Small Sample Properties of Maximum Likelihood Versus Generalized Method of Moments Based Tests for Spatially Autocorrelated Errors

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Abstract

This paper undertakes a Monte Carlo study to compare MLE-based and GMM-based tests regarding the spatial autocorrelation coefficient of the error term in a Cliff and Ord type model. The main finding is that a Wald-test based on GMM estimation as derived by Kelejian and Prucha (2005a) performs surprisingly well. Our Monte Carlo study indicates that the GMM Wald-test is correctly sized even in small samples and exhibits the same power as their MLE-based counterparts. Since GMM estimates are much easier to calculate, the GMM Wald-test is recommended for applied researches.

JEL Code: C12, C21, R10.

Keywords: spatial autocorrelation, hypothesis tests, Monte Carlo studies, maximum likelihood estimation, generalized method of moments.

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1 Introduction

The recent literature on the estimation of processes with spatially autocorrelated errors distinguishes between two estimation principles: maximum likelihood estimation (henceforth MLE; Cliff and Ord, 1981; Anselin, 1988; Lee, 2004) and estimation by the generalized method of moments (henceforth GMM; Conley, 1999; Kelejian and Prucha, 1999, 2002). Whereas MLE relies on normally distributed errors, GMM is flexible with regard to the distribution of the innovations. However, this advantage of GMM comes at the expense of an efficiency loss as compared to MLE.

This paper focuses on the small sample properties of Wald, Likelihood Ratio (LR), and Lagrange Multiplier (LM) tests for processes with spatial autoregressive residuals (SAR). Specifically, we provide a comparison of MLE- and GMM-based tests. Whereas MLE naturally obtains variance-covariance estimates of all parameters including the SAR coefficient, this does not hold true for all GMM-type models. For instance, Kelejian and Prucha (1999) suggest a two-step procedure to estimate the SAR coefficient that does not provide its standard error. Similarly, Conley (1999) proposes a heteroskedasticity and autocorrelation consistent (HAC) non-parametric estimator of the variance-covariance matrix that excludes the SAR coefficient. Kelejian and Prucha (2005b) suggest a HAC-estimator of the variance-covariance matrix of the regression parameters that excludes the SAR parameter as well.[1] In a recent paper, Kelejian and Prucha (2005a) derive the asymptotic distribution of the GMM-based SAR parameter under a set of general assumptions allowing for heteroskedasticity of the error term. In particular, they obtain a consistent estimate of the variance of the SAR parameter upon which a test can be based.

So far, evidence on the relative performance of MLE- versus GMM-based tests for SAR processes seems not to be available.[2] It is this paper's objective to investigate

[1]Conley's (1999) approach works under a different set of assumptions than that one of Kelejian and Prucha (2005b). See Lee (2001) for a related approach.

[2]Burridge (1980), Anselin and Rey (1991), Kelejian and Robinson (1992), Kelejian and Robinson (1993), Anselin, Bera, Florax, and Yoon (1996), Anselin and Florax (1995), Kelejian and Robinson (1997), Anselin (2001), Kelejian and Prucha (2001), Anselin and Moreno (2003), Kelejian and Prucha (2005a) are excellent contributions on testing for spatial correlation, and by no means this list is exhaustive. However, these papers partly do not focus on SAR but on different spatial processes (e.g., Kelejian and Robinson, 1992, Anselin and Moreno, 2003), partly they consider

the size and power of MLE-based versus GMM-based tests against spatially autocorrelated residuals in small samples by setting up an extensive Monte Carlo study. Our primary goal is to derive results that facilitate the choice among available tests for applied researchers.

The next section lays out the data generating process and the Wald-, LR-, and LM-type tests based on MLE and the GMM-based Wald-test. Section 3 describes the design of our Monte Carlo study, whereas section 4 summarizes our main findings with regard to the size and power of the tests. The last section concludes with a summary of our main findings.

2 The Data-Generating Process

Ord (1975) type models with spatially autocorrelated residuals can be formulated in the following way:[3]

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}, \\ \boldsymbol{u} &= \rho \boldsymbol{W}\boldsymbol{u} + \boldsymbol{\varepsilon}, \end{aligned} \tag{1}$$

where $\boldsymbol{y}$ is an $n \times 1$ vector of observations on the dependent variable, with n denoting sample size. $\boldsymbol{X}$ is an $n \times k$ matrix of non-stochastic explanatory variables, where $\lim_{n\to\infty} \frac{1}{n}\boldsymbol{X}'\boldsymbol{X}$ exists and is non-singular. $\boldsymbol{\beta}$ is the $k \times 1$ vector of parameters, and $\boldsymbol{u}$ is an $n \times 1$ vector of spatially autocorrelated disturbances. $\boldsymbol{W}$ denotes a row-normalized spatial weights matrix of size n, ρ is the SAR parameter with $|\rho| < 1$.

only a single test principle (e.g., Kelejian and Robinson, 1992, Anselin, 2001, Kelejian and Prucha, 2001), and partly they focus only on MLE-based tests (e.g., Burridge, 1980, Anselin and Rey, 1991, Anselin, Bera, Florax, and Yoon, 1996, Anselin, 2001) or only on GMM-based estimates (e.g., Kelejian and Robinson, 1993, Kelejian and Prucha, 2005a). Hence, the aim of previous research was not to provide an analysis of a large class of tests against SAR based on MLE versus GMM.

[3] There is an extensive line of research on the estimation and testing of models with endogenous spatial lags (Anselin, 1988, Lee, 2003, Kelejian and Prucha, 1999, Pinkse, Slade, and Brett, 2002, Saavedra, 2003). The GMM-type literature focuses on instrumental variable estimation and the choice of optimal instruments. However, that branch of the literature does not focus on tests against SAR.

Hence, $(\boldsymbol{I}-\rho\boldsymbol{W})^{-1}$ is non-singular and uniformly bounded in absolute value. $\boldsymbol{\varepsilon}$ is an $n\times 1$ vector of errors, $\varepsilon_i \sim i.i.d.(0,\sigma^2)$, with finite second and fourth moments. A comprehensive set of assumptions underlying GMM-estimation is given in Kelejian and Prucha (1999, 2005a).

3 Tests on Spatially Autocorrelated Errors

3.1 Estimators

For estimation and testing, we consider two different principles: MLE and GMM-estimation. MLE assumes normally distributed errors, and the log-likelihood function for the data generating process outlined above is given by (see Anselin, 1988)

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}(\boldsymbol{y}-\boldsymbol{X\beta})'(\boldsymbol{I}-\rho\boldsymbol{W})'(\boldsymbol{I}-\rho\boldsymbol{W})(\boldsymbol{y}-\boldsymbol{X\beta}) + \ln|\boldsymbol{I}-\rho\boldsymbol{W}|. \tag{2}$$

This log-likelihood function can be concentrated to a non-linear function of the spatial parameter ρ only. Hence, optimization can be accomplished by univariate non-linear techniques based on the Nelder-Mead Simplex method (see Lagarias, Reeds, Wright, and Wright, 1998). Standard errors are derived from the inverse of the information matrix (see Anselin, 1988, for details).

In contrast, GMM does not rely on distributional assumptions and the estimators are easy to calculate (see Kelejian and Prucha, 1999, 2005b). The moment conditions related to ρ and σ^2 are given by:

$$E[\frac{1}{n}\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}] = \sigma^2 \tag{3}$$

$$E[\frac{1}{n}\overline{\boldsymbol{\varepsilon}}'\overline{\boldsymbol{\varepsilon}}] = \sigma^2\frac{1}{n}tr(\boldsymbol{W}\boldsymbol{W}') \tag{4}$$

$$E[\frac{1}{n}\overline{\boldsymbol{\varepsilon}}'\boldsymbol{\varepsilon}] = 0 \tag{5}$$

After substituting the first moment condition into the second one and using the notation $\overline{\boldsymbol{u}} = \boldsymbol{W}\boldsymbol{u}$ and $\overline{\overline{\boldsymbol{u}}} = \boldsymbol{W}^2\boldsymbol{u}$, the two moment conditions for ρ can be written as follows:

$$E\left[\frac{1}{n}\left(\overline{\boldsymbol{u}}-\rho\overline{\overline{\boldsymbol{u}}}\right)'\left(\overline{\boldsymbol{u}}-\rho\overline{\overline{\boldsymbol{u}}}\right)\right] = E\left[\frac{1}{n}\left(\boldsymbol{u}-\rho\overline{\boldsymbol{u}}\right)'\left(\boldsymbol{u}-\rho\overline{\boldsymbol{u}}\right)\right]\frac{tr(\boldsymbol{W}'\boldsymbol{W})}{n} \tag{6}$$

$$E\left[\frac{1}{n}\left(\overline{\boldsymbol{u}}-\rho\overline{\overline{\boldsymbol{u}}}\right)'\left(\boldsymbol{u}-\rho\overline{\boldsymbol{u}}\right)\right] = 0. \tag{7}$$

A consistent estimate of the residuals $\boldsymbol{u}$ is obtained by ordinary least squares, ignoring the spatial correlation of the error term. These residuals are plugged into the above moment conditions to obtain a consistent estimate of ρ. For the subsequent exposition, it is useful to write the above two moment conditions in vector form as $\tilde{\boldsymbol{\Gamma}}\begin{pmatrix}\rho\\ \rho^2\end{pmatrix} = \tilde{\boldsymbol{\gamma}}$. The estimate of the spatial correlation coefficient $\tilde{\rho}$ is defined as

$$\tilde{\rho} = \arg\min\left[\left(\tilde{\boldsymbol{\gamma}}-\tilde{\boldsymbol{\Gamma}}\begin{pmatrix}\rho\\ \rho^2\end{pmatrix}\right)'\tilde{\boldsymbol{\Upsilon}}^{-1}\left(\tilde{\boldsymbol{\gamma}}-\tilde{\boldsymbol{\Gamma}}\begin{pmatrix}\rho\\ \rho^2\end{pmatrix}\right)\right], \tag{8}$$

where $\widetilde{\boldsymbol{\Upsilon}}$ is a 2×2 symmetric positive semidefinite (moments) weighting matrix. Kelejian and Prucha (2005a) recently derived the limit distribution of $\widetilde{\rho}$ as well as a consistent parametric estimate of its variance:

$$\sqrt{n}(\widetilde{\rho}-\rho) \sim N(0,\boldsymbol{\Omega}_{\widetilde{\rho}}), \tag{9}$$

They showed that $\boldsymbol{\Omega}_{\widetilde{\rho}}$ can be consistently estimated by:

$$\widetilde{\boldsymbol{\Omega}}_{\widetilde{\rho}} = (\widetilde{\boldsymbol{J}}'\widetilde{\boldsymbol{\Upsilon}}\widetilde{\boldsymbol{J}})^{-1}\widetilde{\boldsymbol{J}}'\widetilde{\boldsymbol{\Upsilon}}\widetilde{\boldsymbol{\Psi}}\widetilde{\boldsymbol{\Upsilon}}\widetilde{\boldsymbol{J}}(\widetilde{\boldsymbol{J}}'\widetilde{\boldsymbol{\Upsilon}}\widetilde{\boldsymbol{J}})^{-1}, \tag{10}$$

where

$$\widetilde{\boldsymbol{J}} = \widetilde{\boldsymbol{\Gamma}}\begin{pmatrix}1\\ 2\widetilde{\rho}\end{pmatrix},$$

$$\widetilde{\boldsymbol{\Psi}} = \left[\widetilde{\psi}_{rs}\right]_{r,s=1,2},\ \widetilde{\psi}_{rs} = \widetilde{\sigma}^4(2n)^{-1}tr[(\boldsymbol{A}_r+\boldsymbol{A}_r')(\boldsymbol{A}_s+\boldsymbol{A}_s')],$$

$\boldsymbol{A}_1 = \boldsymbol{W}'\boldsymbol{W} - diag_{i=1}^{n}\left(n^{-1}tr(\boldsymbol{W}'\boldsymbol{W})\right)$, and

$\boldsymbol{A}_2 = \boldsymbol{W}$,

For one variant of the test, we use $\widetilde{\boldsymbol{\Upsilon}} = \boldsymbol{I}_2$. The Wald-test based on this estimate is referred to as 'Wald GMM'. Kelejian and Prucha (2005a) also showed that inserting a consistent estimate $\widetilde{\boldsymbol{\Psi}}^{-1}$ for $\widetilde{\boldsymbol{\Upsilon}}$ leads to the efficient GMM estimator of ρ. The Wald-test based on the estimate with the efficient GMM estimator is referred to as 'Wald GMM eff.'.

3.2 Test Statistics for $\rho = 0$

The most common test for $H_0 : \rho = 0$ is the Moran I test statistic, which tests for the lack of spatial correlation in the residuals against an unspecified alternative. Under the present assumptions, the Moran I test statistic is equivalent to an MLE-based LM-test (see Kelejian and Prucha, 2001).

In the specified model with SAR-errors, one is able to test directly for $\rho = 0$ against $\rho \neq 0$. For this, we rely on the three familiar asymptotic testing principles, i.e., the Wald-test, the LR-test, and the LM-test. First, we calculate the following statistics regarding the null hypothesis $H_0 : \rho = 0$ by means of the MLE approach:

$$W_{ML} = \frac{\hat{\rho}}{\hat{\sigma}_\rho}, W_{ML} \sim N(0,1), \quad (12)$$

$$LR_{ML} = -2(LL_u - LL_r), LR_{ML} \sim \chi^2_{df=1}, \quad (13)$$

$$LM_{ML} = \frac{1}{n}(\hat{\boldsymbol{u}}'\boldsymbol{W}\hat{\boldsymbol{u}}/\hat{\sigma}^2)^2, LM_{ML} \sim \chi^2_{df=1}, \quad (14)$$

where a '$\widehat{\ }$' refers to ML estimates. $\hat{\sigma}_\rho$ is the estimated standard error which is the diagonal element corresponding to ρ in the inverse of the information matrix; LL_r (LL_u) is the restricted (unrestricted) log-likelihood. In our case LL_r is obtained by an ordinary regression. $df = 1$ is short for 1 degree of freedom.

Under GMM-estimation, a Wald-type test based on the parametric variance estimate following Kelejian and Prucha (2005a) can be employed to test for $\rho = 0$. This test may serve as an alternative to the frequently used Moran I test under GMM (see Cliff and Ord, 1981; Kelejian and Prucha, 2001).

4 Design of Monte Carlo Experiments

The regression part of our model is $\beta_0 + \beta_1 x_i$, where we choose $\beta_0 = \beta_1 = 1$. We assume that x_i is uniformly distributed in the interval $[0, 1]$. In order to generate the weighting scheme, the observation units are randomly placed on a grid of different sizes (see Table 1). The weighting matrix $\boldsymbol{W}$ exhibits typical elements $W_{ij} = e^{-d_{ij}}$, for $i \neq j$ and $W_{ii} = 0$, where d_{ij} is the Euklidean distance between grid points i and j. If $d_{ij} > 7$ we set $W_{ij} = 0$ to limit the memory of the spatial process.

Table 1: Characteristics of Spatial Weights

n	Lattice[a]	Sparseness[b]
100	20×20	0.751
100	32×32	0.889
250	32×32	0.883
500	32×32	0.879
500	45×45	0.935

[a] Only irregular lattices are considered.

[b] Percentage of zero entries in weighting matrix **W**.

We consider two different lattices for sample sizes of $n = 100$ and $n = 500$ to infer the properties of the tests regarding the sparseness of the weighting matrix (see Table 1). The SAR parameter ρ varies between -0.5 and 0.5 in steps of 0.1. We focus on the performance of the three (Wald-, LR-, and LM-type) test statistics based on MLE versus the Wald-test based on GMM estimates. Since the performance of GMM versus MLE is expected to face a trade-off between sample size and deviation from the assumption of normally distributed remainder errors, we consider three sample sizes ($n = 100, 250, 500$) and three different underlying error distributions. Following Kelejian and Prucha (1999) and Anselin and Moreno (2003), we consider a normal, a lognormal, and a mixture of normal and lognormal distributions of the disturbances (referred to as 'mixed normal errors'). In the benchmark case of normally distributed errors, we assume that ε_i are $i.i.d.$ $N(0,1)$ with $i = 1, ..., n$. Alternatively, we use the standardized version of the lognormal distribution. We assume in this case that $\varepsilon_i = [exp(\xi_i) - exp(0.5)]/[exp(2) - exp(1)]^{0.5}$, where the ξ_i are $i.i.d.$ $N(0,1)$. The standardization implies that the ε_i are $i.i.d.$ $(0,1)$. This lognormal distribution is positively skewed. The third distribution is a standardized version of a mixture of normals in which a normally distributed random variable (ξ_i) is contaminated by another (η_i) that has a larger variance; the ξ_i are $i.i.d.$ $N(0,1)$, and the η_i are $i.i.d.$ $N(0,100)$. We assume that $\varepsilon_i = [\lambda_i \xi_i + (1 - \lambda_i)\eta_i]/(5.95)^{0.5}$, where the λ_i are $i.i.d.$ Bernoulli variables with $Prob(\lambda_i = 1) = 0.95$. Also the processes λ_i, ξ_i, and η_i are assumed to be jointly independent. Again the standardization implies that the ε_i are $i.i.d.$ $(0,1)$. This distribution exhibits thicker tails than the normal.

Altogether, we analyze 165 experiments,[4] for each of which we conduct 1000 Monte Carlo runs. The main results can be summarized as follows.

5 Monte Carlo Simulation Results

5.1 Size of the Tests

Tables $2-4$ summarize the rejection frequencies of the considered tests under H_0 using the 5 percent critical value significance level to investigate the size of the tests in small and medium sized samples.

Table 2: Empirical rejection frequencies under H_0, normal (α=0.05; 1000 replications)

n	Sparseness	MLE			GMM	GMM eff.
		Wald	LM	LR	Wald	Wald
100	0.751	0.069	0.040	0.056	0.055	0.053
100	0.889	0.075	0.067	0.057	0.060	0.060
250	0.883	0.054	0.047	0.044	0.049	0.045
500	0.879	0.063	0.065	0.055	0.057	0.056
500	0.935	0.044	0.043	0.040	0.045	0.040

Let us first focus on Table 2, where the errors are drawn from a normal distribution. All considered tests tend to be properly sized even in samples of moderate size, and as sample size increases the rejection frequencies converge to the true ones as expected. In particular, the GMM Wald-test performs as well as MLE-based tests with respect to test size even in small samples.

[4]Three error processes (normal, lognormal, mixed normal), eleven values of the SAR parameter $\rho = [-0.5, -0.4, ..., 0.4, 0.5]$, and three sample sizes ($n = 100, 250, 500$). Additionally, we run all experiments for $n = 100$ and $n = 500$ for alternative lattices.

Table 3: Empirical rejection frequencies under H_0, lognormal (α=0.05; 1000 replications)

n	Sparseness	MLE			GMM	GMM eff.
		Wald	LM	LR	Wald	Wald
100	0.751	0.047	0.032	0.038	0.029	0.048
100	0.889	0.044	0.030	0.033	0.046	0.083
250	0.883	0.056	0.052	0.051	0.049	0.065
500	0.879	0.047	0.043	0.033	0.040	0.046
500	0.935	0.042	0.042	0.039	0.050	0.057

If the errors follow a lognormal distribution (Table 3), the size of the MLE-based tests tends to be slightly lower than in the case of normally distributed errors, specifically in case of the the LM- and LR-MLE based tests. Note, this is not the case for the GMM-based Wald-tests.

Table 4: Empirical rejection frequencies under H_0, mixed normal (α=0.05; 1000 replications)

n	Sparseness	MLE			GMM	GMM eff.
		Wald	LM	LR	Wald	Wald
100	0.751	0.047	0.030	0.039	0.044	0.062
100	0.889	0.049	0.039	0.034	0.049	0.066
250	0.883	0.056	0.046	0.050	0.052	0.078
500	0.879	0.043	0.043	0.047	0.045	0.058
500	0.935	0.054	0.049	0.047	0.057	0.060

The results for mixed normally distributed errors in Table 4 indicate that in small samples ($n = 100$ in our case) the size distortion of the tests is somewhat smaller than for lognormally distributed errors. With more observations, the MLE-based tests perform quite well and exhibit approximately the same size as in the lognormal case. The GMM based test which relies on the asymptotically efficient weighting scheme is oversized. However, with just a few exceptions the GMM-based Wald-test based on the simple weighting scheme is properly sized in all cases and can be recommended for applied research in this respect.

Figures $1-5$ provide further details on the size of the tests at various critical values. These figures correspond to *p-value plots* suggested by Davidson and MacKinnon

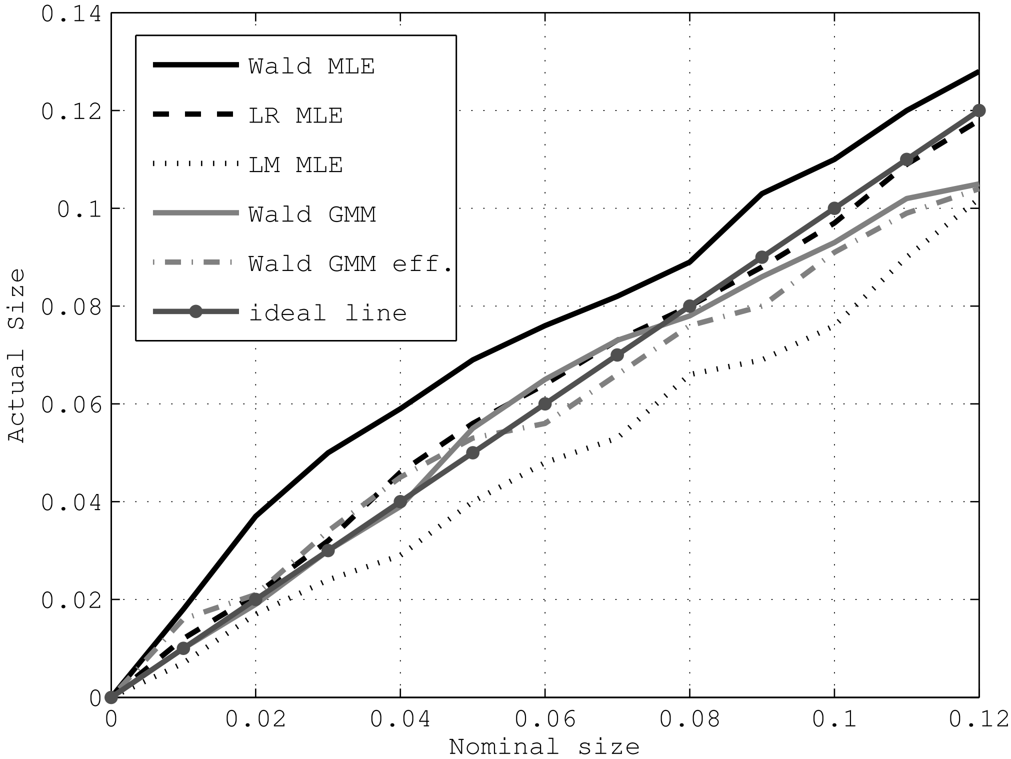

Figure 1: P-value plots for tests regarding $H_0 : \rho = 0$ given normally distributed errors, $n = 100$. Lattice size is 20 by 20.

(1998) for three different sample sizes ($n = 100, 250, 500$). For sample sizes of 100 and 500, we illustrate how an increasing lattice size, which is equivalent to an increasing sparseness of the spatial weighting matrix at given n, changes the size of the various test statistics. Applied researchers typically pay attention only to Type I errors, and hypothesis tests are usually carried out at levels of 10 percent or smaller. Therefore, we focus on values of $p \leq 0.12$ on the abscissa of the plots. Ideally, actual size would coincide with nominal size, reflected by the locus referred to as the *ideal line* in the figures.

The figures indicate that both the GMM-based Wald-test and the MLE-based LR-statistic work very well, even in small samples of $n = 100$. According to Figure 1, the GMM-based Wald-test outperforms the MLE-based LM- and Wald-tests in terms of size, if the sample size is small. The former tends to be undersized, and

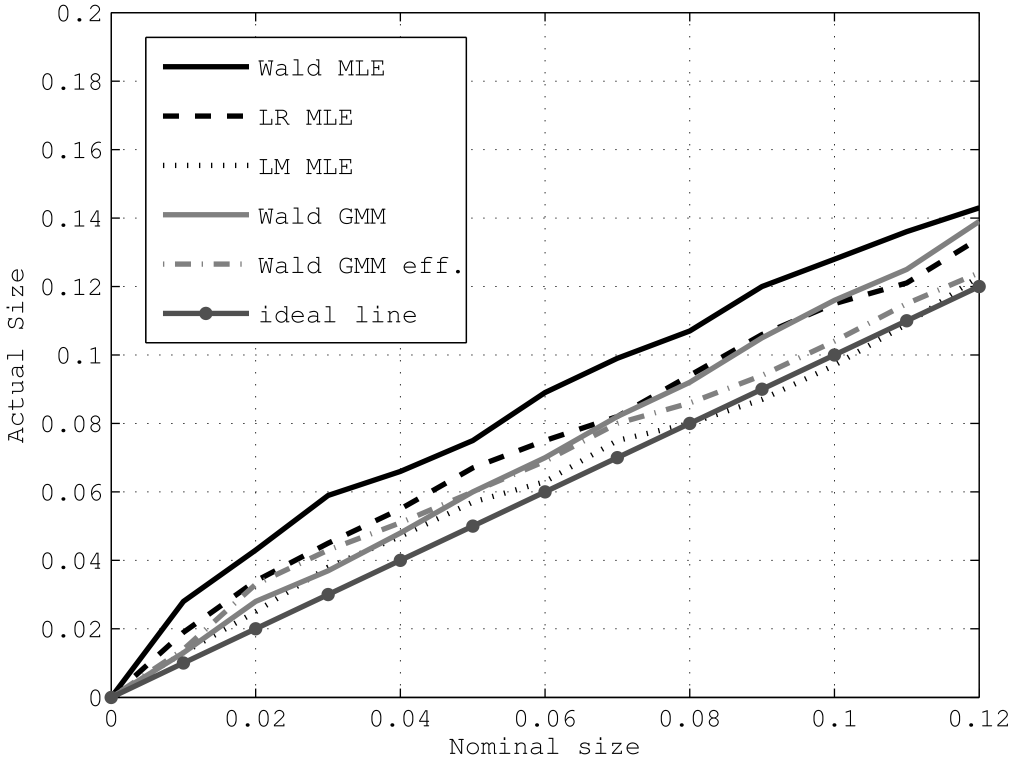

Figure 2: P-value plots for tests regarding $H_0 : \rho = 0$ given normally distributed errors, $n = 100$. Lattice size is 32 by 32.

the latter is oversized. As expected, the performance in terms of actual size of all tests considered in Figures $1 - 5$ improves as we increase the sample size. For this, compare Figures 1, 3, and 5.[5]

The performance of the tests relative to each other is similar in case of lognormally or mixed normally distributed errors (these figures are not reported for the sake of brevity but available from the authors upon request). If anything, the deviations from the ideal line tend to be stronger as we deviate from the normal error distribution. Also, the convergence of the p-value loci for the various tests towards the ideal line is slower as the sample size increases than with normally distributed errors.

[5]Note that the number of observations increases in the same way as the number of cells on the respective lattice. Hence, the sparseness increases as n increases.

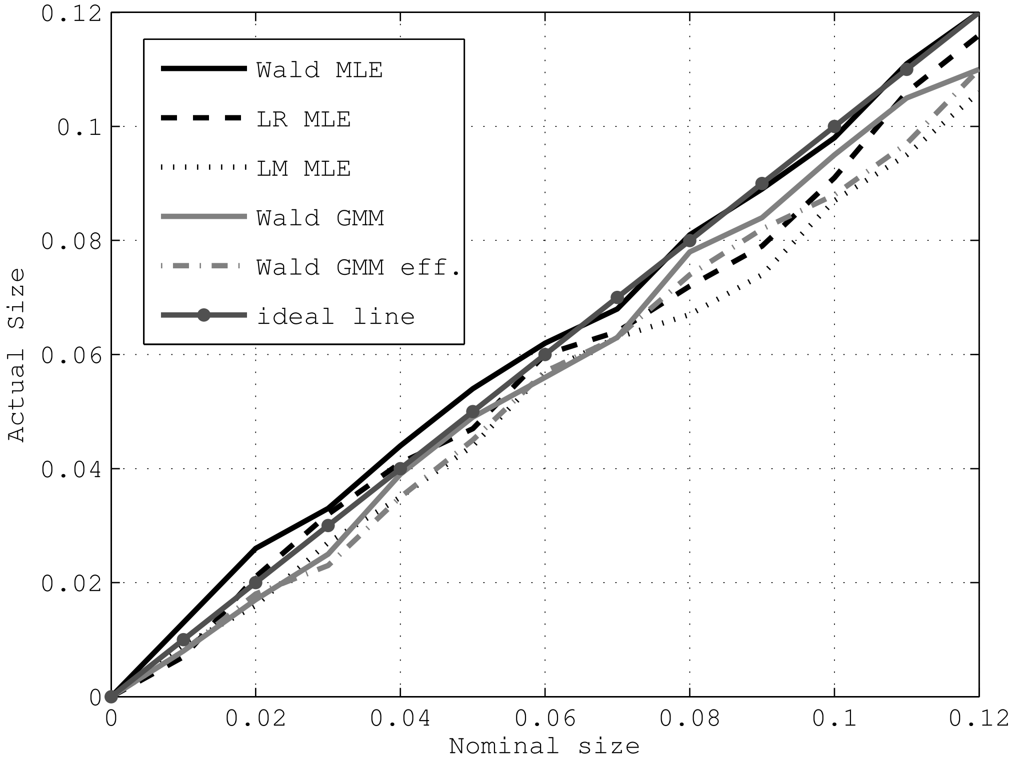

Figure 3: P-value plots for tests regarding $H_0 : \rho = 0$ given normally distributed errors, $n = 250$. Lattice size is 32 by 32.

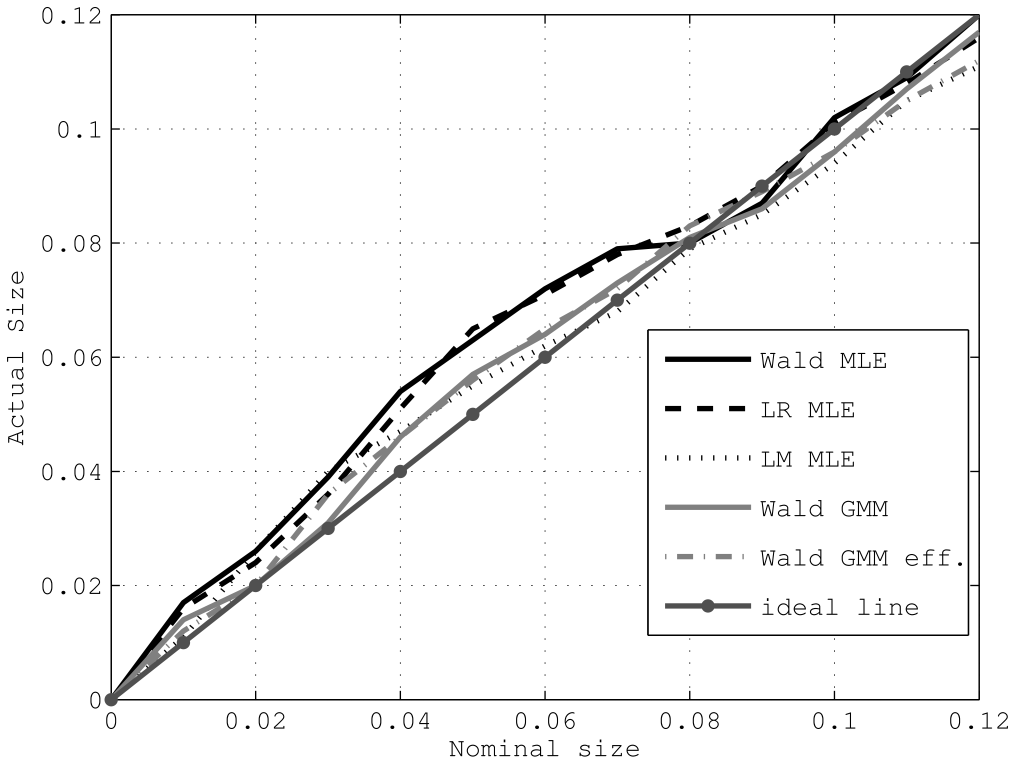

Figure 4: P-value plots for tests regarding $H_0 : \rho = 0$ given normally distributed errors, $n = 500$. Lattice size is 32 by 32.

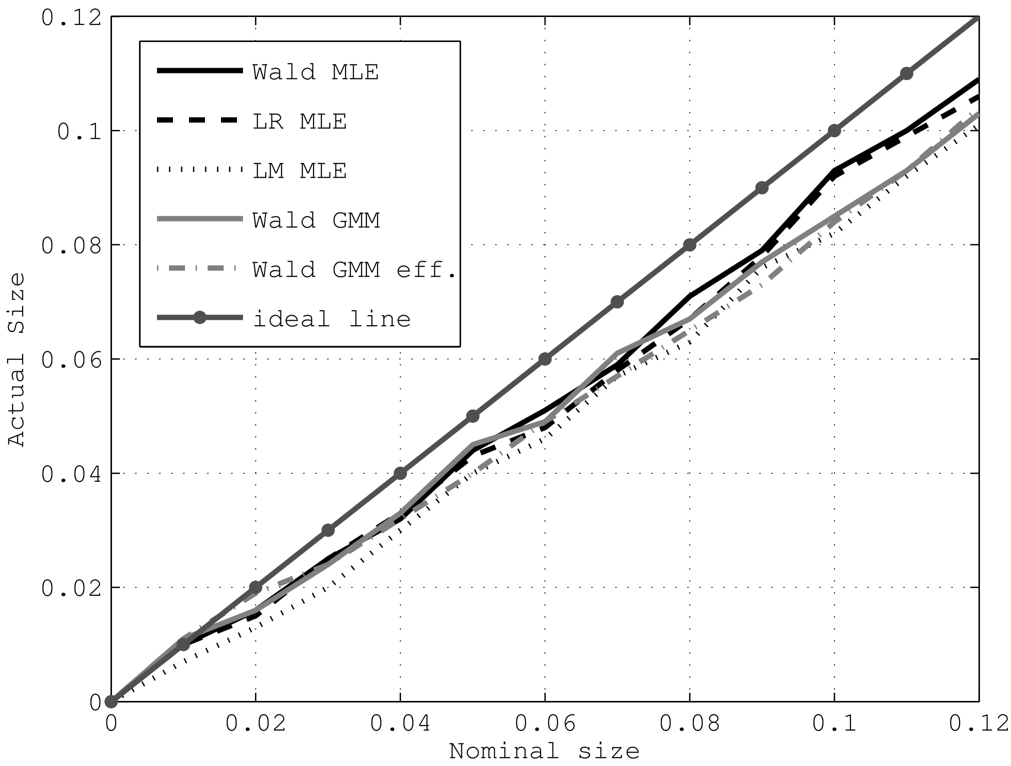

Figure 5: P-value plots for tests regarding $H_0 : \rho = 0$ given normally distributed errors, $n = 500$. Lattice size is 45 by 45.

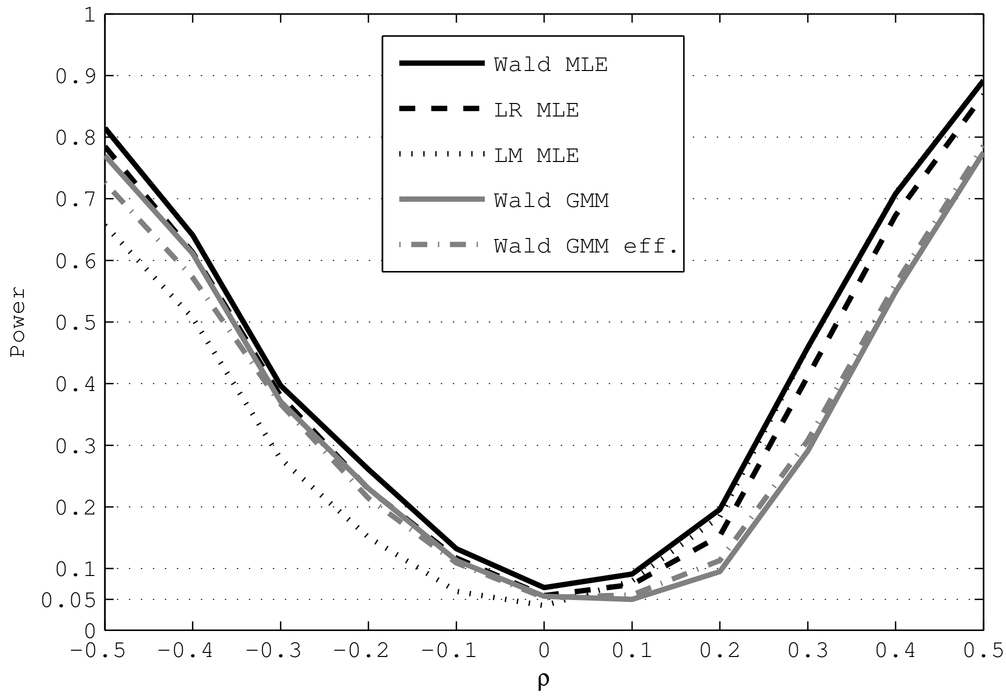

Figure 6: Power function for tests regarding $H_0 : \rho = 0$ at a significance level of 5% given normally distributed errors, $n = 100$. Lattice size is 20 by 20.

5.2 Power of the Tests

We use a nominal size of 0.05 and 11 different values of the spatial autocorrelation parameter ρ in the interval $[-0.5, -0.4, ..., 0.4, 0.5]$ to compare the five considered test statistics with respect to their power. Our findings are illustrated graphically for the case of normally distributed errors in Figures $6-10$, where we plot the power function for all (MLE-based and GMM-based) tests corresponding to a sample size of 100, 250, and 500, respectively. Again, we illustrate how the power of the tests depends on the sparseness of the weighting scheme at given n. For this, compare Figure 6 with Figure 7 and Figure 9 with Figure 10, respectively.

Two observations are worth noting. First, the power functions are asymmetric with respect to ρ. Especially, this is the case in small samples and at a low sparseness of the weighting matrix (see Figure 6). At $n = 100$ the MLE-based LM-test exhibits lower power for negative values of ρ as compared to the other test. If ρ is positive,

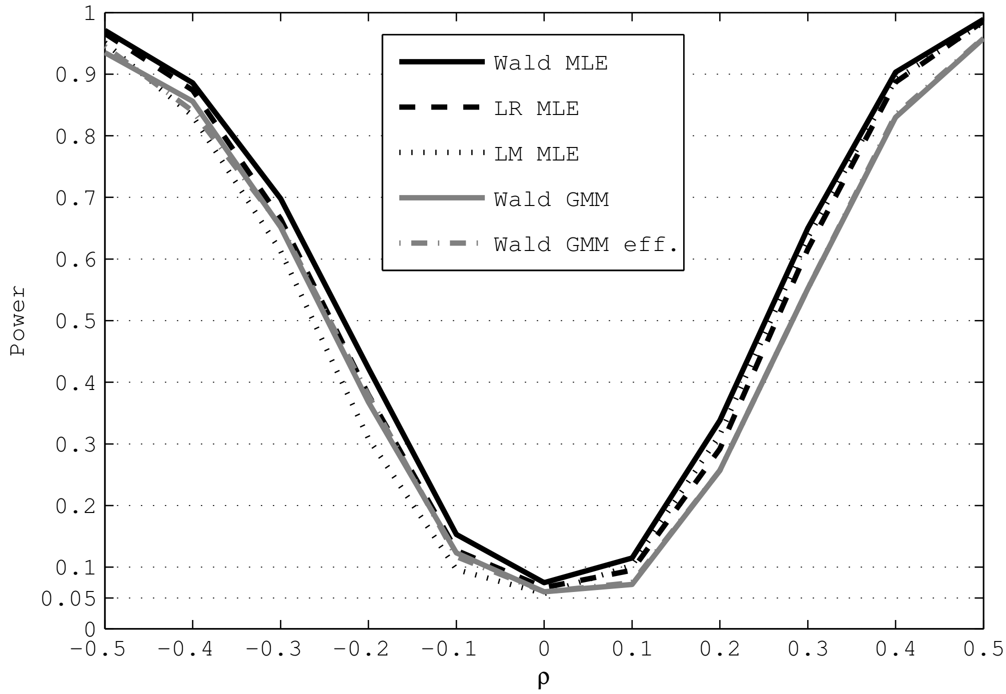

Figure 7: Power function for tests regarding $H_0 : \rho = 0$ at a significance level of 5% given normally distributed errors, $n = 100$. Lattice size is 32 by 32.

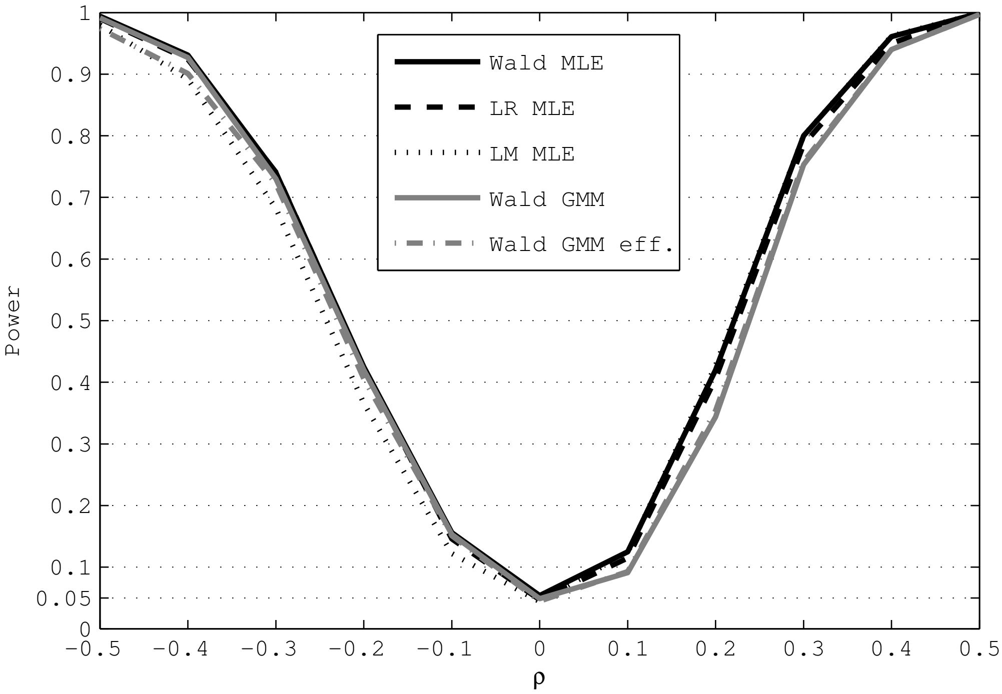

Figure 8: Power function for tests regarding $H_0 : \rho = 0$ at a significance level of 5% given normally distributed errors, $n = 250$. Lattice size is 32 by 32.

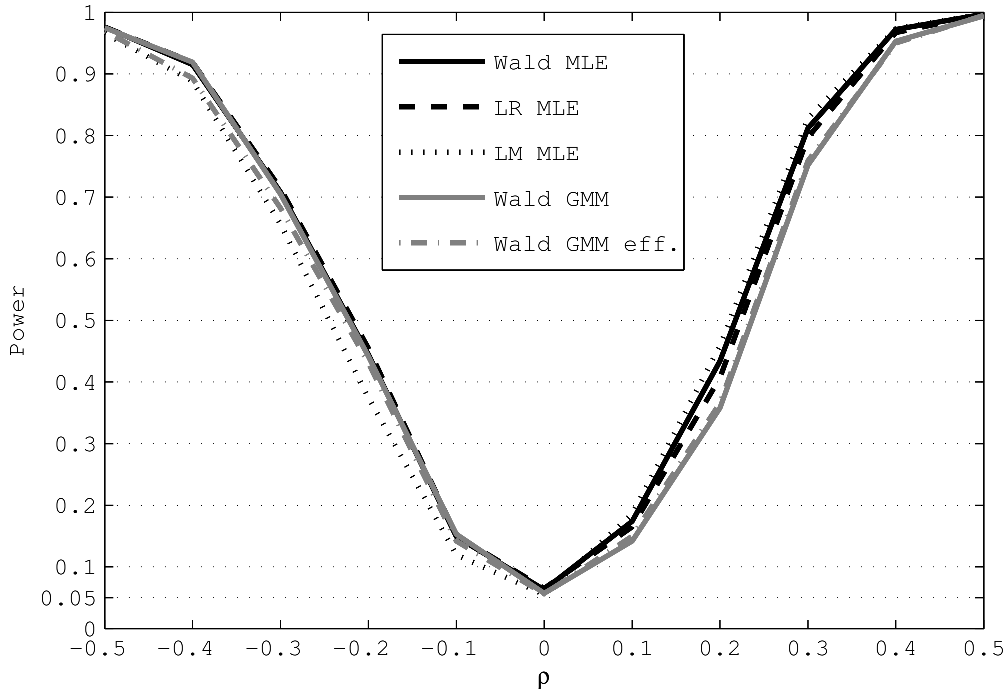

Figure 9: Power function for tests regarding $H_0 : \rho = 0$ at a significance level of 5% given normally distributed errors, $n = 500$. Lattice size is 32 by 32.

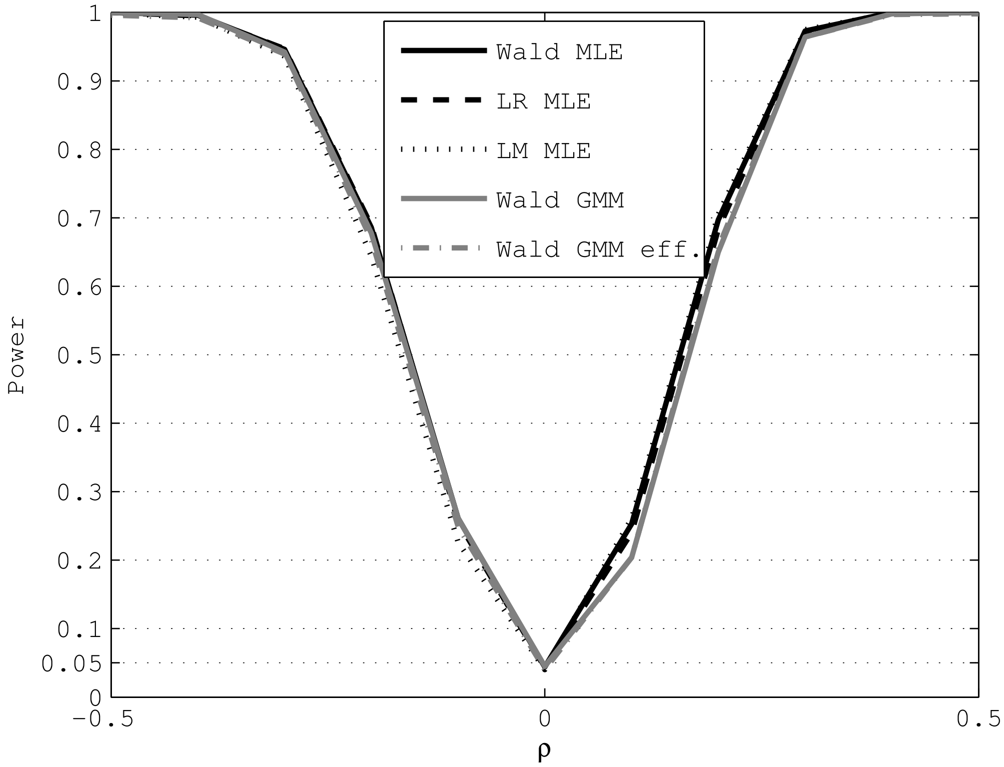

Figure 10: Power function for tests regarding $H_0 : \rho = 0$ at a significance level of 5% given normally distributed errors, $n = 500$. Lattice size is 45 by 45.

the power of the GMM-based tests is somewhat lower. One has to bear in mind, however, that the MLE-based LM-test tends to be undersized and the MLE-based LR-test is oversized in small samples. This fact also contributes to the differences in the power function. Second and not very surprisingly, the power functions of all tests (i) exhibit narrower waists and (ii) become more similar as the sample size n grows larger.

So far, we have focused on normally distributed errors. The power functions for lognormally and mixed normally distributed errors are summarized in Tables 5 and 6. Two results are especially worth paying attention to. First, for lognormally distributed errors (see Table 5) the MLE-based Wald test and the GMM-based Wald-test seem most reliable regarding their size and power in small to medium sized samples. Second, these two tests exhibit higher power than the MLE-based LM-test if ρ is much smaller than zero. Similar findings are obtained in case of mixed normally distributed errors. However, in this case also the MLE-based LR-test performs quite well and exhibits high power even in small samples. The MLE-based Wald- and LR-tests as well as the GMM-based Wald-test work better than the MLE-based LM-test if ρ is much smaller than zero.

Finally, we briefly summarize the importance of the sample size (n), the sparseness of the population of spatial lattices (*Sparseness*, *Sparseness*$\times\rho^2$), the sign and size of the SAR parameter (ρ, ρ^2, $n \times \rho^2$), and the deviation from a normal error distribution (*Lognormal*$\times\rho^2$, *Mixed Normal*$\times\rho^2$) for the power of the tests by means of response surface regressions.[6] The logistically transformed rejection probabilities are employed as the dependent variables throughout. We run separate regression models for each type of the considered test statistics. Table 7 reports the corresponding response surface parameter estimates. Here, we only highlight the most important results.

First, there is an overall negative impact of sample size on the rejection probability. The reason for this is that the tests tend to be oversized in small samples. Second, the power functions are parabolic as indicated by the significantly positive coefficient

[6]Also, we have estimated a set of less parsimonious specifications that includes the main effects of *Lognormal* and *Mixed Normal*. However, it turns out that these determinants do not contribute significantly to the explanation of the rejection probabilities.

Table 5: Power function, lognormal (α=0.05; 1000 replications)[a]

n	ρ	MLE			GMM	GMM eff.
		Wald	LM	LR	Wald	Wald
100	-0.5	0.843	0.678	0.808	0.822	0.729
	-0.4	0.647	0.449	0.605	0.609	0.536
	-0.3	0.444	0.282	0.407	0.412	0.372
	-0.2	0.230	0.126	0.199	0.202	0.228
	-0.1	0.076	0.038	0.068	0.062	0.109
	0	0.047	0.032	0.038	0.029	0.048
	0.1	0.069	0.065	0.051	0.037	0.052
	0.2	0.155	0.157	0.138	0.101	0.114
	0.3	0.398	0.401	0.350	0.280	0.277
	0.4	0.697	0.686	0.646	0.498	0.503
	0.5	0.903	0.887	0.875	0.693	0.712
250	-0.5	0.971	0.953	0.969	0.971	0.943
	-0.4	0.911	0.856	0.901	0.915	0.857
	-0.3	0.773	0.697	0.760	0.773	0.713
	-0.2	0.471	0.378	0.452	0.472	0.429
	-0.1	0.123	0.097	0.113	0.119	0.166
	0	0.056	0.052	0.051	0.049	0.065
	0.1	0.129	0.129	0.119	0.104	0.108
	0.2	0.399	0.417	0.388	0.336	0.314
	0.3	0.810	0.793	0.779	0.690	0.658
	0.4	0.980	0.972	0.972	0.908	0.885
	0.5	1.000	1.000	1.000	0.994	0.985
500	-0.5	0.999	0.999	0.999	0.999	0.995
	-0.4	0.984	0.980	0.984	0.989	0.975
	-0.3	0.945	0.936	0.944	0.948	0.920
	-0.2	0.714	0.667	0.709	0.733	0.682
	-0.1	0.238	0.208	0.239	0.257	0.279
	0	0.042	0.042	0.039	0.050	0.057
	0.1	0.202	0.211	0.186	0.198	0.177
	0.2	0.732	0.746	0.715	0.655	0.637
	0.3	0.987	0.984	0.985	0.959	0.954
	0.4	1.000	1.000	1.000	0.999	0.997
	0.5	1.000	1.000	1.000	1.000	0.997

[a] Sparseness is 0.751 (n=100), 0.883 (n=250), and 0.935 (n=500).

Table 6: Power function, mixed normal errors (α=0.05; 1000 replications)[a]

n	ρ	MLE			GMM	GMM eff.
		Wald	LM	LR	Wald	Wald
100	-0.5	0.811	0.606	0.771	0.797	0.680
	-0.4	0.659	0.457	0.615	0.643	0.544
	-0.3	0.392	0.266	0.368	0.359	0.378
	-0.2	0.183	0.106	0.175	0.171	0.222
	-0.1	0.071	0.032	0.062	0.062	0.107
	0	0.047	0.030	0.039	0.044	0.062
	0.1	0.068	0.067	0.056	0.055	0.071
	0.2	0.193	0.210	0.164	0.157	0.164
	0.3	0.472	0.478	0.426	0.324	0.329
	0.4	0.763	0.717	0.691	0.511	0.518
	0.5	0.913	0.890	0.882	0.702	0.714
250	-0.5	0.973	0.955	0.969	0.980	0.942
	-0.4	0.947	0.891	0.937	0.951	0.884
	-0.3	0.768	0.655	0.752	0.784	0.679
	-0.2	0.401	0.316	0.387	0.405	0.408
	-0.1	0.133	0.115	0.132	0.136	0.192
	0	0.056	0.046	0.050	0.052	0.078
	0.1	0.107	0.111	0.096	0.112	0.093
	0.2	0.465	0.465	0.429	0.372	0.338
	0.3	0.812	0.792	0.780	0.697	0.653
	0.4	0.964	0.960	0.958	0.917	0.894
	0.5	0.994	0.993	0.993	0.983	0.971
500	-0.5	0.997	0.997	0.997	0.998	0.994
	-0.4	0.994	0.994	0.994	0.994	0.988
	-0.3	0.955	0.934	0.951	0.956	0.929
	-0.2	0.698	0.635	0.693	0.714	0.666
	-0.1	0.220	0.180	0.213	0.231	0.246
	0	0.054	0.049	0.047	0.057	0.060
	0.1	0.200	0.220	0.187	0.221	0.192
	0.2	0.776	0.765	0.749	0.674	0.657
	0.3	0.972	0.970	0.967	0.931	0.917
	0.4	0.998	0.999	0.999	0.996	0.988
	0.5	1.000	1.000	1.000	1.000	1.000

[a] Sparseness is 0.751 (n=100), 0.883 (n=250), and 0.935 (n=500).

Table 7: Response surface estimation (α=0.05; 1000 replications)[a]

Variable[b]	MLE			GMM	GMM eff.
	Wald	LM	LR	Wald	Wald
ρ	0.001	0.001	0.001	0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
	0.128	*0.108*	*0.103*	*0.295*	*0.368*
ρ^2	0.960	1.604	0.930	0.064	0.389
	(0.222)	(0.231)	(0.237)	(0.248)	(0.200)
	0.000	*0.000*	*0.000*	*0.797*	*0.054*
n	-34.322	-26.179	-29.497	-22.145	-19.123
	(9.897)	(10.080)	(10.065)	(11.161)	(9.261)
	0.001	*0.010*	*0.004*	*0.049*	*0.041*
$n \times \rho^2$	0.010	0.019	0.016	0.023	0.019
	(0.007)	(0.007)	(0.007)	(0.008)	(0.006)
	0.124	*0.005*	*0.021*	*0.003*	*0.001*
Lognormal $\times \rho^2$	0.054	-1.373	-1.141	-0.990	-2.116
	(1.404)	(1.345)	(1.428)	(1.608)	(1.204)
	0.970	*0.309*	*0.425*	*0.539*	*0.081*
Mixed Normal $\times \rho^2$	-1.288	-2.364	-1.968	-1.959	-2.973
	(1.416)	(1.490)	(1.553)	(1.602)	(1.238)
	0.365	*0.115*	*0.207*	*0.223*	*0.018*
Sparseness	3.626	4.947	4.058	5.595	4.872
	(1.514)	(1.552)	(1.585)	(1.570)	(1.495)
	0.018	*0.002*	*0.011*	*0.000*	*0.001*
Sparseness $\times \rho^2$	68.179	55.969	61.099	48.276	43.720
	(13.090)	(13.213)	(13.140)	(14.639)	(12.129)
	0.000	*0.000*	*0.000*	*0.001*	*0.000*
Constant	-5.272	-6.616	-5.770	-6.984	-6.187
	(1.194)	(1.223)	(1.251)	(1.241)	(1.182)
	0.000	*0.000*	*0.000*	*0.000*	*0.000*
R^2	0.905	0.907	0.899	0.891	0.899

[a] OLS regressions with heteroskedasticity-robust standard errors.

[b] Coefficient estimates; standard errors in parentheses; p-values in italics.

of ρ^2. Third, the size and power of the tests seem independent of the choice among the considered error distributions. Only for the efficient GMM-based Wald-test, the power tends to be significantly lower in case of lognormally or mixed normally distributed errors as compared to normally distributed ones. The sparseness of (irregular) spatial weighting matrices is a very important determinant. Both the size and the power of the tests increase significantly with the sparseness of the lattice.

6 Conclusions

This paper compares MLE-based and GMM-based tests against spatially autocorrelated errors in Ord-type spatial models. In doing so, we consider a test based on the recently derived parametric estimate of the GMM-based variance of the SAR parameter. The small sample properties are investigated in a Monte-Carlo study.

Assuming different spatial processes, previous research tended to point to a better performance of MLE-based tests rather than GMM-based ones, especially, in small samples. However, our findings do not support this view for testing against SAR. Two versions of a Wald-test based on the variance of the SAR parameter under GMM derived by Kelejian and Prucha (2005a) perform as well as MLE-based tests in case of normally distributed errors. The GMM-based Wald-tests tend to perform extremely well irrespective of the underlying error distribution, and they outperform the MLE-based LM-test in terms of both size and power in small to moderately-sized samples for negative values of the SAR parameter. Across the board (i.e., across sample sizes and error distributions), our results support the use of both MLE-based tests and the GMM-based Wald-tests. Regarding GMM-based Wald-tests, in particular that one based on one-step estimates using the simple fixed moments weighting matrix can be recommended for applied researchers even in small sample applications.

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