound). Therefore, the search behavior of the agents will eventually converge as an unemployment spell persists.[7] One would consequently expect the effect of the initial wealth level on the hazard rate to diminish as the unemployment spell progresses. However, a worker with more initial wealth will always have more wealth after any number

[7] The argument ignores the possibility that the existence of lotteries might upset this result. However, simulations of the model suggests that lotteries are virtually never an issue.

of weeks of unemployment than a worker starting with less initial wealth. Thus, the effect on the hazard rate should be negative at all duration points. By assuming a proportional hazard, it is essentially assumed that the initial wealth will have the same impact on the hazard rate at all points during the spell.

However, one can try to include the possibility of different effects of initial wealth at different unemployment spell durations. One way would be to assume a hazard of the following form:

$$\lambda_i(t) = \lambda_0(t) \exp(\alpha(t) k_i) \exp(z_i' \beta), \tag{17}$$

where k_i is individual i's wealth at the beginning of the spell and z_i is the set of co-variates which now excludes the wealth component. $\alpha(t)$ is a set of parameters to be estimated that capture the varying effect of the worker's initial wealth over time. Again, this is re-written to adjust to the fact that spell endings are observed only in discrete intervals. Thus, define $\gamma(t)$ as in (16). Assume for convenience that $\alpha(t)$ is already specified discretely in (17). Then the new log-likelihood function can be written as:

$$\begin{aligned} L(\gamma, \alpha, \beta) \;=\; & \sum_{i=1}^{N} \Big[\delta_i \log\{1 - \exp[-\exp(\gamma(w_i) + \alpha(w_i) k_i + z_i'\beta)]\} - \\ & \textstyle\sum_{t=1}^{w_i - 1} \exp[\gamma(t) + a(t) k_i + z_i'\beta] \Big] . \end{aligned} \tag{18}$$

In the estimations results, this will be referred to as specification 2. Rather than estimating an α-parameter for each week, in order to keep the maximum likelihood estimation tractable, the estimation will be performed for quarterly α's only. This reduces the number of extra parameters to 3.

The set of co-variates in the estimation includes the variables described in table 1 with the exception of the wage variable. Instead of the wage variable, a simple wage regression is performed and the wage residual is included in the set of co-variates. The wage regression includes years of education, gender, years of work experience and job category as explanatory variables. Also included is a set of yearly dummies to account for any year-to-year differences in the hazard. Finally, two multiplicative terms are included to account for any gender differences in the spousal income and children effects. The model estimates 77 parameters for specification 1 and 80 parameters for specification 2. The baseline hazard estimate and

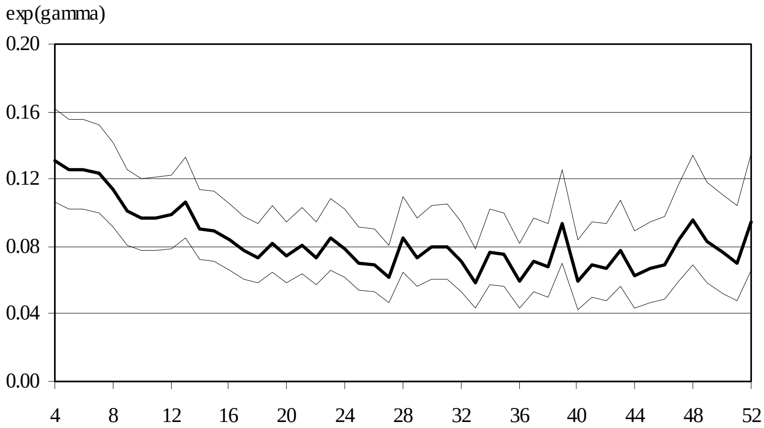
Table 2: α and β estimates

	Specification 1		Specification 2	
	Point Estimate	t-value	Point Estimate	t-value
Years of education	-0.0035	-0.7319	-0.0034	-0.7143
1=Female	-0.0039	-0.1621	-0.0034	-0.0938
Spousal income	1.78e-7	1.3456	1.72e-7	1.1674
(1=Female)*(Spousal income)	-3.73e-7	-2.3363	-3.68e-7	-2.1853
# of children	0.0006	0.8835	0.0006	0.7829
(1=Female)*(# of children)	-0.0004	-0.3782	-0.0004	-0.3324
Years of work experience	-0.0039	-3.1063	-0.0039	-3.0705
1=Upper management	-0.0176	-0.4131	-0.0187	-0.4610
1=Lower management	0.0338	1.0056	0.0337	1.0067
1=Salaried worker	0.0577	2.1849	0.0577	2.1731
1=Skilled worker	0.0069	0.1989	0.0070	0.1940
Wage residual	-0.0005	-1.8382	-0.0004	-1.9034
UI compensation rate	-0.2107	-2.3043	-0.2118	-2.4076
1=Owner of real estate	-0.0140	-0.4975	-0.0141	-0.5198
Net wealth for all weeks	-0.86e-7	-2.1151	-	-
Net wealth for weeks 4-15	-	-	-0.91e-7	-1.8827
Net wealth for weeks 16-27	-	-	-1.82e-7	-2.2056
Net wealth for weeks 28-39	-	-	0.57e-7	0.4938
Net wealth for weeks 40-52	-	-	0.55e-7	0.3097

the yearly dummy estimates for specification 1 are displayed in figures 2 and 3. The two specifications differ only very little for these estimates so only one specification is shown. The remaining estimates are presented in table 2.

Note that the wealth effect is generally negative, which supports the theoretical model. Looking at the overall wealth effect in specification 1, it is seen to be negative and statistically significant. Thus, all else being equal, a worker with a higher level of initial wealth has a lower unemployment hazard rate. Specification 2 breaks down the wealth effect over spell durations. It shows that the effect of more initial wealth is significantly negative for the first two quarters of a spell with the second quarter actually showing a stronger effect than the first quarter. For the last two quarters, there seems to be no effect. Contrary to the model predictions, the actual point estimates for the third and fourth quarters are positive but very insignificantly so. Thus, overall, the results for specification 2 support the

Figure 2: Baseline Hazard Estimate

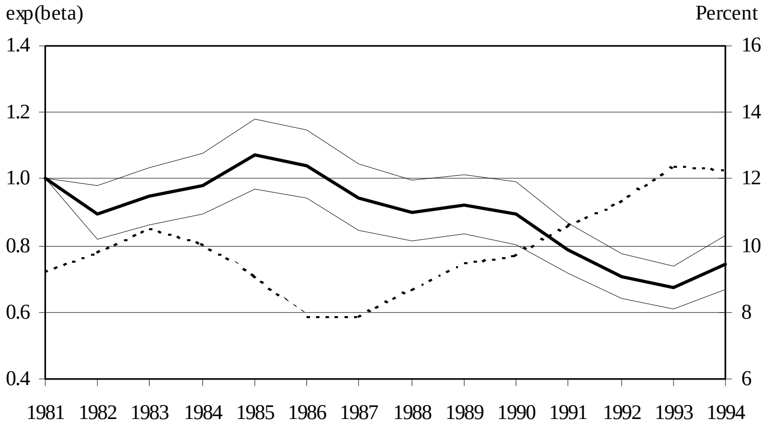


Point estimate drawn in bold with the 95% confidence bounds drawn in thin pen.

idea that wealthier workers dissave at a faster rate than less well off workers and as the unemployment spell persists, workers of different initial wealth levels will begin to exhibit the same search behavior because their wealth levels are converging. The analysis is hardly conclusive evidence of this effect but it is nevertheless suggestive.

Other notable results include the negative effect on the hazard rate from the unemployment insurance compensation rate. This is quite intuitive since the relative difference between unemployment benefits and the future wage level is decreasing in the unemployment insurance compensation rate. Thus, the search intensity should be lower for higher rates of compensations. Also, note that there is a strongly significant difference between men and women with respect to the effect of spousal income: The higher his spouse's income, the more intensely a man will search for a job. For women the effect is directly opposite. A similar effect seems to be true with respect to the number children. However, in this case the estimates do not show strong statistical significance. The signs on years of education and years of work experience are in line with the results in Meyer (1990). Years of work experience is very closely correlated with age in this sample and the result is that the older a worker is, the longer it takes for this worker to find re-employment. In general, the analysis would say that ceteris paribus, unskilled workers experience lower hazard rates than other

Figure 3: Yearly Dummy Estimates and the Danish Unemployment Rate.



The dummy estimates and 95% confidence bounds are drawn in solid lines on the left axis while the unemployment rate is drawn with a dashed line on the right.

job-seeker types except for the upper level managers. However, the only statistically significant effect here is for the salaried workers who have higher hazard rates than any other category. The dummy variable on whether the worker owns real estate or not is included in the analysis in order to pick up on any possible liquidity effects. Essentially, one might be worried that if a sizeable proportion of the worker's wealth is tied up in a house, the worker's wealth may not have the liquidity required to work as a vehicle for consumption smoothing. Of course, if capital markets are sufficiently efficient, it should be possible to borrow against the value of real estate and as such the degree of liquidity of assets should not matter. However, it is not clear that the Danish capital markets were sufficiently efficient during the period studied to remove any such liquidity effects. One would expect the sign on the estimate to be positive if workers could not borrow against the value of their real estate. The actual estimate is negative but not significantly so. Thus, one cannot draw any strong conclusions as to whether liquidity matters or not.

Turning to the baseline hazard estimates in figure 2, the hazard is seen to be decreasing up to the 20th week after which it is more or less constant. Finally, as seen in figure 3, the yearly dummies closely follow the unemployment rate. As the unemployment rate goes up, the labor market gets tighter and one would expect that a given search intensity would yield fewer job offers. As such, the hazard rate should decrease in times of increasing unemployment rates and vice versa. This negative relationship is seen to supported by the empirical analysis.

It is well known that unobserved heterogeneity may seriously affect the results of duration analyses such as that above. Allowing for unobserved heterogeneity and assuming that it takes a multiplicative form, the hazard rate can be written as:

$$\lambda_i(t) = \theta_i \lambda_0(t) \exp(z_i' \beta), \tag{19}$$

where θ_i is the unobserved heterogeneity term. θ_i is assumed to be a random variable and independent of z_i. The resulting log-likelihood expression involves integrating over the distribution of θ_i. By assuming that θ_i is distributed according to the gamma distribution with mean one and variance σ^2, one obtains a nice closed form expression for the log-likelihood expression:[8]

$$\begin{aligned} L(\gamma, \beta, \sigma^2) &= \sum_{i=1}^{N} \log \left\{ \left[1 + \sigma^2 e^{z_i' \beta} \sum_{t=1}^{w_i - 1} e^{\gamma(t)} \right]^{-\sigma^{-2}} - \right. \\ & \left. \delta_i \left[1 + \sigma^2 e^{z_i' \beta} \sum_{t=1}^{w_i} e^{\gamma(t)} \right]^{-\sigma^{-2}} \right\}. \end{aligned} \tag{20}$$

Estimations were performed based on (20) and the unconstrained estimate of σ^2 was zero implying the same estimates as stated in figures 2 and 3 and in table 2.

6 Conclusion

In this paper we study a risk averse worker who moves back and forth between employment and unemployment and thus faces a joint consumption smoothing and job search (leisure smoothing) problem. The main insight is that allowing a worker a savings motive when employed, assuming a capital market, and thereby introducing wealth, affects both search and savings behavior in a fundamental manner. The savings will not smooth consumption perfectly over time, and search effort and thus re-employment prospects are inversely related to wealth, so that, as an unemployment spell stretches out and savings diminishes, the probability that the individual will find a job increases. We conjecture that this result can be

[8] See Meyer (1990) for details on the derivation of this expression.

carried over directly to conclusions about reservation wages (negative duration dependence). Suppose the worker chooses a reservation wage instead of search effort as in our model, then the expected results should show be that a wealthier worker will have a higher reservation wage, which again leads to a lower hazard rate. The analysis in Gomes et al. (2001) suggests that this might well be true.

We also tested some of our main propositions using Danish micro data. First, we find the expected negative association between wealth and the unemployment hazard rate. Second, we find that the effect of the initial wealth on the probability of leaving unemployment is gone after six months of unemployment. This is compatible with the intuitive implications of the model, that workers' wealth levels will converge over unemployment spells and as such their search behavior will converge. This both confirms our theory and encourages further empirical work on search models where risk averse employed workers save because they face a risk of becoming unemployed and where wealth is allowed to affect search behavior.

7 Appendix

Proof of Lemma 1. The value function $V_g(k)$ is the concavification of $V(k)$. Formally, let epi(V) be the epigraph of $V(k)$, i.e., $\text{epi}(V) = \{(k, V) \in [\underline{k}, \bar{k}] \times \mathbb{R} \mid V \leq V(k)\}$, and let conv(epi($V$)) be its convex hull. The concavification of $V(k)$ is then simply the upper bound of the convex hull of V's epigraf, i.e., the $\sup\{V \in \mathbb{R} \mid (k, V) \in \text{conv}(\text{epi}(V))\}$ which is exactly $V_g(k)$. Similarly, $U_g(k)$ is the concavification of $U(k)$. By concavity of V_g and U_g and by strict concavity of $u(c)$ it follows that $V_c(k)$ and $U_c(k)$ are strictly concave. To see this for $V_c(k)$, choose some $k_0, k_1 \in [\underline{k}, \bar{k}]$. Denote $\hat{k}_0 \equiv k_{+1}(k_0; e)$ and $\hat{k}_1 \equiv k_{+1}(k_1; e)$. Furthermore, define $k_\lambda \equiv \lambda k_0 + (1-\lambda) k_1$ and $\hat{k}_\lambda \equiv \lambda \hat{k}_0 + (1-\lambda)\hat{k}_1$ for some $\lambda \in [0, 1]$. Note that $\hat{k}_\lambda$ is not necessarily the optimal choice of next period's wealth given k_λ. Furthermore, it must be that $\hat{k}_\lambda$ is in the feasible set of choices of next period's wealth levels given k_λ. To see this, note that $\hat{k}_0 \leq \min[(1+r) k_0 + w, \bar{k}]$ and $\hat{k}_1 \leq \min[(1+r) k_1 + w, \bar{k}]$. Hence, it must be that $\hat{k}_\lambda \leq \min[(1+r) k_\lambda + w, \bar{k}]$. To show strict concavity, one must show that for all $k_0, k_1 \in [\underline{k}, \bar{k}]$, it must be that $V_c(k_\lambda) > \lambda V_c(k_0) + (1-\lambda) V_c(k_1)$. Thus, we get the following,

$$\begin{aligned}\lambda V_c(k_0) + (1-\lambda) V_c(k_1) &= \lambda \left[u\left((1+r) k_0 + w - \hat{k}_0 \right) + \frac{V_g\left(\hat{k}_0\right)}{1+\rho} \right] + \\ & \quad (1-\lambda) \left[u\left((1+r) k_1 + w - \hat{k}_1 \right) + \frac{V_g\left(\hat{k}_1\right)}{1+\rho} \right] \\ &< u\left((1+r) k_\lambda + w - \hat{k}_\lambda \right) + \frac{V_g\left(\hat{k}_\lambda\right)}{1+\rho} \\ &\leq V_c(k_\lambda),\end{aligned}$$

where the strict inequality follows from strict concavity of $u(\cdot)$ and concavity of $V_g(\cdot)$. The weak inequality follows from optimality. Thus, it must be that $V_c(\cdot)$ is strictly concave. A similar argument applies to $U_c(\cdot)$. $V(\cdot)$ is a convex combination of two strictly concave functions. Consequently, $V(k)$ must be strictly concave.

Note that $U(k)$ need not be concave. By the envelope theorem,

$$U'(k) = s(k) V_c'(k) + (1 - s(k)) U_c'(k).$$

This implies that:

$$U''(k) = s(k) V_c''(k) + (1 - s(k)) U_c''(k) + s'(k) \left[V_c'(k) - U_c'(k) \right].$$

By the first order condition of the optimal search choice, $e'(s(k)) = V(k) - U(k)$, it follows that:

$$U''(k) = s(k) V_c''(k) + (1 - s(k)) U_c''(k) + \frac{\left[V_c'(k) - U_c'(k) \right]^2}{e''(s(k))}. \tag{21}$$

The first two terms on the right hand side of (21) are negative but the last term is positive. Thus, concavity of $U(\cdot)$ does not follow directly. It should be noted that simulations of this model suggest that the last term rarely dominates the two negative terms and as such, $U(\cdot)$ is concave for these simulations. In this case, lotteries are never used. However, as an analytical result one may have to allow for lotteries in the case where $U(\cdot)$ is not concave. ■

Proof of Lemma 2. First we have that $\hat{V}(k) = V(k)$ for all k, since $V(k)$ is concave by Lemma 1, and thus $V_g'(k) = V'(k)$ for all k. Turning to $U_g'(k)$ note that for interior choices of k' and k'' it must be that the optimal solution is characterized by $U'(k') = U'(k'')$. To see this suppose that $U'(k') < U'(k'')$. The original problem is slightly changed to make k'' and α control variables. This is equivalent to the original problem for $k' = (k - (1-\alpha)k'')/\alpha$. Then $U_g(k)$ can be written as

$$U_g(k) = \alpha U\left(\frac{k - (1-\alpha)k''}{\alpha}\right) + (1-\alpha)U(k''). \tag{22}$$

Now consider a slight increase in k_1 holding α fixed:

$$\begin{aligned}\frac{\partial U_g(k)}{\partial k''} &= -\bar{\alpha}^* U'(k')\frac{1-\bar{\alpha}}{\bar{\alpha}} + (1-\bar{\alpha})U'(\bar{k}'') \\ &> 0\end{aligned}$$

Thus, it cannot be that $U'(k') < U'(k'')$ in the optimal solution. A similar argument applies to $U'(k') > U'(k'')$. Thus, for interior choices of k' and k'', the optimal choice of lottery must imply that $U'(k') = U'(k'')$. Now, turn to the formulation of the problem where k' and k'' are the control variables. The first order condition associated with the choice of k' when having been unemployed in the previous period is given by:

$$\begin{aligned}U'(k')\frac{k''-k}{k''-k'} - \frac{1}{k''-k'}U(k'') - \frac{k''-k}{(k''-k')^2}U(k') - \frac{k-k'}{(k''-k')^2}U(k'') &= 0 \\ &\Updownarrow \\ U'(k')(k''-k) - U(k'') - \frac{k''-k}{k''-k'}U(k') - \frac{k-k'}{k''-k'}U(k'') &= 0.\end{aligned}$$

Via the envelope condition (4) this can be re-written as,

$$\begin{aligned}U'(k')(k''-k) - U(k'') &= \frac{k''}{k''-k'}U(k') - \frac{k'}{k''-k'}U(k'') + k\frac{U(k'')-U(k')}{k''-k'} \\ &= \frac{k''}{k''-k'}U(k') - \frac{k'}{k''-k'}U(k'') + kU_g'(k).\end{aligned} \tag{23}$$

The first order condition with respect to the choice of k'' is given by:

$$\begin{aligned}\frac{k-k'}{k''-k'}U'(k'') + \frac{1}{k''-k'}U(k') + \frac{k''-k}{(k''-k')^2}U(k') + \frac{k-k'}{(k''-k')^2}U(k'') &= 0 \\ &\Updownarrow \\ U'(k'')(k-k') + U(k') + \frac{k''-k}{k''-k'}U(k') + \frac{k-k'}{k''-k'}U(k'') &= 0.\end{aligned}$$

Again using the envelope condition (4) this can be expressed as:

$$U'(k'')(k-k') + U(k') + \frac{k''}{k''-k'}U(k') - \frac{k'}{k''-k'}U(k'') + kU_g'(k) = 0. \tag{24}$$

Inserting this into (23), one gets:

$$\begin{aligned} U'(k')(k''-k) - U(k'') - kU_g'(k) &= -U'(k'')(k-k') - U(k') - kU_g'(k) \\ &\Updownarrow \\ U'(k')(k''-k) + U'(k'')(k-k') &= U(k'') - U(k'). \end{aligned}$$

Using the fact that $U'(k') = U'(k'')$ it follows that

$$U'(k') = \frac{U(k'') - U(k')}{k''-k'} = U_g'(k),$$

which then establishes the second part of the lemma. ■

Proof that T is a contraction (Lemma 3). One way of establishing this is by appealing to Blackwell's two sufficient conditions for a contraction mapping; monotonicity and discounting.[9] First, monotonicity is established: Choose some $V_c'(k) \geq V_c''(k)$ and $U_c'(k) \geq U_c''(k)$ for all k. Then Blackwell's sufficient conditions state that it must be that $T(V_c', U_c')(k) \geq T(V_c'', U_c'')(k)$ for all k. By examination of (8) and (9) this is seen to be trivially satisfied. The discounting condition states that it must be that $T(V_c+\lambda, U_c+\lambda)(k) \leq T(V_c, U_c)(k) + \lambda\beta$, for some $0 < \beta < 1$ and some $\lambda \geq 0$. It is seen from (8) that $T_V(V_c+\lambda, U_c+\lambda)(k) = T_V(V_c, U_c)(k) + \lambda/(1+\rho)$. Also, it follows from (9) that $T_U(V_c+\lambda, U_c+\lambda)(k) = T_U(V_c, U_c)(k) + \lambda/(1+\rho)$. Thus, discounting is satisfied for $\rho > 0$, which, by definition, is given. Therefore, it has been established by Blackwell's sufficient conditions that $T(V_c, U_c)$ is a contraction mapping. ■

[9] For a proof of Blackwell's sufficient conditions see for example Stokey and Lucas (1989).

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