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Stochastic efficiency analysis with risk aversion bounds: a simplified approach

J. Brian Hardaker, James W. Richardson, Gudbrand Lien and Keith D. Schumann*

A method of stochastic dominance analysis with respect to a function (SDRF) is described and illustrated. The method, called stochastic efficiency with respect to a function (SERF), orders a set of risky alternatives in terms of certainty equivalents for a specified range of attitudes to risk. It can be applied for conforming utility functions with risk attitudes defined by corresponding ranges of absolute, relative or partial risk aversion coefficients. Unlike conventional SDRF, SERF involves comparing each alternative with all the other alternatives simultaneously, not pairwise, and hence can produce a smaller efficient set than that found by simple pairwise SDRF over the same range of risk attitudes. Moreover, the method can be implemented in a simple spreadsheet with no special software needed.

1. Introduction

Risk assessment requires coming to grips with both probabilities and preferences for outcomes held by the decision maker. Chances of bad versus good outcomes can only be evaluated and compared knowing the decision maker's relative preferences for such outcomes. According to the subjective expected utility (SEU) hypothesis (Anderson *et al.* 1977, pp. 66–69), the decision maker's utility function for outcomes is needed to assess risky alternatives as the shape of the utility function reflects an individual's attitude towards risk. The SEU hypothesis states that the utility of a risky alternative is the decision maker's expected utility for that alternative, meaning the probability-weighted average of the utilities of outcomes.

* Brian Hardaker is Emeritus Professor in the Graduate School of Agricultural and Resource Economics, University of New England, Armidale, New South Wales, Australia. James Richardson is Regents Professor and TAES Faculty Fellow in the Department of Agricultural Economics at Texas A&M University, College Station, Texas, USA. Gudbrand Lien is Senior Researcher at the Norwegian Agricultural Economics Research Institute, Oslo and at the Eastern Norway Research Institute, Lillehammer, Norway. Keith Schumann is Extension Associate in the Department of Agricultural Economics at Texas A&M University, College Station, Texas, USA. The authors are grateful to Jock Anderson and two anonymous referees for helpful comments on earlier drafts.

The SEU hypothesis has been criticised because it has long been recognised that many people do not act consistently with the theory in certain risky choice situations (e.g., Allais 1984). Recently, Rabin (2000) has shown that typical aversion to individual risky prospects with small losses is so great as to be inconsistent with any utility function expressed in terms of the utility of wealth. Such loss aversion, as it is called, implies a failure in asset integration, meaning that people seemingly do not regard small gains and losses as changes in wealth. Evidently the SEU hypothesis is flawed as a behavioural theory of choice (Rabin and Thaler 2001). In prescriptive applications, however, it is clear that loss aversion is irrational because, by the operation of the law of large numbers over many small risky prospects with better than fair odds, it implies forgoing the opportunity of profiting with negligible chance of loss. Moreover, loss aversion often disappears when people are given the opportunity of repeated choice or when the size of the risk faced is increased. We therefore argue that the SEU hypothesis remains the most appropriate theory for prescriptive assessment of risky choices, a view supported by Meyer (2001).

Several attempts have been made to elicit utility functions from relevant decision makers in order to put the SEU hypothesis to work in the analysis of risky alternatives in agriculture. Usually the results have been rather unconvincing (King and Robison 1984; Anderson and Hardaker 2003). Partly to avoid the need to elicit a specific single-valued utility function, methods under the heading of stochastic dominance or efficiency criteria have been developed. Stochastic dominance criteria are useful in situations involving a single decision maker whose preferences are not known precisely and in situations where more than one decision maker might be involved, such as analysing policy alternatives or extension recommendations for a group of many individual decision makers.

A stochastic dominance criterion is a decision rule that provides a partial ordering of risky alternatives for decision makers whose preferences conform to specified conditions about their utility functions (preferences for consequences). There is an important trade-off to be made in conducting a stochastic dominance analysis. The fewer restrictions that are placed on the utility function, the more general applicability the results will have, but the less powerful will be the criterion in selecting between alternatives. Usually, efficiency analysis results in only a partial ordering of alternatives into efficient and dominated sets. The decision maker must then make the final choice from among the members of the efficient set. Criteria that identify small efficient sets usually require more specific information or assumptions about preferences.

Hadar and Russell (1969) and Hanoch and Levy (1969) presented the concepts of first-degree stochastic dominance (FSD) and second-degree stochastic dominance (SSD). By use of FSD it is possible to order alternatives for decision makers who prefer more wealth to less and have absolute risk aversion with respect to wealth, $r_a(w)$, between the bounds $-\infty < r_a(w) < +\infty$ (King and Robison 1984). For SSD it is assumed that decision makers are not risk preferring, that is, that absolute risk aversion bounds are $0 < r_a(w) < +\infty$. This means that SSD accounts for the possibility that some decision makers possess an absolute risk aversion parameter that is so large that the utility of a small difference at the lowest observation is extraordinarily important. In empirical work it is often found that these two forms of analysis are not discriminating enough to yield useful results, meaning that the efficient set can still be too large to be easily manage-able (King and Robison 1981, 1984).[1] Moreover, as noted in relation to loss aversion, allowing for extreme risk aversion is unrealistic. Therefore, there is a case to base analysis on a more restricted range of risk aversion.

Stochastic dominance with respect to a function (SDRF), which was introduced by Meyer (1977), allows for tighter restrictions on risk aversion. For SDRF the absolute risk aversion bounds are reduced to $r_L(w) \leq r_a(w) \leq r_U(w)$, and ranking of risky scenarios is defined for all decision makers whose absolute risk aversion function lies anywhere between lower and upper bounds $r_L(w)$ and $r_U(w)$, respectively. Eliciting from the decision makers (or inferring) the bounds on their risk aversion coefficients may be simpler than eliciting a complete utility function.

As we indicate later in the paper, the computing task for SDRF is somewhat tricky. Although software packages are available (e.g., McCarl 1988, 1990; Goh *et al.* 1989; and Richardson *et al.* 2001), it seems likely that many users will have limited understanding of what is going on and how the alternative programs differ in the way they operate. For example, it seems it is not widely appreciated that the software of McCarl's may give a different (smaller) efficient set to that of Goh *et al.* for reasons to be explained shortly.

In this paper we introduce a more transparent and potentially more discriminating SDRF method, which we call stochastic efficiency with respect to a function (SERF). The name is chosen to distinguish it from conventional SDRF and to indicate that the method works by identifying utility efficient alternatives for ranges of risk attitudes, not by finding (a subset of) dominated alternatives. SERF orders alternatives in terms of certainty equivalents (CE) as a selected measure of risk aversion is varied over a defined range. SERF can be applied for any utility function for which the inverse function can be computed based on ranges in the absolute, relative,

[1] There are third to *t*-th degree stochastic dominance criteria but they are seldom much more discriminating than SSD, and so are not reviewed in the present paper. A review of ordinary stochastic dominance and stochastic dominance with respect to a function is given by Zentner *et al.* (1981). Within the stochastic dominance paradigm, Levy (1992) reviewed the theoretical developments and empirical applications in economics, finance and statistics. For a more recent treatment in an agricultural context, see Robison and Myers (2001).

or partial risk aversion coefficient, as appropriate. Conventional SDRF picks only the pairwise dominated alternatives, so one can expect that pairwise SDRF may not isolate the smallest possible efficient set. By contrast SERF will potentially identify a smaller efficient set than SDRF because it picks only the utility efficient alternatives, comparing each with all the other alternatives simultaneously.

McCarl's Riskroot software extends SDRF analysis beyond the simple pairwise comparisons to produce similar efficient sets to SERF by identifying the breakeven risk aversion coefficients among alternatives. However, SERF is arguably more transparent in application than SDRF through Riskroot, allowing a graphical presentation of results that is readily understood by a wide range of potential users. Further, SERF has the advantage that it can be implemented in a simple spreadsheet with no special software needed.[2]

The paper is structured as follows: section 2 describes the SERF method; the relationship between conventional SDRF and SERF is discussed in section 3; two applications of the SERF method are presented in section 4; section 5 contains a short discussion and concluding comments.

2. The SERF method

Let $U(w)$ be the utility function of a decision maker with performance criterion w (wealth).[3] We assume that the risky alternatives to be compared have uncertain outcomes so values of w are stochastic. Let $f_1(w), f_2(w), \ldots, f_n(w)$ be the probability density functions (PDF) describing the outcomes for n risky alternatives. The corresponding cumulative distribution functions (CDF) are denoted by $F_1(w), F_2(w), \ldots, F_n(w)$. The SEU hypothesis is that $U(w) = EU(w) = \int U(w)f(w)dw = \int U(w)dF(w)$, that is, the utility of any risky alternative is its expected value. Because the exact shape of the utility function is unknown or, in other words, the decision maker's exact risk aversion is unspecified, the problem is solved where the absolute, relative or partial risk aversion function $r(w)$ of the decision maker lies everywhere between lower and upper bounds $r_L(w)$ and $r_U(w)$. So for each risky alternative and for a chosen form of the utility function, the function for utility in terms of risk aversion and the stochastic outcome w is defined as:

[2] A sample MS Excel spreadsheet program for CE calculation under two assumed utility functions is provided in the Appendix table A1. A one-year free license to the MS Excel Add-In, Simetar, which implements SERF, is available from James Richardson by emailing at jwrichardson@tamu.edu. The simulation, risk analysis, and econometrics capabilities of Simetar are described by Richardson *et al.* (2001).

[3] Although wealth, w, is used as the performance criterion in this paper, w can be replaced by x (for loss/gain or transient income), provided the degree of risk aversion applied is consistent with the outcome measure (Anderson and Hardaker 2003).

$$U(w,r(w)) = \int U(w,r(w))dF(w) \approx \sum_{i=1}^{m} U(w_i,\ r(w))P(w_i),\ r_1(w) \le r(w) \le r_2(w) \quad (1)$$

where the second term in equation 1 represents the continuous case and the continuous case is converted to its discrete approximation in the third term for computational purposes. In the discrete case $P(w_i)$ is the probability for states i and there are m states for each risky alternative. Starting with CDF data for a set of risky alternatives, equation 1 implies the following computational steps:

1. Select points on each CDF for a finite set of values of w.
2. Convert each of these w values to its utility using the selected form of utility function and the selected value of the risk aversion coefficient.
3. Multiply each finite utility by its associated probability to calculate a weighted average of the utilities of outcomes.

This discrete function is then evaluated for a sufficient number of discrete points of $r(w)$ to describe the relationship between U and $r(w)$ for each alternative.

The CE of a risky prospect is the sure sum with the same utility as the expected utility of the prospect. In other words, for a given utility function, it is the point mass at which the decision maker is indifferent between the value and the risky outcome. For a rational decision maker who is risk averse (the normal case), the estimated CE is typically less than the expected money value (EMV) and greater than or equal to the minimum value. The difference between the EMV and the CE is the risk premium. Partial ordering of alternatives by CE is the same as partial ordering them by utility values. However, for greater convenience we chose to convert the utilities to CE values by taking the inverse of the utility function:

$$CE(w,\ r(w)) = U^{-1}(w,\ r(w)) \quad (2)$$

The calculation of CE depends on the utility function specified. For example, given an exponential utility function, a specific absolute risk aversion coefficient ($r_a(w)$), and a random sample of size n from a risky alternative w, the estimated CE is defined as:

$$CE(w,r_a(w)) = \ln\left\{\left(\frac{1}{n}\sum_{i}^{n} \exp(-r_a(w)w_i)\right)^{-1/r_a(w)}\right\} \quad (3)$$

as outlined in table A1 in the Appendix. The CE representation is preferred to leaving results in utilities, not only because CE values are easier to interpret than utility values, but also because this method allows inclusion of the EMV of each alternative in cases where $U(w,\ r(w))$ is undefined for $r(w) = 0$.

By this method we end up with a vector of CE values for each of the *n* alternatives calculated for several values of $r(w)$ within the bounds $r_L(w) \leq r_i(w) \leq r_U(w)$. At each $r_i(w)$ only the alternative that yields the highest CE is efficient. The efficient set can be identified over a subset of the full range of $r_i(w)$, as might be required for policy analyses. Alternatives can be ordered using the following rule:

- Only those alternatives which have the highest (or equal highest) CE values for some value in the range of $r(w)$ are utility efficient. All other alternatives are dominated in the SERF sense.

For ease of interpreting the CE results, the CE values of the alternatives can be graphed on the vertical axis against risk aversion on the horizontal axis over the range of $r_L(w)$ to $r_U(w)$. The resulting graph allows ready identification of the efficient set and also provides a visual method to explain how preferences among risky alternatives change over the range of $r(w)$.

In figure 1 the SERF method is used to compare three alternatives simultaneously for all values in the range of $r_L(w)$ to $r_U(w)$, and identifies alternatives 1 and 2 as the utility-efficient set. Alternative 1 dominates over the range of $r_L(w)$ to $r_2(w)$ and alternative 2 dominates for the risk aversion range of $r_2(w)$ to $r_U(w)$. With the SERF method alternative 3 is not utility-efficient as it is dominated by one of the other alternatives at every level of risk aversion.

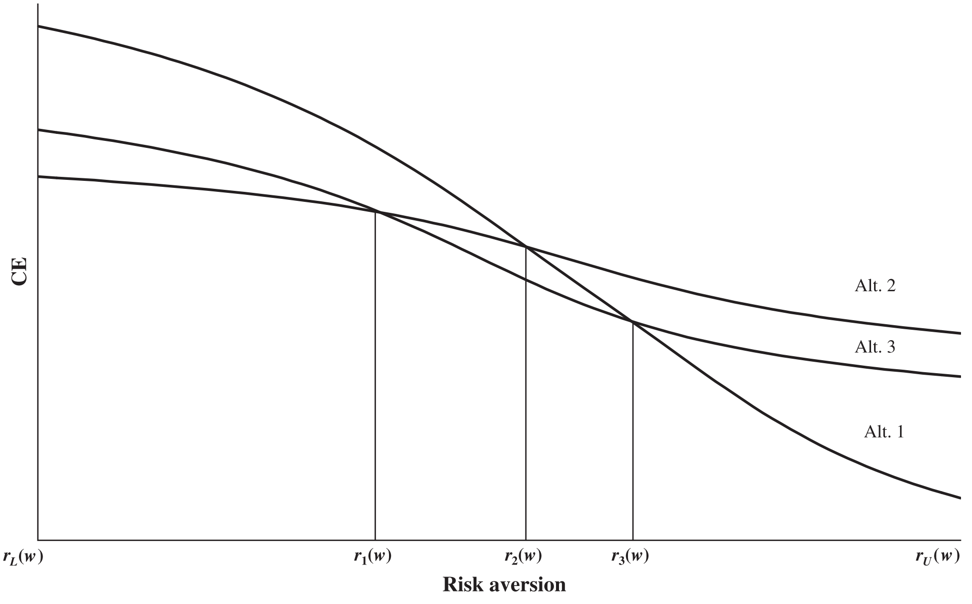

Figure 1 Illustration of SERF for simultaneously comparing three alternatives over risk aversion levels $r_L(w)$ to $r_U(w)$.

Simple pairwise comparison of these three alternatives using SDRF would eliminate none of the three from the efficient set because each curve is crossed by at least one of the other two.

The SERF method can be applied in a spreadsheet program using either the software developed by Richardson *et al*. (2001) or by programming the steps included in Appendix table A1. If the numbers of alternatives being compared is so extensive that a graph is not feasible, it is adequate to build a table of CE values for each alternative calculated over a finite range of the risk aversion co-efficient, such as presented in table A2 of the Appendix. The efficient set found in such a tabular analysis could be graphed if desired.

Drawing on Hammond (1974), McCarl (1988) proposed solving for the risk aversion coefficient where the preference between a pair of efficient alternatives changes. He called the value of the risk aversion coefficient at which the preference changes the breakeven risk root (BRAC). For values of the risk aversion coefficient less than the BRAC, one alternative is preferred and for values greater than the BRAC the other is preferred. With a SERF graph it is simple to identify each BRAC as the values of $r(w)$ where two CE curves cross. The goal seeking function in Excel can be used to find the exact BRAC for two alternatives by varying $r(w)$ to minimise the difference between the two CE values.

The results of a SDRF analysis might depend on the choice of utility function. Although the SERF method can be applied for any utility function for which the inverse function can be calculated,[4] we suggest it will often be best to adopt the CARA function (negative exponential) as a reasonable approximation of the actual but presumably unknown utility function. Such an approximation is appropriate provided that the range of outcomes of risky alternatives being compared is small relative to the decision maker's wealth (Tsiang 1972). An advantage of the CARA function is that, as Anderson and Hardaker (2003) show, the coefficients of absolute risk aversion can be applied to consequences measured in terms of wealth or income. McCarl (1990) has also shown that the CARA function will yield the same results as other functions over small risk aversion intervals.

3. Relationship between SERF and SDRF

The SDRF method involves pairwise comparison of risky alternatives. For a given form of risk-averse utility function, $U(w)$, defined with a risk aversion coefficient within the bounds

$$r_L(w) \leq r_a(w) \leq r_U(w) \tag{4}$$

[4] Examples of some different utility functions are given in Hardaker *et al*. (2004) and Lin and Chang (1978).

the following expression is sequentially evaluated for all values of $r_a(w)$

$$\int [F_2(w) - F_1(w)]U'(w)dw \tag{5}$$

where the cumulative density functions $F_1(w)$ and $F_2(w)$ represent two risky alternatives and $U'(w)$ is the first derivative of $U(w)$. If the minimum of the above expression across all values of $r_a(w)$ in the defined range is positive, then alternative $F_1(w)$ is preferred to $F_2(w)$. That means that the utility (or CE) of $F_1(w)$ is greater than the utility (CE) of $F_2(w)$ for all decision makers whose risk aversion lies within the defined range (for the particular form of $U(w)$ used). If the minimum is zero, some decision maker within the group may be indifferent between the two alternatives. If the minimum is negative, $F_2(w)$ could be preferred to $F_1(w)$. To check whether there is dominance of $F_2(w)$ over $F_1(w)$ in the SDRF sense, $F_1(w) - F_2(w)$ is introduced in the square brackets term in equation 5 in place of $F_2(w) - F_1(w)$ and the evaluation procedure is repeated. After each pairwise comparison, a dominated alternative can be deleted from the set of alternatives to be ordered, but all possible further pairwise comparisons are needed to identify the efficient set.

Equation 5 is equivalent to measuring the difference between utilities of distributions $F_1(w)$ and $F_2(w)$. To show this let the difference in utility between $F_1(w)$ and $F_2(w)$ be

$$\int U(w)f_1(w)dw - \int U(w)f_2(w)dw = \int U(w)[f_1(w) - f_2(w)]dw \tag{6}$$

Applying the change-in-variable technique to integrate, let $dv = f_1(w) - f_2(w)$, $v = F_1(w) - F_2(w)$, and $u = U(w)$. Then, using integration by parts, we write

$$\int U(w)[f_1(w) - f_2(w)]\,dw = U(w)[F_1(w) - F_2(w)]_{-\infty}^{+\infty} + \int [F_2(w) - F_1(w)]U'(w)dw$$

$$= \int [F_2(w) - F_1(w)]U'(w)dw \tag{7}$$

In other words, this method orders the utility of alternatives $F_1(\int U(w)f_1(w)\,dw)$ and $F_2(\int U(w)f_2(w)\,dw)$ within defined bounds of $r_a(w)$. By comparing this method with SERF, as described in section 2, we can see we are making the same comparison, though more directly and informatively than with conventional SDRF.

There is one further difference between SDRF and SERF that may sometimes be important. Using the conventional SDRF approach, it is generally only possible to process the risky alternatives with data points specified for the same set of fractile values. That might require preprocessing of data to

Table 1 Hypothetical example with four alternatives specified for the same set of fractile values

	w			
F(*w*)	Alternative A	Alternative B	Alternative C	Alternative D
0.0	105	50	83	90
0.1	125	100	104	103
0.2	135	128	125	111
0.3	142	145	140	117
0.4	147	152	147	121
0.5	150	157	151	123
0.6	153	162	155	125
0.7	158	171	161	129
0.8	162	183	170	133
0.9	170	207	188	144
1.0	185	230	225	163

get them into this format.[5] By contrast, using the SERF method, there is no need to define the same probability intervals for all alternatives. The method works regardless of how the distribution of returns from each alternative is specified, provided only that there is sufficient information for the expected utility of each to be reliably calculated. For example, SERF will work if some distributions are specified only in terms of moments.

4. Application

In this section, as an example of its application, the SERF method outlined in preceding text is used and compared with the SDRF method using two constructed examples.

4.1 Example 1

The first example is a hypothetical one using four constructed risky alternatives, A to D (table 1). The means of the alternatives vary from about 123 for alternative D to about 153 for alternative B. The overall range of outcomes is from 50 to 230. Both extremes are associated with alternative B. Alternative A has the largest minimum outcome of 105. Figure 2 shows the CDF graphs for each of the alternatives. The empirical CDF for an alternative is developed by sorting the w_i values from low to high, assuming equal probability of occurrence, $P(w_i)$, creating a CDF, $F(w_i)$, by summing the probabilities, and graphing the w_i and corresponding $F(w_i)$ values. The

[5] McCarl's Riskroot program is capable of processing alternatives with different numbers of fractiles.

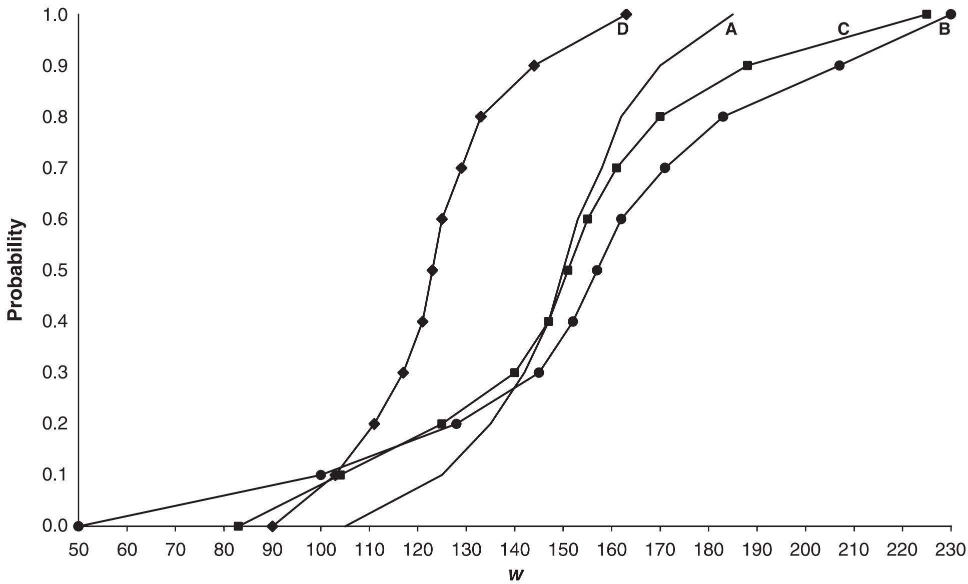

Figure 2 Cumulative probability distributions for alternatives A to D.

CDF is interpreted as the probability that the value for the alternative W will be less than or equal to w_i. For example, alternative B in figure 2, has a 10 per cent chance of being less than 100, a 50 per cent chance of being less that 157, and an 80 per cent chance of being less than 183.

The relation between absolute and relative risk aversion is $r_a(w) = r_r(w)/w$ where w is wealth. Anderson and Dillon (1992) have proposed a rough and ready classification of degrees of risk aversion, based on the relative risk aversion with respect of wealth, $r_r(w)$, in the range 0.5 (hardly risk averse at all) to approximately 4 (very risk averse).[6] The average wealth for the alternatives ranges from 123 to 153 with an overall average of around 150. Then a value of $r_a(w)$ in the range 0.0033–0.0266 corresponds to $r_r(w)$ in the range 0.5–4. In this example we use wider absolute risk aversion bounds, from 0 to 0.035, to give a better illustration of the impact of ranking alternatives. For SDRF analysis, McCarl's Riskroot and the program of Goh *et al*. (1989) were used to rank the four alternatives to compare with the SERF approach.

The SERF approach, when using a negative exponential utility function, resulted in the CE graph shown in figure 3 (the same results as reported in tabular form in Appendix table A2). As can be seen from the graph, the locus of points of highest CE values is comprised of values for alternatives A and B only, so that these two form the efficient set. By comparison, the efficient set

[6] A discussion of choice of risk aversion bounds and consistency of risk aversion across payoff measures is given in Anderson and Hardaker (2003).

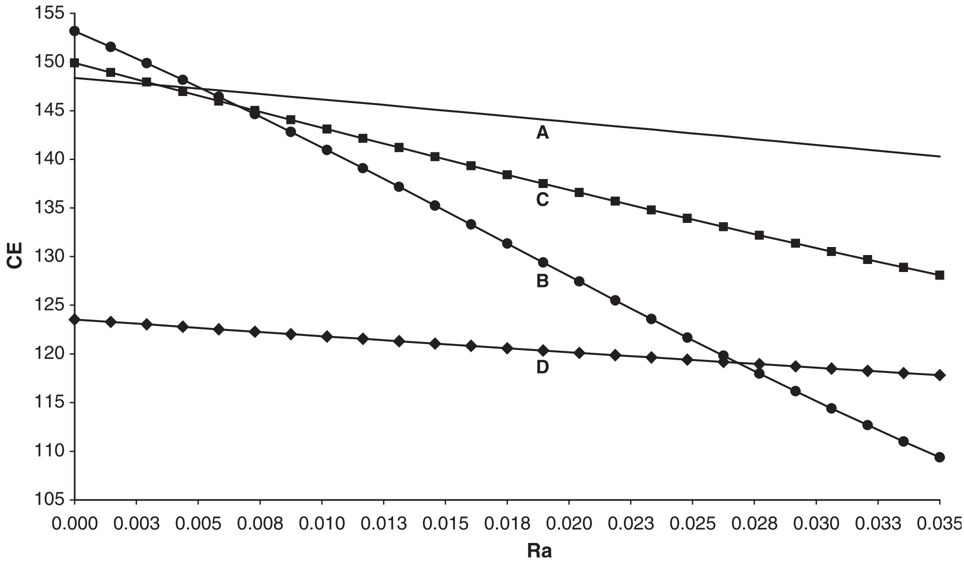

Figure 3 SERF results for alternatives A-D over the absolute risk aversion range of 0.00–0.035, assuming a negative exponential utility function.

derived using the program of Goh *et al.* (1989) with the same utility function and range for $r_a(w)$ as in the SERF analysis is A, B and C. However, for the reason explained in preceding text, Riskroot gives the identical efficient set to that obtained by the SERF approach. The SERF results show that the value of $r_a(w)$ where the CE curves for alternatives A and B cross is $r_a(w) = 0.0052$ (i.e., where $r_r(w) \approx 0.77$), the identical BRAC found using Riskroot.

The reason the Goh *et al.* (1989) program found an efficient set with one more member than SERF and Riskroot is that this program is limited to pairwise comparisons of alternatives only. It seems that (differences in data processing apart) both Riskroot and SERF can be expected to give identical efficient sets from the same set of alternatives, utility function and risk aversion range, with this set being potentially smaller than that found by the Goh *et al.* (1989) program.

As illustrated in figure 3, the SERF approach provides a graphical explanation of how different (groups of) decision makers might rank risky alternatives. As is clear from the figure, subsets of the SERF efficient set can be formed for specific risk aversion levels. Therefore, the SERF efficient set contains only alternative B for decision makers with absolute risk aversion levels less than 0.0052 and it contains only alternative A for decision makers with risk aversion levels greater than 0.0052.[7] From the CE graph in figure 3 we can

[7] Adjusting the lower and upper risk aversion coefficients to −1 and +1, respectively, shows the rankings are consistent over an exceptionally wide range of decision makers' preferences.

also see the rankings of the alternatives for the whole range of risk aversion analysed. In this way SERF is more informative than some SDRF software that may simply report absence of dominance (indifference) between some alternatives.

Mjelde and Cochran (1988) proposed using risk premiums to determine the confidence of decision makers in a particular preferred risky alternative. Subtracting the *CE* for a less preferred alternative (C) from the CE for the dominant alternative (A), yields a utility-weighted risk premium (*RP*) of

$$RP_{A,C,r_i} = CE_{A,r_i(w)} - CE_{C,r_i(w)} \tag{8}$$

at a given risk aversion level ($r_i(w)$). In figure 3 the *RP* between the dominant alternative A and a less preferred alternative C is measured by the vertical distance between the *CE* lines for A and C. In the present case, the *RP* ranges from, for example, 1.83 at $r_i(w) = 0.0075$ to 12.19 at $r_i(w) = 0.035$ or 1.25–8.69 per cent of the respective *CE* values for the preferred alternative A. The RP reflects the minimum sure amount that would have to be paid to a decision maker to justify a switch from alternative A to C. Conversely, the RP between A and C shows the benefits to a decision maker if allowed to move from alternative C to A. For policy analyses the RP values reflect the average risk weighted premiums (losses or gains) to decision makers if forced to operate under a policy not in the efficient set.

The SERF approach was tested with a constant relative risk aversion (CRRA) power function on the same hypothetical example data. This form of function is arguably more applicable in this case where we have defined payoffs in terms of total wealth since, unlike a CARA function, a CRRA function exhibits the generally expected property of diminishing absolute risk aversion. The efficient set was identical to that described and the implied value of $r_r(w)$ where CE curves for alternatives A and B crossover was almost identical ($r_r(w) = 0.64$) to that found using the negative exponential function ($r_r(w) \approx 0.77$).

4.2 Example 2

A second hypothetical example represents net returns from six risky arable crop rotation alternatives, F to K (table 2). The means of these alternatives vary from about 296 for alternative F to about 446 for alternative I. The overall range of outcomes is from 45 to 850. Alternatives H and I have the widest ranges. Alternative K has the largest minimum outcome of 220. Figure 4 shows the CDF graphs for the alternatives.

Use of the Goh *et al*. (1989) software on these alternatives shows both the SSD set and the SDRF set with $r_a(w)$ within the bounds 0 and 0.01 is {I, J, K}. The upper bound on the absolute risk aversion coefficient was

Table 2 Hypothetical example with net returns from six rotation alternatives specified for the same set of fractile values

	Rotation alternative					
$F(w)$	F	G	H	I	J	K
0.0	45	80	140	130	165	220
0.1	158	186	212	252	275	305
0.2	205	230	253	311	325	350
0.3	241	264	296	356	361	378
0.4	272	294	332	394	392	400
0.5	299	326	372	432	421	427
0.6	324	361	416	474	455	449
0.7	353	401	465	522	490	470
0.8	387	449	524	578	525	493
0.9	432	511	612	657	581	522
1.0	540	660	850	800	665	548

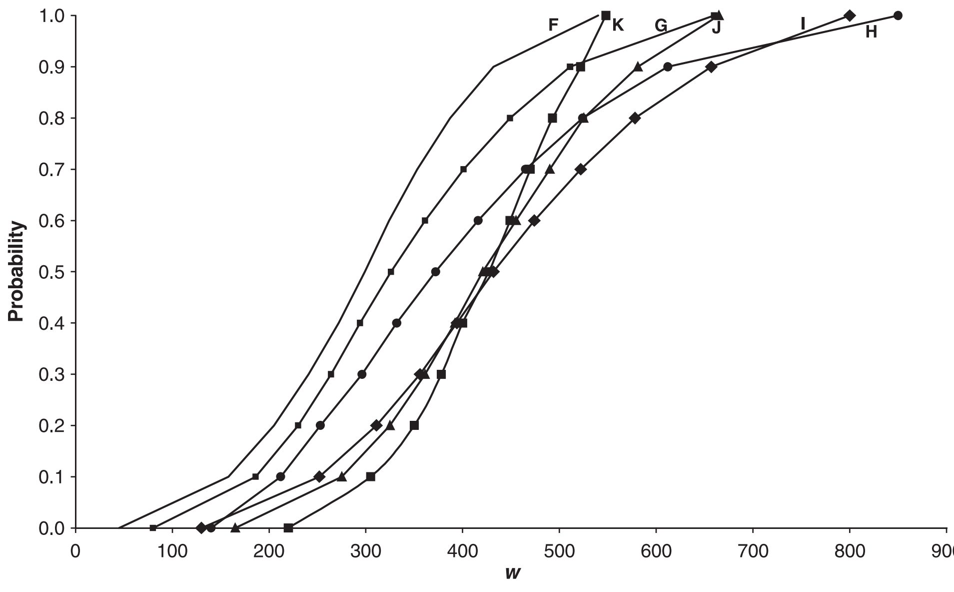

Figure 4 Cumulative probability distributions for rotation alternatives F to K.

based on two basic assumptions. We assume a realistic upper level of relative risk aversion, $r_r(w)$ to be 4. The average wealth level (w) of the six alternatives is around 400, so we get $r_a(w) = 4/400 = 0.01$.

Figure 5 shows the results with the SERF approach using a negative exponential utility function and the same range for $r_a(w)$. As the figure shows, the efficient set is {I, K} over the risk aversion range of 0 to 0.01.

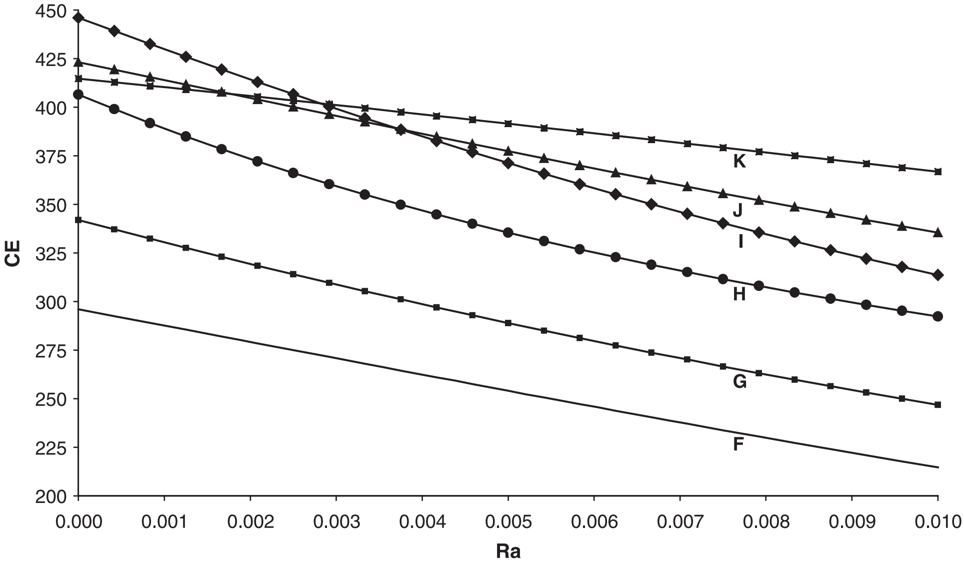

Figure 5 SERF results for the constructed rotation example when using a negative exponential utility function and absolute risk aversion range 0.0–0.01.

The BRAC where CE curves for rotations I and K cross is $r_a(w) = 0.0028$, which is exactly the same as we found with the Riskroot software. As in the previous example, the efficient set is smaller with the SERF method than with the pairwise SDRF method but is identical to that found using Riskroot.

5. Discussion and concluding comments

The advantages of simultaneously comparing many risk alternatives with SERF over SDRF are:

1. SERF can identify a smaller number of alternatives in the efficient set than pairwise SDRF over a given range of risk aversion.
2. SERF provides an ordinal ranking of alternatives at each risk aversion level between the lower and upper risk aversion bounds customarily tested by SDRF.
3. SERF is a one step process that is similar to, but potentially more discriminating than, running an SDRF analysis at all risk aversion levels within the stated bounds of $r_L(w)$ and $r_U(w)$. The graphical presentation of SERF results facilitates the presentation of ordinal rankings for decision makers with different risk preferences.
4. SERF provides a cardinal measure of the decision maker's conviction for preferences among alternatives at each risk aversion level by interpreting the

differences between CE values as risk premiums. The graphical display of ordinal rankings and cardinal preferences by SERF make Mjelde and Cochran's (1988) confidence premiums concept more useful for policy analysis.
5. Unlike the basic SDRF approach, SERF can be used to process data presented in different formats, not only in terms of the same fractile values for all the distributions to be compared.
6. SERF matches Meyer's (1977) original intention of SDRF. He proposed that, 'instead of using restrictions on $U(w)$ to specify groups of agents, we will use restrictions on $r(w)$, which corresponds to restrictions on preferences and they can be used more easily to define groups of agents'. SERF accomplishes this by numerically evaluating CE values for alternatives over many values of $r(w)$ and then graphically displaying ordinal and cardinal rankings for many different groups of agents across a spectrum of risk aversion levels which can be as wide or as narrow as the situation warrants, that is, risk preferring through risk neutral to strongly risk averse, or (usually more realistically) only moderately risk averse within a narrow range.

An advantage of the SERF method over calculating BRAC points with Riskroot is that, in addition to identifying where dominance between two alternatives switches, SERF allows for estimation of the utility-weighted risk premiums between alternatives to provide a cardinal measure for comparing the payoffs between risky alternatives. The graphical display of BRAC points using SERF is a useful feature that extends McCarl's (1988) Riskroot and makes the concept easier to apply for both decision analysis and policy analysis.

In addition to the difference in discriminating power, whether pairwise SDRF and SERF applied using the same form of utility function will give comparable results will also depend on differences in data handling. In particular, the way in which the discrete approximation of continuous functions for a stochastic dominance analysis are handled can affect results.

The SERF method illustrated in the present paper includes all the advantages of SDRF yet is more transparent, is easier to implement, and has a stronger discriminating power. These seem to be powerful advantages that suggest that SERF could extend the power of risk efficiency analysis to practical applications in business and policy decision-making.

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Appendix

Table A1 Spreadsheet example of calculating a certainty equivalent for one risky alternative with one risk aversion coefficient

	A	B	C	D	E	F	G	H	I	J	K
1	Certainty Equivalent Calculated for an Exponential Utility Function						Certainty Equivalent Calculated for a Power Utility Function				
2											
3	Input						Inputs				
4	Risk Aversion Coefficient (RAC)			0.005			Risk Aversion Coefficient (RAC)			0.005	
5							Beginning Wealth (Optional)			0	
6	Certainty Equivalent			147.920596			Certainty Equivalent			148.367312	
7	Formula for the CE calculated in row 6						Formula for the CE calculated in row 6				
8	=IF(ISERROR(C20),C18,IF(ABS(C20)<D11,C17,IF(D4=0,C16,LN(C20/C19)/(-1*D4))))						=IF(ISERROR(I20),I18,IF(ABS(I20)<J11,I17,IF(J4=0,I16,(I20/I19)^(1/J13)-J12)))				
9											
10	Calculations						Calculations				
11	Small Positive Double Precision Number			1.00E-300			Small Positive Double Precision Number			1.00E-300	
12							Beginning Wealth (Wo)			0	
13							Adjusted RAC (RAC')			0.50000	
14							Formula for Adjusted RAC (RAC') row 13		=IF(J4=0,0,1-J4*10^(LEN(ABS(INT(I18)))-1))		
15	Series Statistics			Formulas for series statistics			Series Statistics			Formulas for series statistics	
16	Mean		149.3636	=AVERAGE(CE!A26:A1025)			Mean		149.3636	=AVERAGE(CE!G26:G1025)	
17	Min		100	=MIN(CE!A26:A1025)			Min		100	=MIN(CE!G26:G1025)	
18	Max		195	=MAX(CE!A26:A1025)			Max		195	=MAX(CE!G26:G1025)	
19	Count		11	=COUNT(CE!A26:A1025)			Count		11	=COUNT(CE!G26:G1025)	
20	Sum(exp(-RAC*DATAi))		5.2503	=SUM(C26:C1025)			Sum((DATAi+Wo)^RAC')		133.9867	=SUM(I26:I1025)	
21											
22	Data (up to 1000 observations in rows 26-1025)						Data (up to 1000 observations in rows 26-1025)				
23		Formulas for calculating the values for the first observation in row 26						Formulas for calculating the values for the first observation in row 26			
24		=IF(A26="","",-A26*D4)	=IF(B26="","",EXP(B26))					=IF(G26="","",G26+J12)	=IF(H26="","",H26^J13)		
25	DATA	-RAC*DATAi	exp(-RAC*DATAi)	Observation			DATA	DATAi+Wo	(DATAi+Wo)^RAC'	Observation	
26	100	-0.5000	0.6065	1			100	100	10.0000	1	
27	125	-0.6250	0.5353	2			125	125	11.1803	2	
28	135	-0.6750	0.5092	3			135	135	11.6190	3	
29	142	-0.7100	0.4916	4			142	142	11.9164	4	
30	147	-0.7350	0.4795	5			147	147	12.1244	5	
31	150	-0.7500	0.4724	6			150	150	12.2474	6	
32	153	-0.7650	0.4653	7			153	153	12.3693	7	
33	158	-0.7900	0.4538	8			158	158	12.5698	8	
34	163	-0.8150	0.4426	9			163	163	12.7671	9	
35	175	-0.8750	0.4169	10			175	175	13.2288	10	
36	195	-0.9750	0.3772	11			195	195	13.9642	11	

The calculations for an estimate of the certainty equivalent (CE) illustrated are for an assumed negative exponential utility function and a data set assumed to be a sample of equi-probable observations. The step by step calculations start with specifying a value for the risk aversion coefficient (RAC) level, as in cell D4 of the spreadsheet, and an array of sample data of length *n*, as in range A26:A36, the CE estimate (in cell D6) is calculated by first multiplying each observation by the RAC and '–1', as outlined in range B26:B36. Each of the resulting values is then used as an exponent in the natural exponential function, as in range C26:C36. These values are then added together, as shown in C20. The resulting value (in cell C20) is then divided by *n* (11), and the result is raised to the –1/(RAC) power and the natural logarithm of this value is the estimated CE (in cell D6). The formula for these last calculations is shown in cell A8. If this value is less than the minimum observation or exceeds the maximum observation, then the minimum or maximum value will be the estimated CE, respectively. If the RAC value is zero, the estimated CE will be the mean.

Table A2 Certainty equivalents for alternatives A–D under a negative exponential utility function

	RAC	A	B	C	D
1	0.00000	148.36	**153.18**	149.91	123.55
2	0.00146	148.05	**151.55**	148.92	123.29
3	0.00292	147.73	**149.88**	147.94	123.04
4	0.00438	147.41	**148.17**	146.96	122.79
5	0.00583	**147.08**	146.43	145.98	122.54
6	0.00729	**146.76**	144.64	145.02	122.29
7	0.00875	**146.43**	142.82	144.05	122.04
8	0.01021	**146.10**	140.96	143.10	121.79
9	0.01167	**145.77**	139.08	142.15	121.55
10	0.01313	**145.44**	137.17	141.20	121.31
11	0.01458	**145.10**	135.25	140.27	121.06
12	0.01604	**144.77**	133.30	139.34	120.82
13	0.01750	**144.43**	131.35	138.41	120.58
14	0.01896	**144.09**	129.40	137.50	120.35
15	0.02042	**143.75**	127.45	136.59	120.11
16	0.02188	**143.40**	125.51	135.69	119.87
17	0.02333	**143.06**	123.59	134.81	119.64
18	0.02479	**142.72**	121.69	133.93	119.41
19	0.02625	**142.37**	119.81	133.06	119.18
20	0.02771	**142.02**	117.97	132.20	118.95
21	0.02917	**141.68**	116.17	131.35	118.72
22	0.03063	**141.33**	114.40	130.52	118.49
23	0.03208	**140.98**	112.68	129.70	118.26
24	0.03354	**140.63**	111.01	128.89	118.04
25	0.03500	**140.28**	109.38	128.09	117.81

Maximum for each risk aversion coefficient (RAC) level is highlighted in bold.