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Skill Polarization in Local Labour Markets under Share-Altering Technical Change

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Abstract. This paper considers the "share-altering" technical change hypothesis in a spatial general equilibrium model where individuals have different levels of skills. Building on a simple Cobb-Douglas production function, our model shows that the implementation of skill-biased technologies requires a sufficient proportion of highly educated individuals. Moreover, areas that experiment this kind of technical change will initially exhibit a rise in local skill premia, but such a trend tends to be reverted over time due to labour mobility. Also, when technical progress is such to disproportionately replace middle-skill jobs, the local distribution of skill will exhibit "fat-tails", where the proportion of both highly skilled and low-skilled workers increases. These predictions are consistent with recent existing evidence.

Keywords: share-altering technologies, local skill distribution, local wage premium.
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I. INTRODUCTION

A famous paper by Autor, Levy and Murnane (2003) (ALM henceforth) re-qualified the "skill-biased technical change" hypothesis by noting that, during the last three decades, new technologies have been particularly effective in replacing workers who performed "routine tasks", that is, tasks which require a well-defined set of manual and cognitive abilities. Such new technologies have mostly raised the demand for highly-skilled workers (mainly college graduates) at the expenses of mid-skilled workers (with high-school education), while leaving basically untouched the demand for low-skills, fit to perform non-routine manual tasks such as personal services. This technological trend has generated "polarization" in employment and wages in the US labour market (Autor et al. 2008, Autor and Dorn 2009). Similar "hollowing-out" patterns in wages and employment distribution have been observed in Europe (Goos and Manning 2007, Goos et al. 2009). How do these economy-wide findings reflect in local labour markets?

Recent contributions by Moretti (2010,2011) have argued that labour market outcomes should be evaluated at the local level by exploiting spatial general equilibrium models such as Roback (1982), which account for local price levels. For instance, he observes that the educated are disproportionately located in metropolitan areas characterized by high housing costs. Some analyses of local labour markets have emphasized specific features generated by technical change. Beaudry et al. (2010) have shown that skill-intensive PC technologies have been adopted in cities where highly educated workers were abundant. Also, new developments in urban economics have argued that high levels of local human capital may attract even more educated workers, as in the "rise of the skilled city" story: see Glaeser (2008) for an overview. In particular, Berry and Glaeser (2005) have found that demand for high skilled workers has been rising in initially high skill cities. Such models, however, concentrate only on two types of workers and, for this reason, are unfit to analyze changes in skill-distribution, like rising "polarization" in the labour market. In this perspective, Eeckhout et al. (2010) observe, consistently with the findings of Lin (2011), that large cities exhibit "fat-tails" in the skill distribution. In other words, large cities disproportionately attract more skilled *and* more unskilled workers.

This paper makes a theoretical contribution which accounts for a variety of stylized facts within a simple general equilibrium unified framework, where firms and workers are free to move across areas and markets are perfectly competitive. We argue that the ATM hypothesis on the nature of recent technological change can be given a simple representation by exploiting the idea of "share-altering" technical change. This form of technical change, modelled as a change in Cobb-Douglas share parameters, has been recently re-discovered by Seater (2005), Seater and Peretto (2008) and Zuleta (2008). The idea is rather simple: consistently with observation, most modern innovations seem to raise the share parameter of skilled workers, while keeping constant the *total* labour share. At the same time, when one considers a production function with three types of workers (high, mid, low skills), ATM supports the view that new technologies have disproportionately substituted for "routine" tasks that were mostly occupied by mid-skill workers. We show that our model gives similar implications about endogenous skill-biased technology adoption that Beaudry et al. (2010) derive from the "canonical"[1] CES approach. In particular, skill-biased technologies are adopted where high skills are relatively abundant[2], adoption tends to raise the skill-premium in the short run, but migration tends to equalize the premium economy-wide. More important, by considering three types of skills, our model also predicts that places rich in human capital, by favouring the adoption of skill-biased technologies, tend to exhibit "fat-tails" in the local skill distribution, as observed by Eeckhout et al. (2010). These authors, however, have to resort to a very specific production function, an (additive) Variable Elasticity of Substitution production function, to provide a theoretical justification for their empirical findings.[3]

We also draw additional implications for general equilibrium in local labour markets. Since share-altering technologies can be profitable only when there is an adequate local proportion of skilled workers, it follows that areas rich in amenities which are particularly attractive to the more educated are also likely to qualify for the adoption of such new technologies. Also, even with factor mobility economy-wide, we can have co-existence of different technologies in different areas: in other words, the "technological frontier" can differ across places within the same country. Finally, the quality of skill-mix tends to increase further in areas where such technologies are adopted: in other words, the model predicts new technologies attract a disproportionate inflow of skilled workers, relative to unskilled workers, reinforcing the conditions for future adoption of skill-biased technologies.

The paper is organized as follows. Section II.1 describes the basic Roback model, where skilled individuals exhibit preferences for local amenities that are partly different from those of the mid-skilled and unskilled. Then, in Section II.2, we discuss the implications of share-altering technical change. Section III concludes.

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[1] This terminology is borrowed from Acemoglu and Autor (2010).

[2] This implication is similar to Acemoglu (1998), where investment in skill-complementary technologies depends on the proportion of skilled individuals in the workforce. In Berry and Glaeser (2005), a higher the number of educated residents will generate more skilled entrepreneurs who hire skilled workers. In this perspective, the initial level of city skills crucially determines the future level of local skill demand. Our view however is closest to Beaudry's et al. (2010) "comparative advantage" story for technology adoption.

[3] Eeckhout et al (2010) show in fact that a traditional CES predicts different population sizes across cities that exhibit different TFP, but it implies *the same distribution of skills in every location*.

II. THE MODEL

We consider a standard general equilibrium model, where firms and workers are perfectly mobile across areas: see Roback (1982,1988). The economy is composed of two areas, Area 1 and Area 2, which are endowed with different characteristics, affecting both local productivity and residents' "quality of life". In each area, firms produce an homogeneous good by using "land" and three types of labor: unskilled, medium-skill, skilled.[4] The good is traded competitively across areas. All workers earn a wage and consume both the produced good and residential space, "land". For simplicity, the supply of "land" in each area is taken to be fixed and landowners are absentee. Since firms are assumed perfectly mobile between areas, profits will be equalized across the economy. Similarly, since mobility costs are taken to be zero, workers' utility will be perfectly equalized across areas. For what it concerns individuals' utility, we assume that each area possesses some local characteristics (such as sunshine or crime) that equally affect the quality of life of *all* individuals. However, we postulate that there are other local features that affect the utility of skilled individuals only.[5]

We start by describing the features of spatial general equilibrium in the absence of technical change.

1. The basic framework

The local supplies of skilled, mid-skilled, unskilled labour and land are given, respectively, by $\{n_c^s, n_c^m, n_c^u, \bar{\ell}_c\}$, with $c = \{1,2\}$. We first illustrate firms' optimal behavior. Then, we look at workers, so to characterize the equilibrium in the two areas.[6]

Firms. Firms in area $c = \{1,2\}$ produce an homogeneous good by using land, L_c, and all three labor types, respectively $\{N_c^s, N_c^m, N_c^u\}$, with a Cobb-Douglas technology characterized by constant returns to scale:

$$Y_c = A_Y(Q_c) \cdot L_c^{1-\alpha-\beta-\gamma} \cdot \left(N_c^s\right)^{\alpha} \cdot \left(N_c^m\right)^{\beta} \cdot \left(N_c^u\right)^{\gamma} \qquad (1)$$

where $\alpha + \beta + \gamma \in (0,1)$. The term $A_Y(Q_c)$ denotes the impact of the vector Q_c of local characteristics on firm's productivity. We postulate that the elements of Q_c, q_{ic}, are measured in a way such that

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[4] One can think of skilled workers as college graduates and mid-skill workers as individuals holding high-school diplomas.

[5] This assumption is consistent, for example, with the findings in Carlino and Saiz (2008) for the US, and Dalmazzo and de Blasio (2011) for Italy, and is exploited also in some model extensions in Glaeser (2008).

[6] As in the basic spatial framework introduced by Glaeser (2008), we model both preferences and technology as Cobb-Douglas functions. However, when Glaeser proceeds to investigate local skill-premia, he drops the Cobb-Douglas specification in favour of a CES technology. We show in what follows that this step, under the share-altering hypothesis, is unnecessary.

$\partial A_Y / \partial q_{ic} \geq 0$.[7] Respectively, $\{r_c, p_c\}$ denote the local price of land (rent) and the price of the traded good. In what follows, we will assume that $p_1 = p_2 = 1$. The wage received by a skilled worker in area *c* is denoted by w_c^s, the mid-skilled wage is w_c^m, while the unskilled wage is equal to w_c^u. A competitive firm located in *c* will equate price to marginal cost, which is equivalent to $\vartheta \cdot \left[\frac{A_Y(Q_c)}{r_c^{1-\alpha-\beta-\gamma} \left(w_c^s\right)^\alpha \left(w_c^m\right)^\beta \left(w_c^u\right)^\gamma} \right] = 1$, where $\vartheta \equiv (1-\alpha-\beta-\gamma)^{1-\alpha-\beta-\gamma} \alpha^\alpha \beta^\beta \gamma^\gamma$. Since firms are perfectly mobile across areas, marginal cost must be equal to one in both areas (see Appendix A.1 for details).

Skilled workers. Skilled workers living in area *c* maximize the utility function

$$U_c^s = A_U(X_c) \cdot B_U(Z_c) \cdot L_c^{1-\mu} \cdot Y_c^\mu \qquad (2)$$

subject to the budget constraint $r_c \cdot L_c + Y_c = w_c^s$. Skilled worker's utility (2) includes an "amenity" term $A_U(X_c)$, non-decreasing in the vector of local characteristics X_c, which is common to mid-skilled and unskilled utility (see below). However, skilled utility also includes an additional "amenity" term $B_U(Z_c) \geq 1$, non-decreasing in Z_c. The vector Z_c denotes some additional territorial characteristics that are valuable to skilled individuals, but irrelevant to the welfare of other workers' types. The optimal choice of the consumption bundle generates an indirect utility for a skilled resident in area $c = \{1,2\}$ given by:

$$v_c^s = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot \frac{w_c^s}{r_c^{1-\mu}}, \qquad c = \{1,2\} \qquad (3)$$

where $\eta \equiv (1-\mu)^{1-\mu} \mu^\mu$. In the absence of mobility costs, a skilled worker is indifferent whether to migrate or not whenever the condition $v_1^s = v_2^s = \bar{v}^s$ holds.

Mid-skill and unskilled workers. Mid-skill and unskilled workers in area *c* receive a wage respectively equal to w_c^m and w_c^u. By maximizing utility[8] subject to $r_c \cdot L_c + Y_c = w_c^m$, indirect utility of a mid-skill worker who resides in region *c* is given by $v_c^m = \eta \cdot A_U(X_c) \cdot \frac{w_c^m}{r_c^{1-\mu}}$, with $c = \{1,2\}$. Free

[7] The vector of local characteristics affecting firms' productivity, Q_c, does not necessarily coincide with local characteristics affecting residents' utility.

[8] Mid-skill and unskilled utility has the same structure as skilled utility (2), except for the absence of the amenity term $B_U(Z_c)$.

mobility for mid-skill individuals implies that $v_1^m = v_2^m = \bar{v}^m$. Similar expressions hold for unskilled workers.

Equilibrium (without technical change). The relative rent and wage ratios between areas are given by the following expressions (see Appendix A.1 for details):

$$\log\left(\frac{r_1}{r_2}\right) = \frac{\log\frac{A_Y(Q_1)}{A_Y(Q_2)} + (\alpha+\beta+\gamma)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} + \alpha\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}}{1-\mu(\alpha+\beta+\gamma)}. \tag{4}$$

$$\log\left(\frac{w_1^s}{w_2^s}\right) = \frac{(1-\mu)\cdot\log\frac{A_Y(Q_1)}{A_Y(Q_2)} - (1-\alpha-\beta-\gamma)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} - \left[1-\alpha-\mu(\beta+\gamma)\right]\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}}{1-\mu(\alpha+\beta+\gamma)} \tag{5}$$

$$\log\left(\frac{w_1^m}{w_2^m}\right) = \frac{(1-\mu)\cdot\log\frac{A_Y(Q_1)}{A_Y(Q_2)} - (1-\alpha-\beta-\gamma)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} + \alpha(1-\mu)\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}}{1-\mu(\alpha+\beta+\gamma)} \tag{6}$$

$$\log\left(\frac{w_1^u}{w_2^u}\right) = \frac{(1-\mu)\cdot\log\frac{A_Y(Q_1)}{A_Y(Q_2)} - (1-\alpha-\beta-\gamma)\cdot\log\frac{A_U(X_1)}{A_U(X_2)} + \alpha(1-\mu)\cdot\log\frac{B_U(Z_1)}{B_U(Z_2)}}{1-\mu(\alpha+\beta+\gamma)} \tag{7}$$

These equilibrium expressions have standard interpretations. Equation (4) shows that local characteristics that increase productivity and welfare in Area 1 have a positive effect on rents, relative to Area 2. However, expression (5) emphasizes that relative abundance of local amenities in Area 1 reduces the relative skilled wage in this region. Skilled individuals are ready to accept a lower wages to live in places that have higher quality of life. Finally, equations (6) and (7) show that, if Area 1 is relatively richer in amenities that are mostly appreciated by the skilled, Z_1, the mid-skill and unskilled wage in Area 1 tend to be relatively higher. This occurs because such specific amenities attract skilled workers to Area 1 and, for this reason, mid-skill and unskilled workers become more productive.

Relative population sizes across areas. The equilibrium derived above characterizes the relative prices (rents, wages) across the economy. We now derive the relative equilibrium sizes of skilled, mid-skill and unskilled populations in the two areas. Similarly to Roback (1988), the procedure to determine the equilibrium populations builds on the labour market-clearing conditions

$$n_c^s = N_c^s, \quad n_c^m = N_c^m, \quad n_c^u = N_c^u, \quad c = \{1,2\} \tag{8}$$

and market-clearing in the market for land: we leave the details to Appendix A.2. In equilibrium, the proportion of skilled workers across areas is given by:

$$\log\left(\frac{n_1^s}{n_2^s}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\mu \cdot \log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + [1-\mu(\beta+\gamma)] \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1-\mu(\alpha+\beta+\gamma)} \tag{9}$$

Expression (9) shows that skilled workers will tend to locate in Area 1 when productivity and both types of amenities (X_1, Z_1) in Area 1 are high relative to Area 2. Thus, local characteristics that enhance productivity and welfare are central factors in attracting skilled workers.

Similar calculations show that the proportion of mid-skill and unskilled workers across areas are equal, respectively, to:

$$\log\left(\frac{n_1^m}{n_2^m}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\mu \cdot \log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\mu \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1-\mu(\alpha+\beta+\gamma)} \tag{10}$$

and

$$\log\left(\frac{n_1^u}{n_2^u}\right) = \log\left(\frac{\bar{\ell}_1}{\bar{\ell}_2}\right) + \frac{\mu \cdot \log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right) + \log\left(\frac{A_U(X_1)}{A_U(X_2)}\right) + \alpha\mu \cdot \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)}{1-\mu(\alpha+\beta+\gamma)} \tag{11}$$

Again, higher local productivity (due to Q_1) and general amenities (due to X_1) in Area 1 will bias the location of mid-skill and unskilled workers toward that area. Notice that abundance of local amenities that specifically attract *skilled* individuals (Z_1) will also tend to increase the location of mid-skill and unskilled workers in Area 1. When more skilled workers locate in Area 1, the local productivity of *mid-skill and unskilled* workers will increase, raising the demand for their labor services.

The model has immediate implications about what drives the local skill mix across areas. Consider for example the equilibrium proportion between skilled and mid-skill workers. Since it holds that $\log\left(\frac{n_1^s}{n_2^s}\right) - \log\left(\frac{n_1^m}{n_2^m}\right) = \log\left(\frac{n_1^s}{n_1^m}\right) - \log\left(\frac{n_2^s}{n_2^m}\right)$, equations (9) and (10) can be used to characterize the difference in the skill mix across areas, given by the following expression:

$$\log\left(\frac{n_1^s}{n_1^m}\right) - \log\left(\frac{n_2^s}{n_2^m}\right) = \log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right) \qquad (12)$$

Expression (12) shows that differences in the local proportion between skilled and mid-skill workers only depend on differences in amenities that are *specific to the tastes of the skilled*, such as those included in vector Z_c.[9] Identical conclusions are reached when one considers the proportion between skilled and unskilled individuals. This is a relevant implication for the local "college-share". Indeed, Area 1 will have a higher ratio of skilled vs. mid-skill and unskilled individuals only when endowed, relative to Area 2, with characteristics that are particularly appreciated by the more educated. Consequently, since the local ratio between skilled and mid-skill wages is given by $\frac{w_c^s}{w_c^m} = \frac{\alpha}{\beta} \cdot \frac{n_c^m}{n_c^s}$, when Area 1 is relatively richer in skills, i.e. $\frac{n_1^s}{n_1^m} > \frac{n_2^s}{n_2^m}$, it will also exhibit a *lower* skill premium relative to Area 2, that is, $\frac{w_1^s}{w_1^m} < \frac{w_2^s}{w_2^m}$. Identical considerations hold for the local ratio between skilled and unskilled wages.[10] Thus, the following holds:

Result 1. Skilled labor is cheaper in areas that are relatively rich in amenities which are particularly attractive to the educated.

As argued in what follows, the local capacity to attract skilled individuals may be crucial for the implementation of skill-biased technologies. We will show that, when adopted, such new technologies raise the local concentration of human capital even further. Thus, although local amenities remain a precondition, our conclusions are compatible with the observation of Moretti (2010) that, in the US between 1980 and 2000, changes in the geographical location of skilled workers were mostly driven by changes in their relative demand (i.e., technology), rather than supply (i.e., local amenities).

2. Skill-biased share-altering technical change.

In the last decades, a large body of literature has emphasized the so-called "skilled-biased technical change"[11], with demand for skilled workers growing faster than the pool of educated individuals. This view

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[9] In fact, in this Cobb-Douglas model, local general amenities (X_c) and productivity advantages (Q_c) affect skilled, mid-skill and unskilled individuals in the *same* way and, thus, they are unable to affect the local skill mix. See also Glaeser (2008).

[10] It is immediate to show that, if Area 1 is particularly attractive to skilled workers, it also holds that $\frac{n_1^s}{n_1^u} > \frac{n_2^s}{n_2^u}$, implying $\frac{w_1^s}{w_1^u} < \frac{w_2^s}{w_2^u}$.

[11] In terms of production theory, technical change is skill-biased if it increases the marginal productivity of skilled workers relative to other factors: see Acemoglu (2002, p.785). Typically, representations of skill-

has been refined by some contributions following Autor et al. (2003), which have shown that recent technological advances embodied in information and communication technology are particularly fit to execute "routine" tasks that were previously performed by middle-skill workers: see Autor and Acemoglu (2010) for an overview. This has caused severe consequences to the returns of certain types of skill that we will model as shifts in the shares associated with different types of labour. Indeed, some recent literature on economic growth has investigated technological change as directly represented by changes in share parameters. Seater (2005), Peretto and Seater (2008), and Zuleta (2008), observing the historical fall in the share of raw labour in the US, together with the stability of the share going to labour income, have explored some implications of the "share-altering" technical change hypothesis with Cobb-Douglas production functions.[12]

The share-altering representation of technological change is very convenient in our Cobb-Douglas spatial model. Recent technological advances are biased toward skills but, at the same time, they do not substitute for manual low-skill jobs, such as truck driving. Thus, we can explore the impact of an increase in skilled workers' share that is *exactly matched* by a decrease in the mid-skilled share. Since one can reasonably assume that the low-skill share remains constant, the *overall* income share of labour remains unchanged. In particular, by referring to the Cobb-Douglas technology (1), we suppose that – at date t=0 - a new, share-altering, technology becomes available. The new technology[13] is such that the share of skilled labour α increases by $\Delta \geq 0$, while the share of mid-skill labour β is reduced by the same amount, so that the *total* labour share, $\alpha + \beta + \gamma$, remains constant. Share-altering technical change is thus associated with the following production function:[14]

$$Y_c = A_Y(Q_c) \cdot L_c^{1-\alpha-\beta-\gamma} \cdot \left(N_c^s\right)^{\alpha+\Delta} \cdot \left(N_c^m\right)^{\beta-\Delta} \cdot \left(N_c^u\right)^{\gamma} \qquad (13)$$

The following result, an immediate application of the Envelope Theorem[15], holds true:

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biased technical change rely on CES production functions, as for example in Acemoglu (2002) and Beaudry et al. (2010). For an argument about the (presumed) unsuitability of Cobb-Douglas production functions, see Acemoglu (2003, p.3).

[12] The share-altering hypothesis (not entirely novel, as noticed by Seater 2005), is consistent with several observations about growth history: factor-intensity is determined by factor abundance, the decrease in "raw" labour share while the total labour share does not change over economic development, underdevelopment due to inability to adopt advanced technologies in certain labour markets. Importantly, as shown in Zuleta (2008, p.838), a Cobb-Douglas function with share-altering innovations exhibits an elasticity of substitution greater than one, as commonly assumed in CES representations: see Acemoglu and Autor (2010).

[13] As in Beaudry et al. (2010) and, in general, in this kind of literature, we abstract from implementation costs of new technologies. Such costs are instead considered in the growth analysis by Seater and Peretto (2008) and Zuleta (2008).

[14] Notice that the production function (13) implies that the ratio between the marginal productivity of skilled labour and the marginal productivity of other factors is increasing in Δ. Thus, this form of technical change is consistent with Acemoglu's (2002) definition of skill-biased technical change.

[15] See Appendix A.3 for details of the proof.

Result 2. Each individual firm will find it profitable to adopt the share-altering technology (13) when it holds that $\frac{\partial Y_c}{\partial \Delta} = Y_c \cdot \log\left(\frac{N_c^s}{N_c^m}\right) > 0$. *Under local labour market clearing, this inequality is satisfied when Area c is sufficiently rich in skills, that is, when it holds that:*

$$\frac{N_c^s}{N_c^m} = \frac{n_c^s}{n_c^m} > 1. \tag{14}$$

Result 2 has also a "dual" representation in terms of factor prices. Since it initially holds that $\frac{w_c^s}{w_c^u} = \frac{\alpha}{\beta} \cdot \frac{n_c^u}{n_c^s}$, condition (14) implies that each individual firm will find it profitable to adopt the share-altering technology (13) when the skill-premium $\frac{w_c^s}{w_c^u}$ is lower than $\frac{\alpha}{\beta}$. This has an immediate explanation. When the new technology becomes available at date *t*=0, an area rich in skills (where the ratio $\frac{n_c^s}{n_c^m}$ is high) is characterized by a relatively *low* skill-premium. Thus, the presence of cheap skilled labour in the local labour market may make it convenient to adopt skilled biased technologies: see also Beaudry et al. (2010). Result 2 has another remarkable implication. Suppose that in Area 1 the ratio between skilled and unskilled individuals is greater than 1 while, in Area 2, is less than 1. Then, firms locating in Area 1 will find it profitable to implement the new technology (13), with $\Delta > 0$, while firms locating in Area 2 will stick to the "old" technology, given by (1). Thus, the spatial general equilibrium approach shows that different technologies can coexist across different areas within the *same* economy.

In what follows, we give some results deriving from the share-altering hypothesis which pertain to (i) the size of local skill-premia, and (ii) the skill mix of local populations *across* areas in spatial general equilibrium.

Implications for local skill-premia. When at date *t*=0 condition (14) is respected *only* in Area 1, the local skill-premium must be lower than the one in Area 2. Once adopted, the share-altering technology will have a *direct* positive impact on the local wage-premium: by taking as *given the local skill-mix* $\frac{n_1^s}{n_1^m}$, the ratio $\frac{w_1^s}{w_1^m} = \frac{\alpha + \Delta}{\beta - \Delta} \cdot \frac{n_1^m}{n_1^s}$ is increasing in Δ. Thus, areas where the skilled-biased technology is adopted exhibit – at least, initially - an increase in the local skill-premium. However, since the workforce is mobile, the direct impact on the skill-premium caused by $\Delta > 0$ will be entirely compensated by re-adjustments in the local skill-mix caused by migrations over time. The proof for this claim, as in Glaeser (2008), goes as follows.

Recall that the indirect utility of a skilled worker is equal to $\bar{v}^s = \eta \cdot A_U(X_c) \cdot B_U(Z_c) \cdot \left(w_c^s / r_c^{1-\mu}\right)$, where $\bar{v}^s$ denotes the "reservation utility" of skilled individuals across the economy (with free-mobility). Similarly, the indirect utility of an mid-skilled worker is given by $\bar{v}^m = \eta \cdot A_U(X_c) \cdot \left(w_c^m / r_c^{1-\mu}\right)$, where $\bar{v}^m$ denotes the "reservation utility" of the mid-skill individuals in the economy. Thus, in equilibrium, the wage-premium in Area *c* is given by:

$$\frac{w_c^s}{w_c^m} = \left(\frac{1}{B_U(Z_c)}\right) \cdot \frac{\bar{v}^s}{\bar{v}^m} \qquad (15)$$

Expression (15) shows that the wage-gap between high-skilled and mid-skilled workers depend on amenities that affect skilled utility, and it does *not* depend on technological factors, such as Δ. Similar considerations hold for the wage-ratio of high-skill and unskilled workers. Thus, as will be confirmed in what follows, there must occur re-adjustments in the skill-mix across areas which exactly compensate for the direct effect generated by the adoption of the new technology. The following statement summarizes the conclusions obtained so far:

Result 3 (Skill-premia). The adoption of a skill-biased share-altering technology has a positive effect on the local skill-premium that is, at most, temporary. Re-adjustments in the local skill-mix will entirely compensate the initial positive effect, taking the local skill-premium back to its pre-adoption level.

The prediction that local changes in technology tend to have mostly a transitory effect on local skill premia is consistent with the evidence reported in Beaudry et al. (2010). They observe that during an adjustment to a new technological paradigm, the returns to skill increase most where skill is more abundant but, over the long run, the supply of skill will exert downward pressure on skill premium. However, although local skill premia tend to revert to their pre-implementation level, technological change has *permanent* effects on the distribution of skills within each area. In order to show this, we first derive some spatial general equilibrium implications of share-altering technical change on relative prices across areas. As shown in Appendix A.4, the following result holds:

Result 4 (Relative rents and wages). Share-altering technical change localized in Area 1 will raise rents and wages relative to Area 2.

We can now analyse what happens to equilibrium skill distributions in the two areas when share-altering technological change occurs (all derivations are reported in Appendix A.5). The impact on the ratio between high-skill populations in Area 1 and 2 is given by:

$$\left.\frac{d\log\left(n_1^s/n_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{1}{\alpha}+\frac{\mu\cdot\log\Sigma_0}{1-\mu(\alpha+\beta+\gamma)}>0 \tag{16}$$

Thus, a localised skill-biased technological change will generate a relative increase in the skilled population of that area. The opposite result generally holds for the mid-skilled, which we mainly identify with of high-school educated individuals. Share-altering technical change has the following effect on the such local populations:

$$\left.\frac{d\log\left(n_1^m/n_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{-1}{\beta}+\frac{\mu\cdot\log\Sigma_0}{1-\mu(\alpha+\beta+\gamma)} \tag{17}$$

The sign of expression (17) is ambiguous in principle. However, plausible values of the parameters imply that technological change will generally *reduce* the relative size of the mid-skilled population in Area 1.[16] Low-skilled population will instead *increase* in the area that undergoes technical change:

$$\left.\frac{d\log\left(n_1^u/n_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{\mu\cdot\log\Sigma_0}{1-\mu(\alpha+\beta+\gamma)}>0 \tag{18}$$

Inspection of (16), (17), and (18) immediately shows that skill-biased share-altering technical change will generate thicker tails in the distribution of skills in Area 1, relative to Area 2. This conclusion is summarized in the following:

Result 5 (Local skills distributions). Since it hold that:

$$\left.\frac{d\log\left(n_1^m/n_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}<\left.\frac{d\log\left(n_1^u/n_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}<\left.\frac{d\log\left(n_1^s/n_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}$$

the local implementation of skill-biased share-altering technologies will generate a skill distribution with "fatter tails". The local population of low-skilled and –to a greater extent- the population of highly-skilled increase, relative to the population endowed with middle-skills.

Thus, our simple model has implications consistent with the finding in Eeckhout et al (2010) that large cities have fatter tails in skill-distribution. The reason here is the following. Places that attract highly

[16] With μ=2/3 and $\alpha+\beta+\gamma$=2/3, expression (17) is negative when $\log\Sigma_0<\frac{5}{6}\frac{1}{\beta}$; for example, when β=2/9, (17) is negative when $\log\Sigma_0<\frac{15}{4}$. Thus, if $(\log\Sigma_0)$>0 is not implausibly large, this derivative will have a negative sign. The parameterization of factor-shares is consistent with Mankiw et al. (1992).

skilled individuals tend to grow larger and richer in human capital (as shown above). As a consequence, such locations will be more prone to endogenous adoption of skill-biased technologies that, as documented by Autor et al. (2003) and Goos and Manning (2007), come at a disadvantage for jobs typically occupied by mid-skill individuals. Similar conclusions, at the local level, are reached by Lin (2010,p.565).

Result 5 can be restated in terms of an improvement of the *skill mix* in Area 1. Since it holds[17] that $\left.\frac{d\left[\log\left(n_1^s / n_1^m\right) - \log\left(n_2^s / n_2^m\right)\right]}{d\Delta}\right|_{\Delta\approx 0, \Sigma_0 > 1} = \frac{1}{\alpha} + \frac{1}{\beta} > 0$, skilled-biased share-altering technical change localized in Area 1 will lead to a *further* concentration of skills in that region. Hence, when Area 1 is fit to adopt skilled-biased share-altering technologies (i.e., when condition (14) is satisfied), later on it will be ready to implement *additional technological advances of the same kind*. By contrast, if the innovation could not be profitably adopted in Area 2, this region will remain stuck with the old technology also in the future. This implies that, even within the same country, output will grow in Area 1, while Area 2 stagnates (see also Seater 2005).

[17] Combining (16) and (17) gives $d\left[\log\left(n_1^s / n_2^s\right) - \log\left(n_1^m / n_2^m\right)\right] / d\Delta = d\left[\log\left(n_1^s / n_1^m\right) - \log\left(n_2^s / n_2^m\right)\right] / d\Delta$.

III. CONCLUSIONS

The hypothesis of (skill-biased) share-altering technical change has several implications. The first one, mostly technical, is that we can represent such bias and the reduction in demand for mid-skill individuals even with the simplest Cobb-Douglas production function. Thus, there is no need to use more cumbersome CES technologies, as for the spatial general equilibrium analysis by Glaeser (2008) and the labor-market analysis in Beaudry et al. (2010), or even Variable Elasticity of Substitution functions as in Eeckhout et al. (2010), to model some main stylized facts that have been occurring in local labor markets during the last three decades.

The main contribution of this framework is related to the spatial general equilibrium implications of share-altering technical change. In particular, the model draws some specific conclusions about path-dependency in regional development. Only areas that are sufficiently rich in human capital will be ready to adopt skilled-biased share-altering technological advancements. This is consistent with the "comparative advantage" story of Beaudry et al. (2010), who find that skill abundance is associated with skilled-biased technology adoption and, in particular, adoption is more intense where skills are cheaper. Moreover, our model suggests that, after implementation of new technologies, human capital tends to move more and more from areas that exhibit a relatively poor skill mix, to areas that are *already* rich in educated workers. As a result, there will be further polarization in the composition of the local labour force across the economy. This prediction is consistent with the empirical results reported in Berry and Glaeser (2005) and Glaeser and Gottlieb (2008): areas which are rich in human capital (a pre-condition for the adoption of skilled-biased technologies, here) will attract a disproportionate number of skilled workers. Further, such migratory movements generate "fat-tails" in the skill distribution in places where new technologies are implemented: this is consistent with evidence in Eeckhout et al (2010).

Finally, the model raises some policy questions. The existence of relevant non-linear effects of local skills, documented in Glaeser and Gottlieb (2008), may suggest that there can be returns from pushing skilled workers into *already* skilled areas. However, this would mean to subsidize areas that are rich in human capital. Such a policy seems inequitable and improper, because skilled people tend to move towards skilled places even without government aid. Still, our approach suggests that in an economy characterized by a low average level of education, it may be desirable to concentrate human capital in few specific places to get sort of areas of "excellence"[18], which allow for the adoption of techniques that would otherwise be unprofitable. In our perspective, this kind of intervention can be implemented by subsidizing local amenities that prove to be particularly attractive to educated individuals.

[18] Examples of such policies in the US are given by Glaeser and Gottlieb (2008, p.224-25).

APPENDIX

A.1 Derivation of relative local rents and wages in equilibrium.

Equalization of marginal cost to one in both areas implies that the following condition holds:

$$\left(\frac{r_1}{r_2}\right)=\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right)^{\frac{1}{1-\alpha-\beta-\gamma}}\cdot\left(\frac{w_2^{\ s}}{w_1^{\ s}}\right)^{\frac{\alpha}{1-\alpha-\beta-\gamma}}\cdot\left(\frac{w_2^{\ m}}{w_1^{\ m}}\right)^{\frac{\beta}{1-\alpha-\beta-\gamma}}\cdot\left(\frac{w_2^{u}}{w_1^{u}}\right)^{\frac{\gamma}{1-\alpha-\beta-\gamma}} \qquad \text{(A1.1)}$$

Equalization of skilled utility across areas implies that:

$$\left(\frac{w_1^{\ s}}{w_2^{\ s}}\right)=\frac{A_U(X_2)}{A_U(X_1)}\cdot\frac{B_U(Z_2)}{B_U(Z_1)}\cdot\left(\frac{r_1}{r_2}\right)^{1-\mu} \qquad \text{(A1.2)}$$

Equalization of mid-skill utility and unskilled utility across areas delivers, respectively, the following two expressions:

$$\left(\frac{w_1^{m}}{w_2^{m}}\right)=\frac{A_U(X_2)}{A_U(X_1)}\cdot\left(\frac{r_1}{r_2}\right)^{1-\mu} \qquad \text{(A1.3)}$$

and

$$\left(\frac{w_1^{u}}{w_2^{u}}\right)=\frac{A_U(X_2)}{A_U(X_1)}\cdot\left(\frac{r_1}{r_2}\right)^{1-\mu} \qquad \text{(A1.4)}$$

By substituting (A1.2), (A1.3) and (A1.4) into (A1.1), one obtains expression (4) in the text. Then, exploiting (4) to substitute into (A1.2), (A1.3) and (A1.4) one obtains, respectively, (5), (6) and (7) in the text.

A.2. Derivation of relative population sizes in equilibrium.

Profit maximization for firms located in area *c* implies that the demands for highly skilled labour N_c^s, mid-skilled labour N_c^m, unskilled labour N_c^u, and land L_c are given, respectively, by:

$$N_c^s=\frac{\alpha\cdot Y_c}{w_c^s},\quad N_c^m=\frac{\beta\cdot Y_c}{w_c^m},\quad N_c^u=\frac{\gamma\cdot Y_c}{w_c^u},\quad L_c=\frac{(1-\alpha-\beta-\gamma)\cdot Y_c}{r_c} \qquad \text{(A2.1)}$$

In equilibrium, skilled labour demand N_c^s must be equal to its local supply n_c^s. Also, mid-skill labor demand N_c^m and unskilled labour demand N_c^u must be equal, respectively, to local mid-skill labor supply

n_c^m and unskilled supply, n_c^u. Finally, the local supply of land, $\bar{\ell}_c$, must be equal to the total demand for land, which is given by the sum of land demanded by firms (as from A2.1), plus the land demanded by the skilled workers, equal to $n_c^s \cdot (1-\mu) \cdot \frac{w_c^s}{r_c}$, plus the land demanded by the mid-skill and unskilled workers, respectively, $n_c^m \cdot (1-\mu) \cdot \frac{w_c^m}{r_c}$ and $n_c^u \cdot (1-\mu) \cdot \frac{w_c^u}{r_c}$. Thus, the following four equations constitute a system in $\{Y_c, n_c^s, n_c^m, n_c^u\}$, given the price vector $\{r_c, w_c^s, w_c^m, w_c^u\}$:

$$n_c^s = \frac{\alpha \cdot Y_c}{w_c^s} \qquad \text{(A2.2)}$$

$$n_c^s = \frac{\beta \cdot Y_c}{w_c^s} \qquad \text{(A2.3)}$$

$$n_c^u = \frac{\gamma \cdot Y_c}{w_c^u} \qquad \text{(A2.4)}$$

$$\bar{\ell}_c = \frac{1}{r_c}\left\{(1-\alpha-\beta-\gamma)\cdot Y_c + (1-\mu)\cdot n_c^s \cdot w_c^s + (1-\mu)\cdot n_c^m \cdot w_c^m + (1-\mu)\cdot n_c^u \cdot w_c^u\right\} \qquad \text{(A2.5)}$$

Using (A2.2), (A2.3) and (A2.4) to substitute $\{n_c^s \cdot w_c^s, n_c^m \cdot w_c^m, n_c^u \cdot w_c^u\}$ away in (A2.5), one obtains:

$$Y_c = \frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta+\gamma)} \qquad \text{(A2.6)}$$

which can be substituted back into (A2.2), (A2.3) and (A2.4) to obtain:

$$n_c^s = \frac{\alpha}{w_c^s}\left[\frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta+\gamma)}\right], \quad c = 1,2 \qquad \text{(A2.7)}$$

$$n_c^m = \frac{\beta}{w_c^m}\left[\frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta+\gamma)}\right], \quad c = 1,2 \qquad \text{(A2.8)}$$

$$n_c^u = \frac{\gamma}{w_c^u}\left[\frac{\bar{\ell}_c \cdot r_c}{1-\mu(\alpha+\beta+\gamma)}\right], \quad c = 1,2 \qquad \text{(A2.9)}$$

Thus, the relative population sizes of skilled individuals across areas will be given by:

$$\frac{n_1^s}{n_2^s} = \frac{\bar{\ell}_1}{\bar{\ell}_2} \cdot \frac{r_1}{r_2} \cdot \frac{w_2^s}{w_1^s} \qquad \text{(A2.10)}$$

Taking logs of (A2.10) and using (4) and (5), one obtains equation (9) in the text. Using a similar procedure, one can exploit (A2.8) and (A2.9) to obtain, respectively, (10) and (11) in the text.

A.3. Proof of Result 2.

Share-altering technical change, summarized by $\Delta > 0$, will be adopted by local firms when it has a positive impact on profit, given by $\pi_c = Y_c - r_c \cdot L_c - w_c^s \cdot N_c^s - w_c^u \cdot N_c^u$. By Envelope Theorem, it holds that:

$$\frac{d\pi_c}{d\Delta} = \frac{\partial Y_c}{\partial \Delta} + \left[\frac{\partial Y_c}{\partial L_c} - r_c\right]\frac{dL_c}{d\Delta} + \left[\frac{\partial Y_c}{\partial N_c^s} - w_c^s\right]\frac{dN_c^s}{d\Delta} + \left[\frac{\partial Y_c}{\partial N_c^u} - w_c^u\right]\frac{dN_c^u}{d\Delta} = \frac{\partial Y_c}{\partial \Delta};$$

Thus, if condition (14) holds true, profit is increasing in Δ, making the share-altering technology convenient to adopt.

A.4. Derivation of relative rents and wages under technical change.

We now explore the effects of share-altering technological change localized *only* in Area 1 on relative local prices across the two regions. To this purpose, we will evaluate the results for an *initially* given skill-ratio, N_1^s / N_1^m, set equal to the constant $\Sigma_0 > 1$. Moreover, derivatives in comparative statics results will be calculated by setting $\Delta \approx 0$, that is, evaluating the impact of change by starting with the same production function in both areas.

Since condition (14) is satisfied in Area 1, but not in Area 2, firms in the former region (costlessly) adopt the share-altering innovation, while firms in the latter one continue to use the old technology. Under perfect competition in tradable good production, price (the *numeraire*) equals marginal cost, implying that firms locating in Area 1 will respect the following condition:

$$\vartheta'\left[\frac{A_Y(Q_1)}{r_1^{1-\alpha-\beta-\gamma}\cdot\left(w_1^s\right)^{\alpha+\Delta}\cdot\left(w_1^m\right)^{\beta-\Delta}\cdot\left(w_1^u\right)^{\gamma}}\right]=1, \qquad \text{(A4.1)}$$

where $\vartheta'\equiv(1-\alpha-\beta-\gamma)^{1-\alpha-\beta-\gamma}(\alpha+\Delta)^{\alpha+\Delta}(\beta-\Delta)^{\beta-\Delta}\gamma^{\gamma}$. For competitive firms locating in Area 2, the condition:

$$\vartheta\cdot\left[\frac{A_Y(Q_2)}{r_2^{1-\alpha-\beta-\gamma}\cdot\left(w_2^s\right)^{\alpha}\cdot\left(w_2^m\right)^{\beta}\left(w_2^u\right)^{\gamma}}\right]=1 \qquad \text{(A4.2)}$$

will continue to hold. Free mobility implies that, in equilibrium, firms must make zero profit no matter where they choose to locate. Thus, by combining (A4.1) and (A4.2), one obtains:

$$\frac{r_1}{r_2}=\left[\left(\frac{(\alpha+\Delta)^{\alpha+\Delta}(\beta-\Delta)^{\beta-\Delta}}{\alpha^{\alpha}\beta^{\beta}}\right)\cdot\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right)\cdot\left(\frac{w_2^s}{w_1^s}\right)^{\alpha}\cdot\left(\frac{w_2^m}{w_1^m}\right)^{\beta}\cdot\left(\frac{w_2^u}{w_1^u}\right)^{\gamma}\cdot\left(\frac{w_1^m}{w_1^s}\right)^{\Delta}\right]^{\frac{1}{1-\alpha-\beta-\gamma}} \qquad \text{(A4.3)}$$

By substituting equations (A1.2), (A1.3) and (A1.4) into (A4.3), one obtains that (the log of) the equilibrium rent-ratio between Area 1 and Area 2:

$$\log\left(\frac{r_1}{r_2}\right)=\frac{1}{1-\mu(\alpha+\beta+\gamma)}\left\{\begin{array}{l}\left[(\alpha+\Delta)\cdot\log(\alpha+\Delta)+(\beta-\Delta)\cdot\log(\beta-\Delta)-\log\left(\alpha^{\alpha}\beta^{\beta}\right)\right]+\Delta\cdot\log\left(\frac{w_1^m}{w_1^s}\right)+\\ +\log\left(\frac{A_Y(Q_1)}{A_Y(Q_2)}\right)+(\alpha+\beta+\gamma)\cdot\log\left(\frac{A_U(X_1)}{A_U(X_2)}\right)+\alpha\cdot\log\left(\frac{B_U(Z_1)}{B_U(Z_2)}\right)\end{array}\right\}$$

(A4.4)

where profit-maximization implies that $\left(\frac{w_1^m}{w_1^s}\right)=\frac{\beta-\Delta}{\alpha+\Delta}\left(\frac{N_1^s}{N_1^m}\right)$. Differentiating (A4.4) with respect to Δ, and evaluating the result for $\Delta\approx 0$ and $\frac{N_1^s}{N_1^m}=\Sigma_0>1$, one obtains:

$$\left.\frac{d\log(r_1/r_2)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left(\frac{1}{1-\mu(\alpha+\beta+\gamma)}\right)\cdot\log\Sigma_0 \qquad \text{(A4.5)}$$

Since $\Sigma_0>1$, the sign of expression (A4.5) is positive. Thus, localized skill-biased technical change will increase rents in Area 1 relative to Area 2.

Consider now the impact of the change in the skilled share on the skilled wage-ratio across areas. By exploiting (A1.2), differentiating with respect to Δ, and evaluating the result for $\Delta \approx 0$ and $\frac{N_1^s}{N_1^u} = \Sigma_0 > 1$, one obtains that:

$$\left.\frac{d\log\left(w_1^s / w_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = (1-\mu)\cdot\left.\frac{d\log\left(r_1 / r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left(\frac{1-\mu}{1-\mu(\alpha+\beta+\gamma)}\right)\cdot\log\Sigma_0 > 0 \qquad \text{(A4.6)}$$

Similarly, one can use (A1.3) and (A1.4) to assess the impact of share-altering change on relative mid-skill and unskilled wages. It turns out that the effect is the same as for skilled wages:

$$\left.\frac{d\log\left(w_1^m / w_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left.\frac{d\log\left(w_1^u / w_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1} = \left(\frac{1-\mu}{1-\mu(\alpha+\beta+\gamma)}\right)\cdot\log\Sigma_0 > 0 \qquad \text{(A4.7)}$$

Expressions (A4.6) and (A4.7) confirm that, with labor mobility, local skill premia are unaffected by technical change, as emphasized by (15).

A.3. Populations and share-altering technical change.

We first analyze the impact of skill-biased share-altering technical change on the relative size of the skilled population. It is immediate to show that, in Area 1, skilled population is now given by:

$$n_1^s = \frac{\alpha+\Delta}{w_1^s}\left[\frac{\bar{\ell}_1\cdot r_1}{1-\mu(\alpha+\beta+\gamma)}\right], \qquad \text{(A5.1)}$$

while for Area 2 equation (A2.7) still holds. Hence, with share-altering technical change, the skilled-population ratio is given by:

$$\frac{n_1^s}{n_2^s} = \frac{\alpha+\Delta}{\alpha}\cdot\frac{\bar{\ell}_1}{\bar{\ell}_2}\cdot\frac{r_1}{r_2}\cdot\frac{w_2^s}{w_1^s}. \qquad \text{(A5.2)}$$

Taking the logs of (A5.2), differentiating with respect to Δ, and calculating the resulting expression for Δ≈0, one obtains:

$$\left.\frac{d\log\left(n_1^s/n_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left(\frac{1}{\alpha}\right)+\left.\frac{d\log\left(r_1/r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}-\left.\frac{d\log\left(w_1^s/w_2^s\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{1}{\alpha}+\frac{\mu\cdot\log\Sigma_0}{1-\mu(\alpha+\beta+\gamma)}>0 \quad \text{(A5.3)}$$

which gives expression (16) in the text.

Mid-skilled population in Area 1 is given by

$$n_1^m=\frac{\beta-\Delta}{w_1^m}\left[\frac{\bar{\ell}_1\cdot r_1}{1-\mu(\alpha+\beta+\gamma)}\right], \quad \text{(A5.4)}$$

while for Area 2 equation (A2.8) still holds. Hence, the unskilled population ratio is equal to:

$$\frac{n_1^m}{n_2^m}=\frac{\beta-\Delta}{\beta}\cdot\frac{\bar{\ell}_1}{\bar{\ell}_2}\cdot\frac{r_1}{r_2}\cdot\frac{w_2^m}{w_1^m}. \quad \text{(A5.5)}$$

Differentiating the log of (A5.5) with respect to Δ and calculating the result for Δ≈0, one obtains:

$$\left.\frac{d\log\left(n_1^m/n_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left(\frac{-1}{\beta}\right)+\left.\frac{d\log\left(r_1/r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}-\left.\frac{d\log\left(w_1^m/w_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{-1}{\beta}+\frac{\mu\cdot\log\Sigma_0}{1-\beta-\alpha\mu} \quad \text{(A5.6)}$$

which gives expression (17) in the text.

Since low-skilled workers' share is unaffected we can use (A2.4) for both areas, so to obtain:

$$\frac{n_1^u}{n_2^u}=\frac{\bar{\ell}_1}{\bar{\ell}_2}\cdot\frac{r_1}{r_2}\cdot\frac{w_2^u}{w_1^u}. \quad \text{(A5.7)}$$

Differentiating the log of (A5.7) with respect to Δ and calculating the result for Δ≈0, one obtains:

$$\left.\frac{d\log\left(n_1^u/n_2^u\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\left.\frac{d\log\left(r_1/r_2\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}-\left.\frac{d\log\left(w_1^m/w_2^m\right)}{d\Delta}\right|_{\Delta\approx 0,\Sigma_0>1}=\frac{\mu\cdot\log\Sigma_0}{1-\beta-\alpha\mu}>0 \quad \text{(A5.6)}$$

which gives expression (18).

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