

# PENSION SYSTEMS, AGEING AND THE STABILITY AND GROWTH PACT

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# PENSION SYSTEMS, AGEING AND THE STABILITY AND GROWTH PACT

## Abstract

This paper explores how the Stability and Growth Pact may cope with the future costs of population ageing in the European Union. Clearly, population ageing has forced countries to reform their pension systems, and will continue to do so, both by reducing the generosity of pension arrangements and by switching to funding rather than relying on pure pay-as-you go pension provision. We study how such reforms affect the room for adhering to the Pact, but also how the Pact may induce or hamper the incentives for reform. In our analysis we will draw on recent literature on the Pact and on the pensions and the ageing problem. We will also calibrate a simple model for addressing intergenerational equity.

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## 1. Introduction

This paper aims at addressing the link between two major European policy issues at the heart of the macroeconomic debate. One issue concerns the rising ageing costs over the coming decades and the resulting need for pension reform to cope with these higher expenditures. Starting from a situation in which Pay-As-You-Go (PAYG) pension systems dominate the scene, many politically difficult decisions still need to be taken, although a number of countries have put serious effort into the reform of their pension systems. The other issue concerns the adherence to the EU's fiscal rules. They are enshrined in the Treaty and detailed in the Stability and Growth Pact (SGP). So far, compliance with the Pact has been weak: public debt ratios are on average the same as ten years ago, while the 3% ceiling on the deficit ratio has been frequently violated. Even worse, these violations have taken place under circumstances that are much more favourable in terms of demography than those that lie ahead.

Weak compliance with the original Pact led to its reform in 2005 that accentuated the object of long-term sustainability. Yet, a number of key questions should be posed. If it has already been difficult to adhere to the Pact when ageing costs are on average still relatively moderate, how will compliance with the (revised) Pact be affected when the expected dramatic rise in the ageing costs really sets in? Will the Pact stimulate reforms of pension and health care systems in such a way that the costs can be contained? Is the Pact indeed instrumental in producing long-term sustainability of the public finances? What are the consequences of the Pact for intergenerational equity, i.e. does the Pact provide sufficient incentive to frontload ageing costs, with the aim of easing budgetary pressures to be faced by the subsequent generations?

The remainder of this paper is structured as follows. Section 2 discusses projections of the demography and age-related public spending for the EU-25. Then, in Section 3 we discuss the implementation of the Stability and Growth Pact, both before and after its recent reform, including the links between the SGP and pension reforms and the issues of the implicit pension liabilities and intergenerational equity. While important, the latter two issues are not explicitly addressed in the SGP. Those issues take the centre stage in Section 4, where we set up a model to provide numerical illustrations of public debts, deficits and implicit liabilities under different pension and fiscal arrangements, while paying explicit attention to a transition towards (partial) funding that might be required for intergenerational equity. We link the outcomes explicitly to the restrictions imposed

by the SGP. Further implications of this analysis will be discussed in Section 5, while Section 6 concludes the main body of the paper. The Appendix will present the formal model in full while in the main text the references to it are kept minimal.

## **2. Projections of demography and ageing costs**

For a comprehensive view of the future budgetary pressures, we present in Table 1 figures for demographic variables, future public pensions and other age-related public expenditures in 2004 and 2050 based on Economic Policy Committee and European Commission (2006a, b), which we henceforth refer to as “EPC projections”.

The projections provide the baseline figures, which are based on unchanged policies. That is, they are based on enacted legislation (that may still need to enter into force) and on a commonly agreed upon set of macroeconomic assumptions regarding the labour force, productivity growth and real interest rates. Pension projections are produced by the Member countries using their own, national models. Projections for the other expenditure items are computed by the European Commission. The projections abstract from potential general equilibrium effects. For example, they do not take into account the possible consequences of rising taxes and premium payments on the labour supply. While there are substantial uncertainties about the outcomes, in particular due to potential deviations of the demographic outcomes from their projections or private sector behavioural responses to the future developments, the figures should suffice to demonstrate the financial severity of the ageing problem.

The old-age dependency ratios (columns 4 and 5, Table 1) are projected to increase in all countries, though not by the same amount. In particular, the Mediterranean countries (Italy, Spain, and Greece) exhibit strong increases in the share of the elderly, primarily due to low fertility. This is also the reason why the most recent EU entrants catch up so dramatically with the other countries in terms of ageing. Existing fertility rates are even lower for these countries than for the Mediterranean countries. Over the long run, fertility rates are projected to converge to some extent, although differences will remain.

Differences in the development of pension expenditures, health care and long-term care are determined by differences in demographic changes, as well as differences in the pension and health systems themselves (in particular, the generosity of the benefits).

Pension expenditures in 2004 range from 4.7% of GDP in Ireland to 14.2% in Italy. While the average increase in the EU-25 by 2050 is a relatively moderate 2.2%-points of GDP, which is relatively much less than the increase in the old-age-dependency ratio, it hides large differences among the Member States, from the 5.9%-points decrease in Poland to a 12.9%-points increase in Cyprus.

Health-care and long-term care costs are also expected to rise on average roughly at the same relative speed as the pensions. However, the dispersion across countries is smaller. Finally, educational expenses are projected to decline, in line with the falling share of children in the total population.

While very instructive, this data may still miss the full scale of the ageing problem for several reasons. First, judging the pension burden, one should not only consider the outlays on public pensions, but also the burden of the private, funded pensions. This is of particular importance when those private pensions have (largely) a defined-benefit character. If such pension funds are classified outside the public sector accounts,<sup>4</sup> then their burden on future workers is hidden from the figures shown in Table 1, but it is as real as the burden presented by public sector pensions. Second, the projections are only up to 2050. The reason why this limitation is important is that the ageing problem persists also in the decades following the projection period. While the generation of baby-boomers will largely have passed away by 2050, the old-age dependency ratio remains high, because fertility is projected to remain low (well below the replacement rate) and life expectancy can reasonably be expected to remain high and maybe rise even further. This matters for public policies, because under such a scenario the problem caused by ageing persists indefinitely, even if the old age dependency ratio stabilises at some (high) level.

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<sup>4</sup> As is the case, for example, for the ABP, the Dutch teachers and civil servants pension fund.

**Table 1: old-age dependency ratios and age-related expenditures**

Country (1)	Fertility rate (2)		Old-age depend. ratio (3)		Public pension spend. (%GDP)		Health-care sp. (% of GDP)		Long-term care sp. (% of GDP)		Education sp. (% of GDP)	
	2004	2050	2004	2050	2004	2050	2004	2050	2004	2050	2004	2050
BE	1.6	1.7	26.1	47.2	10.4	15.5	6.2	7.6	0.9	1.8	5.6	5.0
DK	1.8	1.8	22.5	41.9	9.5	12.8	6.9	7.8	1.1	2.2	7.8	7.5
DE	1.4	1.5	26.8	51.7	11.4	13.1	6.0	7.2	1.0	2.0	4.0	3.2
EL	1.3	1.5	26.4	60.4	..	..	5.1	6.8	..	..	3.5	3.1
ES	1.3	1.4	24.6	65.6	8.6	15.7	6.1	8.3	0.5	0.8	3.7	3.1
FR	1.9	1.9	25.2	46.4	12.8	14.8	7.7	9.5	..	..	5.0	4.5
IE	2.0	1.8	16.4	45.2	4.7	11.1	5.3	7.3	0.6	1.2	4.1	3.1
IT	1.3	1.4	28.9	62.2	14.2	14.7	5.8	7.1	1.5	2.2	4.3	3.7
LU	1.7	1.8	21.0	36.1	10.0	17.4	5.1	6.3	0.9	1.5	3.3	2.4
NL	1.8	1.8	20.5	40.6	7.7	11.2	6.1	7.4	0.5	1.1	4.8	4.6
AT	1.4	1.5	22.8	52.4	13.4	12.2	5.3	6.8	0.6	1.5	5.1	4.1
PT	1.5	1.6	24.9	58.5	11.1	20.8	6.7	7.2	..	..	5.1	4.8
FI	1.8	1.8	23.3	46.7	10.7	13.7	5.6	7.0	1.7	3.5	6.0	5.3
SE	1.7	1.9	26.4	40.9	10.6	11.2	6.7	7.7	3.8	5.5	7.3	6.4
UK	1.7	1.8	24.3	45.0	6.6	8.6	7.0	8.9	1.0	1.8	4.6	4.0
CY	1.5	1.5	17.5	43.2	6.9	19.8	2.9	4.0	..	..	6.3	4.0
CZ	1.2	1.5	19.7	54.8	8.5	14.0	6.4	8.4	0.3	0.7	3.8	3.1
EE	1.4	1.6	23.8	43.1	6.7	4.2	5.4	6.5	..	..	5.0	3.6
HU	1.3	1.6	22.6	48.3	10.4	16.9	5.5	6.5	..	..	4.5	3.8
LT	1.3	1.6	22.3	44.9	6.7	8.6	3.7	4.6	0.5	0.9	5.0	3.3
LV	1.3	1.6	23.6	44.1	6.8	5.6	5.1	6.2	0.4	0.7	4.9	3.5
MT	1.7	1.6	19.0	40.6	7.4	7.0	4.2	6.1	0.9	1.1	4.4	3.3
PL	1.2	1.6	18.6	51.0	13.9	8.0	4.1	5.5	0.1	0.2	5.0	3.1
SK	1.2	1.6	16.3	50.6	7.2	9.0	4.4	6.3	0.7	1.3	3.7	2.4
SI	1.2	1.5	21.4	55.6	11.0	18.3	6.4	8.0	0.9	2.2	5.3	4.9
EU-25	1.5	1.6	24.5	51.4	10.6	12.8	6.4	7.9	0.9	1.5	4.6	4.0
EU-15	1.5	1.6	25.6	51.6	10.6	12.9	6.4	8.1	0.9	1.5	4.6	4.0
€ area	1.5	1.6	25.9	53.6	11.5	14.1	6.3	7.8	0.7	1.3	4.4	3.8

Notes: (1) AT = Austria, BE = Belgium, CY = Cyprus, CZ = Czech Republic, DK = Denmark, DE = Germany, EE = Estonia, ES = Spain, EL = Greece, FI = Finland, FR = France, HU = Hungary, IE = Ireland, IT = Italy, LV = Latvia, LT = Lithuania, LU = Luxemburg, MT = Malta, NL = Netherlands, PL = Poland, PT = Portugal, SE = Sweden, SI = Slovenia, SK = Slovak Republic, UK = United Kingdom. (2) Fertility is the number of children per woman of child-bearing age. (3) The old-age dependency ratio is the ratio of the number of 65 years and older divided by the number of people in the age group 15-64. (4) “..” = figure not available.

### **3. The Stability and Growth Pact**

While monetary policy in the euro area is delegated to the ECB, fiscal policy remains in the hands of the national authorities. They should, however, according to the Treaty agreed in Maastricht in 1991, comply with the principle of sound public finances. To ensure this the Treaty prohibits central bank financing to governments, their privileged access to other financial institutions and the bail-out of debts of any public entity with the help of the European Community or its Member States. A bail-out of a Member State in severe budgetary trouble by raising inflation to erode the real burden of its debt servicing costs was also excluded by setting price stability as the primary objective of the ECB. This arsenal of protection measures was still complemented by laying out an Excessive Deficit Procedure (EDP) with reference values for deficits (3% of GDP) and debt (60% of GDP). Yet, some countries believed that all this would still not be sufficient guarantee for the ECB to be able to operate independently and achieve price stability. This fear led the German Finance Minister at the time to come up with a proposal for a so-called Stability Pact that would lead to automatic financial punishments for countries with excessive deficits. However, the eventual agreement was coined the "Stability and Growth Pact" to reflect the view of some Member States (in particular, France) that the task of the EU fiscal framework was also to stimulate economic activity. Moreover, the automatic fines were replaced by a complex procedure of increasing severity of punishments that would only in the end lead to the payment of fines (see Eichengreen and Wyplosz, 1998, for a more detailed discussion on the SGP and its rationales).

#### *3.1. The original Pact*

The Pact basically makes the Treaty-based Excessive Deficit Procedure operational. It is part of secondary law in the EU and can be amended without changing the Treaty on the European Union. It is composed of two components. One arm (the Regulation "on the Strengthening of Surveillance of Budgetary Positions and the Surveillance and Coordination of Economic Policies") aims at preventing excessive deficits by requiring countries to strive for a budget that is close to balance or in surplus in the medium run. This way, countries build up a safety margin that prevents the deficit from rising above its reference level of 3% of GDP when the economy is in a downturn. The safety margin thus allows the automatic stabilizers to do their work (see Buti et al., 1998), unless the economy falls into a very severe recession. As part of this Regulation, euro area members have to submit a "Stability Program" every year, in which they set out its budgetary path

for the coming years and the measures that underpin the projected path. EU countries that have not (yet) adopted the euro, submit similar “Convergence Programs”. The programs are assessed by the Commission, which provides recommendations on their content, after which the ECOFIN Council gives its opinion.

The other regulation is that on “Speeding up and Clarifying the Implementation of the Excessive Deficit Procedure”. This is the corrective arm of the Pact setting out the provisions for adhering to the Treaty based objectives that the deficit should stay below 3% of GDP, while the public debt should remain below 60% of GDP, or be decreasing at a sufficient pace. In practice, only the deficit criterion is enforced through specific measures. If an excessive deficit is detected, the Commission and the ECOFIN Council put in motion the Excessive Deficit Procedure (EDP). The EDP foresees measures of increasing severity the longer the country maintains its excessive deficit. The country may even undergo financial penalties, first in the form of a non-interest bearing deposit, which can be turned into a fine, if the country does not correct the deficit before the prescribed deadline.

### *3.2. Evaluation of the Pact and comparison with reform proposals in the literature*

Motivated by the (expected) problems with the implementation of the SGP, a large number of experts have put forward more specific proposals to change the SGP, including the proposal to abolish the Pact. Fischer et al. (2006) classify the various proposals, though they restrict themselves to proposals made before 20 March 2005, the date when the European Council (2005) approved the outline for the revision of the Pact. Proposals are based on a variety of arguments. One major group of proposals concerns the lack of credibility of the original framework. Their proposals mainly aim at institutional and procedural changes that strengthen the incentives for fiscal discipline. A noticeable example is Wyplosz' (2005) proposal for independent fiscal policy councils, so that the ECOFIN members no longer judge each other's policies. Related proposals were also made in von Hagen (2002) and Fatas et al. (2003). A second group emphasizes that fiscal rules should shift focus to economic growth. For example, Fitoussi (2002) argues that fiscal rules should be contingent on the state of the economy. In particular, he argues in favour of replacing the current deficit rule with a debt rule, which would allow more flexibility in response to an economic downturn. Blanchard and Giavazzi (2004) advocate golden rules or capital budgeting to fix the alleged disincentive for public investments under the SGP. A third group of proposals focuses on long-term



sustainability. Also this group is in favour of more prominence of public debt instead of deficits. An example is Pisani-Ferry (2002). He also advocates more emphasis on the medium run monitoring of comprehensive public finance figures. Calmfors and Corsetti (2004) propose an SGP deficit ceiling that is explicitly dependent on the public debt.

Buiter and Grafe (2004) present the “permanent balance rule”, which allows a higher deficit in response to transfer payments, government consumption and gross capital formation that exceed their permanent (that is, roughly speaking, the average expected future) levels. In their view it is of particular importance that the countries that entered the EU after the turn of the century and where infrastructure is in a deplorable state implying a need for temporarily high investment in public capital, be allowed to run a temporarily higher deficit. More importantly, they consider that the increase in future PAYG public pensions causes the permanent level of pension transfers to exceed current pension transfers, which implies the need for reducing debt, i.e. producing smaller public deficits or even surpluses.

A major virtue of the permanent balance rule is that it is aimed at keeping the tax rate constant (as long as no new information arrives) and that it would thereby help to reduce distortionary losses. A major complication, of course, is that the rule would require projections of budgetary variables long into the future. In addition, it is not clear how it should be enforced.

Although the issue of fiscal sustainability is high on the priority list of a number of these proposals, somewhat surprisingly existing proposals have paid very little, if any, attention to the question how the SGP could be better aligned with the need to reform pension systems. The articles by Beetsma and Debrun (2004, 2007) are somewhat of an exception. They argue that a strict implementation of a deficit rule may hamper structural reforms if such reforms entail large upfront costs. Obviously, this is not the case for all types of structural reforms (such as reforms that involve a widening of the tax base at given tax rates). However, this is certainly the case for some types of reforms, such as a shift of the pension system towards more funding. Beetsma and Debrun's (2004, 2007) variable for upfront cost also includes compensation that needs to be provided to get the support from those who lose out on the reforms.

### *3.3. The revised Pact: medium term objectives and the costs of structural reforms*

The revision of the Stability and Growth Pact was agreed at the European Council in March 2005, after the Pact had been put on hold at the end of 2003 as a result of the failure to follow its formal procedure in the case of the excessive deficits of Germany and France.<sup>5</sup> There was a general recognition that the Pact should handle the protracted slowdown of these economies in an adequate way and that it would make sense to have a Pact that would discourage pro-cyclical fiscal behaviour (by inducing countries to strengthen their public finances during good times in order to have more room for manoeuvre during bad times) and pay more attention to long-run fiscal sustainability (see also Buti and Nogueira Martins, 2006).

The definition of the medium term objective (MTO) was made country-specific, taking account of potential growth and of the debt level, subject to possible revision when major structural reforms are implemented and in any case every four years. For countries that have already adopted the euro or participate in ERM-II, the MTO ranges from a minimum of -1% of GDP for low debt or high potential growth countries to budget balance or surplus for high debt/low potential growth countries. The MTOs are defined net of one-off and temporary measures. As long as a country has not yet reached its MTO, it should achieve an annual reduction in its cyclically adjusted deficit, again net of one-off and temporary measures, of at least 0.5% of GDP. The short-run costs of structural reforms are explicitly recognized for defining the adjustment path to the MTO, but in general not directly for the determination of the MTO itself.

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<sup>5</sup> Apart from the fact that in many Member States deficits exceeded 3% there was a particular issue that triggered the crisis in implementing the SGP in 2003. It was the question as to what should happen when the Member States in excessive deficit have implemented the recommendations given to it, but exogenous factors turn out to be more unfavourable than expected and the deficit therefore does not decline. This was especially relevant for Germany in 2003. The Commission took the view that it was legally obliged to recommend moving to the next stage on the road towards sanctions while Germany, supported by France, wanted to go back to the previous recommendations and revise them (Korkman, 2005, p. 117). This dispute caused a deadlock in the Council as the required qualified majority was not found under the correct legal procedure to any decision. In the subsequent ruling the Court of Justice of the European Communities (2004) considered (among other things) that recommendations given at the earlier stage can indeed be modified by the Council, but this cannot happen without a fresh recommendation from the Commission (paragraph 92). After this clarification to the original SGP the possibility of repeating the steps was made explicit in the revised SGP, i.e. in the modifications to the SGP regulation in question.

### *3.4. Pension system reform and the revised SGP*

Traditionally, only the Netherlands, Denmark and the United Kingdom have featured a substantial funded pension pillar, while in most EU countries pensions are largely unfunded.<sup>6</sup> If strictly applied, a pure PAYG pension system means that contributions into the system exactly match the pension payments. Hence, the presence of such a system neither affects government deficit nor public debt. However, the consequence of a pure PAYG system is that over the coming decades fewer workers would have to finance the pension benefits of an increasing number of retirees. Hence, in the absence of a reduction in pension benefits or an increase in the retirement age, PAYG pension premium payments have to increase substantially. This has two major implications. One is a potentially unfair distribution of the aging costs with future workers at the losing end. The other is that high premium payments may distort the labour supply, although the precise effects depend on the intra-temporal trade off between income and substitution effects and on the scope for inter-temporal substitution in labour supply.<sup>7</sup> In addition to the rise in the number of pensions to be paid out for each working person, also other age-related expenditures, in particular health care and long-term care will increase. While the old usually do make contributions to the health care system, they receive a disproportionate share of its benefits.

The projected rise in the future financial burden on workers in the absence of reform may easily induce governments to take economically unsound measures. Governments may start to finance the rising expenditures by running deficits and issuing debt, thereby potentially violating the rules of the SGP. Alternatively, or in combination, they may resort to one-off measures such as the sale of public assets or substituting explicit debt for off-balance liabilities. For example, in the recent past some countries (France and Germany) received a cash-inflow in return for an increase in future pension liabilities (see Coeuré and Pisani-Ferry, 2005). Empirical evidence suggests that such budgetary gimmickry has been widely applied – see Milesi-Ferretti and Morriyama (2004), von

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<sup>6</sup> An overview of European pension systems is found in European Commission (2006a), pages 52-56. A scheme of the pension reforms in the EU is contained in the European Commission (2006b), pages 28-31. See also Whiteford and Whitehouse (2006).

<sup>7</sup> Disposable (after-tax) income falls. In order to (partially) make up for this reduction in consumption possibilities, individuals increase their labour supply. The substitution effect works in the opposite direction. Leisure becomes relatively cheaper in terms of the foregone net wage. This produces a negative effect on the incentive to work. What is the overall effect of these two forces depends on the specific preferences entertained by individuals. Inter-temporal substitution produces a shift in labour supply from

Hagen and Wolff (2004) and Koen and Van den Noord (2005). However, the reformed SGP at least partly addresses this issue, as the main fiscal indicator is the structural balance net of temporary factors and one-off measures.

In anticipation of the rising ageing costs and to more evenly share the costs of old-age pension provision among the generations, countries have started to introduce both systemic and parametric reforms in their pension systems. Italy, Latvia, Poland and Sweden transformed their old Defined Benefit (DB) PAYG systems to Notional Defined Contribution (NDC) systems where the pension contributions earn an administratively determined return, basically equal (or closely related) to the rate of growth of the wage bill and the notional capital is converted at retirement into an annuity. This rule to a great extent takes care of adjusting pension according to the number of employees and change in longevity. The latter three of these countries also set up mandatory funded tiers with personal accounts. Some countries have introduced personal retirement accounts, often supported by tax incentives (for example, Estonia, Ireland, Germany and Lithuania) or made mandatory to new labour market entrants or specific groups (such as Hungary, Slovakia and Slovenia). Also, the role of funded private supplementary occupational pensions is increasing. Countries are also undertaking parametric reforms to ease the pressure on future public resources. Among those measures, most countries are gradually raising the retirement age, increasing the contribution period for a full pension, linking pensions to improvements in life expectancy, limiting early retirement and introducing bonuses for working longer. Also, indexation of pension benefits has in some countries been shifted from wages to prices (France and Austria). Furthermore, in some countries, pre-funding within or linked to public DB systems remains significant (Finland) or has recently been introduced (Belgium and Ireland). Overall, apart from measures to increase the effective retirement age, pension funding is acquiring a more important role in pension systems, although PAYG remains dominant so far.

Privatisation that (normally) replaces part of the public pension system by a private sector managed fully funded tier triggers a transition during which a stock of assets is built up in the newly established funded pillar, while at the same moment the pensions of the current retirees need to be financed.<sup>8</sup> To ease the transition, the government may start

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periods when premium payments are high (and, thus, the net wage is low) to periods when premium payments are low.

<sup>8</sup> This is the counterpart of the windfall gain that the first generation of retirees receives when a PAYG public pension system is installed for which they did not have to contribute during their working life.

issuing debt, making some of the implicit debt explicit. However, the problem here is that the public deficit and debt increase, while the fall in implicit liabilities due to the reduction of future pension payments from the PAYG pillar is not recognised in the national accounts relevant for the SGP assessments. In addition, in most cases the emerging surplus in the new funded pillar will not be part of the government accounts as, according to the decision by Eurostat (2004), funded defined-contribution schemes should be recorded as part of the private sector.<sup>9</sup>

The transition from the original system to a (partially) funded system outside the government accounts may thus lead to a potential conflict with the rules of the SGP. The preventive arm of the revised SGP takes this into account as follows:

"...The Council acknowledges that special attention must be paid to pension reforms introducing a multi-pillar system that includes a mandatory, fully funded pillar. Although these reforms entail a short-term deterioration of public finances during the implementation period, the long-term sustainability of public finances is clearly improved. The Council therefore agrees that Member States implementing such reforms should be allowed to deviate from the adjustment path towards the MTO, or from the MTO itself. The deviation from the MTO should reflect the net cost of the reform to the publicly managed pillar, provided the deviation remains temporary and an appropriate safety margin to the reference value is preserved. "

In the corrective arm the leeway provided by the revision of the SGP is specified in the following way:

"...Consideration to the net cost of the reform will be given for the initial five years after a Member State has introduced a mandatory fully-funded system, or five years after 2004 for Member States that have already introduced such a system. Furthermore, it will also be regressive, i.e. during a period of five years consideration will be given to 100, 80, 60, 40 and 20 percent of the net cost of the reform to the publicly managed pillar." (European Council, 2005; also ECOFIN Council, 2005).

We should note here that the allowed deviations from the (path to) the MTO and the reference deficit level as a share of the cost are falling over time and restricted to five years only, while transitions during pension reforms typically last for decades (for a detailed presentation of these rules, see European Commission, 2007).

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<sup>9</sup> This decision by Eurostat concerned the defined contribution, funded pension systems that may be managed by the government. It considered that the fund's assets are ultimately owned by the participants, who are the ones that bear the risk associated with the return on the assets. Therefore, they should be classified in the private sector. For the defined benefit schemes an important criterion is the degree of funding. The Dutch occupational defined-benefit system is classified in the private sector as it is fully funded while the Finnish partially mandatory defined-benefit system falls within general government as the degree of funding is only about a quarter.

Moreover, the new SGP provisions can alleviate only a shift from PAYG towards partial privatisation. Other reforms, such as reductions in the generosity of future pension benefits and increases in retirement age are not directly facilitated for the simple reason that they do not lead to short-run costs.<sup>10</sup> However, they are indirectly facilitated by the new SGP through the emphasis it puts on sustainability.

### *3.5. Treatment of implicit liabilities and long-term sustainability*

In revising the SGP it was recognised that implicit liabilities, notably those related to population ageing should be taken into account in setting the MTOs. However, the criteria and modalities were left open for further agreement in the Council. Current practice partly follows the lines of Buiters (1985): based on the projection of increasing expenditures the constant tax rate that covers future expenditure and the servicing of the current debt is calculated and this is compared to the tax rate under current policies. The difference between the two is called the '*sustainability gap*'. Given the expenditure projection, it indicates the permanent budgetary adjustment that ensures that the inter-temporal budget constraint is fulfilled over an infinite horizon.<sup>11</sup> The European Commission (2006) presents its implications as the required primary balance over a specified period in the medium term, adding, however, that setting budget adjustment targets along these lines would require a large adjustment in most countries. Therefore, it argues, along the lines expressed by the European Council (2005), that adjusting future expenditure downwards by implementing structural reforms should be part of a policy package for reaching sustainability.

As part of the preventive arm of the SGP Member States are asked to address long-term sustainability in their Stability and Convergence Programmes. According to the ECOFIN

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<sup>10</sup> While there exist a number of papers that deal with the political-economy effects of population ageing on pension arrangements (for example, Conesa and Krueger, 1999, and Gonzalez-Eiras and Niepelt, 2005), there are only a few papers that also make a formal connection with Stability and Growth Pact. A notable example is Tabellini (2003) who considers "a pension reform that gradually but permanently reduces pension outlays in the future, but immediately cuts social security contributions so as to relax political constraints". He notes that a "transition from a pay-as-you-go towards a fully funded private pension system could have this effect", and further notes that such a reform could violate the SGP, no matter how desirable from an economic point of view.

<sup>11</sup> See also Buiters and Grafe (2004). This method is also used by the Board of Trustees of the US Social Security system (Board of Trustees, 2007). Since 1988 they have reported the measure for an immediate adjustment for balance over a 75 years period. However, as this does not properly reflect the (likely) imbalance at the end of the period, they have from 2003 onwards reported this measure also for the infinite horizon. The latter is the same as the *sustainability gap* referred to here.

Council (2005), setting up the guidelines "...the programmes should outline the countries' strategies to ensure the sustainability of public finances, especially in light of the economic and budgetary impact of ageing population." Further, the ECOFIN Council (2005) says: "The programmes should include all the necessary additional information, both of qualitative and quantitative nature, so as to enable the Commission and the Council to assess the sustainability of Member States of public finances based on current policies.... Programmes should clearly distinguish between measures that have been enacted and measures that are envisaged." By reporting long-term projections under unchanged policies, including those determining the taxes, the submitted data demonstrate how the increasing ageing costs show up in rising explicit debt.

While the corrective arm of the Pact primarily focuses on deficits and pays some attention to explicit debt, implicit pension liabilities are now addressed to the extent that the cost of switching towards a funded pension system allows for a temporary deviation from the 3% deficit ceiling, thereby facilitating a reduction in implicit pension liabilities.

The international statistical community made progress in March 2007 in preparing the proposal for revising the treatment of pension entitlements in national accounts (see Advisory Expert Group on National Accounts (AEG), 2007; also European Commission, 2007, Part II, Section 2.3). It was agreed that the pension liabilities of private corporations will be shown on their balance sheet, as is already the case under International Financing Reporting Standards (IFRS) for (listed) companies. Due to the difficulties in defining pension liabilities under the government social insurance schemes, these will not be shown in the core accounts but new supplementary accounts will be established for this purpose. The concept to be used will be the accrued-to-reference date liability.

The pension entitlements of the government's own employees form a special case as neutrality with respect to private sector employers should require recording it as government debt. This could add roughly 50-60% of GDP to the public debt figures in the European welfare states. However, pension entitlements of government employees may be closer to social insurance schemes and it is therefore foreseen that they would be shown in the supplementary accounts only. The more precise criteria where to draw the line will be set up later. It is admitted that there will be difficult border line cases and international comparisons will be hampered as fully harmonised treatment will not be found, simply because pension systems considerably differ from one country to another.

### *3.6. Intergenerational equity*

While the revised SGP puts more emphasis on fiscal sustainability, it is important to realize that there are many potential sustainable fiscal paths with very different distributive and efficiency implications across generations. In fact, while under rather weak conditions on macroeconomic developments perfect adherence to the SGP would guarantee fiscal solvency, such a policy would only by coincidence ensure an optimal intergenerational distribution of the costs of population ageing (however this may be defined). It would also only by coincidence ensure efficiency through the minimization of the present discounted sum of the losses caused by the distortionary taxes and premium payments needed to finance the (ageing-related) public expenditures. Hence, even if the Pact is never violated, the intergenerational distribution of the ageing costs may be very unequal. For example, if governments choose to address the ageing costs by raising revenues only when increased expenditure materializes, then the current workers, especially the older ones, will benefit at the expense of the younger generations, who will have to bear the increasing financial burden. This burden would be shared more equitably if also benefits are reduced, the effective retirement age is raised and/or the increase in pension contributions is frontloaded.

Establishing the intergenerational distribution of the costs and benefits from pension and other age-dependent arrangements is a difficult matter, let alone what might be a “fair” distribution. A number of factors would reasonably need to be taken into account. First, how long is a cohort expected to receive pension benefits and medical treatment? If future cohorts are expected to live longer and they do not face a corresponding increase in their retirement age then it seems reasonable that they also face a higher contribution rate. Also, one might need to take account of differences in fertility. A generation that produces fewer children per adult should pay more for given benefits or receive less generous pensions (Sinn 2000). Account must also be taken of public investments that have been made or are being made by current workers and from which the future workers will also profit. A proper intergenerational contract would require future workers to make similar public investments or else make larger contributions into the pension system.

Given these complex issues, it is not surprising that the Pact does not address the issues of efficiency and intergenerational distribution. These do not directly affect the functioning of the common monetary policy, while distributional issues are a matter of political preferences expressed and implemented at the national level – affecting the



fundamental principles of national social security is even explicitly excluded from the competence of the European Union (EU Treaty article 137). The problem is that young (those below the eligible voting age) and unborn generations are not naturally represented in the political processes. Hence, they have no vote in decisions affecting the future explicit and implicit debt burden, but they are still affected by these decisions. Indeed, given that those who have to shoulder part of burden cannot decide about its distribution, one can easily imagine a political-economy equilibrium in which the decision about a pension reform (for example, a reduction in benefits or a lengthening of working life) is postponed longer than would be socially optimal, thereby leading a country into a major crisis. This should show that incapacity to cope with intergenerational distribution related to the major public expenditure schemes could grossly breach the principle of sound public finances established in the EU Treaty. It has also certainly been addressed under the SGP, and to an increasing extent under its revision, as reduction of public debt has been called for to prepare for the future expenditure pressures. However, it is clear, as also admitted by the European Commission (2006, p. 21-22), that the SGP does not incorporate intergenerational equity explicitly and systematically. This is what we want to tackle below with the help of a simple model and numerical illustrations.

#### **4. The intergenerational distribution of costs under alternative fiscal rules**

Here we address explicitly the consequences, under population ageing, of different fiscal and pension arrangements for public deficits, public debt, implicit liabilities and the balance of contributions and benefits per cohort. The former two variables are of particular importance for assessing to what extent the arrangements comply with the SGP. The balance of contributions and benefits per cohort is the key for assessing the size of potential economic distortions and the intergenerational distribution of the ageing burden. We illustrate the consequences of a fall in fertility and a rise in life expectancy. An important question that we try to address is whether compliance with the SGP can be consistent with a balanced treatment of successive generations that we shall term *actuarial neutrality*.

##### *4.1 Description of the pension model*

The model that underlies the illustration is presented in the Appendix. It is adapted from Oksanen (2005 and 2006). Here, we just describe its main features. For convenience, we use a partial equilibrium “overlapping-generations” model of total wage cost, pension contributions and public pensions. There are three generations: children, workers and retirees. Workers (or employers on their behalf) contribute to the pension system, while the retirees receive a public pension (and do not pay pension contributions or, for simplicity, other taxes). The pensions can be partly (or fully) financed out of current workers' contributions and they can be partly (or fully) financed out of the pension systems assets (which are consolidated with the assets – or debt – of the rest of the government). The extreme case in which current pensions are completely financed out of current workers' contributions is, of course, the pure PAYG system. The other extreme in which they are fully financed out of the system's assets corresponds to a fully funded system. The discounted pension benefits to be received by the current workers are termed the "implicit pension debt" (IPD – this corresponds to 'accrued-to-reference date liability' in a more general setting) of the public sector.

The consolidated public sector (government plus the pension system) owns (net) financial assets  $A_t$  in period  $t$  (public debt amounts to negative assets). Further, all taxes are levied on the wage bill.<sup>12</sup> The consolidated public sector budget constraint is

$$(1) \quad c_t w_t L_t + (\rho_t - 1) A_{t-1} = \pi_{t-1} w_t R_t + A_t - A_{t-1},$$

where  $c_t$  is the tax rate,  $w_t$  is the (gross) wage rate,<sup>13</sup>  $L_t$  is the “effective” labour supply of workers in period  $t$ ,  $\rho_t$  is the financial market interest rate factor (the interest rate is  $\rho_t - 1$ ),  $\pi_{t-1}$  is the pension accrual rate and  $R_t$  is the “effective” number of elderly. Here,  $L_t = l_t \tilde{L}_t$ , where  $\tilde{L}_t$  is the number of new entrants to the labour force in period  $t$ , and  $l_t$  is the number of years spent in work in period  $t$ , divided by the number of years in work in period 0. Further, since periods refer to generations here, and as the number of years during which a pension benefit is received generally differs from the number of years that individuals pay contributions into the pension system, we define  $\sigma_t$  as the number of years spent in retirement in period  $t$  divided by the number of years spent working in

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<sup>12</sup> For convenience, we label all primary revenues of the public sector as “taxes” even though in our theoretical model they consist mostly of pension contributions. The reason is that primary revenues also include the taxes collected (from wages) to service the initial explicit debt of the government.

<sup>13</sup> Here, and in the sequel, “wage” stands for “gross wage”, that is, the wage before pension contributions are paid.

period  $t-1$ , i.e.  $R_t = \sigma_t L_{t-1}$ . Finally,  $\rho_t = (1+g_t)(1+\mu_t)$ , where  $1+g_t = \frac{w_t}{w_{t-1}} \frac{L_t}{L_{t-1}}$  is the wage bill growth factor and  $\mu_t > 0$  is an exogenous mark-up of the financial market interest factor on the wage bill growth factor. Hence,  $\mu_t$  is not influenced by the demographic shocks that we consider below.<sup>14</sup> The accrual rate as a share of the wage net of contributions,  $\pi_{t-1}^n$ , is set by policy. Based on  $\pi_{t-1}^n$ , one then derives the appropriate accrual rate  $\pi_{t-1}$  as a share of the gross wage  $w_t$  (for the details on the calculation, see the Appendix). Dividing by the total wage bill in period  $t$ , we can also rewrite (1) as:

$$(2) \quad c_t = \pi_{t-1} (l_{t-1}/l_t)(\sigma_t/f_{t-1}) + a_t - (1+\mu_t) a_{t-1},$$

where  $f_{t-1}$  is the fertility rate (hence,  $\tilde{L}_t = f_{t-1} \tilde{L}_{t-1}$ ) and  $a_t = A_t/(w_t L_t)$  are (net) assets as a share of the wage bill.

#### *Pure PAYG and constant debt ratio*

The first rule (PAYG), which maintains the financial position of the public sector (i.e. assets as a share of the total wage bill are kept constant at a level  $\bar{a}$ ), implies the following tax rate (see the Appendix):

$$(3) \quad c_t = \pi_{t-1} (l_{t-1}/l_t)(\sigma_t/f_{t-1}) - \mu_t \bar{a}.$$

Since assets are held constant at their initial level, we have  $\bar{a} = a_0$ , where  $a_0$  are the initial assets as a share of GDP. Hence, the total tax rate consists of a component that covers the pension outlays (the first term) and, hence, keeps the asset position of the pension system at zero, and a component that captures the debt-servicing cost, when public debt as a share of GDP (or the wage bill) is kept constant (the second term). In the case of positive public debt ( $a < 0$ ) this says that each generation has to pay the interest mark-up (above the growth of the wage bill) on the public debt. As for the budget balance, with positive wage growth, it is in surplus (deficit) if government net assets are positive (negative) (see the Appendix).<sup>15</sup>

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<sup>14</sup> Note that for setting up the accounting framework, the expression for the interest rate is merely an identity; however, for the main results below we need to assume that  $\mu_t$  is exogenous.

<sup>15</sup> One could also look at many other alternative rules. One is that a given explicit debt/GDP ratio is generated at some future point in time. The main problem with such a rule is that there are many different ways in which the future target debt/GDP ratio can be achieved. Suppose, for simplicity, that the future target ratio for, say, 2050 equals the current debt/GDP ratio and that society faces a problem of rising

*Actuarial neutrality across generations*

Next, we introduce the policy rule such that in each period, given any demographic shock or change in any policy parameter, the tax rate is set at a constant level that is financially sustainable as long as there is no new shock. If and when a new shock arrives, the same principle is applied. The implied tax rate is found as (see the Appendix):

$$(4) \quad c_t^a = \mu_{t+1}^t \left[ \left( \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} \right) \left( \frac{\pi_{t-1} (l_{t-1}^{t-1} / l_t^t) \sigma_t^t}{(1 + \mu_t^t) f_{t-1}} - a_{t-1} \right) \right] + \frac{\pi_t \sigma_{t+1}^t}{(1 + \mu_{t+1}^t) f_t},$$

where a superscript  $t$  indicates that the variable is based on the shock in period  $t$ . For example,  $\mu_t^t$  denotes the interest mark-up in period  $t$  based on all information available in period  $t$ , while  $\mu_{t+1}^t$  denotes the expected interest mark-up in period  $t+1$  based on all information available in period  $t$ . We define  $\theta$  as the implicit pension debt (*IPD*) as a share of the total wage bill. Hence,

$$\theta_{t-1}^t = \frac{IPD_{t-1}^t}{w_{t-1} l_{t-1}^{t-1}} = \frac{\pi_{t-1} w_t R_t^t / \rho_t^t}{w_{t-1} L_{t-1}^{t-1}} = \frac{\pi_{t-1} (l_{t-1}^{t-1} / l_t^t) \sigma_t^t}{(1 + \mu_t^t) f_{t-1}},$$

and

$$\theta_t^t = \frac{IPD_t^t}{w_t L_t^t} = \frac{\pi_t w_{t+1} R_{t+1}^t / \rho_{t+1}^t}{w_t L_t^t} = \frac{\pi_t \sigma_{t+1}^t}{(1 + \mu_{t+1}^t) f_t}.$$

Hence,  $\theta_{t-1}$  is corrected for the possible change in the retirement age in period  $t$  (for a detailed explanation, see the Appendix). Using these expressions, we can reduce (4) to:

$$(4') \quad c_t^a = \mu_{t+1}^t \left[ \left( \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} \right) (\theta_{t-1}^t - a_{t-1}) \right] + \theta_t^t.$$

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ageing costs. One extreme way to reach the target debt ratio is to leave pension benefits unchanged and raise the pension contribution rate (it could jump to a higher level now and then stay constant, or it could gradually rise, etc.). Another extreme path would leave contribution rates at their current levels and reduce the pension replacement rate. Again, the reduction could be once and for all after which no further reduction follows, or the replacement rate could gradually fall (but at the end to a lower level than under an abrupt reduction). All these paths may imply very different distributions of the ageing costs. Under a once and for all increase in the contribution rate, it is the current workers that start to contribute more, leaving current retirees unaffected. With a once and for all reduction in the replacement rate, current retirees share in the burden of rising ageing costs.

The rule here is termed "*actuarial neutrality*" as each generation of workers, first, contributes to share the burden stemming from the past decisions on pensions and other expenditures and revenues by paying the interest mark-up on the sum of the implicit pension liabilities and the explicit public debt, and, secondly, pays the full present value of its own future pensions.

Further, under this rule the sum of explicit public debt ( $-a$ ) and implicit liabilities as a share of the wage bill, labelled the "total debt ratio", evolves as follows:

$$(5) \quad \theta_t^t - a_t = \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} (\theta_{t-1}^t - a_{t-1}).$$

This equation thus implies that the total debt ratio remains constant as long as  $\mu_t^t = \mu_{t+1}^t$ . In the more general case in which  $\mu_t^t$  may differ from  $\mu_{t+1}^t$ , the total debt in the previous period has to be re-valued by the factor  $(1 + \mu_t^t)/(1 + \mu_{t+1}^t)$ . If implicit liabilities increase, i.e.  $\theta_t^t - \theta_{t-1}^t > 0$ , for example due to longer time in retirement, pension contributions should increase, thereby leading to an offsetting reduction in the public debt. This contributes a positive component to the public sector budget surplus.

The expression for  $c_t^a$  given above is a general expression for the tax rate under actuarial neutrality. It is valid for any subsequent changes, permanent or temporary, in the demography, retirement age, generosity of pensions and interest rate margin  $\mu_t^t$ . Under this rule the balance of pension contributions and benefits of each generation is fully separated from the characteristics and pension policy choices of other generations starting from the point in time when the actuarial neutrality rule was first implemented. This result is quite robust as it allows, for example, any changes in  $\mu_t^t$  as long as they do not depend on the other factors in the formula. To assess the plausibility of this assumption, note that under elementary growth theory the interest rate should depend on demographic factors and the pension system rules. In the model here this is the case: for given  $\mu_t^t$  and  $\mu_{t+1}^t$ , the interest rate goes down with the fall in fertility. Combined with the actuarial neutrality rule that affects government saving in the economy, this may be a plausible approximation of the functioning of the economy.

Note furthermore that our result on the dynamics of the total debt ratio (including the considerations related to  $\mu_t^t$ ) also covers fully funded pension systems: if the system was

initially fully funded it remains so under the actuarial neutrality rule. The rule here is applicable under any initial degree of funding. The degree of funding then evolves as a function of factors in the formula for  $c_t^a$ .

#### *4.2 The calibration*

Our numerical example is largely based on the following stylized calibration taken from Oksanen (2005). The unit period corresponds to 30 years, which is roughly the average child bearing age of women in Europe. It is also roughly the average age difference between a retired person (70) and a worker (40). Throughout, we assume that the annual growth rate of the nominal wage per worker is 3.28%, which stems from a unit real wage growth rate of 1.75% and an inflation rate of 1.5% per annum. The interest rate mark-up over the growth of the total wage bill equals 1.5 percentage points per annum.

The economy starts in period 0 in a steady state in which the number of years at work equals 40 and those in retirement are 18 (hence,  $\sigma_t = 0.45$ ). These numbers are thus used to scale pension contributions and pensions to correspond to realistic numbers, although the formal model works with the 30-year unit period. Also, initially, fertility preserves a constant population, that is, the number of children in a period equals the number of workers. Further, the unit pension is initially set at 55% of the wage after pension contributions (i.e.,  $\pi_{t-1}^n = 0.55$ ), so as to make the initial numbers comparable with those for 2004 in the EPC projections.

In period 1, there is both a 20% fall in fertility (roughly corresponding to a fall from 2.1 children per woman, which is needed to preserve the population, to 1.7 children per woman, which is close to the current average fertility in Europe) and an increase in longevity by three years (this corresponds to one year for each 10 year period). In period 2 there is a further increase in longevity by three years. This way we match quite closely the assumptions on the increase in longevity in the EPC projections for the EU average (an increase in life expectancy (at birth) for males of 6.3 years and for females of 5.1 years between 2004 and 2050).

#### *4.3 The numerical results*

We show the time paths of the most relevant variables under pure PAYG and various policies under actuarial neutrality. We consider two possible policy measures to deal with the rising ageing burden: an increase in the retirement age and a reduction of the

replacement rate; the tax rate is then determined by these factors and the type of policy rule implemented. For the case of actuarial neutrality we also consider a (partial) privatisation of the public pension pillar, where the latter is partly replaced by a mandatory, funded private pillar. Most numbers are expressed in percentages of GDP, assuming that the total wage bill (including pension contributions) is 60% of GDP.

Table 2 displays the time paths of the tax rate and pension expenditures, the public debt, the implicit pension debt (IPD), the total debt and the budget surplus (all as shares of GDP) under the PAYG rule. A negative value for the budget surplus/GDP ratio thus indicates a public deficit. Throughout, the debt and the budget surplus are expressed as ratios of annual GDP. The Appendix describes a procedure to transform thirty-year figures for these variables into annual figures. In the baseline case no policy changes are undertaken, while in the next case the retirement age increases “moderately” so that the working life in period 1 rises to 41 years and in period 2 to 42 years (i.e. it rises by one-third of the increase in life expectancy); under the “large” retirement increase the respective numbers are 41.5 and 43 years (i.e. the retirement age increases by half of the increase in life expectancy). Further, we also consider a reduction in the pension accrual rate  $\pi_{t-1}^n$  as a share of the wage net of pension contributions from 55% to 48%. In Table 3 we report the corresponding cases under actuarial neutrality. In Tables 4 and 5 we also illustrate a partial or full privatisation of the public pension system under actuarial neutrality. A new steady state is always achieved in period 3; the period 4 numbers are merely reported to confirm this. Budget balances and public debt levels that violate the Stability and Growth pact are indicated with bold characters.

Panel 1 of Table 2 shows the results for the baseline PAYG scenario. The reduction in fertility and the two-step rise in longevity produce an increase in pension expenditure from an initial level of 11.9% of GDP to a new steady state level of 17.5% of GDP. Taxes, the bulk of which consist of pension contributions, as a share of the total wage cost (“wage” for short) rise from 21.7% in the beginning to 31.1% in the new steady state. This substantial increase in the tax rate is caused by the failure to take additional measures to contain the ageing costs. Given that the generosity of the pension benefits is untouched, the implicit pension debt rises as a share of GDP. The rise is produced both by the rise in the pension benefit as a share of the gross wage (due to the additional years in retirement) and the fall in fertility. Permanently lower fertility means that the relative IPD increases because the growth of the wage bill is permanently reduced, implying that

a given amount of future pension outlays is discounted at the lower rate. Naturally, an increase in the retirement age alleviates the rise in pension expenditures and the tax rate. In period 1 the tax rate falls because the contribution period has increased while the rise in longevity has not yet materialized (because it concerns those working in period 1 and thus the longevity rise materializes in period 2). Panels 2-3 of Table 2 show the results for moderate and large increases in working life, respectively. Panels 4 – 6 correspond to panels 1 – 3, respectively, assuming that the target net replacement rate set in period 1 and onwards is 48% instead of 55%. Obviously, as next period's pension outlays fall, the implicit pension debt is lower under the lower net replacement rate. The budget surplus is practically unaffected in all these cases. Projected pension expenditure in Panel 5 roughly corresponds to the EPC projection for EU-15.

Table 3 illustrates *actuarial neutrality* as an alternative policy rule. The baseline assumes no change in the generosity of the benefits or in the retirement age. Both pension expenditures as a share of GDP and the tax rate rise in line with the fall in fertility and the rise in life expectancy. However, now the system moves to partial funding. This is achieved by increasing the contributions already in period 1. The rise in the implicit debt ratio induces an equivalent reduction in the public debt ratio (or increase in the public asset ratio), which naturally happens only when the deficit ratio is reduced. Actually, the public budget turns into a surplus from period 1 and onwards in panels 1-3. The surplus is largest in period 1 when the demography is hit by two shocks simultaneously (the rise in workers' life expectancy and the fall in the fertility rate). An increase in the retirement age (panels 2 and 3) reduces the pensions (as a share of the gross wage), thereby lowering the implicit pension debt. This implies that fewer public assets need to be accumulated and, hence, the public sector runs a smaller surplus than under the baseline. Also, a reduction in the accrual rate (panel 4) produces a smaller implicit debt/GDP ratio, thereby requiring smaller surpluses than under the baseline. This effect is further strengthened when the reduction in the accrual rate is combined with the increase in the retirement age (panels 5 and 6).

Panel 5 in Table 3 roughly corresponds to the EPC projection for the EU. The expenditure increase is 2.7%-points of GDP (that is a little higher than in the corresponding pure PAYG scenario because now the 48% net accrual rate leads to somewhat higher pensions made possible by the fall in the tax rate owing to the virtual elimination of the 60% of GDP public debt).



All actuarially neutral scenarios spell out clearly that as long as the demographic change is permanent, the outcome should also be a permanent change in government financial position. Therefore, a strategy in which public assets are depleted when the ageing process has reached its peak is excluded, unless pension accrual is drastically reduced.

Table 4 considers the latter option induced by a partial privatisation (possibly in combination with other measures) in which implicit pension debt is swapped for (explicit) public debt. The partial privatisation is implemented with a one-third reduction in the accrual rate expressed as a share of the gross wage rate. Obviously, compared to the baseline under actuarial neutrality, the partial privatisation leads to a reduction in public pension expenditure as a share of GDP and a lower tax rate as a share of the gross wage. Comparing period 0 with the new steady state, the debt/GDP ratio falls only slightly (panel 1), because the original reduction in this ratio is almost entirely offset by the worsening of the debt/GDP ratio caused by the partial privatisation. We observe that the public debt ratio in period 1 exceeds the 60% limit of the SGP, while the deficit ratio in that period remains just marginally below the 3% limit. Obviously, because a rise in the retirement age lowers the implicit debt ratio, the combination of such a rise with a partial privatisation produces higher public debt and deficit ratios than a partial privatisation alone as a larger share of implicit debt becomes explicit. The deficit ratio now violates the SGP in period 1 (see panels 2 and 3) and so does the debt ratio from period 1 onwards. Adding to this a reduction in the accrual rate leads to a further swap of debt, hence to a worsening of the budgetary figures (see panels 4-6).

**Table 2: Public finances and pensions under PAYG**

Period	0	1	2	3	4	change
<b>1. PAYG – baseline: net accrual rate 55%, fixed retirement age</b>						
pension exp/GDP, %	11.9	11.9	15.9	17.5	17.5	5.6
tax rate	21.7	21.7	28.4	31.1	31.1	9.4
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	305.4	336.3	336.3	336.3	107.8
total debt/GDP, %	288.5	365.4	396.3	396.3	396.3	107.8
budget surplus/GDP, %	-1.9	-1.9	-1.5	-1.5	-1.5	0.4
<b>2. PAYG – net accrual rate 55%, moderate increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.7	14.8	15.9	15.9	4.0
tax rate	21.7	21.3	26.5	28.4	28.4	6.7
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	284.0	304.9	304.9	304.9	76.4
total debt/GDP, %	288.5	344.0	364.9	364.9	364.9	76.4
budget surplus/GDP, %	-1.9	-2.0	-1.5	-1.5	-1.5	0.4
<b>3. PAYG – net accrual rate 55%, large increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.6	14.3	15.1	15.1	3.2
tax rate	21.7	21.1	25.6	27.0	27.0	5.3
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	273.7	289.5	289.5	289.5	61.0
total debt/GDP, %	288.5	333.7	349.5	349.5	349.5	61.0
budget surplus/GDP, %	-1.9	-2.0	-1.5	-1.5	-1.5	0.4
<b>4. PAYG – target net replacement rate 48%; fixed retirement age</b>						
pension exp/GDP, %	11.9	11.9	14.4	15.9	15.9	4.0
tax rate	21.7	21.7	25.8	28.3	28.3	6.6
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	275.9	304.8	304.8	304.8	76.3
total debt/GDP, %	288.5	335.9	364.8	364.8	364.8	76.3
budget surplus/GDP, %	-1.9	-1.9	-1.5	-1.5	-1.5	0.4
<b>5. PAYG – target net replacement rate 48%; moderate increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.7	13.3	14.3	14.3	2.4
tax rate	21.7	21.3	24.1	25.8	25.8	4.1
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	255.9	275.4	275.4	275.4	46.9
total debt/GDP, %	288.5	315.9	335.4	335.4	335.4	46.9
budget surplus/GDP, %	-1.9	-2.0	-1.5	-1.5	-1.5	0.4
<b>6. PAYG – target net replacement rate 48%; large increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.6	12.8	13.6	13.6	1.7
tax rate	21.7	21.1	23.3	24.5	24.5	2.8
public debt/GDP, %	60.0	60.0	60.0	60.0	60.0	0.0
IPD/GDP, %	228.5	246.3	261.0	261.0	261.0	32.5
total debt/GDP, %	288.5	306.3	321.0	321.0	321.0	32.5
budget surplus/GDP, %	-1.9	-2.0	-1.5	-1.5	-1.5	0.4

Notes: (1) The tax rate consists mainly of pension contributions and is expressed as % of the total wage cost. (2) The final column “change” gives the percentage point change from period 0 to the new steady state, except for the budget surplus/GDP ratio, where it gives the percentage point change from period 0 to the peak.

**Table 3: Public finances and pensions under actuarial neutrality**

Period	0	1	2	3	4	change
<b>1. Actuarial neutrality – baseline: net accrual rate 55%, fixed retirement age</b>						
pension exp/GDP, %	11.9	11.9	16.3	18.2	18.2	6.3
tax rate	21.7	26.4	28.4	28.4	28.4	6.7
public debt/GDP, %	60.0	-25.1	-60.5	-60.5	-60.5	-120.5
IPD/GDP, %	228.5	313.5	349.0	349.0	349.0	120.5
total debt/GDP, %	288.5	288.5	288.5	288.5	288.5	0.0
budget surplus/GDP, %	-1.9	2.5	2.3	1.5	1.5	4.4
<b>2. Actuarial neutrality – net accrual rate 55%, increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.6	15.0	16.4	16.4	4.5
tax rate	21.7	25.3	26.1	26.1	26.1	4.4
public debt/GDP, %	60.0	-12.9	-38.4	-38.4	-38.4	-98.4
IPD/GDP, %	228.5	295.8	314.3	314.3	314.3	85.8
total debt/GDP, %	288.5	282.9	275.9	275.9	275.9	-12.6
budget surplus/GDP, %	-1.9	1.8	1.5	0.9	0.9	3.7
<b>3. Actuarial neutrality – net accrual rate 55%, increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.5	14.4	15.5	15.5	3.6
tax rate	21.7	24.7	25.0	25.0	25.0	3.3
public debt/GDP, %	60.0	-6.8	-27.2	-27.2	-27.2	-87.2
IPD/GDP, %	228.5	287.1	297.4	297.4	297.4	68.9
total debt/GDP, %	288.5	280.2	270.2	270.2	270.2	-18.3
budget surplus/GDP, %	-1.9	1.5	1.1	0.7	0.7	3.4
<b>4. Actuarial neutrality – accrual rate reduced to 48%, fixed retirement age</b>						
pension exp/GDP, %	11.9	11.9	14.6	16.3	16.3	4.4
tax rate	21.7	24.6	26.4	26.4	26.4	4.7
public debt/GDP, %	60.0	8.1	-24.4	-24.4	-24.4	-84.4
IPD/GDP, %	228.5	280.3	312.9	312.9	312.9	84.4
total debt/GDP, %	288.5	288.5	288.5	288.5	288.5	0.0
budget surplus/GDP, %	-1.9	0.8	1.3	0.6	0.6	3.2
<b>5. Actuarial neutrality – accrual rate reduced to 48% plus increased working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.6	13.4	14.6	14.6	2.7
tax rate	21.7	23.5	24.3	24.3	24.3	2.6
public debt/GDP, %	60.0	18.8	-4.3	-4.3	-4.3	-64.3
IPD/GDP, %	228.5	264.1	280.9	280.9	280.9	52.4
total debt/GDP, %	288.5	282.9	276.6	276.6	276.6	-11.9
budget surplus/GDP, %	-1.9	0.2	0.6	0.1	0.1	2.5
<b>6. Actuarial neutrality – accrual rate reduced to 48% plus increased working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.5	12.9	13.8	13.8	1.9
tax rate	21.7	23.0	23.2	23.2	23.2	1.5
public debt/GDP, %	60.0	24.1	5.9	5.9	5.9	-54.1
IPD/GDP, %	228.5	256.1	265.4	265.4	265.4	36.9
total debt/GDP, %	288.5	280.2	271.3	271.3	271.3	-17.2
budget surplus/GDP, %	-1.9	-0.1	0.2	-0.1	-0.1	2.1

Notes: see Table 2.

**Table 4: Public finances and pensions under actuarial neutrality: one-third privatisation**

Period	0	1	2	3	4	change
<b>1. Actuarial neutrality – 55% accrual rate</b>						
pension exp/GDP, %	11.9	11.9	10.9	12.1	12.1	0.2
tax rate	21.7	20.6	21.9	21.9	21.9	0.2
public debt/GDP, %	60.0	<b>79.4</b>	55.8	55.8	55.8	-4.2
IPD/GDP, %	228.5	209.0	232.7	232.7	232.7	4.2
total debt/GDP, %	288.5	288.5	288.5	288.5	288.5	0.0
Budget surplus/GDP, %	-1.9	-2.9	-0.8	-1.4	-1.4	-1.0
<b>2. Actuarial neutrality – 55% accrual rate + increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.6	10.0	10.9	10.9	-1.0
tax rate	21.7	19.8	20.3	20.3	20.3	-1.4
public debt/GDP, %	60.0	<b>85.7</b>	<b>68.7</b>	<b>68.7</b>	<b>68.7</b>	8.7
IPD/GDP, %	228.5	197.2	209.5	209.5	209.5	-19.0
total debt/GDP, %	288.5	282.9	278.2	278.2	278.2	-10.3
Budget surplus/GDP, %	-1.9	<b>-3.3</b>	-1.4	-1.7	-1.7	-1.4
<b>3. Actuarial neutrality – 55% accrual rate + increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.5	9.6	10.3	10.3	-1.6
tax rate	21.7	19.4	19.6	19.6	19.6	-2.1
public debt/GDP, %	60.0	<b>88.8</b>	<b>75.3</b>	<b>75.3</b>	<b>75.3</b>	15.3
IPD/GDP, %	228.5	191.4	198.2	198.2	198.2	-30.3
total debt/GDP, %	288.5	280.2	273.5	273.5	273.5	-12.0
Budget surplus/GDP, %	-1.9	<b>-3.5</b>	-1.6	-1.8	-1.8	-1.6
<b>4. Actuarial neutrality – accrual rate reduced to 48%</b>						
pension exp/GDP, %	11.9	11.9	9.7	10.9	10.9	-1.0
tax rate	21.7	19.4	20.6	20.6	20.6	-1.1
public debt/GDP, %	60.0	<b>101.6</b>	<b>79.9</b>	<b>79.9</b>	<b>79.9</b>	19.9
IPD/GDP, %	228.5	186.9	208.6	208.6	208.6	-19.9
total debt/GDP, %	288.5	288.5	288.5	288.5	288.5	0.0
Budget surplus/GDP, %	-1.9	<b>-4.0</b>	-1.5	-2.0	-2.0	-2.1
<b>5. Actuarial neutrality – accrual rate reduced to 48% + increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.6	9.0	9.8	9.8	-2.1
tax rate	21.7	18.6	19.1	19.1	19.1	-2.6
public debt/GDP, %	60.0	<b>106.8</b>	<b>91.4</b>	<b>91.4</b>	<b>91.4</b>	31.4
IPD/GDP, %	228.5	176.1	187.3	187.3	187.3	-41.2
total debt/GDP, %	288.5	282.9	278.7	278.7	278.7	-9.8
Budget surplus/GDP, %	-1.9	<b>-4.4</b>	-2.0	-2.2	-2.2	-2.5
<b>6. Actuarial neutrality – accrual rate reduced to 48% + increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.5	8.6	9.2	9.2	-2.7
tax rate	21.7	18.3	18.4	18.4	18.4	-3.3
public debt/GDP, %	60.0	<b>109.5</b>	<b>97.3</b>	<b>97.3</b>	<b>97.3</b>	37.3
IPD/GDP, %	228.5	170.7	176.9	176.9	176.9	-51.6
total debt/GDP, %	288.5	280.2	274.3	274.3	274.3	-14.2
Budget surplus/GDP, %	-1.9	<b>-4.5</b>	-2.2	-2.4	-2.4	-2.6

Notes: (1) The tax rate consists mainly of pension contributions and is expressed as % of the total wage cost. (2) The final column “change” gives the percentage point change from period 0 to the new steady state, except for the budget surplus/GDP ratio, where it gives the percentage point change from period 0 to the lowest level.

**Table 5: Public finances and pensions under actuarial neutrality: full privatisation**

Period	0	1	2	3	4	change
<b>1. Actuarial neutrality – fixed retirement age</b>						
pension exp/GDP, %	11.9	11.9	0.0	0.0	0.0	-11.9
tax rate	21.7	9.0	9.0	9.0	9.0	-12.7
Public debt/GDP, %	60.0	<b>288.5</b>	<b>288.5</b>	<b>288.5</b>	<b>288.5</b>	228.5
IPD/GDP, %	228.5	0	0	0	0	-228.5
total debt/GDP, %	288.5	288.5	288.5	288.5	288.5	0.0
budget surplus/GDP, %	-1.9	<b>-13.6</b>	<b>-7.1</b>	<b>-7.1</b>	<b>-7.1</b>	-11.7
<b>2. Actuarial neutrality – moderate increase in working life (40-41-42)</b>						
pension exp/GDP, %	11.9	11.6	0.0	0.0	0.0	-11.9
tax rate	21.7	8.8	8.8	8.8	8.8	-12.9
Public debt/GDP, %	60.0	<b>282.9</b>	<b>282.9</b>	<b>282.9</b>	<b>282.9</b>	222.9
IPD/GDP, %	228.5	0.0	0.0	0.0	0.0	-228.5
total debt/GDP, %	288.5	282.9	282.9	282.9	282.9	-2.6
budget surplus/GDP, %	-1.9	<b>-13.5</b>	<b>-7.2</b>	<b>-6.9</b>	<b>-6.9</b>	-11.6
<b>3. Actuarial neutrality – large increase in working life (40-41.5-43)</b>						
pension exp/GDP, %	11.9	11.5	0.0	0.0	0.0	-11.9
tax rate	21.7	8.8	8.8	8.8	8.8	-12.9
Public debt/GDP, %	60.0	<b>280.2</b>	<b>280.2</b>	<b>280.2</b>	<b>280.2</b>	220.2
IPD/GDP, %	228.5	0.0	0.0	0.0	0.0	-228.5
total debt/GDP, %	288.5	280.2	280.2	280.2	280.2	-8.3
budget surplus/GDP, %	-1.9	<b>-13.4</b>	<b>-7.2</b>	<b>-6.9</b>	<b>-6.9</b>	-11.5

Notes: see Table 4.

The final exercise we report in Table 5 is a complete privatisation under actuarial neutrality. The accrual rate is set at zero in period 1, i.e. no public pension will be paid from period 2 onwards. As pension commitments for period 1 are honoured, implicit debt becomes fully explicit, resulting in a public debt ratio of almost 290% of GDP, a deficit of over 13% in period 1 and a new steady state deficit ratio of more than 7%. The deficit in the new steady state is so high because the economy is growing and the debt is magnified as the explicit debt now includes the previous implicit debt. Although the transition only implies a shift of pensions from the public to private domain (under the underlying simplifying assumptions), both numbers imply a gross violation of the SGP.

#### *4.4 Other age-related expenditure*

Above, we have focused on pensions and their financing from the point of view of intergenerational distribution, using actuarial neutrality as the benchmark. This approach can be extended to cover other age-related expenditures as well. Here, we discuss the principles and refer to an application to the euro area or EU-15 average numbers taken from the EPC projections.

While a projection for *unemployment benefits* and *education expenses* might be interesting for some purposes one should consider carefully whether these expenditures indeed belong to an analysis of debt and deficit focused on the distribution of the ageing burden between successive generations. Unemployment benefits as a percentage of GDP might be projected to fall in the long run, but should that enter already the determination of the current tax rate? It might well be more appropriate that the (projected) declining burden of those expenditures be reflected in the tax rate only when (if) they occur.

One may also doubt whether a decline in education expenditure should enter the determination of the debt and deficit paths. If it does, the result could be that *current* workers are allowed to pay lower taxes thanks to a *future* decline in education expenditure. This may not be fair, in particular if the (projected) decline is due to a decline in the number of children, which after all would lead to an increase in the pension burden. The alternative is to view education expenditures as something to be primarily paid by the workers as parents as they occur (helped also by the retirees as grandparents with their contribution to taxes). This would imply that also education expenditure should be kept out of our analysis of the implications of aging for debt and deficit. Naturally, these arguments to leave out these two expenditure items are helpful because their projections are highly uncertain.

Next come *health care and long-term care expenditures*. In 2004 these expenditures are respectively 6.4% and 0.9% of GDP in the EU-15, while their projected increase by 2050 amounts to 1.6% and 0.7% of GDP respectively. These numbers serve here as references, although it should be noted that they are highly uncertain and that the conclusions derived from them are admittedly only tentative, requiring a substantial amount of further work (e.g. Chapters 4-5 in Economic Policy Committee and the European Commission, 2006a).

The key questions are who benefits from these expenditures and who provides the financing. A stylized fact is that roughly half of health care expenditure benefits the working age population (including their children), while the other half benefits the elderly, especially those approaching their final year in life. As the main bulk of public expenditure on long-term care is related to the elderly population, we simply assume that they consume all of it.

Then, we have to make an estimate of the effect of a longevity increase on the use of health and long-term care services by the elderly. One extreme assumption is that they proportionally increase with the number of people over, say, 60 years of age. Another extreme assumption is that a longevity increase does raise these expenditures as the bulk of them are concentrated on a constant number of years before death.

Finally, it is important to note here that public health and long-term care expenditure is normally covered from tax revenues paid also by the elderly population. Thus, the financing of these expenditures differs significantly from that of the pension outlays, which are typically covered by pension contributions paid by workers, but not by pensioners.

Fortunately, our framework above is derived from a more general model that is applicable also to these expenditure items. One assumption that we need to make concerns the ratio between the level of taxable income of the elderly to that of the workers. We set this at 60% having primarily in mind the level of pensions in Europe. This is a simple way to take into account that the elderly pay their taxes out of pensions that are lower than the wages of the workers. The Appendix provides the details of the calculations.

Table 6 reports the results for two scenarios. In both, without loss of generality, the system is initially, until period 0, in steady state, while health and long-term care spending for the elderly are financed out of current taxes. Moreover, the initial public debt is set to zero to focus on the issue at hand.

The effects of ageing on expenditures, taxes on wages and income of the elderly, debt and deficit are spelled out. We do not present the results for the financing of these expenditures from current taxes (analogous to pure PAYG pensions), as taxes would simply follow expenditures (note, however, that the net implicit debt figures in Table 6 would also apply regardless of the financing rule). Instead, the results below are based on the same rule as for pensions: after any change the tax rate is set at a level that is financially sustainable as long as there is no new shock, and *mutatis mutandis*, revised when such a shock arrives. Our framework also computes the implicit debt for each period defined as the capital value of these expenditures benefiting the elderly in the following period. We should note that in this case the policy rule does not result in perfect actuarial neutrality as for pensions. This stems from the assumptions that also the

elderly pay taxes and that the same tax rate is applied to both workers' and pensioners' incomes. For example, if those in working age start to consume a higher amount of health care services than the previous generation (either when still young and working or later when old and having retired), they should pay more and the tax rate should be immediately increased. However, the elderly in the same period will also pay higher taxes even if they would not benefit more. There is no way under the assumptions here to go around this interdependency that makes it impossible to implement full actuarial neutrality. It is important to note that the policy rule implemented here means that the additional tax revenues paid by the elderly in such a situation do not only benefit the younger generation in the same period, but they are mostly used for the redemption of public debt and thereby benefit all future generations.

We do not consider the extreme case that the time as an elderly person and therefore as a net user of health and long-term care public services is a constant proportion of adult life, i.e. that a person's health status fully improves along with longevity increase. Note, however, that also in this extreme case our results would be driven by the decline in fertility because from period 1 onwards each generation is smaller than its predecessor. We also leave aside the other extreme case that the health status is assumed to deteriorate at a fixed age regardless of the longevity increase.

The first scenario (panel 1 of Table 6) assumes that the time as a net user of these services increases by two years in both period 1 and 2 while longevity is assumed to increase by three years. This is perhaps also a relatively optimistic assumption with regard to development of individuals' health status. The result is a stepwise frontloading of tax collection leading to an eventual increase in government assets of 23.3% of GDP (or reduction of debt). Government assets rise because the tax rate immediately jumps to a higher level as soon as the projected expenditure increases, while this increase will materialise much later – in fact, in our simple model expenditure first decreases due to the fall in the number of children. In the second scenario the ratio of elderly to net contributors is assumed to increase moderately. In this case public assets increase to 27% in the new steady state. The third scenario additionally assumes for periods 1 and 2 a pro rata 4% increase in expenditures for both the younger and the elderly per the 30-year unit period. The implied expenditure increase of 2.3%-points of GDP roughly corresponds to the EPC projection for the EU average. Our rule for frontloading taxes leads in this case to an eventual reduction of public debt by 35% of GDP and a budget surplus of 1.4% of



GDP in period 1 and 0.9% in the new steady state. Total debt decreases in these scenarios partly because the elderly always also contribute to the payment of expenditures benefiting the younger generations.

**Table 6: General model applied to health and long-term care expenditure**

Period	0	1	2	3	4	change
<b>1. Small increase elderly/net contributors ratio</b>						
expenditure/GDP, %	7.3	6.9	8.1	8.5	8.5	1.2
tax rate	10.0	10.4	10.5	10.5	10.5	0.5
public debt/GDP, %	0.0	-18.2	-23.3	-23.3	-23.3	-23.3
net ID/GDP, %	52.8	68.4	71.1	71.1	71.1	18.3
total debt/GDP, %	52.8	50.2	47.9	47.9	47.9	-4.9
budget surplus/GDP, %	0.0	0.9	0.7	0.6	0.6	0.9
<b>2. Medium increase elderly/net contributors ratio</b>						
expenditure/GDP, %	7.3	7.0	8.4	8.9	8.9	1.6
tax rate	10.0	10.6	10.7	10.7	10.7	0.7
public debt/GDP, %	0.0	-20.1	-26.9	-26.9	-26.9	-26.9
net ID/GDP, %	52.8	70.2	75.4	75.4	75.4	22.6
total debt/GDP, %	52.8	50.1	48.5	48.5	48.5	-4.3
budget surplus/GDP, %	0.0	1.0	0.8	0.7	0.7	1.0
<b>3. Medium increase elderly/net contributors ratio and 4% increase in expenditure per 30 years</b>						
expenditure/GDP, %	7.3	7.3	9.1	9.6	9.6	2.3
tax rate	10.0	11.3	11.5	11.5	11.5	1.5
public debt/GDP, %	0.0	-27.4	-34.8	-34.8	-34.8	-34.8
net ID/GDP, %	52.8	76.4	82.0	82.0	82.0	29.2
total debt/GDP, %	52.8	49.0	47.2	47.2	47.2	-5.6
budget surplus/GDP, %	0.0	1.4	1.0	0.9	0.9	1.4

Notes: (1) The tax rate consists mainly of pension contributions and is expressed as % of the total wage cost. (2) The final column “change” gives the percentage point change from period 0 to the new steady state, except for the budget surplus/GDP ratio, where it gives the percentage point change from period 0 to the peak.

Obviously, the full consequences of ageing for the public budget under the policy rule introduced in this paper can be calculated by summing the budgetary effects associated with the pension outlays and those associated with health and long-term care expenditures reported in Table 6. This way we can compute the (explicit) public debt, the total debt and the budget surplus as a share of GDP under a large number of combinations. Table 7 just gives one example, combining pensions under actuarial neutrality, a moderate retirement age increase and a net accrual rate reduction to 48% (case 5 in Table 3) and the last scenario for health and long-term care expenditure. The total debt ratio declines relatively little, showing that an unchanged ratio represents a

rough approximation of our policy rule. The conventionally measured government deficit moves from the initial 1.9% of GDP deficit to a surplus of 1.6% for 60 years, and net explicit debt declines by 100% of GDP over two generations. These are large numbers that deserve a careful assessment. Before turning to these implications in Section 5 we first look into the sensitivity of our results with regard to the interest rate assumption.

**Table 7: Overall financial implications of ageing**

Period	0	1	2	3	4	change
expenditure/GDP, %	19.2	18.9	22.5	24.3	24.3	5.1
tax rate	31.7	34.8	35.7	35.7	35.7	4.0
public debt/GDP, %	60.0	-8.6	-39.2	-39.2	-39.2	-99.2
net ID/GDP, %	281.3	340.5	362.9	362.9	362.9	81.6
total debt/GDP, %	341.3	331.9	323.8	323.8	323.8	-17.5
budget surplus/GDP, %	-1.9	1.6	1.6	1.0	1.0	3.5

Notes: (1) The tax rate consists mainly of pension contributions and is expressed as % of the total wage cost. (2) The final column “change” gives the percentage point change from period 0 to the new steady state, except for the budget surplus/GDP ratio, where it gives the percentage point change from period 0 to the peak.

#### 4.5 Sensitivity to the interest rate

The above results depend on the assumed, exogenously determined interest rate margin,  $\mu_t$ , of 1.5% per annum. For example, the required budget surplus in period 1 in Table 7 is 1.6%. Still assuming  $\mu_t$  to be exogenous and constant, this number would become 0.3% for  $\mu_t = 3\%$ , and nearly 4% for  $\mu_t = 0$ , which is at the border of the dynamic efficiency of the economy. These numbers show that the budget surplus is rather sensitive to the assumed interest rate margin, which should therefore be carefully considered in all applications. Note, however, that this sensitivity equally concerns the permanent balance rule and *sustainability gap* indicators.

As we argued earlier (in Section 4.1), the assumption that the interest rate falls along with the fall in the number of workers due to the fall in fertility is plausible. It is plausible even for a small open economy as population ageing takes place all over the world. However, without going into too much detail here, we briefly explore the implications of the alternative assumption that the interest rate itself, rather than its margin over the growth rate of the wage bill, is exogenous at a constant level  $\bar{\rho}$ . Solving

our model under this alternative assumption, the expressions for the tax rate and total debt dynamics become respectively (see the Appendix, Section 4):

$$(4'') \quad c_t^a = \left[ \frac{\bar{\rho}}{g_{t+1}^w f_t} - 1 \right] \left[ \left( \frac{g_{t+1}^w l_{t-1} f_t}{g_t^w l_t f_{t-1}} \right) (\theta_{t-1}^t - a_{t-1}) \right] + \theta_t^t,$$

$$(5') \quad \theta_t^t - a_t = \frac{g_t^w (l_t^t / l_{t-1}) f_{t-1}}{g_{t+1}^w (l_{t+1}^t / l_t^t) f_t} (\theta_{t-1}^t - a_{t-1}),$$

where  $g_t^w = w_t / w_{t-1}$  is the growth factor of the unit wage and where implicit debt as a share of the wage bill is now given by:

$$\theta_{t-1}^t = g_t^w \pi_{t-1} \sigma_t / \bar{\rho} \quad \text{and} \quad \theta_t^t = g_{t+1}^w \pi_t \sigma_{t+1} / \bar{\rho}.$$

The factor in front of  $(\theta_{t-1}^t - a_{t-1})$  on the right-hand side of (5') is the ratio of the growth factors of the total wage bill between  $t-1$  and  $t$  and that between  $t$  and  $t+1$ .

Note in particular that fertility in period  $t$  does not affect the *IPD* in period  $t$ , as the given pension expenditure in period  $t+1$  is now discounted at an exogenously determined interest rate. However, our policy rule feeds it into the determination of the tax rate and thereby into the debt dynamics.

Assuming the same value for the constant interest rate as in the initial steady state in the simulations reported above, the numerical results are remarkably similar for the tax rate, public debt and the deficit. The *IPD* and the total debt are affected because, compared with our earlier results, the interest rate margin now increases to a permanently higher level due to the fall in the growth of the wage bill.

## 5. Implications of actuarial neutrality for pension reforms and the SGP

The framework presented and applied above is quite general as it is merely an accounting framework based on the budget identity of the public pension system. It can be used for discussing the implications of changes in demographic factors and policy rules. Combinations of these changes represent alternative public pension reforms with differing implications for public finances at large. Although the underlying assumptions are simple, the results for actuarial neutrality are quite robust. The model focuses on the

demographic and pension system variables while the interest rate margin above the rate of growth of the economy (and of the wage rate) is exogenous, though not necessarily constant as we for simplicity assumed in the numerical illustrations above. Yet, admittedly, the applications of the framework represent a partial analysis because the behavioural responses of private sector agents (in particular, their labour supply decisions) are unambiguously left outside the present paper. In spite of this, it is apparent that the demographic and pension system variables dominate the results even if the model would comprise some endogenous private sector reactions. Hence, a number of interesting results for policy design can be derived.

### *5.1 Implications for designing pension reforms*

The key purpose of the framework is to highlight the implicit pension debt based on the distinction between pension rights accrued to reference date and those to be accrued in future. In the examples the former was taken as given, i.e. accrued rights were assumed to be well defined and respected. This does not correspond to reality as different interested parties naturally tend to give their own interpretation to the rules and their implications, based on both genuine differences in understanding them and tactical considerations. Yet, the framework here invites for an effort to clarify the application of the rules up to now and arrive at, at least, a range of estimates for the accrued rights. If this is not done, there is a danger that the elderly, having already accrued most of their pension rights, fear that they will lose under any not-so-well-defined reform plan. As their number is increasing, this may block reforms even relating to the *future* accumulation of rights.

The framework here can be compared to the generational accounts developed by Kotlikoff and others (e.g. Kotlikoff, 2002) that aim at revealing intergenerational imbalances by projecting public expenditure by generation under prevailing policies and calculating the required net tax payments for the current population and for all future generations. The framework in the present paper can be seen as making this approach operational, notably for public pensions, and developing the actuarial neutrality formula for spelling out the many different combinations of demographic and economic factors and policy rule options that treat successive generations neutrally.

The present framework also helps to solve a puzzle in the literature on the political economy of pensions: it is well understood since Samuelson (1958) that a pure PAYG

public pension system as an implicit intergenerational contract is superior to a fully funded pension system if the rate of increase in the wage bill exceeds the interest rate (ignoring operating costs and other complications). However, if the interest rate is higher, every narrowly self-interested worker would like to leave the system, place her/his pension savings on the market and receive a higher return on his pension contributions. As the latter is true in a dynamically efficient economy, it becomes difficult to explain the permanence of a public unfunded pension system under the simple assumption of self-interested individuals taking a majority vote on a mandatory PAYG pension system (assuming that the workers still outnumber the pensioners). Razin et al. (2002) go around this by assuming that the cash grant is paid to both workers and retirees. This artificial assumption is crucial for their results (for a critical discussion, see *European Journal of Political Economy*, 2007). Galasso (2006), following Browning (1975), assumes that when they vote on pensions people face a situation in which they lose all their accumulated rights if they do not accept to continue the current system, while if they do accept to continue this system, they will benefit from both the previously accumulated rights and those to be acquired in the future. If the question put to vote is formulated this way, the return on future contribution payments obviously exceeds the market return for workers above a certain cut-off age. However, a choice between such extreme alternatives hardly ever arises in real political processes, not only because most people would consider it brutally unfair, but also because it would tend to make reforms increasingly difficult.

The actuarial neutrality rule offers an alternative framework for setting up the political process for pension reform, as it builds upon the distinction of accrued rights and those to be accrued in future: if an agreement is reached on accrued rights, then current pensioners and older workers should be able to lift their concerns and accept even radical reforms of the rules for future rights and their financing. This would allow the choice of the new rules from a wide range of options. The selected alternative would then gradually replace the original arrangement. Thus, pension reforms can be important even if the government does not default on its implicit debt. Surely, in this case current and future generations will in one form or another pay a tax for servicing the implicit debt at the time of the reform.

The presentation of actuarially neutral combinations for the future can proceed in stages: the first stage could be to calculate the pension contribution rate required if the rules for

benefits and the retirement age are maintained. For the typical European welfare state this leads to quite a significant immediate increase in the contribution rate. This should be used to reveal the true price of the prevailing pension rights, and it can be combined with the observation that if the increase in contribution rates is delayed, future generations would have to pay even higher contributions for the same benefits. Having made this point clearly, one could then consider reducing the benefit rights to be acquired from now onwards, including the replacement rate and the age for retirement. Thus, the trade-off between contributions and benefits from now onwards is made clear for each generation, and made a subject for political choice (possibly raising, as has been recently done in some countries, the accrual rates in the later years of a working life to make the retirement decision neutral in present value terms, and allowing individuals free to decide when to retire).<sup>16</sup>

The actuarial neutrality rule developed above is simple because it is based on a model where successive generations follow each other after each 30-year unit period, i.e. all members of a given generation are born at the same instant. The reality is different and we can only interpret the results representing neutrality for an average-aged worker and an average-aged retiree. As the same tax rate is set for all workers in a given year, and as the demographic change is gradual, there is no way to reach perfect actuarial neutrality for every yearly age-cohort. However, our results can be generalised for annual data and gradual demographic change, and the unavoidable deviations from perfect neutrality could be estimated.

Another easy extension of our model and numerical illustrations is to address the consequences of forecasting errors. Recall first that under actuarial neutrality the decisions for each generation are fully separated from those for the subsequent generations. Thus, the required time span is a generation, i.e. on average 30 years, and uncertainty concerning more distant future does not matter. This already helps, as people

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<sup>16</sup> The rule of actuarial neutrality can also be compared with Musgrave's (1986) 'fixed relative position' rule for determining a fair pension formula, referred to in recent discussion, e.g. by Esping-Andersen et al. (2002). It means that pensions are indexed to the wage rate after pension contributions, while a pure PAYG system is strictly preserved. Although the same indexation rule is applied in the illustrations above, the decisive difference is that the degree of funding is made endogenous here. The Musgrave rule deviates from actuarial neutrality and leads under population ageing to an increasing burden for future generations.

are often (rightly) worried about the accuracy of very long term projections.<sup>17</sup> Yet, projections over the next 30 years may turn out having been wrong. For example, suppose that the actual longevity increase for the next period turns out having been underestimated, something that has happened frequently in reality. Under PAYG, the tax rate of the workers in period 2 will go up to cover the larger amount of pension outlays. Explicit debt will remain constant and, hence, the consequences of the mistake are borne in the period in which it materializes. Under actuarial neutrality, the mistake will also result in a change of the tax rate. However, the increase in the tax rate in period 2 is limited, implying that the mistake also shows up in an increase in the total public debt. Thus, the consequences of the mistake are spread out over all working generations as of period 2. This feature is a clear advantage of an actuarial neutrality system: the system implicitly allows for intergenerational risk sharing by spreading the costs of unexpected shocks over all current and future workers.

Furthermore, the results for actuarial neutrality developed above can be used to find for a given contribution rate and the various other factors the accrual rate and thereby the replacement rate in the next period. We do not present this extension here in detail but note the main, relatively straightforward implications. The emerging rule can be compared to Notional Defined Contribution (NDC) systems where basically the rate of growth of the wage bill, among other factors, determines the pensions. This blueprint of the NDC system deviates from actuarial neutrality to the extent that the retirement age of the subsequent generation affects the growth rate of the wage bill that, in turn, affects the replacement rate for the currently retired. Hence, complete neutrality is not achieved. However, we should not see this as a major deficiency of the NDC system, but recognise that this system goes a long way towards actuarial neutrality when compared with maintaining defined benefits (DB) under a pure PAYG system in the presence of an ageing population (for a more detailed analysis of the NDC system, notably of the transition from DB to NDC and the need for an additional adjustment mechanism to complement the basic blueprint, see Oksanen, 2004, pp. 584-586).

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<sup>17</sup> Projections over 2 to 3 generations are useful to illustrate the very long-term implications of alternative assumptions on ageing and pensions. The uncertainty surrounding those projections is controlled by updating them every 5 to 10 years.

## 5.2 Implications for the Stability and Growth Pact

As noted above, the European Commission exploits the age-related public expenditure projections for analysing the long-term sustainability of the public finances, most recently in European Commission (2006 and 2007). Based on these projections, a constant tax rate as a percentage of GDP required to cover these expenditures in terms of present values is calculated. The difference between this tax rate and the cyclically adjusted current tax rate is called the *sustainability gap*. It gives the immediate once and for all adjustment that guarantees sustainability for given expenditure projection.

In the context of the revision of the SGP in 2005 and its implementation it was well understood, and rightly so, that the *sustainability gap* indicator should not be interpreted as directly implying an MTO, i.e. as a recommendation to increase taxes (or the primary balance) to close the gap. Sustainability was given more emphasis, but it was rightly stressed that it would be equally important to take measures to reduce future expenditures so as to narrow the gap from that end. In addition, even for cases in which the expenditure projection would have reflected an acceptable policy line, closing the gap would have implied an MTO that would require a too ambitious policy revision. There was no readiness for this, partly because it was not yet possible to establish and agree on the criteria and modalities to appropriately take into account the implicit liabilities. For these reasons, for the time being, the medium term objectives for budgetary adjustment were set in the range from  $-1\%$  of GDP to balance or surplus, with a more ambitious target for high debt and low potential growth countries and vice versa (ECOFIN Council, 2005). Making the MTO dependent on debt presumably reflects the aim to reduce the disparities in indebtedness because all countries borrow from a common pool of savings, while high debt countries have shown less discipline in the past. Explicit debt is also more objectively measurable than implicit debt. While putting emphasis on explicit debt obviously does not primarily stem from considerations about intergenerational equity, potential growth could in principle be linked to intergenerational aspects, for example when pensions are indexed to prices and, therefore, decline relative to wages if real growth is higher. However, this factor could already be incorporated in the expenditure projection and should not be counted twice.

The newly established MTOs to be implemented in the next few years do quite safely guarantee sustainability of the (explicit) public debt: they imply a reduction in public debt and there is no doubt that rolling over such borrowing would not encounter any



difficulties in the markets. However, there is a big underlying question: can the governments implement those MTOs in future when ageing related expenditures increase, calling for increasing level taxes (or reduction in some other expenditure items). The *sustainability gap* indicators help in emphasising this key question about sustainability by providing a summary statistic that contains the projected increase in expenditures, and potentially providing arguments for more rapid reduction of debt than implied by the current MTOs to ease the financial burden further into future.

Also the ECOFIN Council (2006) noted that reaching the MTOs in the 2005 programmes based on the revised SGP guidelines would be an important step, but not sufficient. In particular, it called “for further structural reforms and/or budgetary consolidation, in line with the three-pronged strategy to ensure sustainability decided by the Stockholm European Council in 2001, i.e. (i) reducing debt at a fast pace; (ii) raising employment rates and productivity; and, (iii) reviewing and, where appropriate, reforming pension, health care and long-term care systems.” It also called for further work and improvements in methodology in the new round of expenditure projections and the Sustainability Report to be made available in 2009. Actuarial neutrality, focusing on intergenerational equity, provides further rationale and precision for the ambitious policy line expressed by the ECOFIN Council conclusions referred to above.

The European Commission (2006, p. 21-22) explicitly states that so far the analysis for the SGP does not assess intergenerational equity. More precisely, it is not possible to do this systematically as the expenditure projections do not provide the data by age cohort. Also, with regard to the *sustainability gap* indicator, the technical assumption is a constant tax rate. Thus, the incidence of expenditure and taxes on successive generations that differ in terms of their demographic characteristics is not systematically taken into account. Therefore, in most cases for the EU countries, it tends to exaggerate the immediate need for budgetary adjustment to some extent (for an early comparison of the sustainability gap indicators and the actuarial neutrality rule, see Oksanen, 2003).

The results from the illustrations of the (partial) privatisation of the public pension system clearly indicate that a conflict with the SGP rules may arise, but how serious it is depends on several factors. In our stylised examples for one third privatisation and 60%

of GDP initial public debt (Table 4), breaching the rules becomes an apparent risk.<sup>18</sup> If at the same time there are reasons for temporarily high public investment, the conflict would become even more intense, while lower initial debt gives more room for adjustment. We should also note that our figures refer to 30-year averages. Hence, the peak in the yearly deficit would be even higher and it would not happen immediately (or over the first five years) after the privatisation but well after (as the immediate loss in contributions to the first pillar is followed by compounded interest on the debt this produces). However, note also that under a policy of frontloading the taxes to finance the increase in health and long-term care expenditures the budget surplus target is larger, thus giving more leeway to the negative effect on the budget balance of pension system privatisation.

One noteworthy assumption that we made above is that the interest rate does not depend on swapping implicit debt (to be serviced from future taxes) for explicit debt (to be issued on the open market and then serviced from future taxes). Especially under a major privatisation this swap presumably has effects that are left outside our simple model. However, we should note that the direct impact of privatisation on aggregate saving is nil. Yet, as demand for and supply of various assets and liabilities change, secondary effects are likely to occur. Our stylised simple examples highlight the direct effects on the key fiscal variables. Even though the results can be refined, the orders of magnitude should not be seriously disputed.

### *5.3 Measurement and treatment of implicit pension debt and the SGP*

Data on implicit pension debt, which has a prominent place in the present paper, will become available in the coming years. As noted above (Section 3.5) the international statistical community is about to finalise its proposals to set up, in the next SNA/ESA revision, supplementary accounts for public pension liabilities measured as the present value of accrued-to-reference date rights. As regards to the assessment of intergenerational equity, this extension to the national accounts will greatly improve the projections of future public expenditures as then the data will show the distinction

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<sup>18</sup> Note here that the reference scenario is not a pure PAYG mono-pillar system, but a mono-pillar that is first reformed to implement actuarial neutrality. The latter would lead to a reduction of the public debt, while the one third privatisation would turn that around to an increase (compare, for example, panels 5 in Tables 3 and 4, respectively).

between future pensions resulting from rights already accrued and from rights to be accrued later.

The estimates in the future supplementary accounts will then be available for improved analysis of the public finances. As shown in the present paper, actuarial neutrality across generations can be made operational as the rule of constant total debt (adequately modified for additional factors) and this can provide a benchmark for policy decisions. This would provide genuine prominence to debt sustainability in fiscal analysis and notably pull intergenerational equity considerations related to public pensions into the analysis.

At this point it is useful to take stock of the long-standing controversy on concepts and practical problems surrounding implicit pension debt. Firstly, it should be recognised that the IPD should not be used as a stand-alone indicator of the future pension burden in all circumstances. This is shown in an early paper by Franco (1995) in an example in which the IPD does not change even though fertility falls and the next, smaller generation will, under a pure PAYG system, have to pay much higher contributions for the same benefits than their parents. His example assumes a fixed interest rate.<sup>19</sup> This argument is correct. It should, however, not discredit sensible use of the IPD for analysis and policy design. Equations (4'') and (5') in Section 4.5 above are directly applicable to the tax rate and debt dynamics also in this case if the straightjacket of pure PAYG is relaxed. Thus, this example shows again that any statistical indicator is merely an indicator, and its use for policy design always requires an analysis that is coherent and fits the policy issue to be tackled.

Secondly, it is also often argued that its sensitivity to a change in the (perceived) interest rate makes the IPD a problematic variable for fiscal surveillance. However, one should realise that, say, a fall in accrued-to-date pension debt due to an increase in (the perception of) the long-term interest rate does not indicate a fall in the pension burden, all other things equal. We showed above that the IPD still has a prominent place in the analysis: the effect of the change in the interest rate on the IPD determined by past decisions is a capital gain or loss that should be taken into account and that appears in our equations for the tax rate and debt dynamics. In statistical terms this is not different

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<sup>19</sup> This assumption is crucial. If the interest rate falls due to the fall in fertility together with the fall in the growth of the wage bill, as assumed in most of the scenarios in the present paper, the IPD immediately increases.

from the treatment of capital gains and losses under current national accounting rules for long-term fixed interest rate debt. Furthermore, private companies are required by law to report changes in their pension liabilities due to changes in interest rate. So, why should this be more difficult for the government (especially as an employer)?

Thirdly, the IPD as defined here can be compared to *open system pension liabilities*, defined as the present values of projected pension expenditure minus revenue up to infinity. The correspondence between the open system liabilities and the *sustainability gap* indicator has been worked out in the most recent policy documents.<sup>20</sup> Thus, both of them are suitable for sustainability analysis, but the projection for *open system liabilities*, just like the *sustainability gap* indicator, is not sufficient for assessing intergenerational equity: it does not contain the data by age cohort and it does not make the distinction between rights accrued-to-reference date and those to be accrued in future (or, if they are in the data used, this information is suppressed by aggregation). Instead, the IPD as defined here, projected for the future, and based on the same assumptions as the projection for *open system liabilities*, contains the same information plus the distinction according to already accrued and to-be-acrued implicit liabilities. Thus, its information content is always larger, and most importantly, it contains the elements that are indispensable for a systematic treatment of intergenerational equity. Again, a coherent framework is needed to make use of this information.

Furthermore, there is a completely separate controversy as to whether an effort should be made to estimate the (accrued-to-date) IPD, and if so, how it should be related to the explicit public debt. First, under most public pension systems in Europe, implicit pension rights are not backed by explicit contracts. Therefore, it is generally difficult to measure them. For example, how should one deal with the indexation of the expected future pension payments to inflation and or wage levels? If specific indexation rules are not anchored in law, but indexation is simply based on the fact that it has been a regular practice for a long time and/or it is based on moral arguments, can one then assume that the future pension payments are indexed as well? Second, it is often feared that official publication of implicit liabilities may give them an explicit character and thereby make it more difficult to reduce pension expenditure. This could be of particular concern for the pensions for civil servants who often have special rights. Indeed, Coeuré and Pisani-

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<sup>20</sup> European Commission, 2006, Chapter I and Annex I, and 2007, Section 2.3; see also Buti and Nogueira Martins, 2006, and Blanchet and Ouvrard, 2006.

Ferry (2005) argue quite fiercely against including implicit liabilities in their measure of the net value of the government sector (which in their view is the right measure to look at when assessing fiscal sustainability, which, in turn, should be a priority of the SGP in their view). They state that "Implicit liabilities, such as pensions, cannot be aggregated to financial liabilities because they belong to a different class of debt. Governments can default on them without producing a financial crisis and, in fact, default is what pension reform frequently amounts to" and they "therefore propose to use separately the present value of age-related net expenditures for choosing the target for the government net value".

From the concerns expressed by Coeuré and Pisani-Ferry (2005) we can derive one undoubtedly useful conclusion: official publication of implicit liabilities should always be accompanied by an explicit statement that the figures are based on current policy and no rights can be extracted from them. Moreover, baseline figures for implicit liabilities could be complemented by projections based on alternative assumptions about indexation, for example, to give the baseline figures a less definitive character. This would reflect the true nature of the IPD estimates and to a great extent take care of the concerns of the misuse of the officially published figures.

In addition, we would like to emphasise that pension reforms do not necessarily mean that the government is defaulting on its obligations. For more complete information it is useful to keep the accrued-to-date IPD, which can be an object of competing interpretations and even default, separate from decisions with regard to future accumulation of rights. This distinction, as already emphasised above, is the contribution of the framework in the present paper and may help in opening wider options for the accumulation of future pension rights and their financing, after having agreed on an interpretation of the accrued rights.

#### *5.4 Does actuarial neutrality lead to overly ambitious MTOs?*

The public finance targets implied by the actuarial neutrality rule seem very ambitious in view of how public deficits and debt have behaved over the past 30 years in most EU countries. The summary example (Table 7) led to a 100 % of GDP reduction in the government debt ratio and a 1.6 % surplus over 60 years. First, one should ask whether the rationale for this policy is strong enough and, secondly, what exactly should be adjusted in policy and when.

As to the first question, the answer could come from the principle that the rules for such important public policy areas as pensions and health care should be based on clearly expressed motivations for their existence and that their financing should, at least broadly, follow the principles of social *insurance* and thereby there should be a clear link between the costs and benefits for each generation. Unless future benefits are significantly reduced this leads to partial pre-funding in these systems. If they belong to the general government sector, the targets for public debt and deficit should be set such that the surplus in the public pension system is fully integrated with the debt and deficit targets imposed on the entire government sector, as otherwise the pension system surplus is squandered with a deficit in other public policy areas.<sup>21</sup> This means that even if actuarial neutrality were implemented in the public pension system, its effect on intergenerational equity requires that the target for the general government deficit is not, for example, fixed at zero or some other number, as this target would ultimately determine whether the pre-funding of pensions has the intended effect on intergenerational equity. Therefore, the way MTOs should be set under the SGP is of utmost importance, and it is not yet fully settled.

A framework for setting the MTOs should aim at answering the question as to what should be adjusted and at what speed. One should first ask whether the expenditure projections should be taken as realistic and acceptable, at least tentatively, before seriously considering financing these expenditures. This might well be so for our stylised example summarised in Table 7. The pension expenditure projection roughly approximates the average for the euro area countries (that is somewhat higher than for the entire EU). Benefits as a share of wages are significantly reduced from current levels and the retirement age is set to increase. This would, under actuarial neutrality, be satisfied with a small surplus in the government budget. Our projection for health and long-term care expenditure shows a significant increase in line with the average EPC/AGW projections. However, there might well be a risk that current expenditure

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<sup>21</sup> This means that the accumulation of funds in the public pension system should show up as a positive budget balance component and a reduction in net debt. If the public pension system is organised as a separate entity the question arises as to where should it invest. Some authors argue that if pension funds (private or public) invest in government bonds, they are in effect pure PAYG (e.g. Barr, 2004, p.114). This obviously requires that the government always issue new debt (i.e. increases the deficit) for accommodating such investment. In practice, this need not be the case. From the perspective of intergenerational equity, investment by a public (private) pension fund in government debt is neutral if the government net debt is reduced (kept constant) in response to this investment. Bosworth and Burtless (2004) find that at the state level in the US this requirement has been met, while at the national level in

projections will be exceeded as we know that in the past couple of decades these expenditures have increased much faster than now projected for the coming decades (Economic Policy Committee and European Commission, 2006a, p. 121 and 127). Therefore, the extra 1.4%-points to the surplus target due to health and long-term care expenditures (panel 3 in Table 6) could turn out to be insufficient. In this light the current MTOs for the EU on average seem not ambitious enough for actuarial neutrality.

Importantly for many countries, the projected expenditure increase considerably exceeds the EU average. For these countries, whatever quantitative indicator is used to express the imbalance, the focus should first be put on reforming the pensions and health care systems to contain spending growth, and move on to setting proper targets for the debt and deficit only on the basis of reformed rules. In some other countries the projected expenditure can be unrealistically low because the projections are based on a strict application of the rules currently in force, even though they might not be politically sustainable. This can result, for example, from the rule that pensions are indexed to prices only and therefore fall behind real wage growth (an example is Poland where the GDP share of pension expenditure is projected to fall by 5.9%-points). It is not easy to open a speculation on changes in benefit rules but it is also obvious that one should avoid setting the fiscal targets based on unrealistic projections.

While one needs to treat the extreme cases separately, the ambitious budget targets resulting from actuarial neutrality for countries at or around the EU average numbers should not be too easily dismissed. The electorate could choose to go for such a level of benefits even if they had to pay the actuarially neutral price for them. And they could accept to pay that price already now that the rights are acquired, as they would understand that otherwise they would force the next generation to pay even higher pension contributions and taxes on their behalf. Technically speaking, they would then accept to reduce the explicit government debt in order to neutralise the effects (on pension contributions and taxes) of the increase in the implicit debt stemming from their own future benefits.

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OECD countries 60-100% of public pension saving has been offset by larger deficits in other budgetary accounts.

In all events, the question about allowing time for the adjustment to actuarial neutrality is relevant, not only for political economy reasons (finding the necessary majority for the reforms) but also for purely economic grounds based on the cost of adjustment.

Even though the unit period in the framework in this paper corresponds to 30 years, it can without major difficulties be applied to gradual adjustment for truly overlapping yearly age cohorts. The average age retiree is 30 years older than the average age worker, and we can use the framework for setting the parameters for these representative individuals. Then, if the system already implements actuarial neutrality, any changes in demography and possibly in pension system rules should be incorporated gradually, over a 30 years period. This leads to quite slow change and should not lead to adjustment costs that need specific treatment. However, this is in practice not the main argument for gradual adjustment. Instead, for most countries, as their initial conditions are far away from actuarial neutrality, one should ask at what pace the new policy rule should be implemented.

If putting in place the new rules causes tangible adjustment costs, then it can be argued in the spirit of actuarial neutrality that these costs should be shared between the current and future generations. Thereby the new parameters of the pension arrangement should enter into force gradually and, as a result, total debt would, correspondingly, reach a higher level than with immediate adjustment.

A completely different argument may emerge if the induced increase in government saving causes over-saving, or a saving glut, in the economy. This would manifest itself as a rate of interest below the long-term growth rate of the economy, indicating that the economy is dynamically inefficient and consumption could therefore be increased without a negative impact on consumption of future generations. It is controversial whether this is likely to happen in modern economies but if and when these signs would be seen, governments would then have the pleasant duty to increase expenditures or reduce taxes. One could argue that even in this case, notably because the problem with over-saving will most likely only be temporary, it is useful to implement actuarial neutrality in public pensions and move to partial pre-funding (assuming that this follows from the benefit rules), and use other public finance instruments to eliminate the saving glut. This would follow from the idea that it is useful to keep separate the transitional issues from the accepted principles of the pension arrangement.



## 6. Concluding remarks

In this paper we have explored how the Stability and Growth Pact may cope with the future costs of population ageing in the European Union. In anticipation of the rising costs associated with ageing, countries have started to or plan to reform their pension systems, both by reducing the generosity of pension arrangements, increasing retirement age and by switching from pure PAYG pension provision to pre-funding, including reduction of public debt and partial privatisation. We have investigated how such reforms relate to the provisions of the Pact.

Although the Pact, especially after its revision in 2005, clearly aims to ease the financial burden to be shouldered by the future generations, it does not incorporate intergenerational equity explicitly. The simple model in this paper provides such a framework, based on the rule that generations that are identical in terms of demography (longevity and fertility) and retirement age should face the same tax rate for the same level of benefits. This implies a neatly defined benchmark termed *actuarial neutrality*, and it is shown that there exists a wide range of alternative pension arrangements that comply with it. The results provide further rationale and precision for the ambitious policy line widely expressed by the European Union finance ministers and others. They also show that pure PAYG rule does not, in general, comply with actuarial neutrality but rather tends to shift an increasing burden to future generations.

For many countries where ageing related expenditure is projected to increase considerably under current policies, emphasis should be on changing those policies and hence on reducing the expected expenditures. The medium term objective for budget balance should then be set on the basis of the reformed system. However, the EPC projection for the EU average might be a relevant starting point for setting targets for debt and deficit. This average increase already incorporates a significant reduction in the replacement rate and an increase in retirement age, and yet, as a result of the ongoing change in age structure of the population, expenditures increase. Our stylised example above mimicking those figures and also taking into account the projected increase in health-care expenditure (Table 7) shows that actuarial neutrality then implies that the target should be a significant budget surplus for several decades. This is an ambitious target, and the economically optimal and politically acceptable speed to reach it becomes a pressing issue. The framework here may help in coping with the political acceptability as it makes explicit how the burden is shared across generations. If such a policy line is

not accepted, one important conclusion would be that the retirement age has to be increased significantly more than now projected for the EU average.

The revised SGP now recognises the problem with the transitional cost of privatisation. However, the revised rules only allow a limited excess over the 3% of GDP deficit ceiling for a limited period of time. This was presumably motivated by the goal to preserve the SGP as a credible budgetary anchor. The downside of this naturally is, as our results show, that a partial privatisation on a fully actuarially neutral basis of a reformed and sound mono-pillar pension system may not easily be accommodated under the current rules. If, for example, one third of the implicit pension debt is swapped for explicit public debt, the government budget balance should be allowed to deteriorate by 4 to 5 percentage points of GDP relative to the otherwise similar mono-pillar system. Therefore, the risk of breaching the 3% deficit ceiling becomes imminent. Hence, while there might be sound economic reasons for privatisation (e.g. a reduction in the distortionary effects of the pension system on the labour market), it is clear that under otherwise similar policies a country that maintains a mono-pillar system can be much more comfortable with the SGP rules than a country that contemplates and implements a partial privatisation of the system. This means that even though the fundamental principles of pension policies are in the sole competence of the EU Member States, major privatisation plans will (in most cases) have to be treated at the EU level. This is triggered by the rise in the deficit and the explicit public debt that has to be financed on the common financial market. Therefore, a balanced assessment of all relevant factors should probably take into account, for example, the country's financial position (in particular, the debt level) before the reform.

With regard to public pensions and the SGP there will always be the major question about the enforcement of the fiscal rules. Tension between economic rationale that obliges one to look at the highly uncertain long term with deficient data and the simplicity required by the political process will never disappear. Having put in place the short to medium term rules in the late 1990s, after the turn of the century the SGP was gradually extended to pay attention to long-run budgetary sustainability. The provisions in the EU Treaty and the revision of the SGP in 2005 provide the improved legal framework for policies to comply with the principle of sound public finances. However, it is generally acknowledged that improving the implementation of the legal rules is an ongoing process as the issues are complex and adequate data is often lacking. The

framework in the present paper provides some clarification to the issues to be tackled, and the ongoing work of pension actuaries and statisticians to gather estimates on implicit pension liabilities for the national accounts will greatly help in analyzing the issues and designing economically sound reforms.

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# Appendix: a Model for Ageing-Related Public Spending under PAYG versus Actuarial Neutrality

## 1 Preliminaries

Table A1 describes the variables used in the model and its solution, while Table A2 provides definitions and some relations that are useful for the sequel.

**Table A1: Description of variables:**

Symbol	Description
$w_t$	unit gross wage (total wage cost)
$w_t^n$	wage net of pension contributions (= "net wage")
$\gamma_t$	taxable income of the elderly as a share of $w_t$
$\pi_{t-1}$	accrual rate as a share of $w_t$ for pension to be paid out in period $t$
$\pi_{t-1}^n$	accrual rate as a share of the net wage $w_t^n$ for pension to be paid out in period $t$
$c_t$	tax on gross wages and taxable gross income of elderly
$c_t^p$	contribution rate to pensions as a share of gross wage
$c_t^{pn}$	contribution rate to pensions as a share of net wage
$L_t$	new entrants into the labour force
$E_t$	number of newborns
$R_t$	"effective" number of elderly
$l_t$	years spent working in period $t$ / years spent working in period 0
$\sigma_t$	years spent in retirement in period $t$ / years spent working in period $t - 1$
$IPD$	implicit pension debt
$GID$	gross implicit debt (present value of elderly related expenditure in the next period)
$NID$	net implicit debt = $GID$ minus the present value of taxes from the elderly
$A_t$	consolidated net public sector assets
$Q_t$	assets held by the pension sector
$D_t$	government debt
$a_t$	consolidated net public sector assets as a share of the wage bill
$q_t$	pension sector assets as a share of the wage bill
$d_t$	government debt as a share of the wage bill
$z_t$	degree of funding of pension system
$\rho_t$	gross interest rate / interest factor
$\mu_t$	interest mark-up on gross growth rate of wage bill
$h_{e,t}$	public spending, e.g. health care, benefiting the elderly
$h_{a,t}$	public spending, e.g. health care, benefiting children and adults other than the elderly
$\theta_t$	implicit pension debt as share of the wage bill

**Table A2: Definitions and useful relations:**

$$\tilde{L}_t \equiv E_{t-1}$$

$$E_t \equiv f_t \tilde{L}_t$$

$$L_t \equiv l_t \tilde{L}_t$$

$$R_t \equiv \sigma_t L_{t-1} = \frac{\sigma_t}{f_{t-1}} \frac{l_{t-1}}{l_t} L_t$$

$$A_t \equiv Q_t - D_t,$$

$$a_t \equiv A_t / (w_t L_t)$$

$$q_t \equiv Q_t / (w_t L_t)$$

$$d_t \equiv D_t / (w_t L_t)$$

$$a_t = q_t - d_t$$

$$z_t \equiv A_t / IPD_t$$

$$IPD_t = \pi_t w_{t+1} R_{t+1} / \rho_{t+1}$$

$$\rho_t \equiv (1 + \mu_t) w_t L_t / (w_{t-1} L_{t-1}) = (1 + \mu_t) \frac{w_t (l_t / l_{t-1}) \tilde{L}_t}{w_{t-1} L_{t-1}}$$

## 2 The general model

We assume that the economy is in a steady state up to and including period 0. Hence,  $a_0 = a_{-1} = \dots$ ,  $f_0 = f_{-1} = \dots$  and  $l_0 = l_{-1} = \dots$ .

The consolidated (government and pension system) flow budget constraint is given by

$$c_t w_t L_t + c_t \gamma_t w_t R_t + (\rho_t - 1) A_{t-1} = h_{a,t} w_t L_t + h_{e,t} w_t R_t + A_t - A_{t-1},$$

which we can rewrite as

$$c_t \left( 1 + \frac{l_{t-1} \gamma_t \sigma_t}{l_t f_{t-1}} \right) = h_{a,t} + h_{e,t} \frac{l_{t-1} \sigma_t}{l_t f_{t-1}} + a_t - (1 + \mu_t) a_{t-1}. \quad (1)$$

### 2.1 Financing from current taxes

The expenditures in question can be financed from current taxes. This would correspond to a pure PAYG for public pensions. A simple case for debt policy is to keep its GDP share constant. Therefore, total taxes should include a component to cover the interest rate margin over the growth rate of the economy.

### 2.2 A policy rule aimed at equal treatment of generations

Assuming that the economy is in steady state in period 0, the tax rate in  $t = 0$  is:

$$c_0 = \frac{h_{a,0} + h_{e,0} \sigma_0 / f_0 - \mu_0 a_0}{1 + \gamma_0 \sigma_0 / f_0}.$$

In some period  $t$ , there may be one or more shocks, such as a fall in fertility, an increase in life expectancy (of the workers retired in that period). There may also be a change in the number of years in working life. Furthermore, the public expenditures benefiting the adults and elderly per person ( $h_{a,t}$  and  $h_{e,t}$ ), and the projections thereof for the next period may change. Note that these parameters may contain both policy induced elements and perceptions as to how exogenous factors may affect these expenditures per capita (medical technology, etc.). Also, the interest rate mark-up  $\mu_{t+1}$  may change taking effect in the next period. For our numerical application we assume that these changes do not depend on the other variables in the model.

We solve the general model under the rule that working generations that are identical also have to pay the same contributions for the same benefits. This is done by technically assuming that, once they occur, the shocks persist. Therefore, if a shock occurs in some period  $t$ , then we solve the model as if there will be no further shocks in the subsequent periods. We indicate the solution under this assumption with a superscript  $t$ , where relevant.



Changes in the length of the working life may take place in response to demographic and policy changes. These changes in the length of the working life are assumed to take place in the period when the demographic and policy changes happen or are known. Hence,  $l_t^t = l_{t+1}^t = \dots$

Note that the way we solve the model in no way limits the generality of the solution as new shocks may arrive later and a new solution is found in a similar way. The solution is based on the known values of the variables for the period in question and expectations on the values in the next period. We do not address here any specific questions related to the accuracy of expectations. In our setting any error in expectations becomes known in the next period, and the consequences are shared between the then current and future generations as the new decisions are based on the realised values and newly set expected values for the subsequent periods.

We define,

$$\begin{aligned} H_t^t &\equiv h_{a,t} + h_{e,t} \frac{l_{t-1}^{t-1}}{l_t^t} \frac{\sigma_t^t}{f_{t-1}}, & H_{t+1}^t &\equiv h_{a,t+1} + h_{e,t+1} \frac{l_t^t}{l_{t+1}^t} \frac{\sigma_{t+1}^t}{f_t} = h_{a,t+1} + h_{e,t+1} \frac{\sigma_{t+1}^t}{f_t}, \\ K_t^t &\equiv 1 + \gamma_t \frac{l_{t-1}^{t-1}}{l_t^t} \frac{\sigma_t^t}{f_{t-1}}, & K_{t+1}^t &\equiv 1 + \gamma_{t+1} \frac{l_t^t}{l_{t+1}^t} \frac{\sigma_{t+1}^t}{f_t} = 1 + \gamma_{t+1} \frac{\sigma_{t+1}^t}{f_t}, \end{aligned}$$

where, of course,  $l_0^0 = l_0$  and  $\sigma_0^0 = \sigma_0$ .

Hence, based on the shocks up to and including period 1, we can write (1) as:

$$c_1^1 = \frac{H_1^1 + a_1 - (1 + \mu_1^1) a_0}{K_1^1}, \quad c_2^1 = \frac{H_2^1 + a_2 - (1 + \mu_2^1) a_1}{K_2^1}$$

In the absence of further shocks, and assuming that unit benefits and taxable income of elderly are constant over time ( $h_{a,1} = h_{a,2} = \dots$ ,  $h_{e,1} = h_{e,2} = \dots$  and  $\gamma_1 = \gamma_2 = \dots$ ), the economy is in the new steady state as of period 2. Hence,  $a_2 = a_3 = \dots$ ,  $\sigma_2^1 = \sigma_3^1 = \dots$ ,  $\mu_2^1 = \mu_3^1 = \dots$ ,  $H_2^1 = H_3^1 = \dots$  and  $K_2^1 = K_3^1 = \dots$ . The tax rate in the new steady state must equal:

$$\frac{H_2^1 - \mu_2^1 a_2}{K_2^1} = c_2^1 = c_3^1 = \dots$$

The two expressions for  $c_2^1$  together imply:

$$\frac{H_2^1 + a_2 - (1 + \mu_2^1) a_1}{K_2^1} = \frac{H_2^1 - \mu_2^1 a_2}{K_2^1},$$

which implies that  $a_1 = a_2$ . Further, realising that working generations as of period 1 are identical, we have  $c_1^1 = c_2^1$ , which becomes:

$$\frac{H_1^1 + a_1 - (1 + \mu_1^1) a_0}{K_1^1} = \frac{H_2^1 - \mu_2^1 a_1}{K_2^1}.$$

Hence,

$$a_1 = \frac{\frac{H_2^1}{K_2^1} - \frac{H_1^1}{K_1^1} + \frac{(1+\mu_1^1)a_0}{K_1^1}}{\frac{1}{K_1^1} + \frac{\mu_2^1}{K_2^1}}.$$

Substitute this back into the expression for  $c_1^1$  above, which can then be simplified to:

$$c_1^1 = c_2^1 = \frac{H_2^1 + \mu_2^1 [H_1^1 - (1 + \mu_1^1) a_0]}{K_2^1 + \mu_2^1 K_1^1}.$$

In the same way, if a new shock occurs in period 2, we obtain

$$c_2^2 = c_3^2 = \frac{H_3^2 + \mu_3^2 [H_2^2 - (1 + \mu_2^2) a_1]}{K_3^2 + \mu_3^2 K_2^2}.$$

More generally, for a shock up to and including period  $t$ , we have:

$$c_t^t = c_{t+1}^t = \frac{H_{t+1}^t + \mu_{t+1}^t [H_t^t - (1 + \mu_t^t) a_{t-1}]}{K_{t+1}^t + \mu_{t+1}^t K_t^t}. \quad (2)$$

This thus provides the tax rate under the policy rule specified above.

### 2.3 Further calculations

Gross implicit debt in period  $t$  (based on the shocks in period  $t$ ) as a share of the wage bill is calculated as

$$\frac{GID_t^t}{w_t L_t^t} = \frac{h_{e,t+1} w_{t+1} R_{t+1}^t / \rho_{t+1}^t}{w_t L_t^t} = \frac{h_{e,t+1} w_{t+1} \sigma_{t+1}^t L_{t+1}^t / f_t}{(1 + \mu_{t+1}^t) w_{t+1} L_{t+1}^t} = \frac{h_{e,t+1} \sigma_{t+1}^t}{f_t (1 + \mu_{t+1}^t)}.$$

Net implicit debt in period  $t$  (based on the shocks in period  $t$ ) as a share of the wage bill is calculated as

$$\begin{aligned} \frac{NID_t^t}{w_t L_t^t} &= \frac{GID_t^t}{w_t L_t^t} - \frac{c_{t+1}^t \gamma_{t+1} w_{t+1} R_{t+1}^t / \rho_{t+1}^t}{w_t L_t^t} \\ &= \frac{(h_{e,t+1} - c_{t+1}^t \gamma_{t+1}) \sigma_{t+1}^t}{f_t (1 + \mu_{t+1}^t)}. \end{aligned}$$

## 3 Special case: the pension model

For pensions, we have:

$$\gamma_t = 0, \quad h_{a,t} = 0, \quad h_{e,t} = \pi_{t-1}, \quad IPD_t = GID_t = NID_t. \quad (3)$$

### 3.1 Pay-as-you-go

Under our PAYG system, taxes are set such that public debt (or public assets) remains constant over time (i.e.,  $a_t = a_{t-1}$ ). That is, pension contributions are set so as to cover current outlays, while additional taxes are raised to service the outstanding stock of debt. Hence,

$$c_t = \frac{l_{t-1}}{l_t} \frac{\pi_{t-1} \sigma_t}{f_{t-1}} - \mu_t a_{t-1} = c_t^p - \mu_t a_{t-1}. \quad (4)$$

In this case, there is no need to indicate by superscripts the moment of the shock.

#### 3.1.1 Determination of the gross accrual rate for given net accrual rate

To compute the appropriate value of  $\pi_{t-1}$ , we use that both

$$w_t^n = (1 - c_t^p) w_t, \text{ and } w_t^n = \frac{\pi_{t-1}}{\pi_{t-1}^n} w_t.$$

Hence,

$$c_t^p = 1 - \frac{\pi_{t-1}}{\pi_{t-1}^n}.$$

Substituting  $c_t^p = (l_{t-1}/l_t) (\sigma_t/f_{t-1}) \pi_{t-1}$  and solving, we obtain:

$$\pi_{t-1} = \frac{\pi_{t-1}^n}{1 + [(l_{t-1}/l_t) \sigma_t/f_{t-1}] \pi_{t-1}^n}. \quad (5)$$

### 3.2 Actuarial neutrality

Substituting the simplifications (3) into the expression (2) for  $c_t^t$  yields

$$\begin{aligned} c_t^{a,t} &= \frac{\mu_{t+1}^t}{1 + \mu_{t+1}^t} \left[ \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{f_{t-1}} - (1 + \mu_t^t) a_{t-1} \right] + \frac{\pi_t (l_t^t/l_{t+1}^t) \sigma_{t+1}^t}{(1 + \mu_{t+1}^t) f_t} \\ &= \mu_{t+1}^t \left[ \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} \left( \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{(1 + \mu_t^t) f_{t-1}} - a_{t-1} \right) \right] + \frac{\pi_t \sigma_{t+1}^t}{(1 + \mu_{t+1}^t) f_t}. \end{aligned} \quad (6)$$

Further,

$$\begin{aligned} \theta_{t-1}^t &= \frac{IPD_{t-1}^t}{w_{t-1} L_{t-1}^{t-1}} = \frac{\pi_{t-1} w_t R_t^t / \rho_t^t}{w_{t-1} L_{t-1}^{t-1}} = \frac{\pi_{t-1} w_t (l_{t-1}^{t-1}/l_t^t) \sigma_t^t L_t^t / f_{t-1}}{\rho_t^t w_{t-1} L_{t-1}^{t-1}} \\ &= \frac{\pi_{t-1} w_t (l_{t-1}^{t-1}/l_t^t) \sigma_t^t L_t^t / f_{t-1}}{(1 + \mu_t^t) w_t L_t^t} = \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{(1 + \mu_t^t) f_{t-1}} \end{aligned} \quad (7)$$

and

$$\begin{aligned}\theta_t^t &= \frac{IPD_t^t}{w_t L_t^t} = \frac{\pi_t w_{t+1} R_{t+1}^t / \rho_{t+1}^t}{w_t L_t^t} = \frac{\pi_t w_{t+1} (l_t^t / l_{t+1}^t) \sigma_{t+1}^t L_{t+1}^t / f_t}{\rho_{t+1}^t w_t L_t^t} \\ &= \frac{\pi_t w_{t+1} \sigma_{t+1}^t L_{t+1}^t / f_t}{(1 + \mu_{t+1}^t) w_{t+1} L_{t+1}^t} = \frac{\pi_t \sigma_{t+1}^t}{(1 + \mu_{t+1}^t) f_t}.\end{aligned}$$

Using this (twice) in the previous expression for  $c_t^{a,t}$  we get:

$$c_t^{a,t} = \mu_{t+1}^t \left[ \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} (\theta_{t-1}^t - a_{t-1}) \right] + \theta_t^t, \quad (8)$$

which is the tax rate under the policy rule specified here. In the context of the pension model this policy rule can be termed "actuarial neutrality" as each generation of workers pays, first, the interest mark-up on the sum of the implicit pension liabilities and the explicit public debt stemming from the past decisions, valued at the perceived interest mark-up for the future, and, secondly, pays the full present value of their own future pensions. The latter establishes a direct link between the future pensions and current contributions.

### 3.2.1 Total debt under actuarial neutrality

Applying (3) to the general case (1), the tax rate under actuarial neutrality must obey

$$c_t^{a,t} = \frac{l_{t-1}^{t-1} \pi_{t-1} \sigma_t^t}{l_t^t f_{t-1}} + a_t - (1 + \mu_t^t) a_{t-1},$$

which we can combine with the previous expression for  $c_t^{a,t}$ , (8), and (7) to give:

$$\begin{aligned}a_t &= \mu_{t+1}^t \left[ \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} (\theta_{t-1}^t - a_{t-1}) \right] + \theta_t^t - \frac{l_{t-1}^{t-1} \pi_{t-1} \sigma_t^t}{l_t^t f_{t-1}} + (1 + \mu_t^t) a_{t-1} \\ &= \theta_t^t - \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} \left[ \frac{\pi_{t-1} (l_{t-1}^{t-1} / l_t^t) \sigma_t^t}{(1 + \mu_t^t) f_{t-1}} - a_{t-1} \right],\end{aligned} \quad (9)$$

or

$$\theta_t^t - a_t = \frac{1 + \mu_t^t}{1 + \mu_{t+1}^t} [\theta_{t-1}^t - a_{t-1}]. \quad (10)$$

We can see that for a constant  $\mu$  actuarial neutrality implies a constant total debt ratio. In the general case, where  $\mu_{t+1}^t \neq \mu_t^t$ , the total debt in the previous period has to be revalued using the factor  $(1 + \mu_t^t) / (1 + \mu_{t+1}^t)$ . This revaluation should be treated as a capital gain. Obviously, the projected  $\mu_{t+1}^t$  should then also be used in (8) for calculating the required tax rate. We can see directly that an increase in  $\mu$ , implying  $\mu_{t+1}^t > \mu_t^t$ , does not increase the first component

of  $c_t$  proportionally, as  $(1 + \mu_t^t) / (1 + \mu_{t+1}^t) < 1$ . Also,  $\mu_{t+1}^t > \mu_t^t$  implies that the second component,  $\theta_t^t$ , of  $c_t^t$  decreases.

### 3.2.2 Determination of the gross accrual rate for given net accrual rate

The policy instrument is  $\pi_{t-1}^n$ , the accrual rate as a share of the net wage, and we are left with the determination of  $\pi_{t-1}$ .

First, we calculate the outcome that obtains as a result of the shocks in period 1, assuming that there are no further shocks. Setting public debt  $d_t$  to zero, by (6) we get the contribution rate to the pension system for period 1 as:

$$c_1^{a,p,1} = \frac{\mu_2^1}{1 + \mu_2^1} \left[ \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{f_0} - (1 + \mu_1^1) q_0 \right] + \frac{\pi_1 \sigma_2^1}{(1 + \mu_2^1) f_1}.$$

Here,  $a$  and  $p$  in the superscript for the contribution rate indicate that we are considering actuarial neutrality and the contribution rate to pensions (rather than the total tax rate). Using (9) the assets of the pension sector are given by:

$$q_1 = \frac{\pi_1 \sigma_2^1}{(1 + \mu_2^1) f_1} + \frac{1 + \mu_1^1}{1 + \mu_2^1} \left[ q_0 - \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{(1 + \mu_1^1) f_0} \right]. \quad (11)$$

Further, we have, by applying (1) and setting  $d_t$  to zero, that the contribution rate to the pension system in period 2 based on shocks in period 1, is

$$c_2^{a,p,1} = \frac{l_1^1}{l_2^1} \frac{\pi_1 \sigma_2^1}{f_1} + q_2 - (1 + \mu_2^1) q_1 = \frac{\pi_1 \sigma_2^1}{f_1} - \mu_2^1 q_1,$$

because  $q_2 = q_1$ . Substitute into this the expression for  $q_1$ , to give:

$$\begin{aligned} c_2^{a,p,1} &= \frac{\pi_1 \sigma_2^1}{f_1} - \mu_2^1 \left[ \frac{\pi_1 \sigma_2^1}{(1 + \mu_2^1) f_1} + \frac{1 + \mu_1^1}{1 + \mu_2^1} \left( q_0 - \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{(1 + \mu_1^1) f_0} \right) \right] \\ &= \frac{\mu_2^1}{1 + \mu_2^1} \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{f_0} + \frac{\pi_1 \sigma_2^1}{(1 + \mu_2^1) f_1}, \end{aligned}$$

under the assumption that initial pension system assets are zero,  $q_0 = 0$ . This is the new steady state pension contribution rate.

Because  $\pi_1 w_2 = \pi_1^n w_2^n$  and  $w_2^n = (1 - c_2^{a,p,1}) w_2$ , we must have that:

$$\pi_1 = \pi_1^n (1 - c_2^{a,p,1}).$$

Hence, upon substitution, we obtain:

$$\pi_1 = \pi_1^n \left[ 1 - \frac{\mu_2^1}{1 + \mu_2^1} \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{f_0} - \frac{\pi_1 \sigma_2^1}{(1 + \mu_2^1) f_1} \right].$$

Hence, rewriting:

$$\pi_1 = \frac{\left[ 1 - \frac{\mu_2^1}{1+\mu_2^1} \frac{\pi_0 (l_0/l_1^1) \sigma_1^1}{f_0} \right] \pi_1^n}{1 + \frac{\sigma_2^1}{(1+\mu_2^1) f_1} \pi_1^n}.$$

We now calculate the accrual rate as a share of the gross wage corresponding the second shock. By (6), using that  $d_t = 0$ , the pension contribution rate is:

$$c_2^{a,p,2} = \frac{\mu_3^2}{1 + \mu_3^2} \left[ \frac{\pi_1 (l_1^1/l_2^2) \sigma_2^2}{f_1} - (1 + \mu_2^2) q_1 \right] + \frac{\pi_2 \sigma_3^2}{(1 + \mu_3^2) f_2}.$$

Pension fund assets are given by:

$$q_2 = \frac{1 + \mu_2^2}{1 + \mu_3^2} q_1 + \frac{\pi_2 \sigma_3^2}{(1 + \mu_3^2) f_2} - \frac{\pi_1 (l_1^1/l_2^2) \sigma_2^2}{(1 + \mu_3^2) f_1}.$$

For the pension contribution rate in period 3 we have by (6) and substituting  $q_2$  that

$$c_3^{a,p,2} = \frac{\mu_3^2}{1 + \mu_3^2} \frac{\pi_1 (l_1^1/l_2^2) \sigma_2^2}{f_1} + \frac{\pi_2 \sigma_3^2}{(1 + \mu_3^2) f_2} - \frac{\mu_3^2}{1 + \mu_3^2} (1 + \mu_2^2) q_1.$$

In analogy to the case for period 1, we have  $\pi_2 = \pi_2^n (1 - c_3^{a,p,2})$ . Substitute the above expression for  $q_2$  and solve to give:

$$\pi_2 = \frac{\left[ 1 + \frac{\mu_3^2}{1+\mu_3^2} (1 + \mu_2^2) q_1 - \frac{\mu_3^2}{1+\mu_3^2} \frac{\pi_1 (l_1^1/l_2^2) \sigma_2^2}{f_1} \right] \pi_2^n}{1 + \frac{\pi_2 \sigma_3^2}{(1+\mu_3^2) f_2} \pi_2^n},$$

where  $q_1$  is given by (11) with  $q_0 = 0$ .

A similar procedure yields for any period  $t$ ,

$$\pi_t = \frac{\left[ 1 + \frac{\mu_{t+1}^t}{1+\mu_{t+1}^t} (1 + \mu_t^t) q_{t-1} - \frac{\mu_{t+1}^t}{1+\mu_{t+1}^t} \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{f_{t-1}} \right] \pi_t^n}{1 + \frac{\pi_t \sigma_{t+1}^t}{(1+\mu_{t+1}^t) f_t} \pi_t^n}, \quad (12)$$

where  $q_{t-1}$  is given from the previous step as:

$$q_{t-1} = \frac{1 + \mu_{t-1}^{t-1}}{1 + \mu_t^{t-1}} q_{t-2} + \frac{\pi_{t-1} \sigma_t^{t-1}}{(1 + \mu_t^{t-1}) f_{t-1}} - \frac{\pi_{t-2} (l_{t-2}^{t-2}/l_{t-1}^{t-1}) \sigma_{t-1}^{t-1}}{(1 + \mu_t^{t-1}) f_{t-2}}. \quad (13)$$

## 4 A fixed interest rate

We assume that  $\rho_t$  is constant at a level  $\bar{\rho}$ . Using the expression for  $\rho_t$  in Table 2, we have:

$$1 + \mu_t = \bar{\rho} \frac{w_{t-1} L_{t-1}}{w_t L_t}.$$

Making further use of Table 2, we have:

$$\begin{aligned} 1 + \mu_t &= \frac{w_{t-1}}{w_t} \frac{\bar{\rho}}{(l_t/l_{t-1}) f_{t-1}} \\ &= \frac{\bar{\rho}}{g_t^w (l_t/l_{t-1}) f_{t-1}}, \end{aligned} \quad (14)$$

where

$$g_t^w \equiv \frac{w_t}{w_{t-1}}.$$

Finally, we can write the implicit pension debt as a function of the wage bill as:

$$\begin{aligned} \theta_{t-1}^t &= \frac{IPD_{t-1}^t}{w_{t-1} L_{t-1}^{t-1}} = \frac{\pi_{t-1} w_t R_t^t / \bar{\rho}}{w_{t-1} L_{t-1}^{t-1}} = \frac{\pi_{t-1} w_t \sigma_t^t L_{t-1}^{t-1} / \bar{\rho}}{w_{t-1} L_{t-1}^{t-1}} = g_t^w \frac{\pi_{t-1} \sigma_t^t}{\bar{\rho}}, \\ \theta_t^t &= \frac{IPD_t^t}{w_t L_t^t} = \frac{\pi_t w_{t+1} R_{t+1}^t / \bar{\rho}}{w_t L_t^t} = \frac{\pi_t w_{t+1} \sigma_{t+1}^t L_t^t / \bar{\rho}}{w_t L_t^t} = g_{t+1}^w \frac{\pi_t \sigma_{t+1}^t}{\bar{\rho}}. \end{aligned}$$

### 4.1 Pay-as-you-go

Plug the expression for  $\mu_t$  provided by (14) into the expression for the tax rate under PAYG, (4) to give:

$$c_t = c_t^p - \left[ \frac{\bar{\rho}}{g_t^w (l_t/l_{t-1}) f_{t-1}} - 1 \right] a_{t-1}.$$

### 4.2 Actuarial neutrality

Use (14) in (6) to obtain:

$$\begin{aligned}
c_t^{a,t} &= \left[ \frac{\bar{\rho}}{g_{t+1}^w (l_{t+1}^t/l_t^t) f_t} - 1 \right] \left[ \frac{g_{t+1}^w (l_{t+1}^t/l_t^t) f_t}{g_t^w (l_t^t/l_{t-1}^t) f_{t-1}} \left( \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{\bar{\rho} f_{t-1} / [g_t^w (l_t^t/l_{t-1}^t) f_{t-1}]} - a_{t-1} \right) \right] \\
&\quad + \frac{\pi_t \sigma_{t+1}^t}{\bar{\rho} f_t / [g_{t+1}^w (l_{t+1}^t/l_t^t) f_t]} \\
&= \left[ \frac{\bar{\rho}}{g_{t+1}^w f_t} - 1 \right] \left[ \frac{g_{t+1}^w f_t}{g_t^w (l_t^t/l_{t-1}^t) f_{t-1}} \left( \frac{\pi_{t-1} (l_{t-1}^{t-1}/l_t^t) \sigma_t^t}{\bar{\rho} / [g_t^w (l_t^t/l_{t-1}^t)]} - a_{t-1} \right) \right] + \frac{\pi_t \sigma_{t+1}^t}{\bar{\rho} / g_{t+1}^w} \\
&= \left[ \frac{\bar{\rho}}{g_{t+1}^w f_t} - 1 \right] \left[ \frac{g_{t+1}^w l_{t-1}^{t-1} f_t}{g_t^w l_t^t f_{t-1}} (\theta_{t-1}^t - a_{t-1}) \right] + \theta_t^t,
\end{aligned}$$

where we have used that  $l_t^t = l_{t+1}^t$ .

Substitute the expression for  $1 + \mu_t$  into (10) to give:

$$\theta_t^t - a_t = \frac{g_t^w (l_t^t/l_{t-1}^t) f_{t-1}}{g_{t+1}^w (l_{t+1}^t/l_t^t) f_t} [\theta_{t-1}^t - a_{t-1}].$$

Note that the computation of the accrual rate as a share of the gross wage is also affected by the assumption of the constant interest rate. We simply need to substitute (14) (and its appropriate leads and lags) into (12) and (13).

## 5 Correction factor for frequency conversion

The breakdown of figures from the frequency level of a period in our model (a generation) to annual frequency requires the application of a correction factor. Let  $\tau$  index the year (where  $t$  indexes the period at generational frequency, i.e. thirty years). Hence,  $D_{t,\tau}$  is debt in year  $\tau$  of period  $t$  and  $Y_{t,\tau}$  is output in year  $\tau$  of period  $t$ . We define  $PRD_{t,\tau}$  as the primary deficit in year  $\tau$  of period  $t$ ,  $DEF_{t,\tau}$  as the deficit in year  $\tau$  of period  $t$  and  $r_{t,\tau}$  as the interest rate in year  $\tau$  of period  $t$ . Hence,  $\rho_t = \prod_{\tau=1}^{30} (1 + r_{t,\tau})$ . Further,  $d_{t,\tau} \equiv D_{t,\tau}/Y_{t,\tau}$  is the debt - GDP *ratio* in year  $\tau$  of period  $t$  and  $def_{t,\tau} \equiv DEF_{t,\tau}/Y_{t,\tau}$  is the deficit - GDP *ratio* in year  $\tau$  of period  $t$ . Debt dynamics is as follows:

$$\begin{aligned}
D_{t,30} &= (1 + r_{t,30}) D_{t,29} + PRD_{t,30} \\
&= D_{t,29} + (r_{t,30} D_{t,29} + PRD_{t,30}) \\
&= D_{t,29} + DEF_{t,30} \\
&= \dots = \\
&= D_{t-1,30} + \sum_{\tau=1}^{30} DEF_{t,\tau}.
\end{aligned}$$

Hence,



$$\begin{aligned}
\frac{D_{t,30}}{Y_{t,30}} &= \frac{Y_{t-1,30}}{Y_{t,30}} \frac{D_{t-1,30}}{Y_{t-1,30}} + \sum_{\tau=1}^{30} \frac{DEF_{t,\tau}}{Y_{t,30}} \\
&= \frac{Y_{t-1,30}}{Y_{t,30}} \frac{D_{t-1,30}}{Y_{t-1,30}} + \sum_{\tau=1}^{30} \frac{Y_{t,\tau}}{Y_{t,30}} \frac{DEF_{t,\tau}}{Y_{t,\tau}} \\
&= \left( \frac{1}{1+g} \right)^{30} \frac{D_{t-1,30}}{Y_{t-1,30}} + \sum_{\tau=1}^{30} \left( \frac{1}{1+g} \right)^{30-\tau} def_{t,\tau}.
\end{aligned}$$

Assuming that the  $def_{t,\tau}$  are constant at level  $def$  for all  $\tau$ , we have:

$$\begin{aligned}
\frac{D_{t,30}}{Y_{t,30}} &= \left( \frac{1}{1+g} \right)^{30} \frac{D_{t-1,30}}{Y_{t-1,30}} + def \sum_{\tau=1}^{30} \left( \frac{1}{1+g} \right)^{30-\tau} \\
&= \left( \frac{1}{1+g} \right)^{30} \frac{D_{t-1,30}}{Y_{t-1,30}} + \left( \frac{1+g}{g} \right) \left[ 1 - \left( \frac{1}{1+g} \right)^{30} \right] def,
\end{aligned}$$

where we have used that:

$$\sum_{\tau=0}^{29} \left( \frac{1}{1+g} \right)^{\tau} = \left( \frac{1+g}{g} \right) \left[ 1 - \left( \frac{1}{1+g} \right)^{30} \right].$$

Hence,

$$def = \left( \frac{g}{1+g} \right) \left[ 1 - \left( \frac{1}{1+g} \right)^{30} \right]^{-1} \left[ d_{t,30} - \left( \frac{1}{1+g} \right)^{30} d_{t-1,30} \right].$$

Observe that, with a *constant* debt - GDP ratio,  $d$ , this reduces to:

$$def = \left( \frac{g}{1+g} \right) d.$$

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