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Empirical properties of duality theory*

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This research examines selected empirical properties of duality relationships. Monte Carlo experiments indicate that Hessian matrices estimated from the normalised unrestricted profit, restricted profit and production functions yield conflicting results in the presence of measurement error and low relative price variability. In particular, small amounts of measurement error in quantity variables can translate into large errors in uncompensated estimates calculated via restricted and unrestricted profit and production functions. These results emphasise the need for high quality data when estimating empirical models in order to accurately determine dual relationships implied by economic theory.

1. Introduction

Based on the duality theory, it is possible to make appropriate transformations and link parameters of a production function, unrestricted profit function, restricted profit function and/or cost function. Theoretically, these results suggest a researcher may choose an estimation approach based on data availability, ease of estimation or other empirical considerations. For example, Lopez (1984, p. 358) chose to estimate a profit function instead of a cost function because 'it is simpler to estimate, and no endogenous variables need to be used as explanatory variables'. Although a researcher may choose a particular dual approach, such as estimating a profit function, all information about the production or cost functions is presumed to be available via duality. Lau (1976) used Hessian identities to prove that estimates from a restricted profit function or production function can be recovered from an unrestricted profit function and vice versa under the assumption of perfect competition. Thus, uncompensated economic effects may be obtained in three ways: (i) the unrestricted profit function can be estimated and uncompensated economic

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effects calculated directly; (ii) the restricted profit (cost) function may be estimated and converted to uncompensated economic effects using Lau's results; or (iii) the production function may be estimated, and using Lau's matrix identities, uncompensated effects can be calculated.

Although the dual relationship between these functions exists in theory, these properties often fail to hold in empirical applications. Appelbaum (1978, p. 87) concluded that 'finally, we find that the primal and dual do not yield similar implications, a result … which is very disturbing'. Burgess (1975) similarly found that estimates derived from a production function were not consistent with those derived from a cost function. He states 'Since these parameters are crucial in assessing the impact of … policy … we are left with the inescapable conclusion that mild changes in our maintained hypothesis [choosing the primal or dual approach] may lead to dramatic changes in our inferences about economic events' (p. 120).

Some of the differences in primal and dual estimates found by Appelbaum (1978) and Burgess (1975) are likely to rest with factors such as risk and stochastic error. Taylor (1984) showed that dual results might not be achievable if price expectations follow a Markovian structure. Others have also suggested that duality results do not hold for stochastic models (Pope 1980; Pope 1982; Weaver 1983). However, Chambers and Quiggin (1998) illustrated that, in some cases, cost functions can be theoretically derived from stochastic production functions, and Coyle (1999) and Pope and Just (1998) suggested alternative methods of estimating cost and profit functions under risk aversion. Thompson and Langworthy (1989) illustrated that elasticities calculated from primal and dual approaches depend on the choice of functional form. They showed that identical results will never be obtained from primal and dual approaches unless the flexible functional form is self-dual (such as the quadratic) and the underlying data-generating process identically matches the chosen functional form.

However, even in the best-case scenario where there is no risk and the 'correct' functional form is chosen, dual results may still be unachievable. Elasticity estimates, often used for policy analysis, are obtained using a particular dataset. How 'good' does a particular dataset have to be to achieve reliable and consistent results from alternative specifications of the technology? This is exactly the question this research seeks to answer. Although previous research has illustrated the impact of risk, stochastic error and functional form on dual results, little is known about the impact of basic data composition (i.e. measurement error, price variability or sample size) on estimated results from alternative dual specifications. Naturally, econometric parameter estimates are affected by the underlying data and, thus, we might expect that factors such as measurement error and sample size will also influence how well estimates from two dual specifications, such as a cost and profit function, yield the same implication. However, the extent to which parameter estimates from a production function, for example, can be used to make inferences about unconditional price elasticities, typically derived from an unconditional profit function, given a particular dataset, is unknown. If, empirically, one can determine the conditions under which alternative dual specifications do not produce identical estimates of economic effects, such as an uncompensated elasticity, valuable information may be gained regarding the suitability of a particular dataset for certain empirical analyses. If a dataset is not able to meet a particular level of 'quality', calculations of compensated elasticities from uncompensated estimates, as was done by Lopez (1984) and others, may be unreasonable.

In this analysis, we systematically alter data composition, in a Monte Carlo environment, to determine the conditions that a dataset must meet for estimates from a production function, restricted profit function and unrestricted profit function to yield comparable results. Specifically, we examine the impact of measurement error, relative price variability and sample size on dual results. We find that only under ideal conditions, where the estimated functional form exactly matches the data-generating process and the data has no measurement error, estimates from the production, restricted profit and unrestricted profit functions yield the same implications. In general, we find that increases in measurement error in quantities, decreases in relative price variability and, in certain cases, the compounding impact of large sample sizes, leads to a divergence between estimates generated by alternative dual approaches. Increasing price variability improves the ability of unrestricted profit and restricted profit functions to estimate the production technology because a greater degree of information is incorporated into the estimates than when low price variation exists. Increasing sample size, coupled with some degree of measurement error, reduces the ability of production, restricted profit and unrestricted profit functions to produce identical results because least squares estimates are not consistent in the presence of measurement error.

2. Measurement error and price variability

Price and/or quantity data are often not precisely measured due to a myriad of factors, such as aggregation. Although the amount of error in measurement is not typically known, the error may produce poor empirical estimates if not accounted for (Brester and Wohlgenant 1993; Lewbel 1996). In general, measurement error produces inconsistent estimates (Greene 2000). However, it is not known how much measurement error must be present in the data to interfere with the ability of duality to recover the underlying production technology. The amount of measurement error in quantities and prices varies across each dataset, but Morgenstern (1963) found that national income data were measured with a standard error in excess of 10 per cent, and the US Department of Commerce (1988) noted that input and output data for the Food and Kindred Products category were measured with standard errors averaging 8 per cent. Lim and Shumway (1992a, 1992b) illustrated that violations of the maintained hypothesis of profit maximisation can often be explained by data measurement error. Specifically, they determined the minimum level of measurement error required for consistency with maintained hypotheses such as profit maximisation. This research seeks to examine the impact of such measurement error on estimated dual results.

Sample size and price variability are also expected to have a role in recovering the underlying production technology utilising duality. Sample size can accentuate the effect of, say, measurement error on the precision of empirical estimates. As the sample size increases, the asymptotic properties of the least squares estimator are such that parameter estimates should approach their true values and the standard error of the estimator should decline (Greene 2000). Thus, increasing the sample size has the potential to improve estimates if estimation procedures are consistent and efficient and, with the improved estimates, the ability to use Lau's Hessian identities to obtain unrestricted elasticities increases. However, if the estimator is not consistent, as may be the case with measurement error, then larger sample sizes will produce unreliable estimates.

Relative price variability is also important in recovering dual relationships. Quiggin and Bui-Lan (1984) illustrated that insufficient variation in input prices used to estimate cost functions can result in erroneous conclusions when testing the hypotheses of economic efficiency and profit maximisation. Increased price variability facilitates the ability of dual functions to 'map-out' the true underlying technology. As suggested by Quiggin and Bui-Lan (1984), when price variation is low, such as the case with cross-sectional data, estimating cost or profit functions may be inappropriate. In the case of low price variability, the natural solution is to estimate a production function. This research aims to determine how 'low' price variability must be for dual estimates to lead to conflicting results.

3. Methods and procedures

To control extraneous effects that may be involved in empirical estimation, data were generated using Monte Carlo simulation techniques. The firm's primary profit maximisation problem, given a predetermined production technology, was used as a basis for this study. Restricted profit, unrestricted profit and production functions were then estimated from data generated from Monte Carlo simulations. After manipulating Hessian matrices according to Lau's results, estimates from the dual functions were compared to the 'true' Hessian of the underlying technology. Comparisons were made with various levels of price variability, measurement error and sample size.

The quadratic functional form was used to generate the data and to estimate all three functions (unrestricted and restricted profit and production functions), as the goal of this research was to examine the ability of duality to recover underlying technology, not measure how well various functional forms recover a production technology. The sensitivity of estimated coefficients to choice of functional form has been examined elsewhere (e.g. Berndt and Khaled 1979; Chalfant 1984; Shumway and Lim 1993). The effect of choice of functional form on the ability to recover dual results has also been studied (e.g. Thompson and Langworthy 1989). The normalised quadratic form was chosen as both: (i) the true underlying technological data-generating process; and (ii) the flexible functional form used for estimating the profit, restricted profit and production functions. The normalised quadratic was chosen because it allows the Hessian values to be functions of parameter estimates only, and not depend on a particular data point. Lau (1976) also indicated that the quadratic functional form was a logical choice when examining his Hessian identity results empirically. It is noted that alternative functional forms may perform differently in Monte Carlo experiments, however, discussion here is limited to the normalised quadratic. In the following experiment, we present the 'best-case' scenario where the underlying data-generating process exactly matches the estimated functional form.

4. Lau's dual relationships

Lau (1976) illustrated the equivalence of estimates from the unrestricted profit, restricted profit and production functions using Hessian identities. To illustrate these results, we defined a production process consisting of one output and four inputs. The variables Y, R and U represent the production, restricted profit and unrestricted profit functions, respectively. Variable input quantities are defined as x_1, x_2, x_3 and are collectively referred to as x_{1-3}. For simplicity, the input prices were normalised with respect to the output price, P. The last Hessian terms, or output supply effects, were recovered using symmetry and homogeneity properties. Variable input prices, normalised by the output price, are represented by w_{1-3}; x_4 represents the fixed input and w_4 represents the normalised fixed input price.

The production function Hessian matrix is:

$$\begin{bmatrix} \frac{\partial^2 Y}{\partial x_{1-3}^2} & \frac{\partial^2 Y}{\partial x_{1-3}\partial x_4} \\ \frac{\partial^2 Y}{\partial x_4 \partial x_{1-3}} & \frac{\partial^2 Y}{\partial x_4^2} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{bmatrix} \quad (1)$$

where α_{ij} are the second-order derivatives of the production function, Y. Manipulating Lau's results, the unrestricted Hessian from the unrestricted profit function, U, can be obtained by inverting the Hessian from the production function.

$$\begin{bmatrix} \frac{\partial^2 U}{\partial w_{1-3}^2} & \frac{\partial^2 U}{\partial w_{1-3}\partial w_4} \\ \frac{\partial^2 U}{\partial w_4 \partial w_{1-3}} & \frac{\partial^2 U}{\partial w_4^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 Y}{\partial x_{1-3}^2} & \frac{\partial^2 Y}{\partial x_{1-3}\partial x_4} \\ \frac{\partial^2 Y}{\partial x_4 \partial x_{1-3}} & \frac{\partial^2 Y}{\partial x_4^2} \end{bmatrix} \quad (2)$$

If the second-order derivatives of the unrestricted profit function are denoted by β_{ij}, the following identity is implied:

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{12} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{13} & \beta_{23} & \beta_{33} & \beta_{34} \\ \beta_{14} & \beta_{24} & \beta_{34} & \beta_{44} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{bmatrix}^{-1} \quad (3)$$

With the restricted profit function, some inputs are fixed. However, estimates obtained from the restricted profit function, R, can be used to recover unrestricted estimates. Let γ_{ij} represent the second-order derivatives of the restricted profit function. Following Lau, the following identities are defined:

$$\Theta_1 = \left[\frac{\partial^2 R}{\partial x_4^2}\right] = [\gamma_{44}] \quad (4)$$

$$\Theta_2 = \left[\frac{\partial^2 R}{\partial x_4 \partial w_{1-3}}\right] = [\gamma_{14}\ \gamma_{24}\ \gamma_{34}] \quad (5)$$

$$\Theta_3 = \left[\frac{\partial^2 R}{\partial w_{1-3}^2}\right] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{12} & \gamma_{22} & \gamma_{23} \\ \gamma_{13} & \gamma_{23} & \gamma_{33} \end{bmatrix} \quad (6)$$

The restricted profit Hessian matrix can now be directly compared with the unrestricted profit Hessian and the production function Hessian:

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{12} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{13} & \beta_{23} & \beta_{33} & \beta_{34} \\ \beta_{14} & \beta_{24} & \beta_{34} & \beta_{44} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44} \end{bmatrix}^{-1}$$
$$= \begin{bmatrix} (\Theta_3 - \Theta_2' \cdot \Theta_1 \cdot \Theta_2) & -(\Theta_2 \cdot \Theta_1^{-1}) \\ -(\Theta_2 \cdot \Theta_1^{-1}) & -\Theta_1^{-1} \end{bmatrix} \quad (7)$$

Using Lau's results, equation 7 shows that there is a direct relationship between the production function, the unrestricted profit function and the restricted profit function. Thus, estimates from any one of the three forms can be used to determine estimates from the other two, using the Hessian identities.

5. Monte Carlo experiment

The Monte Carlo experiment consisted of five sequential steps as shown in figure 1. First, a particular scenario was defined with a particular level of measurement error, price variability and sample size. Second, data were generated using a predefined technology with the given levels of measurement error, price variability and sample size. Third, restricted profit, unrestricted profit and production functions were estimated. Fourth, Hessians from all three functions were converted to an equivalent level as shown in equation 7. Fifth, the three Hessians were compared for similarities. Steps two to five were then repeated 100 times for a particular scenario of measurement error, price variability and sample size. The results of the Monte Carlo experiment are summarised in step six.

Step 1 Scenario definition

To examine the effects of price variability; measurement error in quantities; measurement error in prices, and sample size on duality, several scenarios were considered. In each scenario, a given level of price variability, measurement error and sample size was chosen. In the analysis, four levels

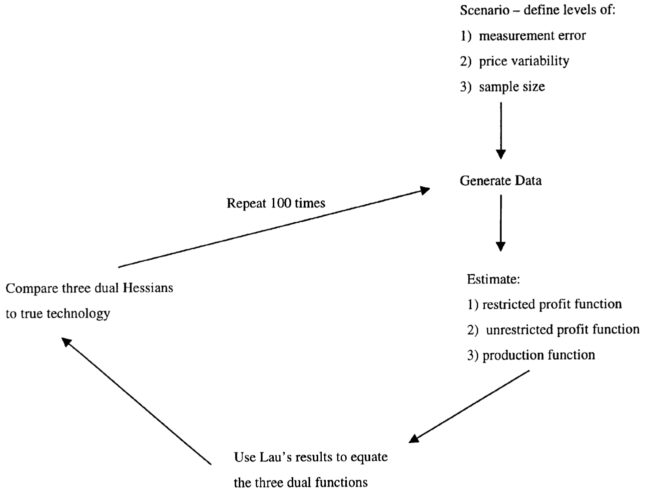

Figure 1 Monte Carlo experiment.

of measurement error in quantities (0.1, 0.5, 1 and 2 per cent); prices (0.1, 0.5, 1 and 2 per cent); and price variability (10, 20, 30 and 40 per cent) were selected. The levels of measurement error in prices, quantities and price variability were calculated as coefficients of variation to provide a consistent level of variation across all variables. The exact calculation of the measures is discussed in the following step. Four levels of sample size were also chosen (50, 100, 250 and 500 observations).

The total number of possible Monte Carlo experiments that would have to be conducted if every combination of every level of measurement error, price variability and sample size were generated is $4^4 = 256$. Each of these scenarios was run and the results are available from the authors upon request. However, to simplify the reporting of the results, an orthogonal fractional factorial design was used such that all main effects could be illustrated (Addelman 1962). The resulting design consists of 16 scenarios. Table 1 shows the different scenarios reported in the experiment.

Step 2 Data generation

To generate data for the analysis as a single output, four input quadratic production function was assumed:

$$Y = \sum_{i=1}^{4} \alpha_i x_i + 0.5 \sum_{i=1}^{4} \sum_{j=1}^{4} \alpha_{ij} x_i x_j \tag{8}$$

Table 1 Scenarios reported from the Monte Carlo experiment

Scenario	Measurement error in quantities*	Measurement error in prices*	Price variability*	Sample size
1	0.10	0.10	10.00	50
2	0.10	0.50	40.00	500
3	0.10	1.00	20.00	100
4	0.10	2.00	30.00	250
5	0.50	0.10	40.00	250
6	0.50	0.50	10.00	100
7	0.50	1.00	30.00	500
8	0.50	2.00	20.00	50
9	1.00	0.10	20.00	500
10	1.00	0.50	30.00	50
11	1.00	1.00	10.00	250
12	1.00	2.00	40.00	100
13	2.00	0.10	30.00	100
14	2.00	0.50	20.00	250
15	2.00	1.00	40.00	50
16	2.00	2.00	10.00	500

*Calculated as coefficient of variation (standard deviation/mean).

where Y is the output quantity, x_i represents the ith input quantity and the α's are parameter values chosen so that economic regularity conditions were met. These parameters were also chosen so that the elasticity matrix contained complement and substitute factors of production as well as elastic and inelastic inputs. These assumed 'true' technology parameters used for the analysis are shown in table 2.[1] To ensure symmetry, $\alpha_{ij} = \alpha_{ji}$ was imposed. The intercept was assumed to be zero, so no output is produced without any inputs. To maintain quasi-concavity, the linear coefficient terms are positive and the diagonal quadratic terms are negative.

Given the production function, the primal profit maximisation problem is:

$$Max\ \pi = P \cdot Y - \sum_{i=1}^{4} w_i x_i \tag{9}$$

where Y is the production function, P is the output price and w_i represents the ith input price. Input and output prices, w_i and P, were exogenously determined by generating observations randomly from independent normal distributions with predefined means and standard deviations. The means and standard deviations were chosen based on a particular coefficient of variation (sample standard deviation divided by the sample mean). We use the coefficient of variation to examine the robustness of duality results to relative price variation.

The first-order conditions of the profit maximisation problem 9 are:

$$\frac{\partial \pi}{\partial x_i} = P(\alpha_i + \alpha_{i1}x_1 + 0.5\alpha_{i2}x_2 + 0.5\alpha_{i3}x_3 + 0.5\alpha_{i4}x_4) - w_i = 0 \quad i = 1,2,3,4 \tag{10}$$

Given the input and output prices, the system of four equations (one for each input first-order condition) was solved simultaneously to obtain input quantities, x_i*. These optimal input quantities were then substituted into the production function, given by equation 8, to obtain optimal output, Y*.

To allow for the introduction of measurement error into the data, the optimal quantities obtained by solving equation 10 and the generated prices were perturbed from their optimal values by random errors with given levels of variation. The optimal values were treated as means and predetermined coefficients of variation were used to generate normally distributed random errors in the variables. Measurement error was defined as the random displacement of the quantities and the prices from their optimal profit-maximising levels, as shown in equations 11 through to 13:

[1] We have used several different specifications of the true technology and the results presented in this paper are consistent across the alternatives we tested.

Table 2 Assumed coefficient values for the production function

Coefficient	Value
α_1	20
α_2	10
α_3	30
α_4	70
α_{11}	−0.9
α_{22}	−0.7
α_{33}	−0.8
α_{44}	−0.3
α_{12}	0.1
α_{13}	−0.37
α_{14}	0.15
α_{23}	0.2
α_{24}	0.1
α_{34}	0.13

$$x_{it} = x_{it}^* + \varepsilon_{1it} \tag{11}$$

$$w_{it} = w_{it}^* + \varepsilon_{2it} \tag{12}$$

$$P_t = P_t^* + \varepsilon_{3t} \tag{13}$$

where $x_{it}*$, $w_{it}*$ and P_t* are the input quantities, input prices and output price, at their optimal profit-maximising levels, $\varepsilon_{1it} \sim N(0, \sigma_{1i}/\mu_{1i})$, $\varepsilon_{2it} \sim N(0, \sigma_{2i}/\mu_{2i})$, $\varepsilon_{3t} \sim N(0, \sigma_3/\mu_3)$ and x_i, w_i and P are input quantities, input prices and output price with measurement error, respectively.[2] The index i represents the ith input price or quantity and the index t represents the observation number.

Step 3 Estimation of dual functions

Once prices and quantities were determined, the restricted profit, unrestricted profit and production functions were estimated using the normalised quadratic functional form. To estimate the restricted profit function, the 4th input, x_4, was treated as the fixed input in the production process. The restricted profit function was estimated in a system with its factor demands:

[2] We have also conducted the analysis with the optimal output quantity, $Y*$, disturbed from its optimal level by a given coefficient of variation. The same conclusions are reached in either case. When measurement error is introduced into the output quantity, the magnitude of the mean squared errors reported in the following sections drastically increases because $Y*$, and thus the resulting coefficient of variation in measurement errors, is large. Therefore, for ease of exposition, we do not perturb optimal output quantity from its optimal level. Results for the alternative specification are available from the authors.

$$R = \sum_{i=1}^{3} \gamma_i w'_i + \gamma_4 x_4 + 0.5 \sum_{i=1}^{3} \sum_{j=1}^{3} \gamma_{ij} w'_i w'_j + \sum_{i=1}^{3} \gamma_{i4} w'_i x_4 + 0.5 \gamma_{44} x_4^2 \qquad (14)$$

$$x_k = \gamma_k + \sum_{i=1}^{3} \gamma_{ik} w'_i + \gamma_{4k} x_4 \qquad k = 1, \cdots, 3$$

where R is restricted profit, γ are coefficients to be estimated, w_i is the ith input price normalised on the output price, P (to impose homogeneity), x_4 is the fixed input quantity and $\gamma_{ij} = \gamma_{ji}$ to impose symmetry.

In the unrestricted profit function, all factors are variable. Thus, the 4^{th} input is a variable input in the estimation of the unrestricted profit function. The following unrestricted profit function and its factor demands were estimated:

$$U = \sum_{i=1}^{4} \beta_i w'_i + 0.5 \sum_{i=1}^{4} \sum_{j=1}^{4} \beta_{ij} w'_i w'_j \qquad (15)$$

$$x_k = \beta_k + \sum_{i=1}^{4} \beta_{ik} w'_i \qquad k = 1, \cdots, 4$$

where U is unrestricted profit, β are coefficients to be estimated and $\beta_{ij} = \beta_{ji}$ so that symmetry is imposed.

The estimated production function has the same form as equation 8, because we chose the 'best-case' scenario where the same functional form was used to generate the data and estimate the dual functions. The dual functions were estimated as a system using ordinary least squares with cross-equation restrictions. No error structure was generated between equations and the measurement errors in quantity variables were independently distributed, thus the ordinary least squares estimator is the appropriate choice.[3]

Step 4 Conversion of dual functions to equivalent basis using hessian identities

The Hessian of the production function, equation 8, can be used to determine the Hessian of the unrestricted profit function, equation 15. As a basis for

[3] There are econometric methods that can be used to obtain consistent estimates in the presence of measurement error. However, in practice it is difficult to ascertain which variables have measurement error and, thus, correcting for the problem is complex. Furthermore, even if measurement error is suspected, by performing non-parametric tests such as the one suggested by Varian (1985), there are often no appropriate instrumental variables available to alleviate the problems associated with measurement error (Greene 2000).

comparison, the assumed true production function coefficients, the estimated production function coefficients and the estimated restricted profit function coefficients were converted to the long-run unrestricted profit Hessian because economists are typically interested in examining the elasticities that are generated from the unrestricted Hessian. This conversion is found in equation 7.

Step 5 Comparison of Hessians

Hessian estimates derived from the estimated production, restricted and unrestricted profit functions were each compared to the Hessian derived from the true production function. To quantify how similar the estimates were, a mean squared error (MSE) was calculated. Each error was calculated as the difference between the true value and the estimated value of each unique element of the Hessian matrix (10 unique values in this case). The mean of the squared errors was then calculated for each Hessian at each repetition of the experiment. The MSE for each of the three functions was:

$$\mathrm{MSE}_i = \left(\sum_{j=1}^{10}(t_j - e_j)^2\right)/10 \tag{16}$$

where t_j are the 10 unique true Hessian values and e_j are the unique Hessian estimates from the restricted profit, unrestricted profit function and production functions (i = restricted profit/unrestricted profit/production function). Since 100 repetitions are performed in the simulation, a distribution of MSE is generated, with the median value of the MSE distribution being used as the basis of comparison.

Step 6 Summary of results

To summarise the results of all 256 Monte Carlo experiments, an ordinary least squares translog regression was estimated where the median MSE from each scenario was assumed dependent upon the level of measurement error, price variability and sample size, as shown in equation 17:

$$\log(\text{median MSE}_i) = \mathrm{f}(\log(\mathrm{MEQ}_i), \log(\mathrm{MEP}_i), \log(\mathrm{PV}_i), \log(\mathrm{SS}_i)) \tag{17}$$

where MEQ is the level of measurement error in the quantities, MEP is the level of measurement error in the prices, PV is the level of price variability, SS is the sample size, and the subscripts $i = 1$, 2 and 3 denote the restricted profit, unrestricted profit and production functions. The translog function can provide a second-order Taylor series approximation to any unknown function and allows the estimation of interaction effects between variables.

6. Results

In the Monte Carlo analysis, measurement error, price variability and sample size impact dual results through their influence on coefficient estimates. In the analysis it is important to make the distinction between the impact of the measurement error, price variability and sample size on: (i) coefficient estimates; and (ii) duality. The influence of these data composition factors on estimated coefficients is well known. However, the extent to which these factors hinder the ability to use duality reliably to achieve equivalent results is unknown. In particular, we illustrate, through the Monte Carlo experiments, that very small errors in estimated coefficients translate into very large errors when attempting to use duality to recover uncompensated effects from restricted profit and production functions.

Table 3 shows the median MSE from the Monte Carlo analysis for each of the 16 experimental scenarios shown in table 1. The last column of table 3 reports the median MSE for the unrestricted profit function. As shown by the duality equivalence in equation 7, the reported MSE values for the unrestricted profit function reflect the direct impact of measurement error,

Table 3 Comparison of true uncompensated effects to uncompensated effects derived from estimated restricted profit, production and unrestricted profit functions*

	Estimating uncompensated effects		
Scenario	Restricted profit MSE†	Production function MSE†	Unrestricted profit MSE‡
1	0.252	2.002	0.041
2	48.995	5021.100	2.729
3	0.012	1.328	0.005
4	34.650	2721.400	2.742
5	428.550	3094.700	2.811
6	157.860	34.625	0.440
7	1957.000	5181.900	2.845
8	0.958	15.876	0.249
9	8518.400	4844.900	2.975
10	2.868	23.152	0.495
11	3469.100	2765.600	3.292
12	0.699	15.861	0.308
13	189.450	209.850	1.005
14	13200.000	2727.900	3.503
15	3.000	11.738	1.170
16	24402.000	7629.300	3.978

*Reported values are the median mean standard error (MSE) from 100 repetitions in a Monte Carlo experiment.
† Estimates from restricted profit and production functions were converted to uncompensated estimates via Lau's duality results and then MSE values were calculated by comparison with true uncompensated effects.
‡ Estimates from the unrestricted profit function were directly compared with true uncompensated effects to calculate MSE values.

price variability and sample size on estimated coefficients alone. That is, unrestricted profit functions coefficient estimates are compared to the true values of the inverted Hessian of the production function. Because the estimated coefficients from the unrestricted profit function are directly compared with the true parameter values, the MSE values reported for the unrestricted profit function do not give any indication of the impact of data composition on duality. Thus, the MSE estimates for the unrestricted profit function can be used as the baseline case from which to separate the impact of measurement error, price variability and sample size on coefficient estimates from the impact of these factors on dual results.[4] As table 3 indicates, for most of the 16 scenarios, the unrestricted profit function recovered the true underlying technology relatively well. In fact, the average unrestricted profit function median MSE for all 16 scenarios was 1.787. As it appears that the unrestricted profit function recovered the true underlying technology quite well, one might assume that duality could be used to accurately determine the compensated effects from the restricted profit function or the production function. This assumption would appear to be wrong.

The middle columns of table 3 report the MSE values for the restricted profit and production functions. To calculate these MSE values, the restricted profit and production functions were estimated first. Estimates from these regressions were then converted to uncompensated estimates via duality, as shown in equation 7. Finally, MSE values were calculated by comparing these estimated uncompensated effects, derived from restricted profit and production function estimates, to the true uncompensated effects. First, it is worthwhile to note that we empirically verified Lau's Hessian identities (i.e. $\text{MSE} = 0$) when no measurement error is present in the data. However, when a very small amount of measurement error was introduced into the data, specifically measurement error in the quantity variables, dual results began to deteriorate rapidly. For example, a comparison of MSE values for scenarios one through to four, where the measurement error in quantities was 0.10 per cent, to the MSE values in scenarios 13 through to 16, where the measurement error in quantities was 2 per cent, indicates that increases in quantity measurement error decreased the abilities of the production and restricted profit functions to recover uncompensated effects. The MSE values reported in table 3 indicate that the restricted profit and

[4] The MSE for the restricted profit function and the production function can be interpreted as the ability of one function (i.e. the restricted profit or production function) to achieve dual results implied by an alternative specification of the technology (i.e. the unrestricted profit function). In contrast, the MSE for the unrestricted profit function can be interpreted as the impact of measurement error, price variability and sample size on estimated coefficients alone. Obviously, at the levels of measurement error, price variability and sample size used in this Monte Carlo experiment, estimated coefficients are hardly influenced, whereas dual results are greatly influenced.

production functions recovered the true uncompensated effects very poorly for most scenarios.[5]

For all 16 scenarios reported in table 3, the median MSE values for the restricted profit and production functions were greater than the median MSE values for the unrestricted profit function. Even in cases in which the median MSE values for the unrestricted profit function were relatively small, MSE values for the two dual functions were quite large (e.g. scenarios two, four, five, six, eight, 11, 13, 14 and 16). Thus, small deviations between true and estimated coefficient estimates in the restricted profit and production functions (due to measurement error) translated into very large differences in true and estimated uncompensated effects. These results imply that, in the presence of a measurement error even as low as 0.01 per cent (e.g. scenario two), one could arrive at conclusions quite contrary to those implied by the actual technology if the restricted profit or production function were used to make inferences regarding uncompensated effects.

To better summarise the effects of measurement error, price variability and sample size on the reported MSE values, equation 17 was estimated for each dual function. Changes in measurement error, price variability and sample size explained over 97 per cent of the variation in median MSE values for the unrestricted profit, the restricted profit and production functions. First, we consider the effects of data composition on the production and restricted profit functions. Data in table 4 indicate that results between the production and restricted profit functions are similar except for the effect of sample size. For example, except for sample size, all variables that are statistically significant in the production function regression are also statistically significant in the restricted profit function regression. In addition, except for sample size, the signs of all statistically significant variables are identical across both equations. This indicates that the effects of relative price variability, sample size and measurement error in quantities generally work in the same direction in the production function and the restricted profit function equations. Measurement error in prices had no statistically significant effect in either equation.

In contrast, the regression explaining the mean squared error for the unrestricted profit function had many more statistically significant variables than either the restricted profit or the production function regressions (table 4). Furthermore, results indicated that many of the significant variables had opposite signs than in either the production function or restricted profit function estimations. To explore the practical implications of these regressions further, figures 2, 3 and 4 were constructed.

[5] The analysis could also be conducted with the restricted profit or production functions as the basis of comparison. Results are similar for all cases. The function used for the basis of comparison recovers the true technology quite well, whereas the two alternative specifications diverge from the true values after the dual Hessian conversions.

Table 4 Effects of measurement error, price variability and sample size on MSE[†]

Variable	Restricted profit function[‡]	Production function[‡]	Unrestricted profit function[§]
Constant	16.658***[¶]	7.721***	13.011
	(3.990)[††]	(1.765)	(1.408)
Log measurement error in quantities (MEQ)[‡‡]	5.683***	2.916***	2.039***
	(0.662)	(0.293)	(0.233)
Log measurement error in prices (MEP)[‡‡]	−0.084	−0.024	0.111
	(0.662)	(0.293)	(0.233)
Log price variability (PV)[‡‡]	−7.206***	−4.604***	0.088
	(1.146)	(0.507)	(0.404)
Log sample size (SS)	−0.158	0.896**	−1.464***
	(0.986)	(0.436)	(0.348)
MEQ squared	0.469***	0.297***	0.042**
	(0.051)	(0.023)	(0.018)
MEQ, MEP interaction	−0.047	−0.000	−0.081***
	(0.040)	(0.018)	(0.014)
MEQ, PV interaction	−0.946***	−0.378***	−0.143***
	(0.085)	(0.038)	(0.030)
MEQ, SS interaction	0.257***	0.328***	−0.064***
	(0.051)	(0.022)	(0.018)
MEP squared	0.006	−0.002	0.051***
	(0.051)	(0.023)	(0.018)
MEP, PV interaction	0.048	−0.001	0.111***
	(0.085)	(0.038)	(0.030)
MEP, SS interaction	0.005	0.001	0.054***
	(0.051)	(0.022)	(0.018)
PV squared	−0.481**	−1.440***	1.335***
	(0.240)	(0.106)	(0.085)
PV, SS interaction	−0.701***	−0.7444***	0.589***
	(0.108)	(0.048)	(0.038)
SS squared	0.036	0.012	0.162***
	(0.089)	(0.039)	(0.031)
R^2	0.97	0.98	0.99

[†] Dependent variable for each regression is the log median MSE from 100 repetitions of a Monte Carlo experiment.
Sample size for each regression = 256.
[‡] Estimates from restricted profit and production functions were converted to uncompensated estimates via Lau's duality results and then MSE values were calculated by comparison with true uncompensated effects.
[§] Estimates from the unrestricted profit function were directly compared with true uncompensated effects to calculate MSE values.
[¶] One, two, and three asterisks indicates significance at the 0.10, 0.05, and 0.01 levels, respectively.
[††]Numbers in parentheses are standard errors.
[‡‡]Measured as coefficient of variation (standard deviation/mean).

Figure 2 illustrates the effect of an increase in sample size on the production, restricted profit and unrestricted profit functions with measurement error in prices and quantities each set at 1 per cent. The dotted lines represent MSE when relative price variability is 20 per cent and the solid lines

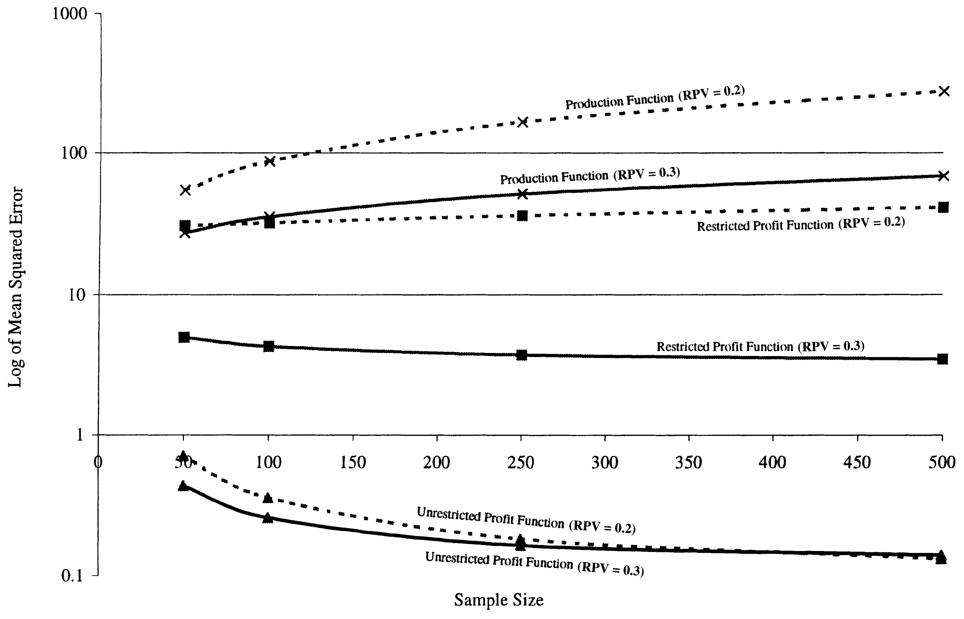

Figure 2 The effect of Relative Price Variation (RPV) and sample size on mean squared error.

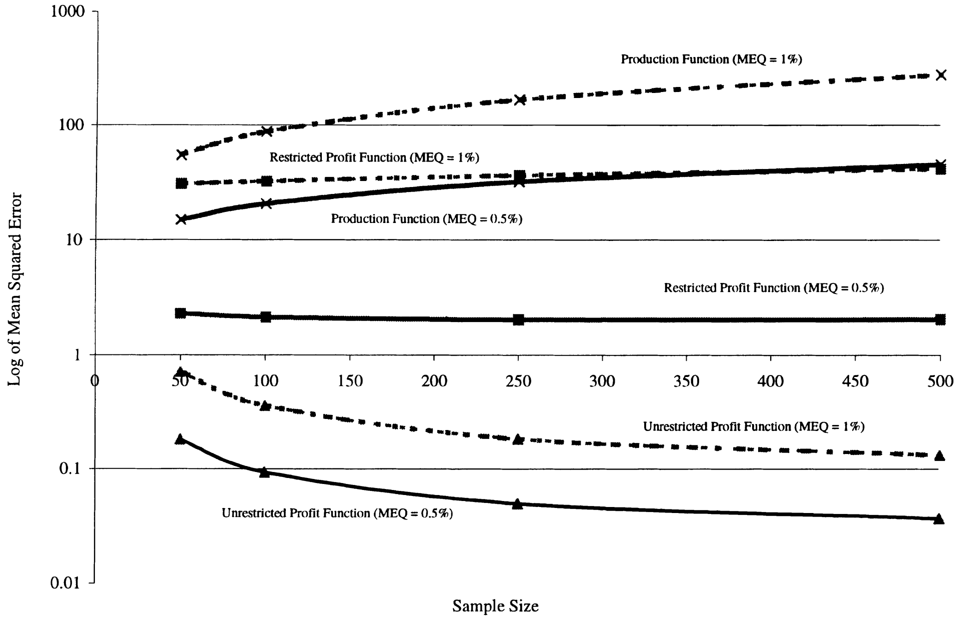

Figure 3 The effects of measurement error in quantities (MEQ) and sample size on mean squared error.

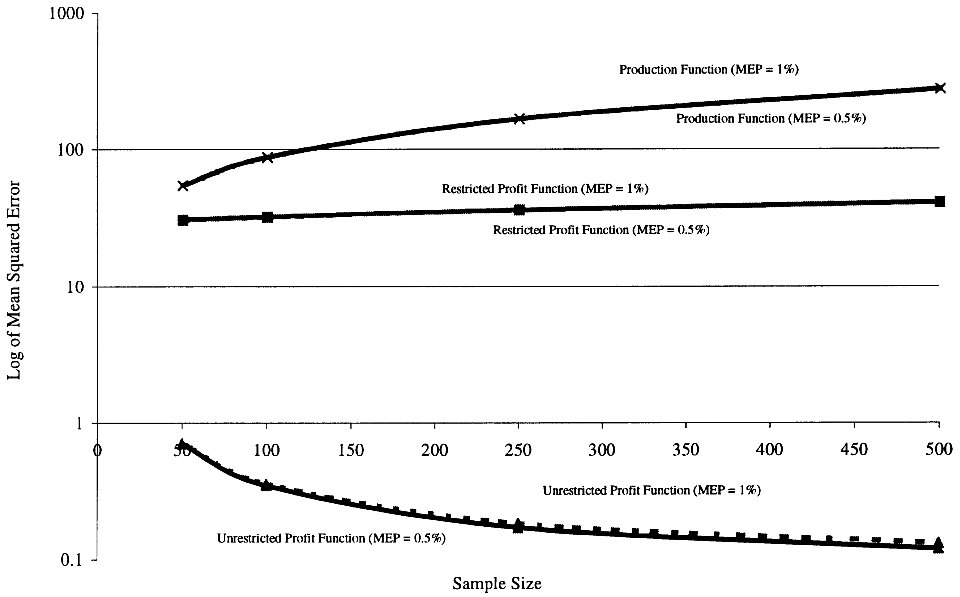

Figure 4 The effect of measurement error in prices (MEP) and sample size on mean squared error.

represent the MSE when relative price variability is 30 per cent. Results indicate that sample size has differing impacts on the MSE calculated for each function. Increases in sample size were associated with a decrease in MSE for the unrestricted profit function at both levels of relative price variability and the restricted profit function with relative price variability of 30 per cent. However, increased sample size was associated with increased MSE for the production function and the restricted profit function with relative price variability of 20 per cent. The interaction of increased measurement error, decreased relative price variability and sample size typically results in an expectation that increased sample size leads to smaller error. It is a well-known econometric property that when parameter estimates are inconsistent estimated coefficients do not approach true values and, hence, the mean squared error does not converge to zero. A quantitative estimate of the measurement error, sample size interaction is reported in table 4.

Figure 2 also illustrates that increased relative price variability significantly improves the ability of the restricted profit function and the production function to achieve uncompensated results. Increases in price variability, at least for this particular functional form, allows the restricted profit and production function to better trace out the true underlying technology when duality is utilised.

The effect of measurement error in quantities on MSE, when relative price variability is 20 per cent and measurement error in prices is 1 per cent, is illustrated in figure 3. Decreases in quantity measurement error from 1 to 0.5 per cent substantially reduce the MSE for all three functions. In contrast, measurement error in prices has little impact on the median MSE for the production and the unrestricted and restricted profit functions (figure 4). Perhaps the impact of measurement error is dampened by normalisation of the prices. Alternatively, measurement error in prices may not have influenced unrestricted and restricted profit function results because of the particular price levels and coefficients of variation chosen for this analysis.

Further analysis of the estimated models can aid in interpreting the influence of measurement error, price variability and sample size on estimated dual relationships. The estimated coefficients can be used to predict MSE values, given various data composition factors. The coefficients imply that the restricted profit function estimates uncompensated effects more accurately than the production function at all levels of measurement error, price variability and sample size used in the analysis.

Because the unrestricted profit function estimates are directly comparable with the true underlying technology, we were able to estimate the influence of measurement error, price variability and sample size on estimated coefficients alone (not on duality). In the case of 1 per cent measurement error in prices and quantities, 30 per cent relative price variability and a sample size of 100 observations, the median MSE for the unrestricted profit function was 0.26. If there was a 1:1 relationship between the influence of data composition on: (i) coefficient estimates; and (ii) the ability to achieve identical dual results, we would expect the predicted MSE for the restricted profit function and production function to be close to 0.26. However, the estimated MSE for the restricted profit and production functions at the same levels of measurement error, price variability and sample size was 2.32 and 40.66. Thus, our results indicated, for the particular technology used in this analysis, that measurement error, price variability and sample size had a small influence on parameter estimates. However, when these small errors in parameter estimates were compounded via matrix manipulations to achieve dual results, very large deviations occurred between actual and true values. In this simple exercise, in which the estimated functional form exactly matched the underlying data-generating process, duality relationships performed very poorly. In fact, uncompensated estimates calculated from the restricted profit and production functions led to economically inconsistent implications relative to the true uncompensated estimates.

7. Implications

To further examine the deviation of empirical duality properties from duality theory, we calculated three other statistics in addition to the MSE measure: (i) percentage of curvature violations (PCV); (ii) percentage of statistically significant variables (PSV); and (iii) percentage of economic violations (PEV). PCV was calculated by checking curvature (i.e. calculating the eigenvalues) of the uncompensated Hessian for the restricted profit, unrestricted profit and production functions from each iteration of the Monte Carlo experiment. The mean number of curvature violations was calculated by adding up the totals from each iteration. Similarly, PSV was calculated by summing the number of statistically significant coefficients and dividing the total by 1000 (100 Monte Carlo repetitions × 10 unique Hessian terms). To calculate PEV, the number of economic violations was totalled from each Monte Carlo iteration. An economic violation occurred if the estimated coefficient implied the input was a substitute when the true coefficient was a complement and vice versa. An economic violation also occurred if the estimated coefficient implied that

Table 5 Percentage of curvature violations (PCV), significant variables (PSV) and economic violations (PEV) in 100 iterations of a Monte Carlo experiment

	Restricted profit function*			Production function*			Unrestricted profit function†		
Scenario	PCV	PSV	PEV	PCV	PSV	PEV	PCV	PSV	PEV
1	1.0	48.6	9.0	99.00	11.60	43.50	4.0	59.5	3.5
2	98.0	40.8	56.2	98.00	99.80	49.40	100.0	39.0	60.4
3	0.0	89.8	0.7	97.00	18.70	33.40	0.0	90.0	0.0
4	100.0	13.7	52.4	95.00	98.20	52.00	99.0	0.6	59.3
5	99.0	27.7	56.6	95.00	98.30	51.10	99.0	1.4	60.3
6	53.0	23.6	28.8	95.00	16.40	49.10	54.0	9.9	20.4
7	100.0	18.2	54.4	100.00	99.80	47.40	98.0	11.2	59.4
8	19.0	19.9	18.1	100.00	16.80	46.20	13.0	14.7	16.5
9	100.0	7.8	53.6	100.00	99.80	48.40	100.0	1.6	62.0
10	50.0	15.0	32.4	99.00	21.90	46.80	39.0	13.5	22.4
11	100.0	9.9	53.2	95.00	92.80	52.70	98.0	1.2	58.4
12	28.0	37.5	24.9	99.00	30.40	45.50	36.0	38.9	18.1
13	72.0	18.4	42.9	97.00	47.40	49.10	83.0	9.5	27.3
14	100.0	9.3	53.6	95.00	98.20	52.70	98.0	2.4	57.8
15	78.0	18.9	44.3	99.00	31.20	47.40	73.0	9.8	32.1
16	100.0	7.6	52.4	100.00	99.80	48.40	100.0	0.8	56.8

*Estimates from restricted profit and production functions were converted to uncompensated estimates via Lau's duality results and then MSE values were calculated by comparison with true uncompensated effects.
†Estimates from the unrestricted profit function were directly compared with true uncompensated effects to calculate MSE values.

the input was own-price elastic (inelastic) when the true coefficient was own-price inelastic (elastic).

For the 16 scenarios in table 1, table 5 reports the PCV, PSV and PEV for the three dual functions. As with the MSE, the PCV, PSV and PEV for the unrestricted profit function were calculated without any matrix inversions, whereas estimates from the restricted profit and production functions were converted to uncompensated effects (see equation 7) before PCV, PSV and PEV were calculated. Interestingly, the restricted profit function performed almost as well as the unrestricted profit function for the PCV and PEV measures (see all 16 scenarios in table 5). Using a paired *t*-test, the hypothesis that the mean PCV for the unrestricted and restricted profit functions across all 256 scenarios were equal could not be rejected. Although the PCV was statistically equivalent for the restricted and unrestricted profit functions, a *t*-test implied that the unrestricted profit function had a higher mean PSV and a lower PEV than the restricted profit function.

The uncompensated Hessian derived from the production function violated curvature more frequently than the other two dual functions. At measurement errors as low as 1 per cent (e.g. scenario one), curvature was violated 99 per cent of the time for the production function. Although the production function frequently violated curvature, coefficient estimates were statistically significant more often than was the case with the restricted and unrestricted profit functions. This was especially true for larger sample sizes (scenarios two, four, five, seven, nine, 11, 14 and 16). In addition, the production function produced a significantly higher number of economic violations than the restricted and unrestricted profit functions.

Regressions similar to those reported in table 4 were estimated with the dependent variables PEV, PCV and PSV rather than the logged MSE. These results indicate that as errors in measuring input quantities increase, the PEV and PCV increase for unrestricted Hessian estimates calculated from the restricted profit, unrestricted profit and production functions. Concurrently, the PSV decreases for the restricted and unrestricted profit functions. Measurement errors in prices have little influence on PCV, PSV and PEV for unrestricted Hessian estimates calculated from all three functions. Increases in sample size increase PSV and decrease the PEV and PCV for each model. Finally, price variation tends to slightly decrease the PCV and PEV for each model. Increases in price variation have a greater influence on PSV for the unrestricted Hessian estimates calculated from the restricted and unrestricted profit functions.[6]

[6] These regression results are available from the authors.

8. Conclusions

Duality allows researchers to recover production technology parameters using several different approaches. At a theoretical level, several studies have illustrated the equivalence between many dual economic relationships. Economists have used these theoretical equivalence results to justify solving applied problems using the most empirically convenient function. However, the present study indicates that dual relationships between profit and production functions, which theoretically hold, may perform poorly in applied research when data are subject to the frailties of measurement error, limited price variability and/or sample size.

This study employed Monte Carlo techniques to empirically examine the dual relationship between the restricted profit, unrestricted profit and production functions. Lau's (1976) Hessian identities were used to compare estimates obtained using these three functions. Data were generated using an assumed true production technology. Estimates from the restricted profit and production functions were converted to the uncompensated Hessian using the dual results provided by Lau (1976) and were compared with the true values of the uncompensated Hessian. Under ideal conditions (no measurement error), the converted values were identical to the true values, thus confirming the theoretical dual results. However, these dual identities failed to hold when conditions that regularly occur in applied work were introduced. Increases in measurement error in quantities, decreases in relative price variability and, in certain cases, increases in sample size, adversely affected the ability of the production and restricted profit functions to recover the true uncompensated effects. Although it is well known that measurement error and sample size affect estimated coefficients, we showed that very small errors in coefficient estimates translated into very large errors when attempting to calculate uncompensated effects from restricted profit or production function estimates.

Some research has concluded 'that the most efficient way of estimating the product supply or factor demand is to derive them from the empirical production function, rather than the reverse' (Mundlak 1996, p. 437). While such a task is mathematically possible, our results indicate that such calculations are likely to produce undesirable results when dealing with fragile empirical data. Similarly, our results imply that if attempting to estimate uncompensated effects, it would be most desirable to estimate the unrestricted profit function rather than to use duality to recover the uncompensated results from the production or restricted profit functions. Uncompensated results obtained through matrix inversions are likely to be inaccurate unless input quantities are measured accurately. Furthermore, this analysis indicates that the restricted profit function recovers uncompensated effects more accurately than the production function when quantity measurement error is present. In addition, uncompensated effects can be recovered more successfully from the restricted profit function if there is a reasonable degree of price variability.

The analysis indicates that small amounts of measurement error produce results inconsistent with duality theory. Consequently, using highly aggregated or secondary data to estimate dual relationships may be problematic. Analysts should also be cautious when using datasets that contain variables that are not easily measured or are measured by a proxy variable. Finally, measurement error problems may arise when aggregating production variables without regard for potentially heterogeneous quality. In summary, we found that the dual relationships between the restricted profit, unrestricted profit and production functions rarely held under conditions typically found in real-world datasets, indicating that the choice of which approach to use in applied research may not be as clear as suggested by theoretical mathematical relationships.

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