Growth and Output Fluctuations*

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Abstract

This paper sheds new light on the interaction between growth and uctuations. Our approach is different from the literature in that we analyse how endogenous uctuations are affected by a faster productivity growth in the long run. Main results: (i) expansion (or contraction) occurs more (or less) frequently, (ii) expansion becomes milder but contraction severer, (iii) the amplitude of uctuations becomes larger, (iv) the variance of output changes ambiguously, indicating a non-monotonous relation. We also investigate how an R&D subsidy alter the nature of output uctuations and re-examine its effect on technological change in the presence of recurrent cycles.

Key words: expectations, uctuations, growth, learning-by-doing, innovations.
JEL Classi cation: E30, O30, O40.

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1 Introduction

There is a considerable literature in macroeconomics on the interaction between business cycles and GDP growth (e.g. see Saint-Paul, 1997). However, growth theory has only recently begun to revisit the way in which cycles in potential output and long-run GDP growth are related (see Aghion and Howitt, 1998, ch.8). Further research is needed to understand a number of important policy issues, such as the effect on output fluctuations of growth-promoting public policies, e.g. subsidising education or R&D.

The present paper attempts to shed new light on some aspects of the long-run growth-output cycle interaction. There are three ways to model this interaction: (i) with endogenous long-run growth and exogenous output fluctuations, one can examine how the former is affected by changes in the nature of cycles; (ii) with exogenous growth and endogenous cycles, one can explore how changes in the growth rate can alter the properties of fluctuations; and (iii) one can model growth and cycles jointly as endogenous phenomena. Clearly approach (iii) is the most general, but both (i) and (ii) can offer useful insights into the underlying bi-directional causal links between cycles and growth.

The present paper starts with approach (ii) by constructing the model where growth is driven by exogenous labour productivity improvement and cycles arise due to endogenous technological change. This allows us to provide answers to questions such as: does a faster productivity growth make expansion (or contraction) more or less likely to happen? ; or how does it affect the amplitude and volatility of cycles? . Moreover, we also analyse how R&D subsidies alter these characteristics of output fluctuations. This line of inquiry is new in growth theory, and our model provides clear-cut (and intuitive) answers to these questions.

Another novel feature of this paper is that we re-examine the effect of research subsidies on the rate of technological progress in the presence of endogenous cycles. In the literature, R&D subsidies are shown to have an unambiguous positive effect on it. However, this strong prediction is obtained in an equilibrium where research intensity is constant (see e.g. Grossman and Helpman (1991)). We will demonstrate that this prediction is modified, i.e. a change in the rate of technological change becomes ambiguous in a cyclical equilibrium.

In the growth-cycle literature, an important (but controversial) result is that recession can promote technological change (and growth). Two main reasons are the lower opportunity cost of research in recession (Aghion and Saint-Paul (1991)) and the cleansing effects of recession when inefficient firms are driven out of the market (see Caballero and Hammour (1994)). In our model, the pattern of changes in the rate of technical progress matches this hypothesis, i.e. it is higher in contraction than in expansion. The central mechanism for this result is technology diffusion, which links research and manufacturing activities. This is fundamentally different from the two reasons mentioned above and complement those studies.

We generalise our model to approach (iii) by introducing learning-by-doing, thereby providing a feedback effect from cycles to productivity growth. This produces a surprising result that labour productivity can grow faster in contraction. This comes against the "conventional wisdom" that recession is bad for learning-driven growth.

In addition to these new features, our model exhibits the patterns of output fluctuations which are consistent with the data. It is typically observed that there are asymmetries of the business cycle (see Sichel, 1993). For example, the absolute size of contraction is larger than that of expansion. The real business cycle models, which are typically linear in disturbance terms, cannot explain these asymmetries. In contrast, our non-linear model offers a possible reason for such asymmetries, which is different from the explanation of earlier studies, such as DeLong and Summers (1988).

In the literature, studies which adopt approach (i) include Aghion and Saint-Paul (1991), Caballero and Hammour (1994), Van Ewijk (1997) and Stadler (1990), who show that stochastic fluctuations have permanent effects on growth. In particular, the first two studies stress the possibility that recession promotes innovation-driven growth.

Studies which adopt approach (ii) include Aghion and Howitt (1998), Andolfatto and MacDonald (1998) and Helpman and Trajtenberg (1996) who assume that exogenous major technological innovation drives growth. However, their analysis is effectively limited to a detailed examination of one deterministic cycle. This approach is reasonable if one is interested in the short term or the medium term at best. In contrast, this paper analyses the *long-term* impact of faster labour productivity growth on cycles in an economy which experiences a series of stochastic (endogenous) technological shocks.

Approach (iii) is taken by Cheng and Dinopoulos (1992) and Corriveau (1994). The main underlying mechanism is the reallocation of workers between the research and production sectors. However, as Aghion and Howitt (1992) point out, it is unlikely that this mechanism can account for large output cycles, as the research sector typically employs only 2 or 3% of the total labour force in developed economies. This shortcoming is overcome in recent studies by Amable (1995) and Li (1997) (as well as in the present paper). However, the Amable and Li models have the defect that although output grows in waves rather than in a smooth exponential fashion, output never falls. That is, both these studies can explain growth recessions, but not actual output cycles. In this paper, by contrast, we explain how growth can cause recurrent cycles of rising and falling output.

The plan of the paper is as follows. In Section 2, we set out the basic structure of the model. Section 3 shows that a stable 2-cycle equilibrium exists and examine some characteristics of output fluctuations and technological change. Section 4 analyses how these characteristics change as labour productivity grows faster, and the effects of R&D subsidies are analysed in Section 5. Labour productivity growth is endogenised in Section 6 and Section 7 summarises main results.

2 The Model

Our model extends the "quality-ladder" growth model of Aghion and Howitt (1992) in two important ways. First, it generates expectation-driven endogenous fluctuations of output due to *entry* and *exit* of firms following technological innovation. This is consistent with Davis and Haltiwanger (1992) who stress entry and exit of firms over output cycles. This modelling approach is also consistent with some recent studies, including Campbell (1997) who empirically supports the hypothesis that technological shocks are a significant source of economic fluctuations. Second, the reallocation of workers between production and R&D sectors plays no role in generating cycles in our model. The original model of Aghion and Howitt (1992) exhibits endogenous output cycles (as well as growth). But they do not stress this aspect as an explanation of output fluctuations precisely because the labour reallocation was the main mechanism of such cycles.

2.1 Consumers and Final Output

There are a xed number of consumers, $L > 0$, who are in nitely-lived and supply one unit of labour service at each moment. They consume a homogeneous nal output and are risk-neutral, so that the rate of interest, ρ, is constant.

Final output is produced in a competitive environment, using intermediate goods which are differentiated in quality and variety. The aggregate production function is

$$Y_{tm} = q_m \int_0^{n_m^p} x_{tm}(i)^{\alpha}\, di, \qquad 1 > \alpha > 0. \tag{1}$$

where q_m denotes the quality level of intermediate goods $x_{tm}(i)$ which is indexed from 0 to n_m^p (the superscript p for $p$roduction). The subscript m stands for the mth quality innovation, and the quality index rises by a factor $\lambda > 1$ following each innovation, i.e. $q_m = \lambda q_{m-1}$.[1] Inputs become obsolete once higher quality intermediate goods are produced. Since a single producer supplies a single variety, n_m^p is equivalent to the number of local monopolists. As we will see, n_m^p does not grow unlike, e.g., Romer (1990). Instead, it determines the level of output, and its changes cause output uctuations. Since the nal output sector is competitive, the demand function for $x_{tm}(i)$ is equivalent to the familiar marginal condition:

$$p_{tm}(i) = \alpha x_{tm}(i)^{\alpha - 1} \tag{2}$$

where $p_{tm}(i)$ is the price of an intermediate good.

[1] The time subscript t is used for variables, like x_{tm}, which grow between two successive innovations in equilibrium, but not for others, like q_m,which are constant for that sub-period.

2.2 Technology Diffusion

To produce variety inputs with quality q_m, firms have to achieve a technological breakthrough through R&D. We use n_m (without the superscript p) to denote the number of research firms, which aim to invent their own brand of inputs with quality q_m. For example, if the ith firm succeeds in research, it creates a blueprint for the ith variety of quality q_m.

Cohen and Levinthal (1989) argue that R&D not only generates new knowledge but also develops a firm's ability to *imitate* new process or product innovations, because R&D facilitates the absorption of knowledge created elsewhere. This suggests that in our context, the first successful launch of the new variety of quality q_m by one firm makes it easier for other firms, which have engaged in R&D, to invent their own brands. This form of technology diffusion is consistent with the observation that the market of innovative goods is often characterised by entry of new firms with different specifications or even brand image. Furthermore, the study of Cohen and Levinthal (1989) implies that firms which have *not* conducted R&D (i.e. those other than n_m firms) find it increasingly difficult to create variety inputs of the same quality. This may be because engaging in R&D itself generates tacit knowledge that is vital to successful variety differentiation, and such knowledge may be gained only by actually engaging in research.

Therefore, if a quality innovation occurs and the latest technology is diffused amongst the active research firms in the form of variety innovation, we will have

$$n_m = n^p_{m+1}. \tag{3}$$

This may be better understood by looking at Table 1 where the mth time interval means

Time Intervals	$\cdots$	m	$m+1$	$m+2$	$\cdots$
Input Production n^p	$\cdots$	n^p_m	n^p_{m+1}	n^p_{m+2}	$\cdots$
R&D n	$\cdots$	n_m	n_{m+1}	n_{m+2}	$\cdots$

Table 1: The number of firms in manufacturing and R&D sectors.

the time period during which the state-of-the-art inputs have quality q_m. In the mth interval, n^p_m firms are producing inputs of quality q_m, and n_m firms are engaged in R&D aimed to invent inputs of quality q_{m+1}. In the $(m+1)$th interval, n^p_{m+1} firms will produce inputs of quality q_{m+1}. Since these firms will have conducted R&D in the previous interval, equation (3) should hold once technology diffusion completes. For simplicity, we assume that technology diffusion is costless and instantaneous, i.e. equation (3) always holds.

2.3 Intermediate Products

Turning to the description of production of intermediate products, we focus on the case of "drastic" innovation. This means that monopolistic producers are not constrained by potential competition from previous incumbent producers.[2] We also make the standard assumption in the literature that no incumbent firms conduct R&D[3]

The production function of inputs is $x_{tm}(i) = A_t l_m(i)$ where $l_m(i)$ is the number of workers employed and $A_t = e^{at}$, $a > 0$, denotes exogenous labour productivity. Given this technology and the demand function (2), firms producing intermediate goods maximise

[2] The condition for this is $\lambda \geq \alpha^{-\alpha}(n^p_m/n^p_{m-1})^{1-\alpha}$. See Aghion and Howitt (1998, pp.74-75) for derivation.

[3] This is due to the so-called replacement effect. See, e.g., Tirole (1988, p.392).

pro ts $\pi_{tm}(i) = [p_{tm}(i) - w_t/A_t]x_{tm}(i)$. The rst-order condition is

$$\frac{w_t}{A_t} = \alpha^2 x_{tm}(i)^{\alpha-1}, \tag{4}$$

which implies $x_{tm}(i) = x_{tm}$ for all i s.

It is assumed that labour is used only for producing intermediate goods. This assumption removes the possibility of the labour reallocation between the production and R&D sectors. Using symmetry, the full-employment of workers requires

$$L = \frac{n_m^p x_{tm}}{A_t}. \tag{5}$$

Using (2), (4) and (5), it is straightforward to verify that a rm producing a variety input of the state-of-the-art quality earns a ow pro t of

$$\pi_{tm} = \alpha(1-\alpha) q_m \left(\frac{A_t L}{n_m^p}\right)^{\alpha} \tag{6}$$

until its product becomes obsolete.

It is instructive at this stage to de ne normalised (or detrended) output as

$$y_m \equiv \frac{Y_{tm}}{q_m e^{\gamma t}} = L^{\alpha}\left(n_m^p\right)^{1-\alpha} = L^{\alpha} n_{m-1}^{1-\alpha}, \qquad \gamma \equiv \alpha a, \tag{7}$$

using (1) and (5). It shows that the level of output crucially depends upon the number of rms in the product market (and the R&D sector). If the number of rms entering the market is constant over time, there are no uctuations in normalised output (although Y_{tm} rises in a step-wise manner due to quality innovations). On the other hand, if $n_m^p = n_{m-1}$ oscillates, normalised output rises and falls over recurrent cycles.

2.4 R&D

In conducting R&D, firms pay a fixed research cost in terms of final output, $D_t = de^{\gamma t} q_m$, $d > 0$, for an infinitesimal time period. If this cost is incurred, the ith firm invents the ith variety input of quality q_{m+1} according to a Poisson arrival rate of $h(n_m) = \varphi n_m^{\eta - 1}$, $\varphi > 0$, $1 \geq \eta > 0$.[4] The dependence on n_m captures the familiar duplication (or congestion) effect in research, and $\eta - 1$ measures its degree. Therefore, the economy-wide Poisson arrival rate is $h(n_m) n_m = \varphi n_m^{\eta}$.

We use V_{tm} to denote the expected present value of profits earned by firms which invented new variety products of quality q_m. It grows at a rate of γ and variety inputs become obsolete due to an extra input innovation with an arrival rate of φn_m^{η}. Hence, the Bellman equation defining V_{tm} is

$$\rho V_{tm} = \pi_{tm} + (\gamma - \varphi n_m^{\eta}) V_{tm}. \tag{8}$$

The expected benefit of engaging R&D aimed at the $(m+1)$th innovation is $\varphi n_m^{\eta} V_{tm+1}$, since firms take into account the possibility of technology diffusion due to rival firm's success in research. Free entry ensures

$$\varphi n_m^{\eta} V_{tm+1} = D_t. \tag{9}$$

2.5 Perfect Foresight Equilibrium (PFE)

Using (3), (6), (8) and (9), we obtain the equilibrium condition:

$$n_m = \left(\frac{\Gamma}{\rho - \gamma + \varphi n_{m+1}^{\eta}} \right)^{\frac{1}{\alpha - \eta}} \equiv f(n_{m+1}) \tag{E}$$

[4] Parameterisation of research technology facilitates the presentation without affecting the main results.

Time Intervals	$\cdots$	$m-1$ (E)	m (C)	$m+1$ (E)	$m+2$ (C)	$\cdots$
Input Production n^p	$\cdots$	n_e	n_c	n_e	n_c	$\cdots$
R&D n	$\cdots$	n_c	n_e	n_c	n_e	$\cdots$

Table 2: The number of rms in the input production and R&D sectors; (C) and (E) stand for contraction and expansion respectively.

where $\Gamma = \alpha (1-\alpha) L^{\alpha} \lambda \varphi / d$. Equation (E) determines n_m as a function of the number of research rms in future. In the (n_{m+1}, n_m) plane, (E) is downward-sloping for $\alpha > \eta$ (solid line) and upward-sloping for $\alpha < \eta$ (dotted line) as Figure 2 shows.

For $\alpha < \eta$, there are two steady states, stable and unstable. But the stable steady state has an unrealistic feature that a higher interest rate promotes R&D activity, despite the fact that the value of innovation falls. Therefore, we take $\alpha > \eta$ as more plausible, and the rest of the paper focuses on this case.

3 2-Cycle PFE ($\alpha > \eta$)

3.1 Existence

Proposition 1 *There exists a stable 2-cycle PFE de ned by*

$$n_e = f(n_c), \quad n_c = f(n_e), \quad n_e > \overline{n} > n_c, \tag{10}$$

if and only if $|f'(\overline{n})| > 1$ *where* $\overline{n} = n_m = n_{m+1}$.

Proof. See Appendix A.

This proposition says that n_e (expansion) and n_c (contraction) alternate following each innovation. Table 2 shows how the number of rms engaging in input production and R&D oscillates as technological breakthroughs occur. In the mth interval, each of n_c monopoly rms produces its own brand of variety inputs, so that normalised nal output is $y_c = L^\alpha n_c^{1-\alpha}$ (see equation (7)). This is a contraction phase. As regards research rms, they expect that there will be a relatively small number of rms engaging in R&D, n_c, in the $(m+1)$th interval. Hence, the risk of obsolescence of inputs of quality q_{m+1} is low, so that the value of the $(m+1)$th innovation is high. This induces a large number of rms, n_e, to engage in research in the mth interval.

The arrival of the $(m+1)$th innovation ushers in an expansionary period. During this interval, n_e differentiated inputs are produced with normalised nal output $y_e = L^\alpha n_e^{1-\alpha}$, since n_e rms have engaged in R&D in the previous interval. Research rms anticipate that the number of research rms in the $(m+2)$th period will be relatively large, n_e, so that the risk of obsolescence is also large. As a result, a small number of rms, n_c, conduct R&D in the $(m+1)$th interval. We have now established the following proposition, which is depicted in Figure 1.

Proposition 2 *In the 2-cycle PFE, the normalised output oscillates between* y_e *and* y_c*,* $y_e > y_c$.

3.2 Characteristics of Output Fluctuations

The ow probability of a transition from contraction to expansion is φn_e^η, and the ow probability of a reverse transition is φn_c^η (see Table 2). Thus, the economy will be in an expansionary phase for a fraction, $H_e = n_e^\eta / \left(n_e^\eta + n_c^\eta\right)$, of its entire life-time, and the complementary fraction is spent in contraction. Note that $n_e > n_c$ implies $H_e > 1/2$.

Thus, the following proposition follows.

Proposition 3 *In the 2-cycle PFE, the economy experiences expansion more frequently than contraction.*

In other words, contraction is quickly followed by expansion, but contraction takes time to arise after expansion. This prediction on asymmetry of business cycle is consistent with the data if we interpret expansion as a phase from a trough to the following peak and contraction as the reverse phase. For example, according to National Bureau of Economic Research, the average months of expansion in the US is 43 months for 1945-1991, whereas contraction took just 11 months on average.[5] Although the dates of peaks and troughs depend on their de nition, the same observation generally seems to hold for other developed economies.

One could also interpret expansion and contraction in our model as representing peaks and troughs. Under this interpretation, our model also exhibits features consistent with what Sichel (1993) called deepness asymmetry of troughs and peaks. This basically means that troughs are deeper than peaks are tall (p. 225).[6] To show this, we rst de ne the average of normalised output as $y^* = H_e y_e + (1 - H_e) y_c$. Using this, one can easily derive the following:

$$y_e - y^* = \theta (1 - H_e), \qquad y^* - y_c = \theta H_e \tag{11}$$

where $y_e - y^*$ and $y^* - y_c$ are the size of expansion and contraction respectively. Since $H_e > 1/.2$, the next proposition follows.

[5] This is taken from its website at http://www.nber.org/cycles.html.

[6] See also Kontolemis (1997) for evidence of G7 economies and the references therein.

Proposition 4 *In the 2-cycle PFE, the size of contraction is strictly larger than that of expansion.*

3.3 Characteristics of Technological Progress

To calculate the rate of technical progress in each phase of business cycle, we follow Aghion and Howitt (1992, p.336). Suppose that expansion prevails forever. Then, quality q_m would grow at the average rate of $\varphi n_c^\eta \ln \lambda$, since innovation occurs at a Poisson rate of φn_c^η (see Table 2). Similarly, it would grow at $\varphi n_e^\eta \ln \lambda$, if contraction continues forever. Therefore, the expected growth rate of output in each phase is

$$g_e = \varphi n_c^\eta \ln \lambda + \gamma, \qquad g_c = \varphi n_e^\eta \ln \lambda + \gamma. \tag{12}$$

Since $n_e > n_c$, the following proposition follows.[7]

Proposition 5 *In the 2-cycle PFE, output grows and technology advances faster in contraction than in expansion on average, i.e.* $g_c > g_e$.

This result arises, since technology diffusion links the production and R&D sectors (see equation (3)), and consequently, oscillation of the number of research rms is countercyclical. Although the result of Proposition 5 itself is not new, the mechanism essentially differs from two popular arguments for recession being bene cial for growth. The rst argument is based on the lower opportunity cost of R&D in recession (Aghion and Saint-Paul (1991)), and the second argument concerns the cleansing effect of recession (Caballero

[7] Another important asymmetry of the business cycle concerns steepness , meaning that downturns are steep but upturns are gradual. This asymmetry is not captured by out model, since growth in downturn is typically negative, whereas g_c is positive.

and Hammour (1994)). Later we also show that once labour productivity growth is endogenised, γ too can be higher in contraction.

We de ne the average growth rate of output over the entire business cycle as $g = H_e g_e + (1 - H_e)\, g_c$. Using this and (12), it is easy to verify that

$$g - g_e = \varphi\,(1 - H_e)\,(n_e^{\eta} - n_c^{\eta}) \ln \lambda, \qquad g_c - g = \varphi H_e\,(n_e^{\eta} - n_c^{\eta}) \ln \lambda \tag{13}$$

which measure the extent of relative changes in the growth rate in expansion and contraction. Since $H_e > 1/2$, we obtain the next proposition.

Proposition 6 *In the 2-cycle PFE, the absolute size of a rise in growth in contraction is strictly larger than that of a fall in growth in expansion.*

This contrasts with Proposition 4, highlighting asymmetric responses of growth and level of output over the business cycles.

4 Effects of Labour Productivity Growth

Now we examine how a higher rate of labour productivity growth affects the properties of output uctuations and technological progress. A higher a (hence γ) may result from, for example, public policy on on-the-job training or education.

4.1 Number of Firms

Proposition 7 *As labour productivity grows faster, entry of new rms into R&D (production) is encouraged in contraction (expansion) and discouraged in expansion (contraction),* *i.e.*

$$\frac{\partial n_e}{\partial \gamma} > 0, \qquad \frac{\partial n_c}{\partial \gamma} < 0. \tag{14}$$

Proof. See Appendix B.

An intuitive account can be obtained by examining the effective discount rate $\rho - \gamma + \varphi n_i^\eta$, $i = e, c$, in (E), with which monopoly producers capitalise future pro t ows. A higher γ lowers it and thereby increases the value of innovation, tending to induce entry into R&D, irrespective of the state of the economy. This is the direct capitalisation effect. For the indirect effect, consider the entry decision of n_e research rms in the mth time interval in Table 2. They use the effective discount rate $\rho - \gamma + \varphi n_c^\eta$ to calculate the value of innovation, since there are n_c research rms in the $(m+1)$th interval. The indirect effect is realised through a rise in n_c (due to the direct capitalisation effect), which increases the risk of obsolescence. This tends to reduce the value of innovation, discouraging entry of rms into R&D. In contraction, the direct capitalisation effect dominates the indirect obsolescence effect, increasing the number of research rms n_e. The reverse happens in expansion, reducing n_c.

4.2 Output Fluctuations

Having established Proposition 7, we immediately have the following.

Proposition 8 *As labour productivity grows faster, (i) the level of output increases further in expansion, but falls further in contraction (i.e.* $\partial y_e/\partial \gamma > 0$ *and* $\partial y_c/\partial \gamma < 0$); *and (ii) the amplitude of uctuations,* $\theta = y_e - y_c$*,increases.*

Proof. (i) This is evident from (7) and (14). (ii) This is evident from result (i). ■

Result (i) is fairly intuitive. A higher productivity growth promotes competition in expansion with more variety inputs created. This increased input specialisation raises productivity of the nal output sector in that state. The exact opposite happens in contraction, leading to result (ii). Note, however, that result (ii) of Proposition 8 does not necessarily imply more volatile uctuations, since volatility also depends on the average time length of expansion and contraction.

Proposition 9 *As labour productivity grows faster, (i) the economy spends more in expansion and less in contraction; and (ii) the size of contraction* $(y^* - y_c)$ *increases, but that of expansion* $(y_e - y^*)$ *may rise or fall.*

Proof. (i) $\partial H_e / \partial \gamma > 0$ is evident from (14). (ii) It follows from (11), result (i) of this proposition and result (ii) of Proposition 8. ■

Result (i) agrees with our intuition and casual observation that a fast growing economy is on average in expansion more frequently than a slowly growing one. It is a good aspect of a faster productivity growth. Result (ii), on the other hand, shows its negative side: contraction becomes severer and expansion may be milder. Besides, (11) implies that the size of contraction relative to the amplitude of uctuations $((y^* - y_c)/\theta = H_e)$ rises and the same measure for expansion $((y_e - y^*)/\theta = 1 - H_e)$ falls unambiguously as γ rises. This shows that a faster growing economy is hit by more frequent and milder expansion and less frequent and severer contraction, at least in a relative sense.

There are several possible explanation of asymmetries of the business cycle mentioned above. For example, DeLong and Summers (1988) refer to asymmetric price adjustment in expansion and contraction. Propositions 8 and 9 suggest that labour productivity growth is another possible important factor in determining the degree of such asymmetries.

The volatility of output uctuations can be measured by the variance of normalised output:

$$\sigma_y^2 = H_e (y_e - y^*)^2 + (1 - H_e) (y_c - y^*)^2 = H_e (1 - H_e) \theta^2 \tag{15}$$

where the second equality uses (11). A higher growth rate makes the amplitude of uctuations (θ) larger, tending to increase the variance. On the other hand, $H_e (1 - H_e)$ is decreasing in γ (since $\partial H_e / \partial \gamma > 0$ and $H_e > 1/2$). This represents the fact that a faster growing economy experiences on average more expansion whose absolute size is smaller than contraction. The net change is ambiguous, indicating the possibility of a non-monotonous relationship. The same prediction is also made by Michelacci (1997) who treats growth as endogenous and uctuations as exogenous the opposite approach to that adopted here.

4.3 Technological Progress

The expected growth rate in each phase of uctuations is given in (12). Thus, Proposition 7 implies the following.[8]

Proposition 10 *As labour productivity grows faster, the rates of output growth and technological advance become higher in contraction, but they change ambiguously in expansion.*

This is somewhat surprising, because this implies that public policy which raises γ, such as education, is unambiguously translated into a higher growth *only* in contraction in

[8] Given (12), Proposition 7 also leads to the result that a higher γ causes the magnitude of a rise in growth in contraction ($g_c - g$) to get larger, but that of a decline in growth in expansion ($g - g_e$) changes ambiguously.

which the economy spends less time. Moreover, it turns out that the average growth rate over the entire business cycle ($g = H_e g_e + (1 - H_e)\, g_c$) ambiguously changes with γ.[9]In this sense, it is possible that resources employed for public policy to stimulate growth could be wasted in a cyclical economy. This result sharply contrasts with the non-cyclical equilibrium with $\overline{n} = n_m = n_{m+1}$. In this case, the average growth rate is $\varphi \overline{n} \ln \lambda + \gamma$ where $\overline{n}$ is strictly increasing in γ (see Figure 2). This demonstrates that policy implications obtained in the non-cyclical equilibrium do not carry over to the cyclical equilibrium. This is particularly important, since the literature conducts most of policy analysis using non-cyclical models despite the fact that a real economy exhibits large and undamped fluctuations.

5 Industrial Policy

This section explores how an R&D subsidy affects the characteristics of output fluctuations. We also re-examine its effect on technological progress, which is much discussed in the literature.

5.1 Output Fluctuations

The government subsidises a fraction $1 > s > 0$ of research costs and it is financed through lump-sum tax. Under this assumption, free entry condition in R&D (9) is replaced with $\varphi n_m^\eta V_{tm+1} = (1 - s)\, D_t$, and we have $\Gamma = \alpha (1 - \alpha) L^\alpha \lambda \varphi / d (1 - s)$ in (E). It is assumed that $s = 0$ initially.

[9]A change in the variance of growth rate turns out ambiguous. Thus, we cannot analytically confirm empirical finding of a negative correlation between the mean of growth rate and is volatility (see Ramey and Ramey (1995)).

We rst consider the state-dependent policy rule, i.e. the policy is placed *either* in expansion *or* in contraction. This will generate some intriguing results and help interpret the effect of permanent research subsidies.

In Table 2, there are n_c research rms in expansion. Hence, if R&D is subsidised in that state, the policy affects only $n_c = f(n_e)$ in (10). Similarly, n_e rms are active in research in contraction, and hence, an R&D subsidy in that state alters only $n_e = f(n_c)$ in (10).

Proposition 11 *In the 2-cycle PFE,*

1. *a research subsidy in expansion increases* n_c*, but decreases* n_e*; and*

2. *a research subsidy in contraction increases* n_e*, but decreases* n_c*.*

Proof. See Appendix C.

The rst part of results (1) and (2) says that research conducted in the phase when the policy is used is encouraged. This is because R&D subsidies reduce costs and stimulate R&D. This is the direct cost-reduction effect, which is familiar in the literature.

The second part of results (1) and (2) are new. An R&D subsidy placed in one state adversely affects research in the other state where the policy is not used. An intuition is as follows. Consider the case where R&D is subsidised in expansion (result (1)). Firms conducting R&D in *contraction* capitalise future pro t ows at the effective discount rate $\rho - \gamma + \varphi n_c^{\eta}$. Since φn_c^{η} rises (due to the rst part of result (1)), the value of innovation to those rms falls, discouraging research with a lower n_e. A similar explanation holds for result (2).

Therefore, an R&D subsidy in contraction generates the same effects as a higher γ on output uctuations and technological progress, and the same policy in expansion has the exactly opposite effects. The state-dependent rule, therefore, results in a trade-off in the sense that a subsidy in contraction (expansion) enables the economy to spend more (less) in expansion but with a larger (smaller) amplitude uctuations (see subsection 4.2).

This might make one wonder if the once-and-for-all subsidy is superior. Unfortunately it is not, and one may even argue that it is inferior to the state-dependent rule, since its effects on n_e and n_c turn out ambiguous. To show this, note that the once-and-for-all policy generates both effects (1) and (2) in Proposition 11. The direct cost-reduction effect tends to increase n_e and n_c, but the indirect obsolescence effect tends to reduce them. Whether the net effect is positive or negative is not predictable without knowing parameter values.

5.2 Technological Progress

An important policy implication in innovation-driven growth models is that subsidies encourage research, thereby raising the long-run growth. This prescription is widely accepted among policy makers and rarely questioned in the literature. However, this result is obtained in a stationary state when research intensity is constant. Thus, we next re-examines the issue in a more realistic environment of business cycle.

Given Proposition 11 and equation (12), it is easy to establish the following.

Proposition 12 *In the 2-cycle PFE,*

1. *a subsidy in expansion promotes technological progress in an expansionary phase,*

but discourages it in contraction; and

2. *the reverse holds if a subsidy is applied in contraction.*

Therefore, the state-dependent policy rule always generates asymmetric changes in technological advance, implying a trade-off between a higher R&D intensity in one state and a lower intensity in the other state. Furthermore, because of this asymmetric response, the average rate of technological progress over the entire business cycle changes ambiguously if either type of the state-dependent rule is followed.

Note also that a change in the expected rate of technological innovation still remains ambiguous even if the policy is once-and-for-all. This is because the policy generates both effects (1) and (2) of Proposition 12, making changes in n_e and n_c ambiguous. This result comes in a marked contrast with a stationary equilibrium in which technology advances at the rate of $\varphi\overline{n}\ln\lambda$. As Figure 2 shows, since Γ is increasing in s, a higher s shifts upward the curve representing (E) with $\overline{n}$ unambiguously increasing.

The policy implication in the stationary equilibrium does not necessarily extend to the cyclical equilibrium, which is arguably more realistic. This suggests that some policy implications related to R&D subsidies in the literature may need to be treated with some cautions.[10]

[10] Another important question concerns a normative issue, i.e. whether R&D should be taxed or subsidised in the presence of endogenous uctuations. Unfortunately, it turned out impossible to analytically examine the issue. Although analysis can be carried out in the stationary case, it is less relevant in our context.

6 Endogenous Labour Productivity Growth

So far, we assumed that labour productivity grows at a given rate of a. This section generalises the model by endogenising a through learning-by-doing. It is assumed that $\dot{A}_t = \delta \left(n_m^p\right)^{\beta} x_{tm}$, $\delta > 0$, $1 > \beta \geq 0$. The presence of n_m^p captures the inter-industry learning spillover effect due to, for example, the movement of workers between rms producing different varieties. The parameter β indicates the strength of such externality. Using this assumption and (5), we can rewrite the rate of labour productivity growth as

$$a\left(n_m^p\right) \equiv \frac{\delta L}{\left(n_m^p\right)^{1-\beta}} = \frac{\delta L}{n_{m-1}^{1-\beta}} \equiv a\left(n_{m-1}\right). \tag{16}$$

The equilibrium condition (E) does not change except for γ being replaced with $\gamma\left(n_m\right) = \alpha a\left(n_m\right)$. Because $\gamma'\left(n_m\right) \leq 0$, it is still true that the curve representing (E) is downward-sloping for $\alpha > \eta$, as in Figure 2. Therefore, Proposition 1 applies again and a stable 2-cycle PFE can arise with n_m oscillating between n_e and n_c.

Proposition 13 *In the 2-cycle PFE, the rate of labour productivity growth is faster in contraction than in expansion.*

Proof. $n_e > n_c$ and $a'\left(n_m\right) < 0$ imply $a\left(n_e\right) < a\left(n_c\right)$. ■

This result runs against the conventional wisdom that recession is harmful for productivity growth driven by learning-by-doing, since it generally means lower output and hence a smaller opportunity to learn. Although we do not introduce the role of government explicitly, this result suggests the possibility that public stabilisation policy may depress the long-run growth prospect (see Stadler, 1990).

A key to understanding this result is the distinction between *inter*- and *within*-industry learning effects. Since contraction involves a smaller number of variety inputs, the inter-industry learning effect is weak. This tends to make the growth rate lower in contraction than in expansion. On the other hand, output of each variety is larger for a given labour force in contraction when the number of variety inputs is smaller (see (5)). As a result, learning occurs more intensively in contraction within each input industry, tending to make the growth rate higher in contraction. If the inter-industry learning effect is relatively weak ($1 > \beta \geq 0$), its negative effect in contraction is more than offset by the positive within-industry learning effect. The opposite happens in expansion.

However, a caveat is in order. Like most of growth models, we assume full-employment. If unemployment is introduced, the positive within-industry learning effect in contraction (when unemployment is higher) is relatively small. This consideration tends to weaken the result. But this intriguing result may not necessarily be merely a theoretical possibility. For example, using the NBER productivity data set for the US, Malley and Muscatelli (1996) find little empirical support for the case that learning-by-doing in temporary expansion increases total factor productivity. This finding may be interpreted as reflecting the positive effect of contraction (and the negative effect of expansion) identified above. A useful extension to the present paper would be further empirical research which discriminates between learning effects within and across sectors to investigate the link between fluctuations and learning-driven growth.

7 Conclusion

This paper has extended the current literature on long-run growth-output cycle interactions in three important ways. First, our quality-ladder growth model relies on the expectations about the degree of competition in both production and R&D sectors to generate cyclical uctuations, but does not rely on implausibly large labour reallocations between these sectors as the cycle-generating mechanism, unlike existing growth models. Second, and more importantly, we have shown how changes in long-run growth can have a permanent impact on the structure of output uctuations. Our main results are the following: as labour productivity grows faster (i) the amplitude of output uctuations increases; (ii) output expansion (contraction) occurs more (less) frequently but becomes milder (severer); and (iii) the volatility of output changes in an ambiguous way, indicating a non-monotonic relationship between the long-run growth rate and output volatility. It was also shown that the same (reverse) results were obtained if R&D is subsidised in contraction (expansion).

Third, we demonstrated that technology can advance faster in contraction than in expansion due to technology diffusion. Moreover, the extended model displays the surprising result that labour productivity can also grow faster in periods of output contraction, even if learning-by-doing is the sole source of such productivity growth. Thus, our model complements the existing literature on growth and cycles, which mainly focuses on the lower opportunity research cost and the cleansing effect of recession. Fourth, we re-examined the effect of R&D subsidies on technological progress in the cyclical equilibrium. It turned out that the state-dependent policy rule results in a trade-off between a higher R&D intensity in one state and a lower intensity in another, and the once-and-for-all policy shift generates ambiguous results. This contrasts with the literature which predicts that the policy generates a strong and positive effect on technological change.

Appendix A: Proof of Proposition 1

Consider the second-iterated function of (E), $n_m = f^2(n_{m+2})$. We have $f^{2'}(n_{m+2}) = f'(f(n_{m+2}))\, f'(n_{m+2}) = f'(n_{m+1})\, f'(n_{m+2}) > 0$, since $f'(.) < 0$. Moreover, $f^2(0) > 0$ and there exists $n^{\#}$ such that $n^{\#} > \overline{n}$ and $n^{\#} > f^2\left(n^{\#}\right)$, since $f^2(+\infty) = \left(\frac{\Gamma}{\rho-\gamma}\right)^{\frac{1}{\alpha-\eta}} < \infty$ where $\rho > \gamma$ (see Figure 3). Thus, the curve representing the second-iterated function must cross the 45° line at odd times. However, it is well known that since $f(n_{m+1})$ is monotonically decreasing in n_{m+1}, the model does not exhibit k-cycle with $k \geq 3$ or chaotic time trajectories and it can generate 2-cycles only (see e.g. Baumol and Benhabib (1989)). Thus, the curve cannot cross the 45° line more than three times. It follows that an asymptotically stable 2-cycle PFE exists (i.e. $|f^{2\prime}(n_i)| < 1$, $i = e, c$) if and only if parameters are such that $1 < |f^{2\prime}(\overline{n})| = [f'(\overline{n})]^2$.

Appendix B: Proof of Proposition 7

Step 1: Totally differentiating the system of two equations in (10) yields

$$\begin{bmatrix} 1 & -f'(n_c) \\ -f'(n_e) & 1 \end{bmatrix} \begin{bmatrix} \partial n_e/\partial\gamma \\ \partial n_c/\partial\gamma \end{bmatrix} = \begin{bmatrix} f_\gamma(n_c) \\ f_\gamma(n_e) \end{bmatrix} \tag{17}$$

where $f'(n_i) \equiv \partial f(n_i)/\partial n_i = -\Delta n_j^\alpha \left(\frac{n_j}{n_i}\right)^{1-\eta} < 0$, $\Delta = \frac{\varphi\eta}{(\alpha-\eta)\Gamma} > 0$, and $f_\gamma(n_i) \equiv \partial f(n_i)/\partial\gamma = \frac{n_j^{1+\alpha-\eta}}{(\alpha-\eta)\Gamma} > 0$, $i, j = e, c$, $i \neq j$, after rearrangement. The determinant of the Jacobian matrix is $|J| = 1 - f'(n_c) f'(n_e) = 1 - \Delta n_c^\alpha \Delta n_e^\alpha > 0$, since $1 > f^{2'}(n_e) = f'(f(n_e)) f'(n_e) = f'(n_c) f'(n_e)$ (see Appendix A). By Cramer s rule,

$$\frac{\partial n_e}{\partial\gamma} = \frac{f_\gamma(n_c)}{|J|}(1 - \Delta n_c^\alpha), \qquad \frac{\partial n_c}{\partial\gamma} = \frac{f_\gamma(n_c)}{|J|}(1 - \Delta n_e^\alpha). \tag{18}$$

Step 2: Since $f'' > 0$, $|f'(\overline{n})| > 1$ and $n_c < \overline{n}$, we have $|f'(n_c)| > 1$. This fact and $|J| > 0$ implies $|f'(n_e)| < 1$. Now de ne $\widehat{n}$ and $\widetilde{n}$ such that $\widehat{n} = f(\widetilde{n})$ and $1 = |f'(\widetilde{n})| = \Delta\widehat{n}^\alpha \left(\frac{\widehat{n}}{\widetilde{n}}\right)^{1-\eta}$ (see Figure 2). But $\left(\frac{\widehat{n}}{\widetilde{n}}\right)^{1-\eta} < 1$ due to $f'(.) < 0$ and $|f'(\overline{n})| > 1$. Thus, $\Delta\widehat{n}^\alpha > 1$. Moreover, $|f'(n_e)| < 1$ implies $\widehat{n} < \widetilde{n} < n_e$, which in turn implies $\Delta n_e^\alpha > 1$. This fact and $|J| > 0$ lead to $\Delta n_c^\alpha < 1$. Now the proposition is fully established.

Appendix C: Proof of Proposition 11

Totally differentiating the system of two equations in (10) yields

$$\begin{bmatrix} 1 & -f'(n_c) \\ -f'(n_e) & 1 \end{bmatrix} \begin{bmatrix} \partial n_e/\partial s \\ \partial n_c/\partial s \end{bmatrix} = \begin{bmatrix} f_s(n_c) \\ f_s(n_e) \end{bmatrix} \tag{19}$$

where $f_s(n_i) \equiv \partial f(n_i)/\partial\gamma|_{s=0} = \frac{f(n_i)}{\alpha-\eta} > 0$, $i = e, c$ after rearrangement and the terms in the Jacobian matrix are de ned in Appendix B.

A Subsidy in Expansion: In this case, $f_s(n_c) = 0$. By Cramer s rule,

$$\frac{\partial n_e}{\partial s} = \frac{f'(n_c) f_s(n_e)}{|J|} < 0, \qquad \frac{\partial n_c}{\partial s} = \frac{f_s(n_e)}{|J|} > 0. \tag{20}$$

A Subsidy in Contraction: In this case, $f_s(n_e) = 0$. By Cramer s rule,

$$\frac{\partial n_e}{\partial s} = \frac{f_s(n_c)}{|J|} > 0, \qquad \frac{\partial n_c}{\partial s} = \frac{f'(n_e) f_s(n_c)}{|J|} < 0. \tag{21}$$

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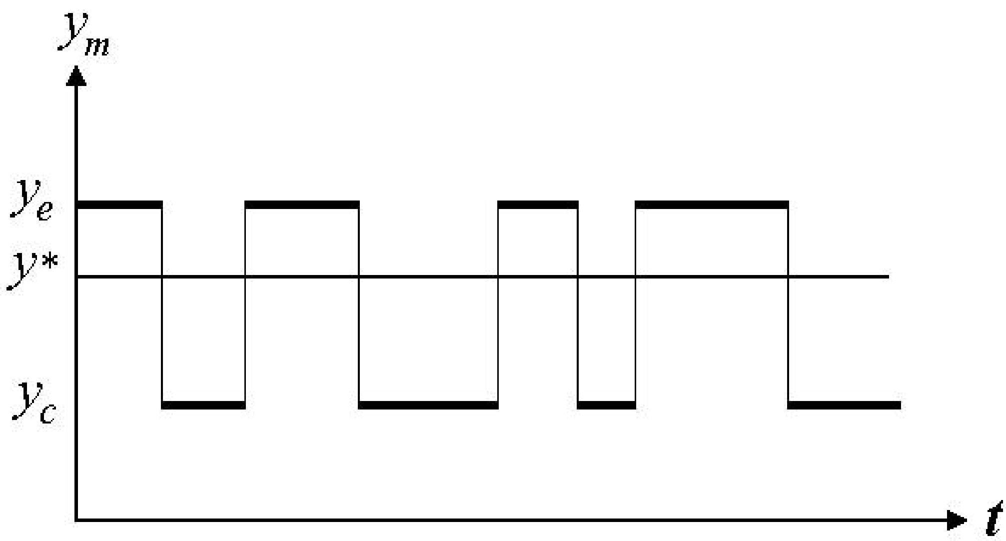

Figure 1: Fluctuations of normalised output.

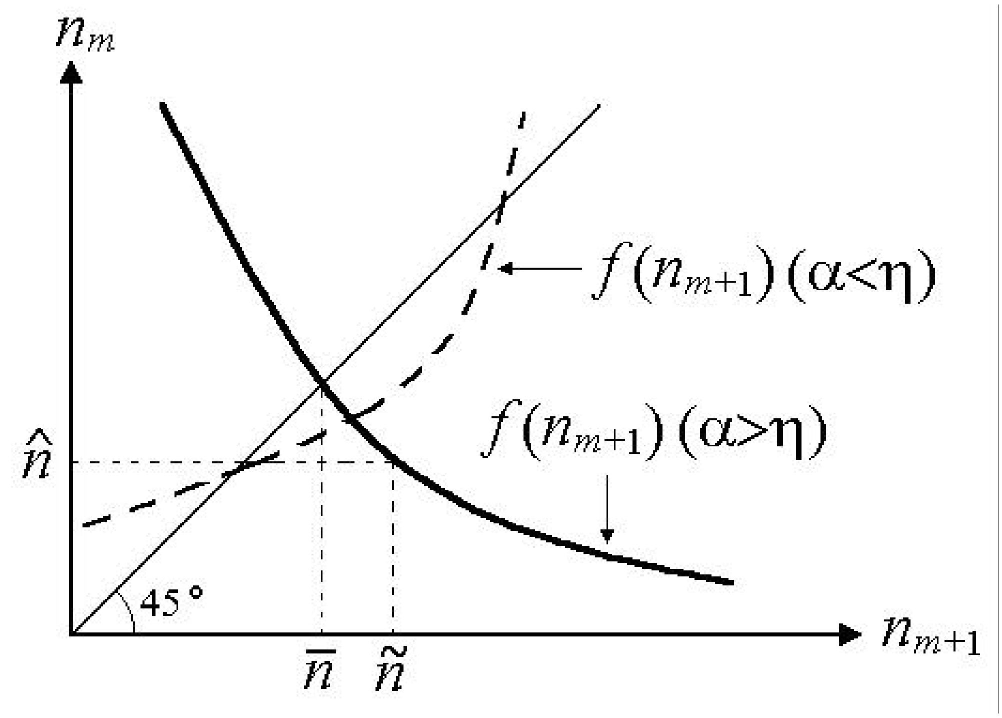

Figure 2: Perfect foresight equilibrium.

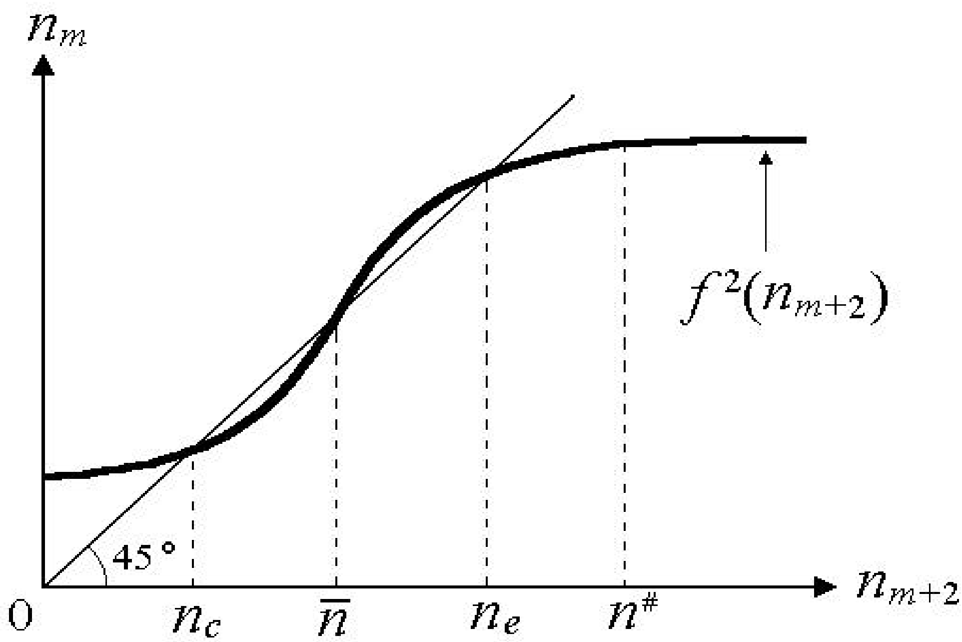

Figure 3: Two-cycle PFE for $\alpha > \eta$.