Investing in Biogas: Timing, Technological Choice and the Value of Flexibility from Inputs Mix

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Abstract

In a continuous-time framework we study the technology and investment choice problem of a continuous co-digestion biogas plant dealing with randomly fluctuating relative convenience of input factor costs. Input factors enter into the productive process together mixed according to a given initial rule. Being inputs relative convenience stochastically evolving, a successive revision of the initial rule may be desirable. Hence, when the venture starts the manager may or may not install a flexible technology allowing for such option. Investment is irreversible and flexibility is costly. The problem is solved determining in the light of future prospects the optimal revision and then playing backward fixing the investment timing rule.

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Keywords: factor proportions, technological choice, flexibility, real options, alternative energy source.

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1 Introduction

Consider a product which is produced by mixing together two input factors according to a given rule. Such product may be provided getting on forever with the initial productive mode or by switching to a revised rule as soon as future changes in the relative input convenience makes it worth still keeping the option to reverse. This flexibility option does not come for free and its cost depends on the "distance" between the initial and the revised rule. Thus, for a given starting mode a set of technologies providing with the option to differently revise such rule are available at different costs. Assume such cost is sunk in nature. The problem for the manager is then the choice of the technology maximizing the value of the venture according to future prospects.

This kind of problem may arise in different situations. Biogas plants provide methane by the anaerobic digestion of biomass, both residual as in the case of manure or sewage, municipal waste, by-products from agriculture and energy crops. The design of the plant operations and the final biogas yield depends on the composition of the feedstock to be fermented into the digester [2,3,7]. Technological progress allows today for digesters able to process almost any biodegradable material and process simultaneously two or more input materials. Needless to say that the choice of the feeding mixture plays a crucial role to reduce the costs of the biogas produced [6,14,21]. However, the relative economic convenience of a factor is affected by different sources of uncertainty such as market, regulatory and technological uncertainty. Under changing circumstances, a technology allowing a revision of the initial diet is clearly an advantage. But such technology may also be more expensive to install. The plant manager must then decide under which conditions such investment is worth. Or consider flexible fuel engines (FFEs) which run with blends of different proportions of gasoline and either ethanol or methanol.[1] Randomly fluctuating fuel prices and changes in transport regulations due to environmental concerns may justify the adoption of more costly FFEs or the R&D investment on more flexible engines. Finally, changing perspective and focusing on the role of flexibility for vertical arrangements affecting firm integration, one may think of the initial rule as a determined vertical structure where only part of the input factor required is outsourced.[2] The question is

[1] A complete description is available at http://en.wikipedia.org/wiki/Flexible_fuel.

[2] As discussed in Bernard et al. (2008) by keeping in-house some portion of input

then how to rearrange such structure in the light of unpredictable changes in the outsourcing convenience and however holding the option to switch back to the original set-up. In this respect, solving the problem sketched above would represent a generalization of the model proposed by [18].[3]

The value of flexibility and its role on investment under uncertainty and irreversibility have been deeply investigated in the last two decades. For example, [15,16,23] apply option theory to assess the value of flexibility on manufacturing. In particular, [16] evaluates, using a stochastic dynamic programming model, options available in a flexible production process and considers the effects of switching costs. [23] analyses the investment on a technology allowing for the production of a k-variety of products with no cost at the switching nodes. Capacity constraints are considered and their model also allows for temporary shut down and restart operation. [15] highlights the relationship between technology and capacity choice. From this perspective, the authors study output flexibility determining in a stochastic frame first the degree of flexibility in the technology and then the capacity to be installed.

In this paper, differently from previous contributions where the standpoint has been mainly represented by product, process and volume flexibilities, we propose to investigate the relationship between the value added by flexibility and the choice of adjustments to the initial productive or organizational mode.[4]

For the sake of a better illustration of the model and convinced that our analysis may shed new light on investment in renewable energy we will develop the analysis of a biogas plant operation.

We propose a continuous-time model considering the optimal choice prob-

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production, a firm may be able to avoid the loss of control of the entire vertical production process and/or the quick obsolescence of a specific know-how embodied in some inputs. Their paper is available at http://econ.lse.ac.uk/~sredding/papers/ift.pdf.

[3] Recent contributions provide a theoretical analysis of partial outsourcing by considering levels of vertical integration which vary continuously over the unit interval [1,24]. However, in these models the selection of the degree of vertical integration is still seen as an irreversible step.

[4] To characterize the operating technology, the scholars quoted above use the concept of a "mode of operation" to describe a mutual exclusive flexibility, i.e. "invest" vs "wait to invest", "use gas" vs. "use oil", or " continuous operation" vs "shut down" or vs. "abandon project" and so on [5,16].

lem of both entry timing and revision of the initial feedstock composition whenever it makes sense according to economic conditions. At time zero, the manager determines the timing of investment in a plant where the technology installed allows the revision of the initial diet. The technology is chosen in order to optimally revise the initial mixture according to future prospects about a randomly fluctuating input factor convenience. Once investment has been undertaken, the plant provides biogas exploiting the most convenient diet while the manager always hold the option to switch to an alternative diet as soon as it is worth.

The paper reminder is organized as follows. In the next section the basic set-up is presented. In section 3 and 4 we respectively determine the value of flexibility and the optimal adjustment policy. In section 5 we solve for the timing of the investment in the optimal technology. In section 6 by numerical simulations and graphical illustrations we provide additional insight on the problem letting uncertainty and cost parameters vary. The last section concludes.

2 The basic set-up

A biogas plant consists, in general, of two main components: a digester (or more digesters) and a gas holder. The digester is a water proof container where the fermentable mixture is introduced in the form of slurry. For the sake of simplicity we assume that to feed the digester in order to produce 1 m^3 of biogas a mixture of two types of materials is needed as input factor.[5] We denote by D^1 such mixture and we assume that it is initially composed by a share, $\alpha \in (0, 1)$, of a biological material which market price is c_t, and a share, $1 - \alpha$, of material which price is d_t.[6] Indicating with p_t the market

[5] Raw material to feed the digester may be obtained from a variety of sources such as livestock and poultry wastes, night soil, crop residuals, paper wastes, aquatic weeds, water hyacinth and seaweed. Yet, residues from the agricultural sector such as spent trawl, hay, cane trash, corn maize and plant stubble, etc. Succulent plant material produces more gas than dried material and hence materials like brush and weeds need semi-drying. We simplify the analysis considering two composite inputs: one formed by dry material and the other by liquid material. In this respect, experience has shown that the raw-material ratio to water must be 1:1 [2,3,9,19].

[6] This assumption could be justified by the existence of regulative or technological constraints which impose to start with a defined diet. In many cases, the price of the inputs is

price of $1\ m^3$ of biogas and assuming perfect substitutability[7] between the two inputs, the instantaneous profit function when D^1 is adopted can be expressed as:[8]

$$\begin{aligned} p_t - C_1 &= p_t - [\alpha c_t + (1-\alpha)\, d_t] \\ &= p_t - d_t + \alpha\,(d_t - c_t) \end{aligned}$$

Further, depending on the relative economic convenience of each material with respect to the other, we allow for a successive revision of D^1 obtained by costlessly switching[9] to D^2 if the latter turns out to be more profitable. In D^2 the shares are adjusted and are respectively given by α' and $1-\alpha'$, where $\alpha' = \gamma\alpha$ with $\gamma \in [0, 1/\alpha]$.[10] Then, once the adjustment parameter γ has been chosen, the instantaneous profit function under D^2 is:[11]

$$\begin{aligned} p_t - C_2 &= p_t - [\alpha' c_t + (1-\alpha')\, d_t] \\ &\equiv p_t - d_t + \gamma\alpha\,(d_t - c_t) \end{aligned}$$

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the opportunity cost faced by the plant holder for disposing of such raw materials to accomplish with regulations. For instance, according to Commission Regulation 208/2006/EC, this is the case with manure [10,14,21].

[7] As suggested by [6], in the appendix A.3 we open a window on the imperfect substitutability case where alfa indicates the share of total cost dedicated to input c_t.

[8] As said before, production of biogas is inefficient if fermentation materials are too diluted or too concentrate. To maintain the right total solid concentration, water may be added to the slurry before the anaerobic action starts. Therefore, without loosing in generality, we may assume perfect substitutability between the inputs, adding the cost of water to the cost of one of them [9,19,22].

[9] Switching costs are essentially related to the time needed for the production process to resume to the standard performance after a change in the diet. To consider the presence of other switching costs would only add complexity to the analysis without giving more insight.

[10] [2] analyses crop rotation as a different example of changing diet to optimise biogas production.

[11] Note that by this assumption one may rearrange over all the feasible range:

$$\gamma = 0 \to \alpha' = 0 \to C_2 = d$$
$$\gamma = \frac{1}{\alpha} \to \alpha' = 1 \to C_2 = c$$

We also consider that operation of the biogas project can be temporarily suspended, under both the regimes D^1 and D^2, when the instantaneous profit falls below a maintenance cost m_t. This could represent the per period maintenance expenditure the firm incurs to keep the project ready to be resumed. If operation restarts, a reactivation cost, F_R, must be paid. Finally, at a cost equal to $F_S > F_R$ we provide the manager also with the option of scrapping the project previously mothballed.[12]

Taking D^1 as given[13] and accounting for the option to switch between diets and for the option to mothball the project, the instantaneous profit function is equal to:

$$\begin{aligned} \pi_t &= \max\left\{-m_t,\ \max\left[\left(p_t - C_1\right), \left(p_t - C_2\right)\right]\right\} \\ &= \max\left\{-m_t,\ p_t - d_t + \max\left[\alpha\left(d_t - c_t\right), \alpha'(d_t - c_t)\right]\right\} \end{aligned} \tag{1}$$

To simplify the analysis, we assume that the market price of a unit of biogas is certain and taken as given,[14] i.e. $p_t = p$, the maintenance cost, m_t, is constant and equal to m, and finally that the price of the input d_t is constant and equal to $d > 0$. The price of the other input c_t is stochastic[15] and randomly fluctuates according to the trendless geometric Brownian motion[16]

$$\frac{dc_t}{c_t} = \sigma dz_t \tag{2}$$

where σ is the volatility of the market price and dz_t is the increment of a Wiener process satisfying the conditions $E\left[dz_t\right] = 0$, $E\left[dz_t^2\right] = dt$. Finally, we

[12] Note that we consider a plant of fixed output flow, then all costs are expressed per unit of output.

[13] This assumption could be justified by the initial market price of the two factors and/or by the existence of regulative or technological constraints which impose to start with a defined diet.

[14] The price of 1 m^3 of biogas may be constant due to regulation and/or to trading Renewable Energy Certificates (RECs). See Directive 2001/77/EC on the promotion of electricity from renewable energy sources in the internal electricity market.

[15] We may alternatively assume that d_t is uncertain while c_t is not. However, this would not influence our conclusions.

[16] By this simple form we are practically assuming that the uncertainty driven by technological and regulative change is processed on the market place and reflected by input price dynamics. A more general GBM with Poisson jumps capturing technological and regulative shocks affecting c_t, may be considered without adding substantial insight to our results.

set $F_R = p \cdot T$. This could be justified assuming that it takes T time-periods as "time-to-resume" the operation and that, even bearing the inputs' cost, any unit of outcome can be produced and sold over that period.[17]

3 A flexible input mixture technology

Accounting only for the option to switch between diets, by (1) we get that D^1 is adopted only if $C_1 < C_2$. Conditionally on the choice of γ, this relation holds when $c_t > d$ if $\gamma > 1$ or when $c_t < d$ if $\gamma < 1$. In both cases, the plant manager produces biogas with the initial diet D^1 and keeps open the option to switch to D^2. On the contrary, if $C_1 > C_2$ it is optimal to adopt D^2 (this holds when $c_t < d$ with $\gamma > 1$ or when $c_t > d$ with $\gamma < 1$), knowing that however it is possible to switch back to D^1.

Therefore, to asses the value of investing in a biogas plant with flexible diet for the digester, we must then distinguish between two scenarios or in other words between the two possible directions in which flexibility could be valuable. Under the first scenario a revision of the initial diet D^1 is desirable if c_t falls under d by choosing $\gamma \in [1, 1/\alpha]$. If this is the case it is in fact optimal to adopt a D^2 with a larger share of the biodegradable material which cost is c_t. Under the second scenario instead, if c_t rises above d the initial D^1 needs to be adjusted decreasing α and then rearranging for a more convenient D^2, i.e. by choosing $\gamma \in [0, 1]$.

Furthermore, if c_t is too high and the technology adopted is not flexible enough to allow for the needed adjustment, the plant manager may also consider to suspend temporarily the production and later decide to resume or abandon the project.

Finally, since the decision to increase or decrease the presence of a factor in the diet is state-contingent, the two scenarios may be seen as symmetric.[18]

[17] If the reactivation cost due to the needed "time-to-resume" is given by the revenue foregone, it is easy to show that $F_R = \int_0^T pe^{-r\tau} d\tau = (1 - e^{-rT})\frac{p}{r}$. Hence, given that $e^{-rT} \simeq 1 - rT + ...$, it follows $F_R \simeq pT$.

[18] If one sets as original diet $D_1 = (\frac{1}{2}, \frac{1}{2})$, by fixing $\gamma = \frac{3}{2}$ the plant manager holds the option to switch to $D_2 = (\frac{3}{4}, \frac{1}{4})$. However, in the state $c < d$, if one sets $D_1 = (\frac{3}{4}, \frac{1}{4})$, by fixing $\gamma = \frac{2}{3}$ the plant manager may switch to $D_2 = (\frac{1}{2}, \frac{1}{2})$. Therefore, in both states the plant manager may switch on and back between the same two diets.

We proceed then the analysis assuming d as *numeraire*[19] and derive the value of the flexible technology for $c_t > d$. As said before, in this case a diet D^2 is available for coping with uncertainty on c_t and the plant manager can hedge against the input prices volatility by increasing the share of the input c_t, i.e. fixing the adjustment parameter $\gamma \in [1, 1/\alpha]$.

3.1 The value of the flexible technology

Since for $\gamma \in [1, 1/\alpha]$, the condition $C_1 < C_2$ holds when $c_t > d$, the value of the value of the biogas plant functioning with the original diet D^1 is given by the solution of the following dynamic programming problem [11,12,13]:

$$\frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2 V^{\left(D^1\right)}(c_t,\alpha,\gamma)}{\partial c_t^2} - rV^{(D^1)}(c_t,\alpha,\gamma) = -\left[p - d + \alpha\left(d - c_t\right)\right] \quad \text{for} \quad d < c_t < c_M \qquad (3)$$

and with diet D^2:

$$\frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2 V^{\left(D^2\right)}(c_t,\alpha,\gamma)}{\partial c_t^2} - rV^{(D^2)}(c_t,\alpha,\gamma) = -\left[p - d + \gamma\alpha\left(d - c_t\right)\right] \quad \text{for} \quad c_t < d \qquad (4)$$

where $V^{\left(D^1\right)}$ and $V^{\left(D^2\right)}$ are respectively the value of the plant under diet D^1 and D^2, c_M is the level of c_t where mothballing the project is optimal and r is the riskless interest rate.[20] In addition, we have to consider the value of the plant when production is temporally suspended, this is given by the following differential equation:

$$\frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2 V^{(S)}(c_t,\alpha,\gamma)}{\partial c_t^2} - rV^{(S)}(c_t,\alpha,\gamma) = -m \quad \text{for } c_R \le c_t \le c_S \qquad \text{(4bis)}$$

where $V^{(S)}(c_t,\alpha,\gamma)$ is the value of the plant when the project is mothballed. The levels of c_t at which the project may be resumed or abandoned are respectively given by c_R and c_S. The general solution of the differential equations (3), (4) and (4bis) takes respectively the form:

$$V^{(D^1)}(c_t,\alpha,\gamma) = \frac{p-d}{r} + \alpha\frac{(d-c_t)}{r} + \widehat{A}_1 c_t^{\beta_1} + \widehat{A}_2 c_t^{\beta_2} \quad \text{for } d < c_t < c_M \qquad (5)$$

[19] Setting $d = 1$ would not affect our results.

[20] An interest rate incorporating a proper risk adjustment can be used taking the expectation with respect to a distribution of c_t adjusted for risk neutrality [8].

$$V^{(D^2)}(c_t, \alpha, \gamma) = \frac{p-d}{r} + \gamma\alpha\frac{(d-c_t)}{r} + \widehat{B}_1 c_t^{\beta_1} \quad \text{for } c_t < d \tag{6}$$

and

$$V^{(S)}(c_t, \alpha, \gamma) = -\frac{m}{r} + \widehat{M}_1 c_t^{\beta_1} + \widehat{M}_2 c_t^{\beta_2} \quad \text{for } c_R \le c_t \le c_S \tag{6bis}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) - r$. In (5) the term $\frac{p-d}{r} + \alpha\frac{(d-c_t)}{r}$ indicates the present value of producing biogas forever using D^1, $\widehat{A}_2 c^{\beta_2}$ represents the value of the option to switch to D^2 and $\widehat{A}_1 c^{\beta_1}$ is the value attached to the temporary suspension. Instead in (6), $\frac{p-d}{r} + \gamma\alpha\frac{(d-c_t)}{r}$ is the present value of producing biogas forever adopting D^2, while $\widehat{B}_1 c^{\beta_1}$ is the value of the option to switch back to D^1.[21] Finally, in (6bis), $-\frac{m}{r} + \widehat{M}_1 c_t^{\beta_1} + \widehat{M}_2 c_t^{\beta_2}$ is the value of the firm when operation is mothballed. The first term represents the present value of the flow of suspension costs. The second term is the value of the option to abandon the project and the third term is instead the value of the option to reactivate the biogas production.

To determine the constants $\widehat{A}_1$, $\widehat{A}_2$, $\widehat{B}_1$, $\widehat{M}_1$, $\widehat{M}_2$ and the critical levels c_M, c_R, c_S at every switching point value-matching and smooth-pasting conditions must be satisfied. At $c_t = d$

$$\begin{aligned} V^{(D^1)}(d, \alpha, \gamma) &= V^{(D^2)}(d, \alpha, \gamma) \\ V_c^{(D^1)}(d, \alpha, \gamma) &= V_c^{(D^2)}(d, \alpha, \gamma) \end{aligned}$$

then at $c_t = c_M$

$$\begin{aligned} V^{(D^1)}(c_M, \alpha, \gamma) &= V^{(S)}(c_M, \alpha, \gamma) \\ V_c^{(D^1)}(c_M, \alpha, \gamma) &= V_c^{(S)}(c_M, \alpha, \gamma) \end{aligned}$$

and finally at[22] c_R and c_S

$$\begin{aligned} V^{(S)}(c_R, \alpha, \gamma) &= V^{(D^1)}(c_R, \alpha, \gamma) - F_R \\ V_c^{(S)}(c_R, \alpha, \gamma) &= V_c^{(D^1)}(c_R, \alpha, \gamma) \\ V^{(S)}(c_S, \alpha, \gamma) &= -F_S \\ V_c^{(S)}(c_S, \alpha, \gamma) &= 0. \end{aligned}$$

[21] Note that under D^2 the general solution to (4) should have the form $V^{(D^2)}(c_t, \alpha) = \frac{p-d}{r} + \gamma\alpha\frac{(d-c_t)}{r} + \widehat{B}_1 c_t^{\beta_1} + \widehat{B}_2 c_t^{\beta_2}$. However, as $c_t \to 0$, the option to switch to D^1 is valueless and should go to zero. But this holds only if $\widehat{B}_2 = 0$.

[22] Note that if $c_R < d$ then the value-matching and smooth-pasting conditions should be $V^{(S)}(c_R, \alpha, \gamma) = V^{(D^2)}(c_R, \alpha, \gamma) - F_R$, $V_c^{(S)}(c_R, \alpha, \gamma) = V_c^{(D^2)}(c_R, \alpha, \gamma)$.

The system of eight equations provides a complete frame for the analysis of the project operation. The real options literature posits that the availability of strategic options always increase the value of a project.[23] Hence, all the constants $\widehat{A}_1$, $\widehat{A}_2$, $\widehat{B}_1$, $\widehat{M}_1$, $\widehat{M}_2$ must be non-negative [11].

However, according to empirical evidence once set up biogas projects are rarely mothballed or abandoned. This may be due to both low maintenance and scrapping costs and long time to resume operations. In fact, we state and prove

Proposition 1 *Provided that* $\frac{m}{r} \leq F_S$ *and* T *is high enough, as* $F_S \to 0$ *and* $m \to 0$ *the option to mothball, to reactivate and to abandon may be neglected.*

Proof. See section A.1 in the appendix for the proof and a deep discussion. ■

If the value attached to such options has small impact on the investment choice then only the flexibility driven by the option to switch between the two diet regimes should matter. This in turn allows reducing the system above to the first two equations which solution is given by:

$$\begin{aligned} \widehat{A}_2 &= (\gamma-1)A = (\gamma-1)\frac{\alpha}{r\left(\beta_1-\beta_2\right)}d^{1-\beta_2} \geq 0 \\ \widehat{B}_1 &= (\gamma-1)B = (\gamma-1)\frac{\alpha}{r\left(\beta_1-\beta_2\right)}d^{1-\beta_1} \geq 0 \end{aligned}$$

Substitution into (3) and (4) finally gives

$$V^{(D^1)}(c_t,\alpha,\gamma) = \frac{p-d}{r} + \alpha\frac{(d-c_t)}{r} + (\gamma-1)Ac_t^{\beta_2} \quad \text{for } d < c_t < \infty \tag{7}$$

$$V^{(D^2)}(c_t,\alpha,\gamma) = \frac{p-d}{r} + \gamma\alpha\frac{(d-c_t)}{r} + (\gamma-1)Bc_t^{\beta_1} \quad \text{for } c_t < d \tag{8}$$

Note that $\widehat{A}_2$, $\widehat{B}_1$ are positive as long as $\gamma > 1$. For $\gamma = 1$, the options to switch on and back between D^1 and D^2 are not available and their value is null as it should be.

Finally, as evident from (7), for the investment decision to be sensible we assume $p > (1-\alpha)d$. In fact, as it will become clear later, this is a necessary condition for the existence of a positive time trigger for the investment.[24] Formally this requires to introduce a restriction on the set of D^1 such that $p/d > 1-\alpha$.

[23] See [13] (chs. 6 and 7) for a exhaustive discussion.

[24] A similar analysis can be developed also for $\gamma \in [0,1]$ where $C_1 < C_2$ holds when

4 Optimal γ with a flexible diet technology

When the current cost c_t is such that $d < c_t < \infty$, the manager's problem is to define the optimal γ by which to revise D^1 in order to benefit from a fall of c_t in the future. In other words, the plant is producing biogas by using the original diet D^1 but the manager holds the option to switch to a new diet D^2 if c_t fluctuates below d. In D^2, where $\gamma > 1$, the presence of the factor c_t is increased. This implies that the decision to increase or decrease the presence of a factor in the diet is state-contingent.

The optimal γ must maximize (8) minus the cost of setting up such a flexible productive technology:

$$\gamma^* = \arg\max NPV(c_t, \alpha, \gamma) \quad s.t.\ \gamma > 1 \qquad \text{for } d < c_t < \infty$$

where $NPV(c_t, \alpha, \gamma) = V^{(D^1)}(c_t, \alpha, \gamma) - I(\alpha, \gamma)$, and $I(\alpha, \gamma)$ is the sunk cost of developing the flexible biogas plant which allows to revise the diet from D^1 to D^2.

Being our analysis focused on the cost of the flexibility, we model $I(\alpha, \gamma)$ as an increasing cost-to-scale Cobb-Douglas quadratic in $(\gamma - 1)$, i.e.:[25]

$$I(\alpha, \gamma) = K(\alpha) \frac{(\gamma - 1)^2}{2} \tag{9}$$

where $K(\alpha)$ is a unit installation cost accounting for the storage capacity of both the gas-holder and the digester.[26]

$c_t < d$. However, it should be noted that having assumed $p > (1 - \alpha)d$, the expected net present value over the range $c_t < d$ is higher than over $c_t > d$. This may induce as desirable diet revision the extreme $\gamma = 0$ where the sole input d is used. In this case the value of the option to mothball, to reactivate and to abandon should vanish even faster due to the option to switch diets potentially allowing for a complete hedge against fluctuations on c_t. This is an issue which would deserve further attention but it is beyond the scope of this paper.

[25] A fixed investment cost K_0 independent on γ may be included. However, [10,17] respectively report that scale economies and technological progress have importantly reduced such cost over the last decade.

[26] The digester reactors can be constructed by using brick, cement, concrete and steel, while the gas holder is normally an airproof steel container. For [20] the diet composition influences the cost of the storage capacity and, in particular, the cost of the digester capacity.

The function $I(\alpha,\gamma)$ is convex on γ and satisfies $I(\alpha,1)=0$, $I_\gamma(\alpha,\gamma)>0$ when $\gamma>1$ and $I_\gamma(\alpha,1)=0$. In other words, the cost of setting up a flexible technology is normalized to zero for $\gamma=1$, and it increases according to the initial diet (D^1) and the "distance" between D^2 and D^1.[27] Being $[1,1/\alpha]$ the feasible range for γ, to guarantee some symmetry of the cost for revising D^1 over that set, we assume that the organization cost is $K(\alpha)=k\frac{\alpha}{1-\alpha}$ with $k\in R_+$.[28]

Finally, the assumption of a quadratic cost function is a matter of realism. Without convexity we would get extreme outcomes, i.e., either the use of d or c_t as sole input ($\gamma=0$ or $\gamma=1/\alpha$). Convexity provides the most realistic view to accommodate non-modal choices.[29] Given (9) we state

Proposition 2 *The optimal flexible technology when investing at $c_t\in(d,\infty)$ is*

$$\gamma^*=\begin{cases}1+\frac{A}{K(\alpha)}c_t^{\beta_2} & \text{for } \widehat{c}\leq c_t<\infty\\ \frac{1}{\alpha} & \text{for } d<c_t<\widehat{c}\end{cases}\tag{10}$$

where $A=\frac{\alpha}{r(\beta_1-\beta_2)}d^{1-\beta_2}$ and $\widehat{c}=\left(\frac{k}{A}\right)^{1/\beta_2}$.

Proof. See section A.2 in the appendix for the proof. ■

Proposition (2) shows that if c_t is high it is worth to have less flexibility. In fact, as $c_t\to\infty$, it is easy to verify $\gamma^*\to 1$. This means that as c_t rises it becomes less likely have a fall which magnitude is sufficient to justify an investment in flexibility. In other words, it does not make sense to invest in technology which flexibility is probably going not to be exploited. Note that for $\widehat{c}\leq d$ the manager always chooses $\gamma^*<\frac{1}{\alpha}$.

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[27] An efficient anaerobic digestion requires that both the liquefaction and gasification steps are properly balanced. In fact, when methane bacteria are absent, the digestion process may start only by liquefying the material. On the other hand, if liquefaction occurs at a faster rate the resultant accumulation of acids may inhibit the process as well [9,19].

[28] Note that $K(\alpha)=k\frac{\alpha}{1-\alpha}$ is assumed to capture the greater cost of operating with a digester when the initial diet is mainly based on one of the two input factors.

[29] Note that the quadratic form has been assumed for simplicity. A more general form for the investment cost such as $I(\alpha,\gamma)=K(\alpha)\frac{(\gamma-1)^\phi}{\phi}$ with $\phi>1$ may be assumed. However, this would only add complexity without altering the results obtained in this paper.

Further, by simply substituting (10) into $NPV(c_t, \alpha, \gamma)$ we rearrange the state-contingent net present value of the adopted technology as

$$NPV(c_t, \alpha, \gamma^*(c_t)) = \begin{cases} \frac{p-C_1}{r} + \frac{1}{K(\alpha)} \frac{(Ac_t^{\beta_2})^2}{2} & \text{for } \widehat{c} \leq c_t < \infty \\ \frac{p-C_1}{r} + \frac{1}{K(\alpha)} (Ac_t^{\beta_2} - \frac{1}{2}) & \text{for } d < c_t < \widehat{c} \end{cases} \tag{11}$$

5 The optimal timing of investment

In this section we derive the value of the option to invest in the plant producing biogas with flexible diet for the digestor as well as the optimal timing rule. In the specific we assume that once installed the plant produces biogas using the original diet D^1, while keeping the flexibility to move to a new one, D^2, every time c_t fluctuates above (below) the cost of the other input d. Hence, for any given diet D^1, it makes sense to assume that the plant manager fixes the optimal diet revision (i.e. how much to diverge from D^1), at the time the investment in the flexible biogas plant is undertaken.

Denoting by $F(c_t)$, the value of the option to invest in the plant, this is given by the solution of the following dynamic problem:

$$\frac{1}{2}\sigma^2 c_t^2 \frac{\partial^2 F(c_t)}{\partial c_t^2} - rF(c_t) = 0 \tag{12}$$

which general solution is

$$F(c_t) = H_1 c_t^{\beta_1} + H_2 c_t^{\beta_2}. \tag{12bis}$$

and where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of $\phi(\beta)$.

Let consider first the option to invest in the region $d < c_t < \infty$ where $\gamma^* > 1$. Since for $c_t \to \infty$ the value of the option to invest in such a technology, $F(c_t)$, should vanish, the boundary condition $\lim_{c_t \to \infty} F(c_t) = 0$ is required. It follows that for such condition to hold H_1 must be null and the general solution should take the form

$$F(c_t) = H_2 c_t^{\beta_2} \quad \text{for} \quad d < c^* < c_t \tag{13}$$

where c^* is the threshold where it is efficient to activate the technology.

As standard in the optimal investment literature the constant H_2 and the optimal investment trigger c^* can be derived attaching to (13) the following matching value and smooth pasting conditions:

$$F(c^*) = NPV(c^*, \alpha, \gamma^*(c^*)) \tag{14}$$

$$F'(c^*) = NPV_c(c^*, \alpha, \gamma^*(c^*)) \tag{15}$$

where NPV is given by (11).[30]

Since by (11), within the region $d < c_t < \infty$ we may get two optimal solutions for the diet adjustment parameter γ^*, the value of the option to invest $F(c_t)$, the constant H_2 as well as the optimal investment trigger c^* must be evaluated separately in the two subsets bounded by $\widehat{c} > d$. In particular we may get:

Proposition 3 *1) If $d < c^* < \widehat{c}$, the optimal investment trigger for the flexible technology is given by*

$$c^* = \frac{\beta_2}{\beta_2 - 1}\left[\frac{p-(1-\alpha)\,d}{\alpha} - \frac{r}{2\alpha K(\alpha)}\right] \qquad \text{where} \quad c^* \in (d, \widehat{c}). \tag{16}$$

2) If $c^* \geq \widehat{c}$, the optimal investment trigger for the flexible technology is given by the solution of the following implicit equation

$$c^* = \frac{\beta_2}{\beta_2 - 1}\left[\frac{p-(1-\alpha)\,d}{\alpha} - \frac{r}{2\alpha K(\alpha)}(Ac^{*\beta_2})^2\right] \qquad \text{where} \qquad c^* \in [\widehat{c}, \infty) \tag{17}$$

Proof. See below for (the proof) and a deep discussion. ■

Let consider first the case where $d < c^* \leq \widehat{c}$. Since in this range the plant manager will invest in a technology adopting $\gamma^* = 1/\alpha$, substituting this value into (14) and (15) after some substitutions we obtain

$$c^* = \frac{\beta_2}{\beta_2 - 1}\left[\frac{p-(1-\alpha)\,d}{\alpha} - \frac{r}{2\alpha K(\alpha)}\right]$$

$$H_2 = \left[\frac{p-(1-\alpha)\,d - c^*}{r} + \frac{1}{K(\alpha)}(Ac^{*\beta_2} - \frac{1}{2})\right] c^{*-\beta_2}$$

[30] Totally differentiating $F(c^*)$ one obtain $F'(c^*) = NPV_c(c^*, \alpha, \gamma^*(c^*)) + NPV_\gamma(c^*, \alpha, \gamma^*(c^*))\frac{d\gamma^*}{c_t}$. But being γ optimally chosen then $NPV_\gamma(c^*, \alpha, \gamma^*(c^*)) = 0$.

Now, define $g(c_t) = 2[\frac{p-(1-\alpha)d-\alpha c_t(1-\frac{1}{\beta_2})}{r}]K(\alpha)$. Conditions such that $c^* \in (d, \widehat{c})$ require that $c^* < \widehat{c}$ is satisfied iff $g(\widehat{c}) < 1$, and $d < c^*$ iff $g(d) > 1$.

Finally, by (13) and (14), the optimal value of the option to invest is given by

$$F(c_t) = \begin{cases} \left[\frac{p-(1-\alpha)d-\alpha c^*}{r} + \frac{1}{K(\alpha)}(Ac^{*\beta_2} - \frac{1}{2})\right]\left(\frac{c_t}{c^*}\right)^{\beta_2} & \text{for } c^* < c_t < \widehat{c} \\ \frac{p-C_1}{r} + \frac{1}{K(\alpha)}(Ac_t^{\beta_2} - \frac{1}{2}) \quad \text{for } c_t \leq c^* & \end{cases} \tag{18}$$

On the contrary, if $c^* \geq \widehat{c}$, the plant manager invests adopting $\gamma^* = 1 + \frac{Ac_t^{\beta_2}}{K(\alpha)}$. Again, substituting this value into (14) and (15) after some manipulation we obtain:

$$\begin{aligned} \left(Ac^{*\beta_2}\right)^2 &= 2K(\alpha)\left[\frac{p-(1-\alpha)d}{r} - (1-\frac{1}{\beta_2})\frac{\alpha}{r}c^*\right] \\ H_2 &= \left[\frac{p-(1-\alpha)d-\alpha c^*}{r} + \frac{1}{K(\alpha)}\frac{(Ac^{*\beta_2})^2}{2}\right]c^{*-\beta_2} \end{aligned}$$

where c^* is the solution of an implicit equation. Although the equation should be solved numerically, it is easy to note that it has two positive solutions for the investment trigger c^*. This is quite clear in figure 1 where $f(c_t) = (Ac_t^{\beta_2})^2$. Then, for an optimal solution one needs $F''(c^*) > 0$. Checking, it is easy to realize that only the highest trigger satisfies this condition.[31]

In this case, the value of the option to invest is given by

$$F(c_t) = \begin{cases} \left[\frac{p-(1-\alpha)d-\alpha c^*}{r} + \frac{1}{K(\alpha)}\frac{(Ac^{*\beta_2})^2}{2}\right]\left(\frac{c_t}{c^*}\right)^{\beta_2} & \text{for } c^* < c_t \\ \frac{p-C_1}{r} + \frac{1}{K(\alpha)}\frac{(Ac_t^{\beta_2})^2}{2} & \text{for } \widehat{c} \leq c_t \leq c^* \end{cases} \tag{19}$$

Now, note that since $\widehat{c} = \left(\frac{k}{A}\right)^{1/\beta_2}$ it follows that $f(\widehat{c}) = k^2$. It is then easy to show that a sufficient condition for $c^* \geq \widehat{c}$ is $g(\widehat{c}) \geq k^2$. See figure 2. To fix conditions for the existence of the triggers will become crucial to interpret

[31] Further, as the cost of the input c_t decreases, we assume c_t sufficiently high to guarantee that the first trigger met is always the highest.

the outcomes of numerical simulations contained in the next section.

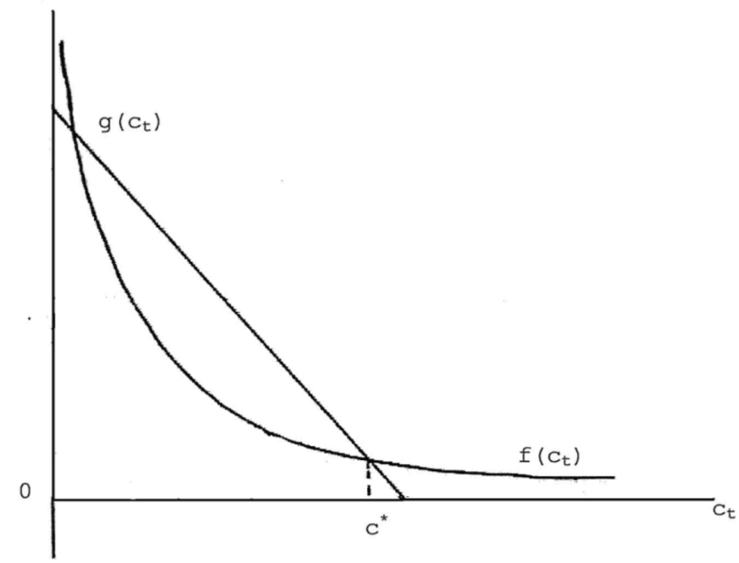

Figure 1

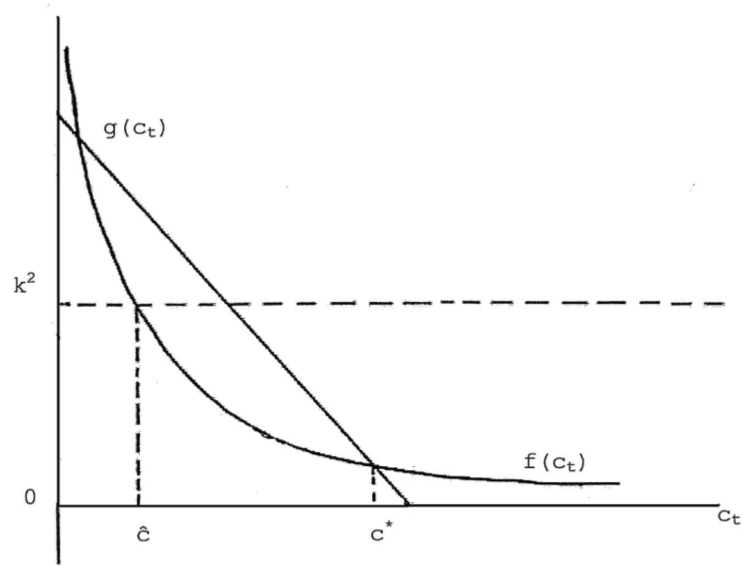

Figure 2

6 Numerical simulations

The scenario chosen is given by a biogas plant under a 1 MW capacity using a Combined Heat and Power (CHP) system to burn biogas and produce electricity.[32] Output price[33] and maintenance/operating cost[34] are chosen consistently with evidence from Italy. To show the properties of the time triggers defined above, we fix as initial diet,[35] $D^1 = (0.3; 0.7)$, and then study the effect of changes in the volatility (σ) and investment cost (k) on the optimal thresholds.

In table 2, we check the effect of input price uncertainty on the timing for investment for three different levels of uncertainty (σ). Fixing the output price net of operating cost, $p = 1.3$, we note that on both intervals, $(d, \widehat{c})$ and $[\widehat{c}, \infty)$, c^* increases as σ soars.[36] In expected terms, this implies that starting from a $c_t > c^*$, the investment occurs earlier as volatility decreases. Allowing for a lower price, $p = 1.2$, this outcome is still confirmed. This result is in line with the conventional insight in the real option literature positing that as uncertainty on future prospect increases the value of the option to invest increases as well. This in turn implies that the exercise of the option should be postponed to benefit from information collection and reduce regret for rushing. In table 3, letting k increase the effect of σ on the thresholds is again confirmed. In table 2, one may note that for $p = 1.3$, the optimal adjustment is $\gamma^* = 3.333$ with $\sigma = 0.2$, while it importantly decreases with $\sigma = 0.1$ where $\gamma^* = 1.33$. This is justified by the need for hedging against uncertainty. The investor requires a technology allowing for a substantial change in the original diet in order to be flexible enough to

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[32] Utilised for CHP production 1 m^3 of biogas provides 21 MJ. By the equivalence 3.6 MJ=1 kWh and, for instance, allowing a thermal efficiency of 89%, 1 m^3 of biogas corresponds to 5.18 kWh electricity. For further details see http://en.wikipedia.org/wiki/Cogeneration.

[33] In Italy, according to legislative decree 159/2007, for plants under a 1 MW capacity a 0.3 euro/kWh rate is paid on the electricity provided. The rate includes benefits from RECs' trade.

[34] [4] reports maintenance/operating costs in a range of 0.025 to 0.040 euro/kWh depending on the plant scale. Thus, not accounting for the options to mothball and abandon the project makes sense.

[35] See [6] assessing performances for different feed regimes.

[36] Since 1 m^3 of biogas corresponds to 5.18 kWh and a kWh is paid 0.3 euro, we get $p = 1.554$ euro/m^3 gross of operating costs.

respond to fluctuations in the input relative convenience. This advantage clearly comes at a greater cost. In fact, as shown in table 3, γ^* decreases as k increases and one needs to trade off benefit and cost of having a flexible technology. However, as illustrated by all cases in both table 2 and table 3, the flexible technology we have drawn is a desirable device against uncertainty in that as uncertainty increases the $NPV(c^*, \alpha, \gamma^*(c^*))$ increases as well.

To complete our analysis and make clearer the way our model works we discuss two cases with plots. Take $\alpha = 0.3$, $k = 1$, $d = 1$, $r = 0.07$, $\sigma = 0.2$. In this case $\widehat{c} = 1.073036703 > d = 1$. This means that both regions $(d, \widehat{c})$ and $[\widehat{c}, \infty)$ exist. The thresholds should be respectively given by $c^* = 1.018652523$ and $c^* = 0.9544704060$. As one can note the former solution is correctly elicited in that $1 < 1.018652523 < 1.073036703$. On the contrary, $0.9544704060 < 1$, and this contradicts conditions for the existence in that c^* should belong to $[\widehat{c}, \infty)$. This implies that the threshold for the exercise of the option to invest in the region $[\widehat{c}, \infty)$ is given by the lowest value in the interval and then $c^* = \widehat{c}$. In figure 3 we plot $g(c_t)$ and $f(c_t)$ to illustrate this case.

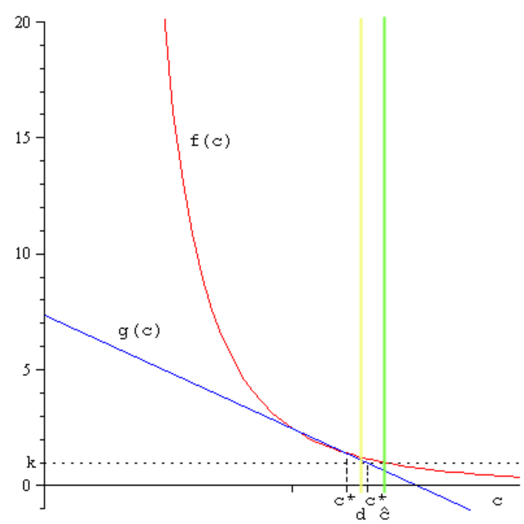

Figure 3

Note in table 3 that $\widehat{c} < d$ for $\sigma = 0.1$ and for every σ when $k = 5$ and $k = 10$. This implies that the time trigger for the exercise of the option to invest should belong to the region (d, ∞) and is given by the solution to (17). This is not always the case as one can see for $\sigma = 0.3$ and $k = 5$, $k = 10$ where $c^* \notin (d, \infty)$ and then $c^* = 1$. Finally, for $\sigma = 0.3$ and $k = 1$, we have $\widehat{c} > d$ but a solution to (17) does not exist at all and any trigger is defined on the region $\widehat{c} \leq c_t$.

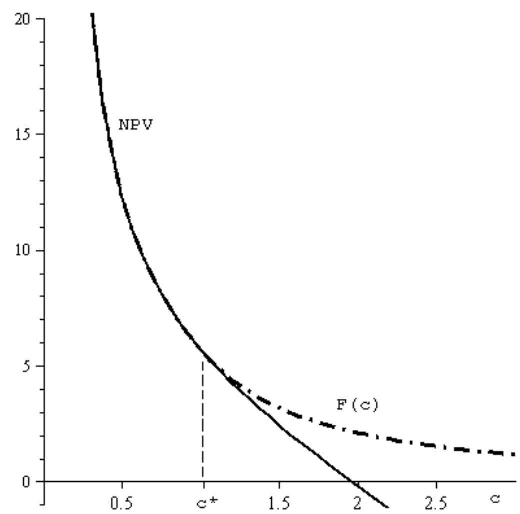

Figure 4

In Figure 4 we plot the net present value $NPV(c_t, \alpha, \gamma^*(c^*))$ and the value of the option to invest $F(c_t)$. At the optimal trigger, $c^* = 1.018652523$, determined by imposing smooth-pasting, the two curves are tangent. If the price of the input, c_t, is below the trigger it is optimal to invest, otherwise one should wait. This is effectively illustrated by figure 4 where up to the optimal trigger, $NPV(c_t, \alpha, \gamma^*(c^*))$ lies below the dash-dot line representing $F(c_t)$. However, even if this seems to completely resemble to the standard finding in the real option literature, we want to stress that the net present value function, $NPV(c^*, \alpha, \gamma^*(c^*))$ is defined only at c^* where the optimal adjustment, $\gamma^*(c^*)$, is chosen.[37]

Now, let analyse the case where $\alpha = 0.3$, $k = 5$, $d = 1$, $r = 0.07$, $\sigma = 0.2$, $r = 0.07$. Here, $\widehat{c} = 0.3499719715 < d = 1$. This means that γ^* is always lower than $1/\alpha$ and $c^* \in (d, \infty)$. The optimal threshold is $c^* = 1.153039880$. See figure 5 to have a general picture. As above in figure

[37] We remind that the decision to increase the presence of a factor in the diet is state-contingent.

6 we plot $NPV(c_t, \alpha, \gamma^*(c^*))$ and $F(c_t)$ and the same discussion applies.

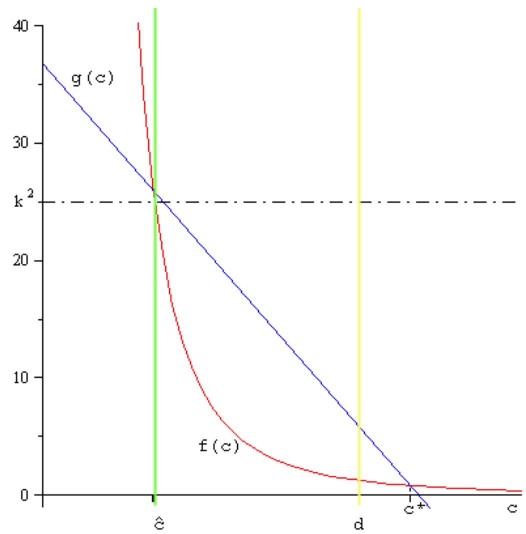

Figure 5

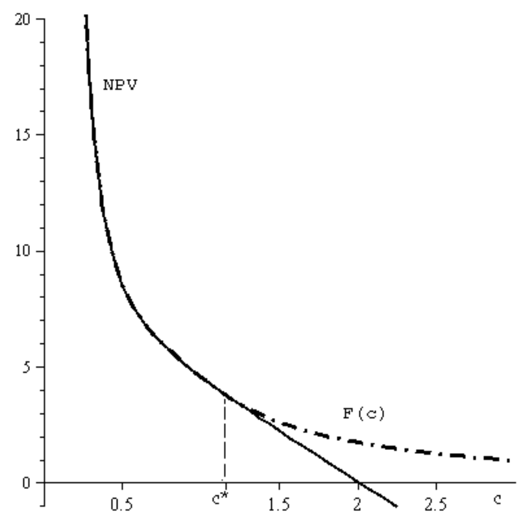

Figure 6

Now, let clarify the point raised in the previous section on the non-optimality of a second solution to (17). In figure 7 we plot $NPV(c_t, \alpha, \gamma^*(c'))$ and $F(c_t)$ relative to the second solution $c' = 0.345336790$. At c' the two curves are tangent but as shown in the figure, $F(c_t)$ intersects $NPV(c_t, \alpha, \gamma^*(c'))$

before and lies below $NPV(c_t, \alpha, \gamma^*(c'))$ from that point on. This is an evident contradiction in that it would imply an earlier exercise of the option and allow us to characterize c' as not optimal.

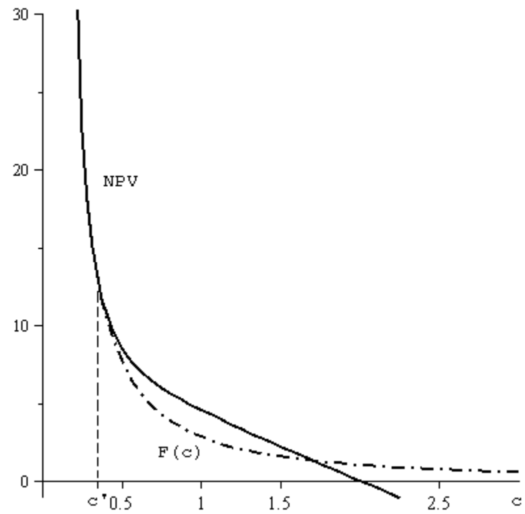

Figure 7

Table 1: Betas for $r = 0.07$

σ	0.1	0.2	0.3
β_1	4.274917218	2.436491673	1.843709624
β_2	−3.274917218	−1.436491673	−0.843709624
$\frac{\beta_2}{\beta_2-1}$	0.7660773416	0.5895738077	0.457615241

Table 2: Simulating for a change in p change $\alpha = 0.3,\ k = 1,\ d = 1,\ r = 0.07$

σ	0.1		0.2		0.3	
RANGE	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$
$\widehat{c}$	0.8412203934		1.073036703		1.738737671	
$p = 1.3$						
c^*	–	1.527972123	1.018652523	1.073036703	1	–
$\pi(c^*)$	–	0.1416083631	0.2944042431	0.2780889891	0.3	–
$\gamma^*(c^*)$	–	1.330445451	3.333333333	3.333333333	3.333333333	–
$NPV(c^*,\alpha,\gamma(c^*))$	–	2.046375372	5.553453968	5.139366510	6.840089657	–
$p = 1.2$						
c^*	–	1.262171830	1	–	1	–
$\pi(c^*)$	–	0.1213484510	0.2	–	0.2	–
$\gamma^*(c^*)$	–	1.617885368	3.333333333	–	3.333333333	–
$NPV(c^*,\alpha,\gamma(c^*))$	–	1.815359799	4.272465089	–	7.902232939	–

Table 3: Simulating for a change in k with $\alpha = 0.3,\ p = 1.3,\ d = 1,\ r = 0.07$

σ	0.1		0.2		0.3	
RANGE	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$	$d < c_t < \widehat{c}$	$\widehat{c} \leq c_t$
$k = 1$						
$\widehat{c}$	0.8412203934		1.073036703		1.738737671	
c^*	–	1.527972123	1.018652523	1.073036703	1	–
$\pi(c^*)$	–	0.1416083631	0.2944042431	0.2780889891	0.3	–
$\gamma^*(c^*)$	–	1.330445451	3.333333333	3.333333333	3.333333333	–
$NPV(c^*, \alpha, \gamma(c^*))$	–	2.046375372	5.553453968	5.139366510	6.840089657	–
$k = 5$						
$\widehat{c}$	0.5146102617		0.3499719715		0.2580985265	
c^*	–	1.531330113	–	1.153039880	–	1
$\pi(c^*)$	–	0.1406009661	–	0.2540880360	–	0.3
$\gamma^*(c^*)$	–	1.065615659	–	1.420867610	–	1.744208408
$NPV(c^*, \alpha, \gamma(c^*))$	–	2.013198174	–	3.819610741	–	4.879120880
$k = 10$						
$\widehat{c}$	0.4164458732		0.2160101205		0.1134990327	
c^*	–	1.531938254	–	1.166522537	–	1
$\pi(c^*)$	–	0.1404185238	–	0.2500432389	–	0.3
$\gamma^*(c^*)$	–	1.032765197	–	1.206948814	–	1.372104204
$NPV(c^*, \alpha, \gamma(c^*))$	–	2.008279393	–	3.663820152	–	4.582417583

7 Conclusions

In this paper we analyses the effect of flexibility on decision-making from a novel perspective. In a dynamic uncertain frame, we model a problem where inputs are substitute but differently from [15] they need to be mixed together to provide output. The choice of the technology is taken in the light of the option to adjust the initial rule if economic circumstances require it. The option to switch between two combinations of the input factors adds value to the project in that it provides a device for hedging against fluctuations in the input relative convenience. The "distance" between the initial rule and a desirable alternative is a technologically feasible but costly requirement. This allows contributing with an innovative analysis where the extent of initial investment is traded off with the advantage in terms of profit smoothing coming from the flexible technology optimally chosen.

A Appendix

A.1 On the negligibility of option to mothball, reactivate and abandon

We substitute (5), (6) and (6bis) in the conditions needed for characterizing a flexible technology. Rearranging, we obtain the following system:

$$(\widehat{A}_1 - \widehat{B}_1)d^{\beta_1} + \widehat{A}_2 d^{\beta_2} = 0 \tag{a}$$

$$(\widehat{A}_1 - \widehat{B}_1)\frac{\beta_1}{\beta_2}d^{\beta_1} + \widehat{A}_2 d^{\beta_2} = (1-\gamma)\,\frac{\alpha d}{r\beta_2} \tag{b}$$

$$(\widehat{A}_1 - \widehat{M}_1)c_M + (\widehat{A}_2 - \widehat{M}_2)c_M \tag{c}$$

$$= -\frac{p - d(1-\alpha) - \alpha c_M + m}{r}$$

$$(\widehat{A}_1 - \widehat{M}_1)\frac{\beta_1}{\beta_2}c_M + (\widehat{A}_2 - \widehat{M}_2)c_M = \frac{\alpha c_M}{r\beta_2} \tag{d}$$

$$(\widehat{A}_1 - \widehat{M}_1)c_R + (\widehat{A}_2 - \widehat{M}_2)c_R \tag{e}$$

$$= -\frac{p(1-rT) - d(1-\alpha) - \alpha c_R + m}{r}$$

$$(\widehat{A}_1 - \widehat{M}_1)\frac{\beta_1}{\beta_2}c_R + (\widehat{A}_2 - \widehat{M}_2)c_R = \frac{\alpha c_R}{r\beta_2} \tag{f}$$

$$\widehat{M}_1 c_S + \widehat{M}_2 c_S = \frac{m}{r} - F_S \tag{g}$$

$$\widehat{M}_1\frac{\beta_1}{\beta_2}c_S + \widehat{M}_2 c_S = 0 \tag{h}$$

From (g) and (h) $\widehat{M}_1$ and $\widehat{M}_2$ should have the same sign. Suppose now that $\frac{m}{r} < F_S$. This would imply that $\widehat{M}_1$ and $\widehat{M}_2$ should be negative. This makes economic sense considering that if $\frac{m}{r} < F_S$ then the suspension regime is always preferred to the abandon. In other words, the option to scrap the project is never considered by the manager and could be dropped out of the problem. Finally, consider $F_S \to \frac{m}{r}$. By the boundary condition $\lim_{c_t} \widehat{M}_1 c_t^{(\gamma>1)\beta_1} + \widehat{M}_2 c_t^{(\gamma>1)\beta_2} = 0$ it follows that $\widehat{M}_1 \to 0$, $c_S \to \infty$ and $\widehat{M}_2 > 0$. Hence, for F_S sufficiently high (e.g. $F_S \geq \frac{m}{r}$) the opportunity of abandoning the project is never taken ($c_S \to \infty$).

Now, we consider the subsystem including (c), (d), (e) and (f). Define

$$\begin{aligned} W(c_t) &= V^{(D^1)}(c_t,\alpha,\gamma) - V^{(S)}(c_t,\alpha,\gamma) \\ &= Q_1 c_t^{\beta_1} + Q_2 c_t^{\beta_2} + \frac{p - d(1-\alpha) - \alpha c_t + m}{r} \end{aligned} \tag{A.1.1}$$

where $Q_1 = (\widehat{A}_1 - \widehat{M}_1)$ and $Q_2 = (\widehat{A}_2 - \widehat{M}_2)$. The value matching and smooth past conditions can be rearranged in terms of $W(c_t)$ as follows:

$$W(c_M) = 0, \; W(c_R) = pT \tag{A.1.2}$$

$$W_c(c_M) = 0, \; W_c(c_R) = 0 \tag{A.1.3}$$

The function $W(c_t)$ can be drawn as shown in figure 8. Heuristically, we have adjusted Q_1 and Q_2 until $W(c_t)$ has become tangent to the line pT in c_R and to 0 in c_M. Hence, $Q_1 \geq 0$, $Q_2 \leq 0$, $W_{cc}(c_M) > 0$ and $W_{cc}(c_R) < 0$. Subtracting (4bis) by (3) we get

$$\frac{1}{2}\sigma^2 c_t^2 W_{cc}(c_t) - rW(c_t) = -\left[p - d(1-\alpha) - \alpha c_t + m\right] \tag{A.1.4}$$

Evaluating (A.1.4) using conditions in (A.1.2) and (A.1.3) we get first

$$\frac{1}{2}\sigma^2 c_M W_{cc}(c_M) = -\left[p - d(1-\alpha) - \alpha c_M + m\right] > 0$$

which implies that

$$c_M > \frac{p - d(1-\alpha) + m}{\alpha} \tag{A.1.5}$$

and second

$$\frac{1}{2}\sigma^2 c_R W_{cc}(c_R) - rpT = -\left[p - d(1-\alpha) - \alpha c_R + m\right] < -rpT$$

from which it follows that

$$c_R < \frac{p(1-rT) - d(1-\alpha) + m}{\alpha} < c_M \tag{A.1.6}$$

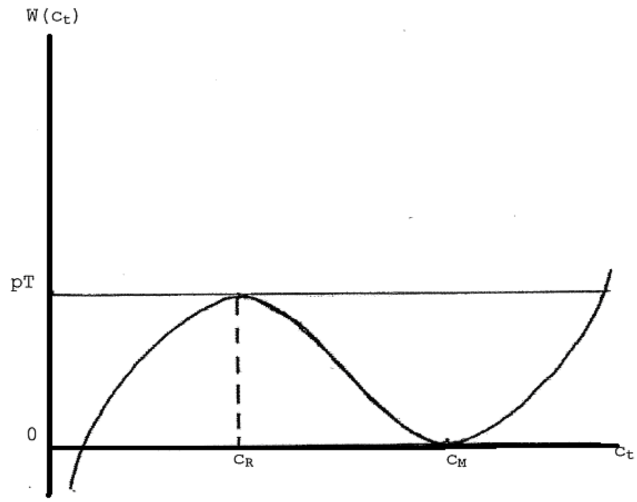

Figure 8

Note that as $T \to 0$, both $c_R$and c_M converge toward the same limit $\frac{p-d(1-\alpha)+m}{\alpha}$. On the contrary, if $T > \frac{p-d(1-\alpha)+m}{rp}$, the project is never resumed as $c_R < 0$. For a complete analysis, suppose T changes by dT. Differentiating the conditions in (A.1.2) we get[38]

$$\begin{aligned} W_{Q_1}(c_M)dQ_1 + W_{Q_2}(c_M)dQ_2 &= 0 \\ W_{Q_1}(c_R)dQ_1 + W_{Q_2}(c_R)dQ_2 &= pdT \end{aligned}$$

We solve the system substituting for $W_{Q_1}(c_M) = c_M$, etc., and we derive as solutions:

$$\begin{aligned} dQ_1 &= -\frac{pdT}{c_M\left(\frac{c_R}{c_M}\right)^{\beta_2} - c_R} < 0 \\ dQ_2 &= \frac{pdT}{c_R - c_M\left(\frac{c_R}{c_M}\right)^{\beta_1}} > 0 \end{aligned}$$

A brief comment on these results is needed. First, we know now that as T increases $\widehat{A}_1 \to \widehat{M}_1$. This means that as T increases the value of reactivation

[38]Note that by (A.1.3) $W_c(c_M^{(\gamma>1)})dc_M^{(\gamma>1)} = W_c(c_R^{(\gamma>1)})dc_R^{(\gamma>1)} = 0$.

vanishes and the value of option to suspend ($\widehat{A}_1$) consistently converge to the value of the option to abandon ($\widehat{M}_1$). As shown above as $\widehat{M}_1 \rightarrow 0$ in that scrapping is not convenient the option to suspend is valueless as $\widehat{A}_1 \rightarrow 0$. Second, as T increases $\widehat{M}_2 \rightarrow \widehat{A}_2$. As the value of the option to reactivate is vanishing the only option which is sensible to consider as $c_t \rightarrow 0$ is the option to switch to D^2 ($\widehat{A}_2 > 0$). To analyse the changes induced by dT on the thresholds, we differentiate the smooth-past conditions in A.1.3. In c_M this yields:

$$W_{cc}(c_M)dc_M = -(\beta_1 c_M dQ_1 + \beta_2 c_M dQ_2)$$

Note that since $W_{cc}(c_M) > 0$, we must have $dc_M > 0$. This implies that as T increases the suspension threshold c_M rises. This can be justified by the option to reactivate losing value.

On the contrary, in c_R

$$W_{cc}(c_R)dc_R = -(\beta_1 c_R dQ_1 + \beta_2 c_R dQ_2)$$

Here, being $W_{cc}(c_R) < 0$, it must be $dc_R < 0$. This makes sense considering that as T increases the option to reactivate should be exercised only if the convenience of the input cost, i.e. c_t low enough, covers the reactivation cost pT. As shown above, for T sufficiently high this could never be the case.

We investigate now the effect that changes in m may have on the time triggers. It easy to show that as m increases for (A.1.5) and (A.1.6) to hold respectively both c_M and c_R should rise. One can easily see that as m increases the option to suspend is less interesting and it is worth to exercise it only for high value for c_t. On the contrary, having suspension become more costly, under that regime the plant manager would prefer to rush reactivation. Last, note that the two limits in (A.1.5) and (A.1.6) corresponds to the marshallian thresholds to which in the absence of uncertainty ($\sigma \rightarrow 0$) c_R and c_M converge.[39]

If the options to mothball and to abandon are available, an optimal strategy makes sense as long as $c_M < c_S$. We have shown above that as T rises c_M increases. This is due to the option to mothball losing value in that the reactivation is more costly. It follows that the cost opportunity of scrapping the project decreases. Hence, it may exists T such that $c_M = c_S = \overline{c}$ which would imply that the option to mothball may be completely neglected. In

[39] See [11] for a complete analysis of a similar problem.

this case, summing (c) to (g) and (d) to (h)

$$\widehat{A}_1\overline{c}^{\beta_1} + \widehat{A}_2\overline{c}^{\beta_2} + \frac{p - d(1-\alpha) - \alpha\overline{c}}{r} = -F_S$$
$$\widehat{A}_1\beta_1\overline{c}^{\beta_1} + \widehat{A}_2\beta_2\overline{c}^{\beta_2} - \frac{\alpha\overline{c}}{r} = 0$$

and rearranging one would get

$$\widehat{A}_1\overline{c}^{\beta_1} = -\frac{\beta_2}{\beta_2 - \beta_1}\frac{p - d(1-\alpha) + rF_S}{r} + \frac{\alpha\overline{c}}{r}\frac{\beta_2 - 1}{\beta_2 - \beta_1}$$

Now, suppose $F_S \simeq 0$ and m is small enough $(m \to 0)$. This implies that even if the reactivation has become more costly and $c_M \to c_S$ it will still be worth not to abandon $(c_S \to \infty)$ and keep the option to mothball.

Finally, suppose that the manager considers sensible only the exercise of the option to switch between D^1 and D^2. By the discussion above the system reduces then to

$$-\widehat{B}_1 d^{\beta_1} + \widehat{A}_2 d^{\beta_2} = 0$$
$$-\widehat{B}_1\frac{\beta_1}{\beta_2}d^{\beta_1} + \widehat{A}_2 d^{\beta_2} = (1-\gamma)\frac{\alpha d}{r\beta_2}$$

This is easy to solve and we get

$$\widehat{A}_2 = (\gamma - 1)\frac{\alpha d^{1-\beta_2}}{r(\beta_1 - \beta_2)} > 0$$
$$\widehat{B}_1 = (\gamma - 1)\frac{\alpha d^{1-\beta_1}}{r(\beta_1 - \beta_2)} > 0$$

A similar analysis can be developed also for the case $\gamma \in [0, 1]$.

A.2 Optimal $\gamma(> 1)$

Since $\widehat{A}_2 = (\gamma - 1)\frac{\alpha}{r(\beta_1 - \beta_2)}d^{1-\beta_2} = (\gamma - 1)A > 0$, the optimal level of flexibility is given by

$$\begin{aligned} \gamma^* &= \arg\max NPV(c_t, \alpha, \gamma) \qquad \text{(A.2.1)} \\ &= \arg\max\left[\frac{p - (1-\alpha)d - \alpha c_t}{r} + (\gamma - 1)Ac_t^{\beta_2} - \frac{K(\alpha)}{2}(\gamma - 1)^2\right] \end{aligned}$$

The FOC is given by

$$Ac_t^{\beta_2} - K(\alpha)(\gamma - 1) = 0 \tag{A.2.2}$$

and the SOC is always satisfied. From A.2.2 it turns out that:

$$\gamma^* = \begin{cases} 1 + \frac{Ac_t^{\beta_2}}{K(\alpha)} & \text{for } \widehat{c} \leq c_t \\ \frac{1}{\alpha} & \text{for } d < c_t < \widehat{c} \end{cases} \tag{A.2.3}$$

where $\widehat{c} = \left(\frac{k}{A}\right)^{1/\beta_2}$.

A.3 Imperfect substitutability

Consider a plant using a Cobb-Douglas technology mixing two different types of materials to produce 1 m^3 of biogas. To minimize cost, the following problem must be solved:

$$c(w,x) = \min\left[w_1 x_1 + w_2 x_2\right] \quad \text{such that} \quad x_1^{\alpha} x_2^{1-\alpha} = 1$$

where x_1 and x_2 are the quantity of the two inputs, α, $1-\alpha$ the output elasticities and w_1 and w_2 the unit input prices.

Solving the problem gives conditional demand functions for both factors and the cost function:

$$\begin{aligned} x_1(w_1,w_2) &= \left[\frac{\alpha}{1-\alpha}\frac{w_2}{w_1}\right]^{1-\alpha} \\ x_2(w_1,w_2) &= \left[\frac{\alpha}{1-\alpha}\frac{w_2}{w_1}\right]^{-\alpha} \\ c(w_1,w_2) &\equiv K w_1^{\alpha} w_2^{1-\alpha} \end{aligned}$$

where $K = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$.

Setting q as unit output price, to maximize profits is equivalent to maximize $\frac{q}{c(w_1,w_2)}$. Taking the logarithm and rearranging the objective function becomes

$$\begin{aligned} \pi &= p - [\alpha c + (1-\alpha)d] \\ &= p - C_1 \end{aligned}$$

where $p = \ln\frac{q}{K}$, $c = \ln w_1$ and $d = \ln w_2$. This is to prove that the analysis we propose may easily apply also to this scenario.

References

[1] L.H.R. Alvarez, R. Stenbacka, Partial Outsourcing: A Real Options Perspective, International Journal of Industrial Organization 25 (2007) 91-102.

[2] T. Amon, B. Amon, V. Kryvoruchko, A. Macmuller, K. Hopfner-Sixt, V. Bodiroza, R. Hrbek, J. Friedel, E. Potsch, H. Wagentristl, M. Schreiner, W. Zollitsch, Methane Production Through Anaerobic Digestion of Various Energy Crops Grown in Sustainable Crop Rotations, Bioresource Technology 98 (2007) 3204-3212.

[3] T. Amon, B. Amon, V. Kryvoruchko, W. Zollitsch, K. Mayer, L. Gruber, Biogas Production from Maize and Dairy Cattle Manure - Influence of Biomass Composition on the Methane Yield, Agriculture, Ecosystems and Environment 118 (2007) 173-182.

[4] A. Boschetti, Buona redditività dalla produzione di biogas, Informatore Agrario 1 (2006) 42-43.

[5] M.J. Brennan, E.S. Schwartz, Evaluating Natural Resource Investments, Journal of Business 58 (2) (1985) 137-157.

[6] F.J. Callaghan, D.A.J. Wase, K. Thayanithy, C.F. Forster, Continuous co-digestion of cattle slurry with fruit and vegetables wastes and chicken manure, Biomass and Bioenergy 27 (2002) 71-77.

[7] D.P. Chynoweth, Biomethane from energy crops and organic wastes, in: International Water Association (Ed.), Anaerobic Digestion 2004, Anaerobic Bioconversion ... Answer for Sustainability, Proceedings 10th World Congress, vol. 1, Montreal, Canada. www.ad2004montreal.org, 2004, pp. 525–530.

[8] J.C. Cox, S.A. Ross, The valuation of options for alternative stochastic processes, Journal of Financial Economics 3 (1976) 145-166.

[9] E.J. Da Silva, Biogas generation: developments, problems and tasks: an overview,. in: Food and Nutrition Bulletin, Conference on the State of the Art of Bioconversion of Organic Residues for Rural Communities, Guatemala City (Guatemala), 13 Nov 1978 / UNU, Tokyo, Japan, supplement (UNU), no. 2, 1979, pp. 84-98.

[10] L. Devenuto, A. Ragazzoni, Il biogas è un affare se la filiera è corta, Informatore Agrario 18 (2008) 29-33.

[11] A.K. Dixit, Entry and Exit Decisions under Uncertainty, Journal of Political Economy 97 (1989) 620-638.

[12] A.K. Dixit, The Art of Smooth Pasting, Harwood Academic Publishers, 1993.

[13] A.K. Dixit, R.S. Pindyck, Investment Under Uncertainty, Princeton University Press, Princeton, N.J., 1994.

[14] P.A. Gerin, F. Vliegen, J. Jossart, Energy and CO2 balance of maize and grass as energy crops for anaerobic digestion, Bioresource Technology 99 (2008) 2620-2627.

[15] H. He, R.S. Pindyck, Investments in flexible production capacity, J. Econom. Dynam. Control 16 (1992) 575–599.

[16] N. Kulatilaka, N., Valuing the flexibility of flexible manufacturing systems, IEEE Transactions on Engineering Management 35 (1988) 250-257.

[17] H. Mæng, H. Lund, F. Hvelplund, Biogas plants in Denmark: technological and economic developments, Applied Energy 64 (1999) 195-206.

[18] M. Moretto, G. Rossini, Vertical Integration and Operational Flexibility, Department of Economics working papers, University of Bologna, (2008), http://www2.dse.unibo.it/wp/631.pdf and FEEM Nota di Lavoro 37 (2008).

[19] National Academy of Sciences, Methane generation from human, animal, and agricultural wastewater, Washington, DC, 1977.

[20] S. Rubab, T.C. Kandpal, A methodology for financial evaluation of biogas technology in India using cost functions, Biomass and Bioenergy 10 (1996) 11-23.

[21] A. Schievano, G. D´Imporzano, F. Adani, Substituting energy crops with organic wastes and agro-industrial residues for biogas production, Journal of Environmental Management doi:10.1016/j.jenvman.2009.01.013 in press (2009).

[22] R.B. Singh, Bio-gas plant, generating methane from organic wastes, Gobar Gas Research Station, Ajitmal, Etawah (U.P.), India, 1971.

[23] A.J. Triantis, J.E. Hodder, Valuing flexibility as a complex option, Journal of Finance 45 (1990) 549–565.

[24] L.M. Wang, L.W. Liu, Y.J. Wang, Capacity Decisions and Supply Price Games under Flexibility of Backward Integration, International Journal of Production Economics 110 (2007) 85-96.

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