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Patrick Plötz

Uncertainty in Diffusion of Competing Technologies and Application to Electric Vehicles

Fraunhofer ISI

Abstract

The diffusion of innovations is an important process and its models have applications in many fields, with particular relevance in technological forecast. The logistic equation is one of most important models in this context. Extensions of this approach as the Lotka-Volterra model have been developed to include the effect of mutual influences between technologies such as competition. However, many of the parameters entering this description are uncertain, difficult to estimate or simply unknown, particularly at early stages of the diffusion. Here, a systematic way to study the effect of uncertain or unknown parameters on the future diffusion of interacting innovations is proposed. The input required is a general qualitative understanding of the system: is the mutual influence positive or negative and does it apply symmetrically to either technology? Since the parameters enter the problem via a set of coupled non-linear differential equations, the approach proposed here goes beyond simple Monte-Carlo-like methods where the result is an explicit function of the parameters. The methodology is developed in detail and applied the case of three types of upcoming electric vehicle propulsion technologies. The findings indicate that competition between electric vehicles and mild hybrid vehicles implies a slow decline of the latter. The approach can easily be generalised to include other initial conditions, more technologies or other technological areas to find stable results for future market evolution independent of specific parameters.

Key words

diffusion of innovations; logistic equation; competition; electric vehicles; Monte Carlo methods

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1 Introduction

The diffusion of new products and technologies into markets has many interesting applications and models for this diffusion process have long been studied [1]. An important class of models with many successful applications in science and technology is based on the logistic differential equation [2]. Quite often, mutual influence of different innovations or technologies is important. Typical examples include the competition of similar innovations, the substitution of old technologies by new ones, or mutual support. More generally, ref. [3] distinguishes six qualitative patterns of innovations interaction: pure competition, predator-prey, symbiosis, parasitic, symbiotic (loss-indifferent), and no-competition. To model these effects, systems of coupled logistic equations have been introduced [4], [5], [6], [7]. Typically, two mutually interfering technologies are modelled. But also larger systems of coupled logistic equations have been studied to cope with the interaction of more than two technologies [8], [9]. Similar steps have been taken (much earlier) in mathematical biology to model many interacting species competing for limited resources such as food or living space (see for example [10], [11] and references therein).

However, in applications for the diffusion of innovations or new technologies, many of the parameters required for modelling are uncertain, difficult to estimate, fluctuating in time or simply unknown. These factors in parameter estimates have not yet been addressed systematically in logistic models of technology diffusion. The goal of the present work is to address this issue in more detail. In engineering models for specific systems and using system-specific (often physical) parameters, it is common to vary input parameters and check the stability of the overall behaviour against these variations (sensitivity analysis) or to use many variations of input parameters and look at the statistical moments (mean and standard deviation) of the outcomes. The parameters in more abstract models (often called top-down models) are often more difficult to estimate and a systematic way (beyond fitting existing diffusion paths when sufficient data is available) of studying the effect of uncertainty or lack of knowledge in these approaches is still missing for technology diffusion.

The present work proposes to include the interaction of innovations via coupled logistic equations and to take a few simple assumptions and to ask whether some model results are almost independent of the choice of parameters. These results are then stable against parameter changes and thus (if the very basic assumptions for mutual influence are valid) highly probable to occur. Or put differently: The specific parameters are yet unknown and difficult to estimate, they also depend on the choices and decisions of the different actors involved (manufacturers and consumers), but are there some developments certain given a few very basic assumptions? To answer this question will be attempted in the present paper for the example of interacting vehicle propulsion techniques.

The paper is structured as follows. The model and a few illustrative examples will be introduced in section 2. The method for coping with parameters in the formulation of the set of coupled equations is elaborated in more detail in section 3. Then this method will be applied to the case of interacting vehicle propulsion technologies (section 4), that is battery electric vehicles (BEV), Plug-in hybrid vehicles (PHEV) and hybrid electric vehicles (HEV), including the variation of system parameters. Section 5 will discuss the present approach and set it into context. The present work will be summarized in section 6.

2 Logistic Diffusion Model for Competing Technologies

2.1 Setup

The simplest model to describe market diffusion is the logistic equation

$$\frac{\mathrm{d}}{\mathrm{d}t} N(t) = rN\left(1 - \frac{N}{N_{\max}}\right), \tag{1}$$

and its well-known solution, the logistic function

$$N(t) = \frac{N_{\max}}{1 + \left(\frac{N_{\max} - N_0}{N_0}\right) \mathrm{e}^{-r(t-t_0)}} \tag{2}$$

Here, $N(t)$ denotes the number of adopters at time t, r is the growth rate, $N_{\max}$ the maximal number of adopters and N_0 is the number of adopters at initial time t_0. Equation (1) is a simple way to describe market diffusion of a *single technology* in a top-down manner. It contains only a few parameters (the initial market share of that technology, the growth rate and the maximum value of market penetration). Empirically, these parameters can either be fitted to existing time series of logistic growth [12], [13] or by comparison with similar technologies and general boundaries.

A drawback of the simple model of eq. (1) is the disregard of all other technologies some of which might have significant impact on the technology under consideration. This is unrealistic in many situations where a technology is replacing an existing one in direct competition or where it (maybe partially) depends on the evolution of other technologies. The case of technological substitution has lead to the extension of the logistic growth model to include a second technology [4], [5], [6], [7].

The more complex interactions of more than two technologies can also be modelled with logistic equations [8], [9]. The logistic growth model of eq. (1) has to be extended to a system of D coupled equations for D interacting technologies. This set of coupled logistic equations reads:

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}N_1(t)=r_1N_1\left(1-\frac{N_1}{N_{\max 1}}-\alpha_{12}N_2-\alpha_{13}N_3-\ldots-\alpha_{1D}N_D\right)\\
&\frac{\mathrm{d}}{\mathrm{d}t}N_2(t)=r_2N_2\left(1-\frac{N_2}{N_{\max 2}}-\alpha_{21}N_1-\alpha_{23}N_3-\ldots-\alpha_{2D}N_D\right)\\
&\ldots\\
&\frac{\mathrm{d}}{\mathrm{d}t}N_D(t)=r_DN_D\left(1-\frac{N_D}{N_{\max D}}-\alpha_{D1}N_1-\alpha_{D2}N_2-\ldots-\alpha_{DD}N_D\right).
\end{aligned} \tag{3}$$

If all couplings between the different technologies vanish, i.e. for $\alpha_{ij}=0\ \forall i,j=1,\ldots,D$, the system of equation (3) describes D independent technologies with their own growth rates r_i and maximum values $N_{\max,i}$. But a non-vanishing coupling between the different technologies describes their interaction and leads to a modified mutually influenced evolution of the different technologies as is more realistic for interdependent technologies. In this sense, the set of equations (3) represents an extension of the single-technology logistic growth of equation (1) to several coupled technology evolutions. For the sake of brevity, equations (3) can also be written as

$$i=1\ldots D:\quad \frac{\mathrm{d}}{\mathrm{d}t}N_i(t)=r_iN_i\left(1-\sum_{j=1}^{D}\alpha_{ij}N_j\right),\ \text{where } \alpha_{ii}=\frac{1}{N_{\max,\,i}} \tag{4}$$

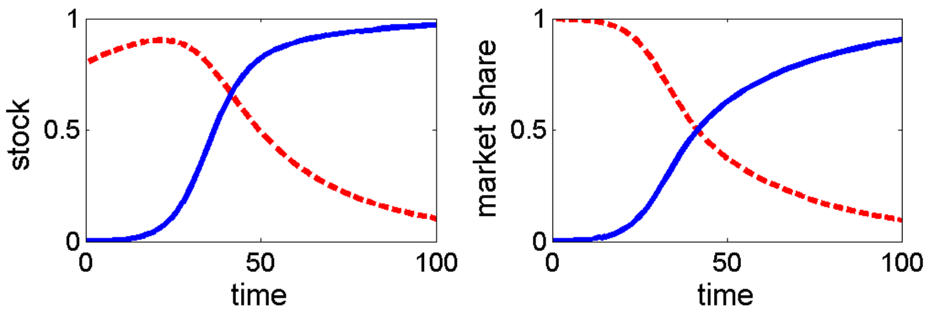
This can be summarized in matrix notation as $\mathrm{d}\vec{N}(t)/\mathrm{d}t=\vec{r}\vec{N}(1-\alpha\vec{N})$ with a matrix of interaction parameters α and a vector of growth rates $\vec{r}$. These equations are well-known in mathematical biology where they serve to model the growth of different species competing for limited resources.

2.2 Examples

In order to illustrate the model introduced in the previous section, two examples will be discussed in the following: technology substitution (one technology is replaced by another one) and competition among different web browsers (more than two technologies). The diffusion of different electric vehicle technologies will be discussed in much more detail in section 4.

The simplest system beyond a single technology is the interaction of $D = 2$ technologies in the process of market diffusion. This case has been studied with generalised logistic equations elsewhere [14]. Here, only the example for the substitution of one technology by another is given to illustrate the set of equations (3) (see Figure 1). The left panel of Figure 1 shows the absolute values of $N_1(t)$ and $N_2(t)$ representing the number of units sold (in arbitrary units), whereas the right panel shows the normalised values $N_i(t)/\Sigma_i N_i(t)$, corresponding to market shares.

Figure 1: Technology substitution as an example evolution for two interacting technologies

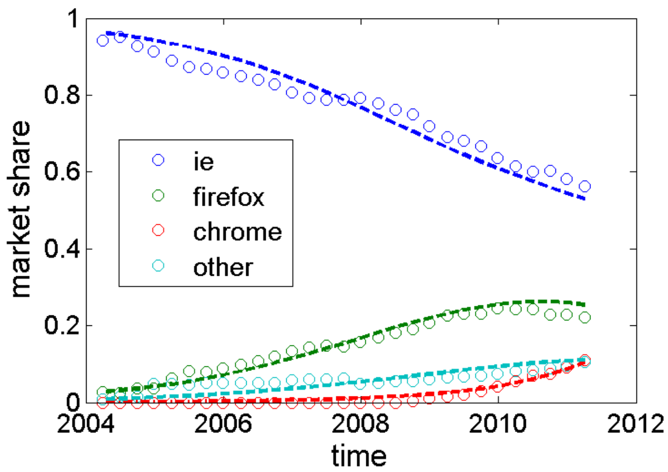


Description: The left panel shows the absolute stock values of the two fictitious technologies and the right panel shows the market shares (normalized fraction of total stock) for the same two fictitious technologies.

Both panels in Figure 1 show that the system of equations (3) is able to produce asymmetric market diffusion curves and goes clearly beyond single logistic growth models. The parameters for the generation of Figure 1 read: Initial market share $\vec{N}(0) = (0.8,\ 0.001)$, growth rates $\vec{r} = (0.05,\ 0.25)$, both have a theoretical market share of 100%, i.e. $\alpha_{11} = \alpha_{22} = 1$ and are interacting with $\alpha_{12} = 1.5$ and $\alpha_{21} = 0.25$.

To demonstrate the usefulness of the present approach and to give a second non-trivial example, the time evolution of market shares of web browsers will be examined. In this case, several browsers are on the market competing for customers (usually referred to as “users”) and their market shares are rather well documented [See http://marketshare.hitslink.com/browser-market-share.aspx?qprid=0 and summarised in 15]). Several web browsers can be distinguished and 3 web browsers with the largest market shares in recent years have been selected: Microsoft Internet Explorer, Mozilla Firefox, and Google Chrome. Other web browsers (such as Safari or Opera and others) have been summarised as ‘others’. The time evolution of the market shares are shown in Figure 2, together with a simple fit from a $D=4$ model of competing technologies. For the fit, 10,000 random values for the growth rates and interaction coefficients have been tried. Shown in Figure 2 is the combination minimising the sum of squared deviations between fit and measured values. Figure 2 is intended only as an illustrative example of several interacting technologies, the actual fit or parameter values are not the focus if the present paper.

Figure 2: Market shares of several web browsers over time and fit with coupled logistic equations



Description: Shown are the measured market shares (circles) and a simple fit with coupled logistic equations as in eq. (3)

3 Method of Random Parameters in Coupled Logistic Equations

The aim of the present section is to discuss the method proposed here in more detail. Mathematically, I propose to study a set of differential equations with random parameters and problem-specific constraints for these parameters. Applied to the case of competing vehicle technologies, the set of differential equations are the coupled logistic differential equation from eq. (3). The random parameters will be the interaction matrix elements α_{ij}, and the problem specific symmetry can be a symmetric competition $\alpha_{ij} = \alpha_{ji}$ or approximate relations such as $\alpha_{ij} \ll \alpha_{kl}$. Using this input, one would solve the set of differential equations for many parameters combinations and discuss the mean (or median) of the set of solutions together with the standard deviations or other measures characterising the resulting set of solutions to the differential equations under consideration.

The approach presented is similar to several existing methods and a few comments are in place. The use of random numbers for uncertain or unknown parameters is tightly connected to Monte Carlo methods. There seems to be no common definition of Monte Carlo methods or Monte Carlo simulations (see, e.g., [16] and [17] for a discussion). However, standard examples include the computation of definite integrals (for various applications in mathematics, physics, etc.) or computation of possible outcomes when using different input variables (typical for engineering). Thus, if one refers to 'Monte Carlo method' as "varying parameters for a result that is an explicit function of the parameters", then the present method is not a Monte Carlo method. However, if 'Monte Carlo method' should refer to "using random numbers for solving scientific or engineering problems" the present method certainly is a Monte Carlo method.

Let us explain this in more detail. A typical problem for the application of Monte Carlo methods would be the following. The quantity to be determined, x, can be computed from a set of parameters $p_1,...,p_n$ and possibly on time t. The general functional form reads

$$x = f(p_1,...,p_n,t), \tag{5}$$

where f is an explicit function of the all its variables. Thus when, all parameters are given, the solution x can directly be obtained. A Monte Carlo method would then vary the input parameters according to suitably chosen random distributions, such as $P_1(p_1),...,P_n(p_n)$ or a joint probability distribution $P_0(p_1,...,p_n)$, and straightforwardly obtain a distribution of results $P(x)$. In this 'explicit' Monte

Carlo approach no integration of differential equations or solution of implicit equations is required.

In contrast to an explicit function, the present approach requires the (numerical) integration of a set of differential equations with coefficients that take random values but are fixed during every integration. It is thus a random coefficient differential equation which differs from stochastic differential equations where the randomness enters the differential equation via a time-dependent random variable. The mathematical problem reads

$$\frac{\mathrm{d}n}{\mathrm{d}t} = f(n, p_1, \ldots, p_n, t) \tag{6}$$

Thus, this (system of) ordinary differential equations has to be solved for many parameter values and one obtains a distribution of solution functions $P(n(t))$. This distribution can then be characterised for example by its moments. In this context it is important to note that the time evolution with mean values of random parameters is generally not equal to the mean of time evolutions with random parameters [18]. By requiring the solution of a set of differential equations many times, the present method is more demanding than a static Monte Carlo procedure where the result is an explicit function of the input parameters. For this reason, other methods have been designed and used to deal with random coefficient differential equations (see [19], [20] for an example including growth models).

Furthermore one needs to mention that many other systems are simulated by the use of random numbers as matrix entries. These methods are usually referred to as random matrix theory, with wide applications in physics [21], [22] and finance [23]. However, these systems, and in particular their dynamics, are linear. Albeit linear does not mean simple (Quantum chaos [24] is a typical example for complicated time evolutions in a linear system). But the system of differential equations studied here is non-linear. For this reason, the methods of random matrix theory (which usually give spectral statistics) cannot be applied, because the non-linearity of the system make the matrix' eigenvalues useless to describe the system's dynamics. However, coupled systems of non-linear growth models with coupling coefficients modelled as random matrices have been studied in mathematical biology where the logistic growth or Lotka-Volterra models were initially developed (see [11], [25] and references therein). But in these biological systems, very large number of species are studied (i.e., $D \gg 1$ in the present notation) and statistical methods can be successfully applied. But for the case of interest here, competition and interaction of technologies, only a small number of technologies show relevant interaction with each other. Usually two to four technologies give an adequate picture of the mutual influence between well-defined and distinct technologies. Therefore $D > 1$ but $D \gg 1$ does not hold and statistical methods cannot be used.

To summarise, coupled logistic equations are an established tool to study the evolution of mutually interacting market shares (or species population in biology) and the inclusion of random parameters reflects uncertainties, measurement errors or simply lack of knowledge often encountered in real systems. The use of approximate symmetries and reduction to the relevant number of technologies allows to model system specific features and avoids arbitrariness of the method.

4 Application: Electric Vehicle Market Dynamics

4.1 Characteristics of the Electric Vehicle Case

The rest of the present paper will focus on market diffusion of electric and conventional vehicle technologies. To this end, these technologies and their qualitative mutual influence will shortly be described. The following subsections will then be dedicated to a detailed analysis of the relevant parameters and the effect of parameter variations.

The future vehicle technologies to be considered include: hybrid electric vehicles (HEV), plug-in-hybrid electric vehicles (PHEV), and battery electric vehicles (BEV). Table 1 gives an overview of how the system is adopted to the three competing technologies.

Table 1: Three competing propulsion technologies and required specific parameters

Index	Vehicle	Approx. Battery Size	Parameter
i = 1	HEV	2 kWh	$r_1, N_{max,1}, N_1(0)$
i = 2	PHEV	12 kWh	$r_2, N_{max,2}, N_2(0)$
i = 3	BEV	24 kWh	$r_3, N_{max,3}, N_3(0)$

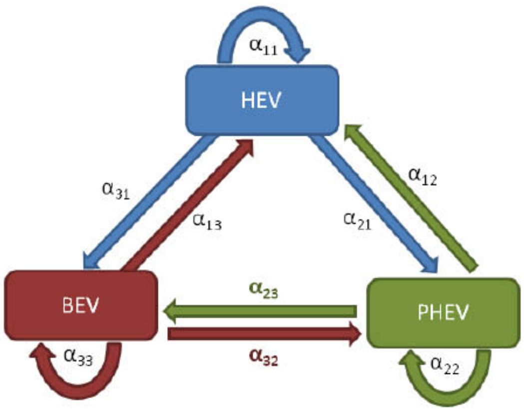
The general choice of interaction parameters as positive or negative in the present case of three vehicle technologies is summarised in Table 2.

The system of interactions is schematically displayed in Figure 3. Please note, that this is not meant as a system dynamics model since the scheme in Figure 3 does not show the non-linearity of the system of equations used here. However, the mutual interaction and general meaning of the interaction matrix elements α_{ij} is easily understood from Figure 3. Furthermore, the two negative matrix elements that represent mutual support are shown in bold face.

Table 2: Choice of interaction parameter signs for electric vehicle technologies

Coupling	Example
j inhibits i: $\alpha_{ij} > 0$	HEV and BEV
j supports i: $\alpha_{ij} < 0$	BEV and PHEV
j independent of i: $\alpha_{ij} = 0$	BEV and horse carriage

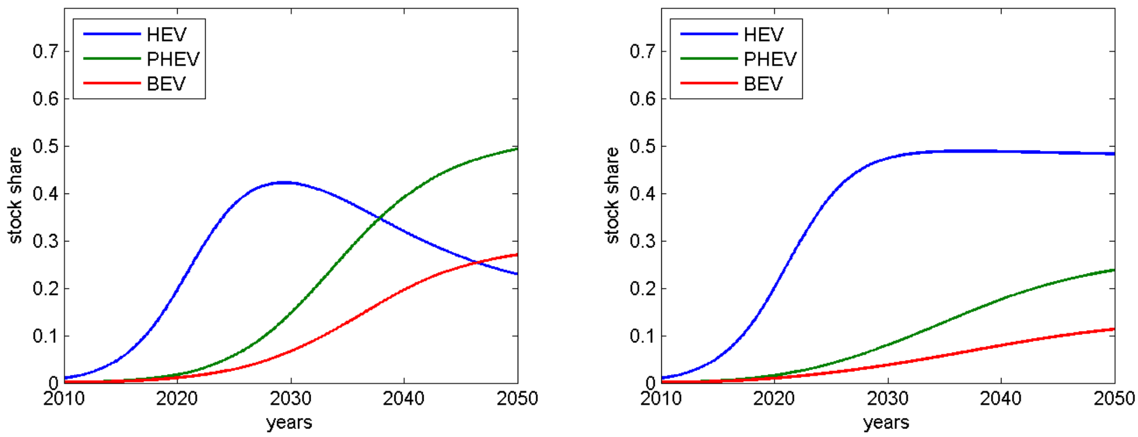
Figure 3: Scheme of the interaction parameters for combined logistic modelling of alternative vehicle technologies



Below, Figure 4 gives *two examples* of the dynamics that can be generated by the model. The first row shows the results with the same initial parameters but different interaction parameters. The basic parameters are $\vec{r} = (0.35, 0.3, 0.25)$, $\vec{N}_{max} = (1/2, 1/2, 1/4)$ and the initial values read $\vec{N}_0 = (0.01, 0.001, 0.001)$. These assumptions are only exemplary values and detailed parameters will be discussed below. These are chosen for a strong competition between HEVs and PHEVs/BEVs in the left column of Figure 4 ($\alpha_{12} = \alpha_{13} = 0.8$, $\alpha_{21} = \alpha_{31} = 0.5$ and $\alpha_{23} = \alpha_{32} = -0.5$), and for a weak competition in the right column ($\alpha_{12} = \alpha_{13} = 0.1$, $\alpha_{21} = \alpha_{31} = 1.0$, and $\alpha_{23} = \alpha_{32} = -0.1$) of Figure 4.

Clearly, Figure 4 demonstrates that the interaction included in the model is able to generate complex dynamics highly similar to actual EV market scenarios presented elsewhere [26]. Furthermore, the set of coupled logistic equations allows to model the mutual influence of several technologies in their respective market share evolution.

Figure 4: Two examples for the possible dynamics of competing propulsion technologies



Description: Shown are exemplary dynamics for strong competition (left panel, parameters in the text) and the weak competition (right panel, parameters given in the text).

4.2 Detailed Parameter Estimates

After giving several examples for the possible dynamics, the relevant parameters will now be determined as far as possible. The focus will lie on sales share since the notion of direct competition applies straightforwardly to this situation (a potential users has to decide between different existing options in buying one of them).

For the initial sales ones faces the problem that PHEVs are currently not available in Germany and there is only limited data for the US. For the US, data from [27] has been used and the monthly mean available has been extrapolated to the full year 2011. The relative values are obtained anticipated 11 million light duty vehicles sales in the US for 2011. For the sales values, it will be assumed that PHEVs will make up about half the BEV values, similar to the US. Table 3 summarises the initial values for market shares of the three vehicle technologies for the US and German market. Following Table 3, the initial values for the German market shares are given by $\vec{N}_0 = (36.6,\, 0.9,\, 1.90)\cdot 10^{-4}$.

The growth rates of the different technologies have to be estimated as well. Since the market is relatively young the HEV sales are strongly fluctuating between the years. Table 4 shows the absolute and relative sales in Germany and the US. These lead values for growth of absolute sales $N_i(t)$ and relative sales $n_i(t)$ with average annual growth factors p_i and growth rates r_i given by

$$p_i = \left(\frac{N_i(2010)}{N_i(2010-T)} \right)^{1/T} - 1, \quad \text{and} \quad r_i = \ln p_i \, .$$

Table 3: Current market shares of the three vehicle technologies for the US and German market

	US Market 2011 [27]		German Market 2010 [28]	
	Absolute sales	Relative sales	Absolute sales	Relative sales
HEV	261545	2.378%	10661	0.366%
PHEV	4920	0.045%	*270**	*0.009%**
BEV	8254	0.075%	541	0.019%
* PHEVs are not explicitly given in German vehicle sales statistics and numbers had to be estimated (shown in italics)				

Table 4: Sales and growth rates of HEVs for the US and German market

Current US HEV Market [29]			Current German HEV Market [28]		
	Absolute sales	Relative sales		Absolute sales	Relative sales
2000	9350	0.056%	2005	3589	0.107%
2010	274210	2.384%	2010	10661	0.366%
Growth p	40.2%	45.5%	Growth p	24.3%	27.9%
Growth rate r	0.34	0.38	Growth rate r	0.22	0.25

Table 4 shows that sales have increased in both countries, however at different rates. Since PHEVs and BEVs are only entering the markets right now, their growth rates have to be estimated. We assume that growth rates will increase due to higher availability of HEVs and strong governmental support to reach greenhouse gas emission targets. Accordingly, the vector of growth rates reads $\vec{r} = (0.75, 0.6, 0.45)$. The assumption on growth rates has some uncertainty and will be relaxed below (cf. section 4.4).

The maximal theoretical market shares are chosen as $\vec{N}_{max} = (1.0, 0.8, 0.4)$. HEVs could directly substitute all existing cars (which would imply a theoretical stock share of 100%), whereas PHEV are more expensive and – due to their characteristics – are not meaningful as vehicles for long trips only. Lastly, the limited range of BEVs and their high purchase price implies a limited theoretical market share.

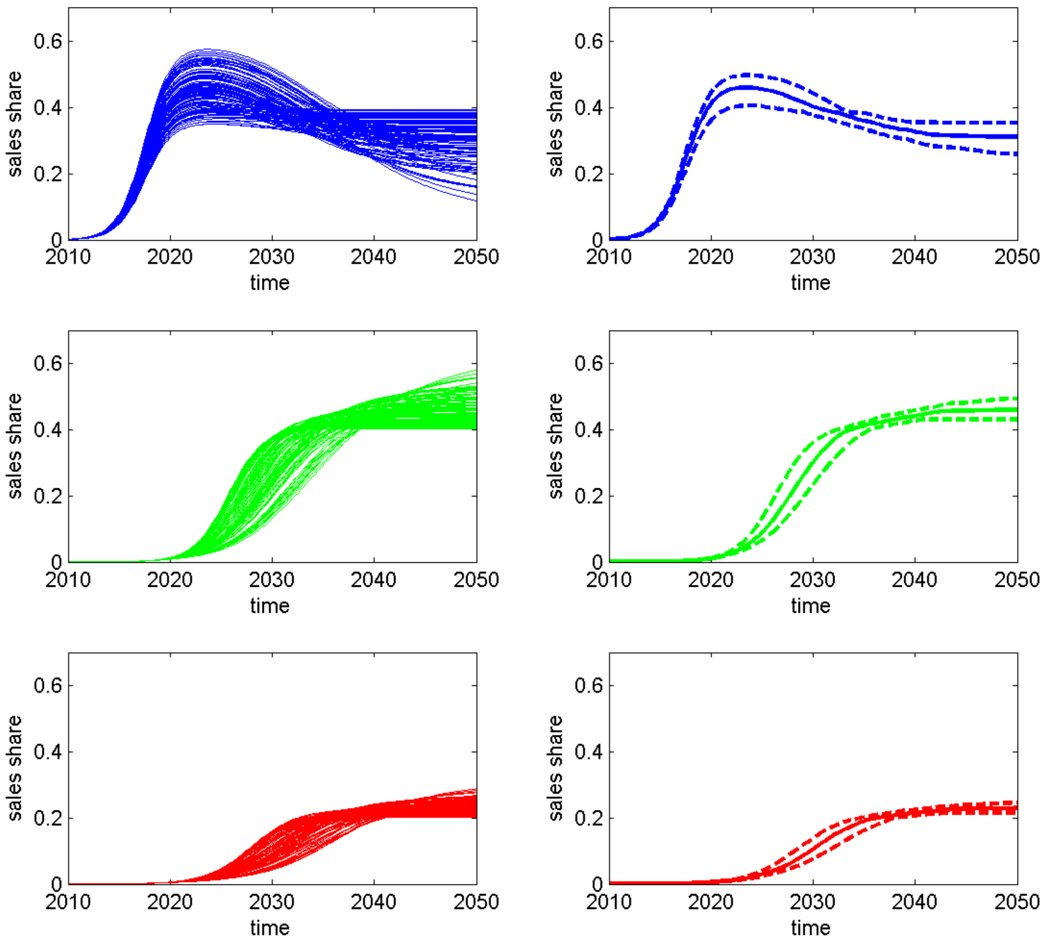
Let us now turn to the interaction matrix elements, reflecting the way the different technologies influence each other. Since mild hybrids (HEV) are very different from PHEVs and BEVs, they compete directly with the latter two and the interaction coefficients between HEV and PHEV/BEV are chosen symmetrically, i.e. $\alpha_{1i} = \alpha_{i1}$. However, large sales of PHEV and BEV imply a high demand for batteries and the resulting economy of scale (as well as technological learning) lead to a decrease of battery costs. This in turn reduces the price of both PHEVs and BEVs. Since the battery of a BEV is typically about twice the size of a PHEV battery we choose $\alpha_{23} = 2\alpha_{32}$. Additionally, the competition between HEVs and both new propulsion vehicles PHEV and BEV should be similar, such that $\alpha_{12} = \alpha_{13}$. Altogether, only 2 free parameters remain α_{12} and α_{23}.

4.3 Results for Varying Coupling Strengths

The last section has been used to fix the initial conditions for the set of ordinary differential equations (3) describing the evolution of market shares under mutual influence. However, the specific values for interaction matrix elements are difficult to estimate (or might vary in time) and we applied certain constraints on these parameters, such as symmetry and possible range of values. In the following we will use random numbers for the parameters and study the resulting time evolutions statistically. That is, we take the two remaining interaction parameters to be uniformly distributed random numbers from the interval [0; 0.5]. The interval is chosen such that the actual strength of the interaction is not as strong as the market diffusion (the signal) itself. The uniform distribution has been chosen as the simplest assumption.

Figure 5 shows the result of 100 solutions to the non-linear system of logistic equation describing the interaction of (top to bottom) HEVs, PHEVs, and BEVs in the market diffusion. The left column displays the 100 individual solutions and the right column shows the median (solid line) as well as the lower and upper quartile (dashed lines). For each computation, all parameters have been as described in the previous section and the two remaining interaction matrix elements have been chosen randomly from uniform distributions.

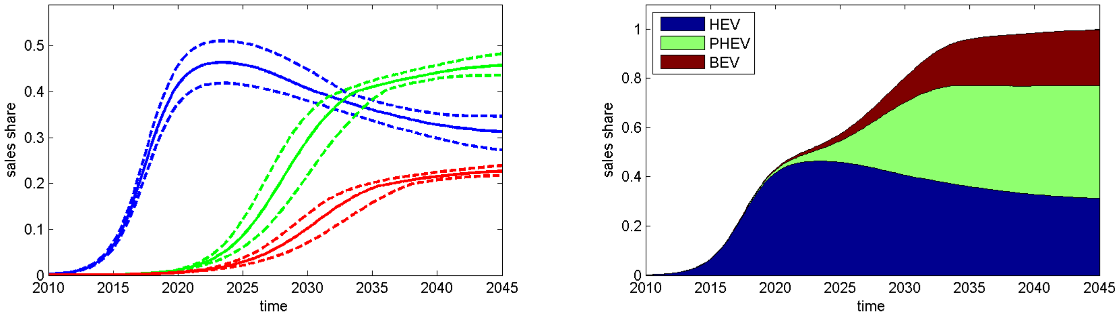
Figure 5: Dynamics of competing propulsion technologies under interaction parameter variation (shown are 100 iterations)



Description: The left panel shows all evolutions of market shares, i.e. the 100 different solutions to the set of differential equations. The right panel shows a statistical summary of all solutions (Solid line: median, dashed lines first and third quartile). The three technologies are HEV (blue, top row), PHEV (green, centre row), and BEV (red, bottom row). Parameters as discussed in section 4.2 and the two free parameters have been chosen as uniform random numbers from [0; 0.5].

We find that the sales share of HEVs increase quickly up to 40 – 60% by around 2025 but decrease after that point of time. Additionally the electric vehicles PHEV and BEV take more time to gain relevant market shares but reach stable values between 2020 and 2030. Depending on the particular values of the randomly chosen interaction matrix elements, the dynamics of market shares vary. However, studying the right column reveals that the overall range of possible outcomes does not strongly depend on the randomly chosen interaction matrix elements. Figure 6 shows the same result on a larger statistical basis of 1024 solutions of the set of coupled logistic equations with the same parameters as in Figure 5.

Figure 6: Dynamics of competing propulsion technologies under interaction parameter variation (summary of 1024 iterations).



Description: The left panel shows the evolution of market shares (Solid line: median, dashed lines first and third quartile) for HEV (blue), PHEV (green), and BEV (red). The right panel shows is a stacked area plot of the median market share with the same colours. Parameters as in Figure 5.

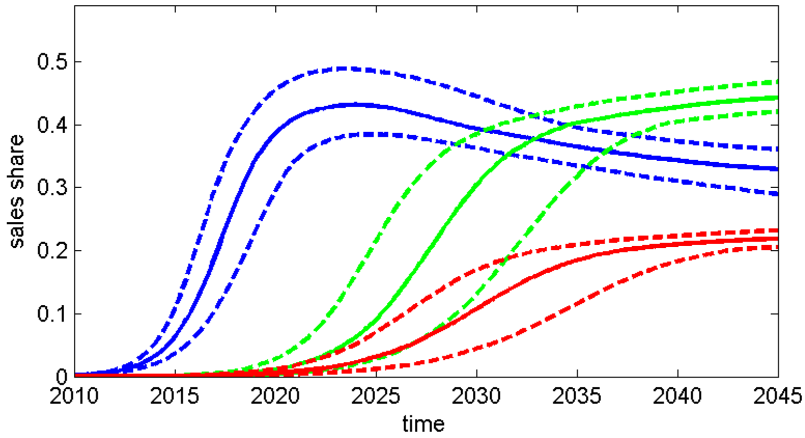
Figure 6 confirms the outcome of Figure 5, showing the same dynamics of market shares (HEV in blue, PHEV in green, BEV in red; solid line – median, dashed lines – upper and lower quartile). The following qualitative features of the market share evolution under the given assumptions are therefore stable against variation of the interaction parameters: HEVs form a crossover technology, their market share rises quickly but declines later on; PHEVs and BEVs become important later on but reach stable market shares with higher shares for PHEVs.

4.4 Variation of Growth Rates

In the previous section, we fixed all initial values and growth rates as well the relation between different interaction matrix elements. We only randomly varied the values of the two remaining free parameters. Let us relax the assumptions for the growth rates in the present paragraph.

We allow the growth rates to vary according to a Gaussian distribution with the growth rates from section 4.2 as mean and a normalised standard deviation $\sigma = 0.25$. That is we use $r = r_0[1 + N(0,\sigma)]$ where r_0 is original growth rate from section 4.2 and $N(\mu,\sigma)$ is a normal distribution with mean μ and standard deviation σ. Figure 7 shows the summarised result of 4096 computations with this variation of growth rates and all other parameters as in the previous section.

Figure 7: Dynamics of competing propulsion technologies including variation of growth rates (summary of $N = 4096$ iterations)



Description: Shown is the evolution of market shares (solid line: median, dashed lines first and third quartile) for HEV (blue), PHEV (green), and BEV (red). Parameters as in Figure 5 and additional variation of growth rates as discussed in the present section.

We find that the variation of growth rates leads to broader statistical range of solutions but does not alter the overall qualitative behaviour. We conclude from this and the previous section that under the given initial conditions and the assumed mutual influence of the three vehicle technologies, a quick rise of HEVs in market shares with later decline and take over by PHEVs and BEVs are very likely. The details of the market share dynamics such as maximum growth rate, equilibrium market share, time of acquiring higher market shares may vary, but overall features are independent if the modelling details.

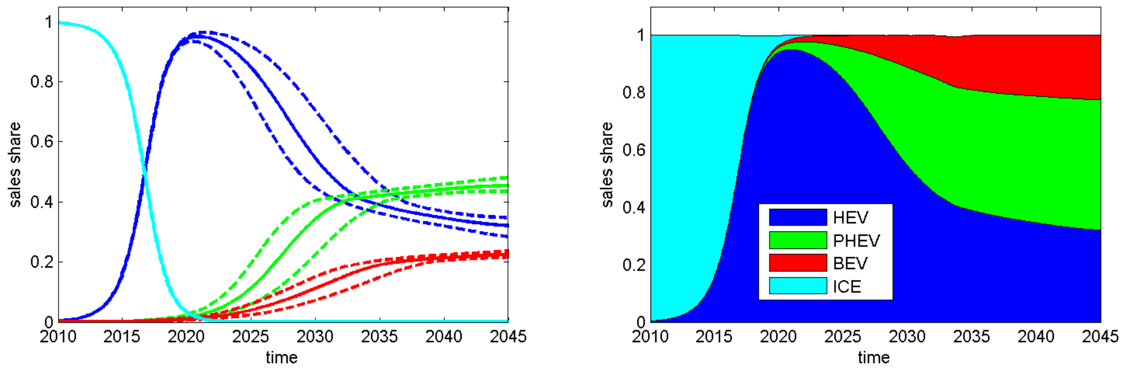
4.5 Inclusion of Combustion Engine Vehicles

The previous sections showed that – under the assumptions used – the two electric vehicle technologies PHEV and BEV will eventually win over the HEVs even though the latter are given a head start (have higher initial growth rates). A valid objection to the analysis presented above is that the presently dominating technology, the internal combustion engine vehicle (ICE), will support the market diffusion of HEVs (due to their similarity) but has not yet been included. The present section therefore will include ICEs in order to understand their effect on the qualitative results found above.

In general, under the assumption of only weak interaction, the inclusion of ICEs into the analysis should not lead to qualitative changes. If the influence of ICEs on the other technologies is limited, they simply account for the part missing in 100% when summing the market shares of the various technologies. In fact, one would expect the ICEs to lead to a larger statistical spreading but not to qualitative changes. This expectation is confronted with numerical simulations including ICEs in Figure 8.

For Figure 8 all parameters have been left as before but new interaction matrix elements have of course been added. The coupling from ICE to BEV and PHEV has been chosen just as the coupling from HEV to BEV and PHEV. Specifically, the growth rates are the same as before with the additional rate for ICEs strongly shrinking $\vec{r} = (0.75, 0.6, 0.45, -0.90)$. The initial values for the market share are also the same, but now adding up 100%, i.e. $N_0 = (36.6, 0.9, 1.9, 9960.6) \cdot 10^{-4}$. We use the same interaction parameters and the additional ones are chosen such that ICE and HEV support each other $\alpha_{14} = \alpha_{41} = \alpha_{23}/20$ but the ICE faces very strong competition with the electric vehicles: $\alpha_{42} = 2\alpha_{12}, \alpha_{24} = \alpha_{12}/2$ and the same for α_{43}. In Figure 8, the same colour code as before is used (blue: HEV; green: PHEV; red: BEV) and the ICE has been added in cyan. The medians for the evolution of market shares for each technology from 4096 computations are shown as solid lines and the first and third quartile as dashed lines. Additionally, the right panel of Figure 8 shows a stack plot of the median market share evolutions.

Figure 8: Market share evolution for $D = 4$ vehicle types, statistical summary of $N = 4096$ simulations

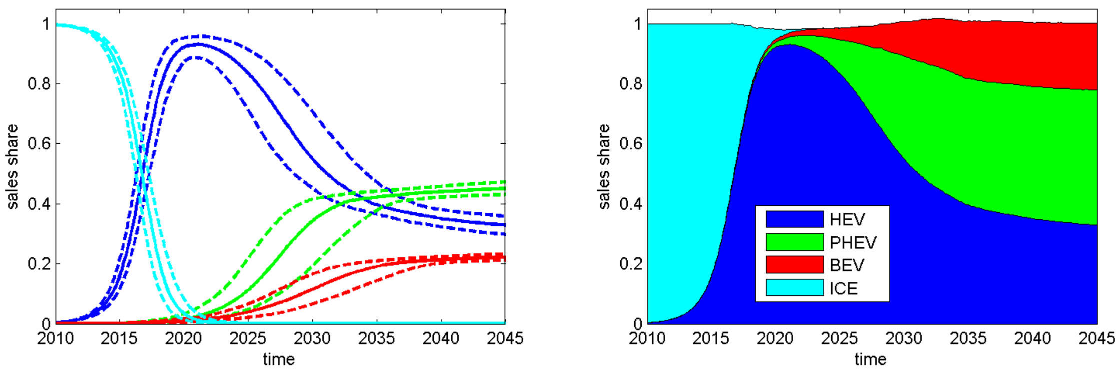


Description: The left panel shows the evolution of market shares (solid line: median, dashed lines first and third quartile) for HEV (blue), PHEV (green), BEV (red), and ICE (cyan). The right panel shows is a stacked area plot of the median market share with the same colours (parts missing to 1 derive from rounding errors). Parameters are given in the text. All parameters as in Figure 5 and additional parameters for ICEs as discussed in the present section.

Figure 8 shows that the ICEs quickly loose market share, early on in favour of HEVs and later on also for PHEVs. This clear decline in market share is not surprising considering the large negative growth rate used for ICEs and the fierce competition as encoded in the interaction matrix elements. Compared to the results from the previous sections, we observe that the mutual support between ICEs and HEVs allows the latter to gain much larger market shares than before (cf. Figure 7). However, this is also accompanied by much clearer maximum in the time evolution of the market shares including a stronger loss of market shares after the maximum. Again, PHEVs and BEVs reach relevant market shares later than HEVs but the relation is the same as in earlier results (cf. Figure 7).

As before, we include the effect of varying growth rates using Gaussian distributed deviations from the original growth rates. Figure 9 shows the results from 1024 numerical solutions of the set of four coupled logistic equations with $r = r_0[1+N(0,\sigma)]$ where r_0 is original vector of growth rates and $\sigma = 0.15$.

Figure 9: Market share evolution for $D=4$ vehicle types ($N=1024$) incuding variation of growth rates.



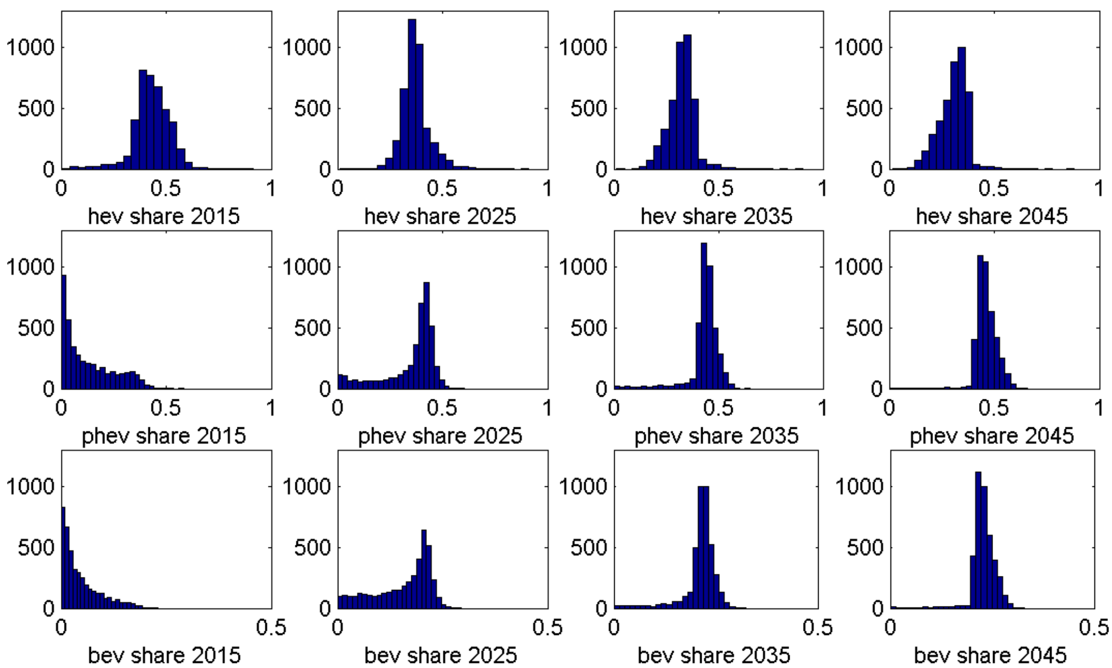
Description: The left panel shows the evolution of market shares (solid line: median, dashed lines first and third quartile) for HEV (blue), PHEV (green), BEV (red), and ICE (cyan). The right panel shows is a stacked area plot of the median market share with the same colours (parts missing to 1 derive from rounding errors). All parameters as in Figure 9 and additional variation of growth rates as discussed in the text. From Figure 9 we observe that the variation of growth rates in addition to the randomly chosen free interaction parameters leads to slightly larger statistical spread but now significant changes compared to Figure 8.

To summarise, the inclusion of ICEs in the analysis leads to a more pronounced maximum in the time evolution of the HEV market share but confirms the qualitative outcome that HEVs are a crossover technology that the market will later be dominated by PHEVs and BEVs.

5 Discussion

In contrast to static applications of random numbers to estimate the possible outcomes, we used random numbers for creating many different *dynamics* and study the possible time evolutions. These are, of course, “outcomes” but here the time evolution is crucial, in particular for the rise and fall of hybrid electric vehicles. The method we used is sometimes referred to as random coefficients differential equations. To summarise, usually the result of Monte-Carlo methods is a probability distribution of numbers, whereas here we obtained a distribution of functions (to characterise the distributions we showed the median and first and third quartile). To give an example, Figure 10 shows the distribution of market shares for the simulation from section 4.4 at four different instants of time. The first shows the distribution of market shares for HEVs in the years (from left to right) 2015, 2025, 2035, and 2045. The second row shows the distribution of market shares for PHEVs and the third row for BEVs at the same instants of time.

Figure 10: Time evolution of market share probabilities for three different vehicle technologies, HEV, PHEV, and BEV (cf. sec. 4.4 for parameters)



Description: The rows show the absolute number of market shares (top: HEV, centre: PHEV and bottom: BEV), the columns are for four different years 2015, 2025, 2035, and 2045 (from left to right).

One can clearly see how the distributions evolve in time. The medians and quartiles shown in Figure 5 to Figure 9 are a first step to characterise the evolution of these time dependent market share distributions.

6 Summary and Conclusion

With the rise of electric vehicles, many scenarios for their market development have been proposed. These differ widely in their assumptions and results, but all of them rely on specific choices for underlying parameters. However, many of the required parameters and their future evolution are difficult to estimate or almost unknown. Here, we used a method in order to determine stable results for the future market evolution independent of specific parameters choices. Technically, this question was answered by using random choices for a few model parameters and system-specific constraints. The latter system specific constraints such as symmetry of the mutual influence and signs of the parameters make the results non-arbitrary and effectively include system-specific features. Our findings indicate that competition between plug-in-electric vehicles and mild hybrid vehicles leads to slow decline of market share for the latter.

The presented method basically is an application of random coefficient differential equations in the field of technological forecast. Further research in the direction presented here should include a stability analysis as is common on the field of non-linear dynamics. This would not give the time-dependent solutions of the set of coupled logistic equations, but reveal parameter ranges leading to non-trivial steady-state solution. Furthermore, the present analysis neglected any detailed technological parameters (such as a changing fuel prices or future battery prices) as is typical for logistic growth models. But further analysis should compare the present results to the predictions of a more technical approach (cf. [30] for a possible combination of logistic models and technical or physical system properties).

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