The Australian Journal of Agricultural and Resource Economics, 45:4, pp. 623–640

Earmarking of pollution charges and the sub-optimality of the Pigouvian tax[†]

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One approach to internalising a negative externality of economic activity is to impose a Pigouvian tax equal to the marginal cost of the externality. However, this approach overlooks the possibility that the tax revenue can be earmarked to correct the externality directly, i.e. financing the environmental protection projects. It is found that a pure Pigouvian tax is usually not an optimal policy. This issue is examined in both partial and general equilibrium, static and dynamic settings. Certain conditions for justifying a pure Pigouvian tax or a fully earmarked tax scheme are developed.

1. Introduction

One of the textbook approaches to internalise a negative externality of economic activities is to impose the so-called Pigouvian tax on such activities. The standard Pigouvian solution calls for 'a tax (subsidy) per unit on the externality-generating activity equal to its marginal external damage (benefit)' (Baumol and Oates 1988, p. 55). However, this standard version of the Pigouvian tax overlooks the question of how the tax revenue is used. Some economists propose the use of such revenues in correcting pre-existing distortions. This idea is called the 'double-dividend hypothesis', and suggests that increased taxes on polluting activities can improve environmental quality and simultaneously enhance economic efficiency through reducing distorting taxes (Pearce 1991).[1] However, no-one has examined earmarking of the tax revenue for environmental projects.

[†] The author wishes to thank Warwick McKibbin, Ben Smith, Yiping Huang, Kanhaiya Singh, Alexandra Sidorenko, two anonymous referees and the Editor for helpful comments.

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[1] Fullerton and Metcalf (1997) argue that if the tax only replaces current command and control regulations, there is no first dividend. Goulder *et al.* (1996) point out that the second dividend is not generally guaranteed, because the revenue-recycling effect may well be

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This is surprising because it is a common approach in practice. Examples are the Superfund in the United States, the pollution levy system in China, and pollution fee schedules in some European countries. It is even more surprising because in his classic work *The Economics of Welfare*, Pigou (1920) presented at least two different revenue schemes. On the one hand, he assumed externality taxes were fiscal taxes equivalent to the damage imposed; on the other hand, he indicated that they were to be earmarked for a special purpose. In his later work (Pigou 1928), he clarifies that the taxation of negative externalities raises revenue to be spent on the provision of positive externalities. Strangely, many environmental economists neglect this aspect of Pigou's argument, although they often assign his name to a pollution charge. Perhaps Mikael Skou Andersen (1994) is the only exception.

According to the above discussion, at least three taxation programs are to hand. First, under a *pure pollution tax scheme*, suggested by the conventional Pigouvian tax, the revenue from pollution taxes is used as a transfer to the public, with no spending on the environment. Second, and in contrast, a *(fully) earmarked pollution levy scheme* uses all revenue for environmental purposes. Finally, a *general tax-income scheme* does not impose any prior restrictions on spending.

This article attempts to examine these arrangements in both partial and general equilibrium settings, and to find the optimal pollution tax scheme. It is found that the usual Pigouvian tax might not be optimal if the possibility of earmarking is considered. It also provides a theoretical ground for the common approach in practice of employing fee schedules that generate revenues sufficient to cover the cost of public pollution-abatement programs. The earmarking of pollution charges may help to achieve sustainable development in a dynamic setting.

The article is organised as follows: the next two sections analyse these schemes in static settings; then the discussion moves into a dynamic general equilibrium analysis; and the final section summarises the findings.

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eroded by the tax-interaction effect. However, their numerical result does show that, if the proceeds of an environmental tax are used to reduce pre-existing taxes, the gain from the revenue-recycling effect is larger than the loss from the tax-interaction effect, implying a net gain in addition to environmental improvement. Similar results can be found in studies by Goulder *et al.* (1999), Parry *et al.* (1996) and Parry and Williams III (1999). However, Bovenberg and Goulder (1996) report that the substitution of environmentally-motivated taxes for traditional income taxes involves a gross cost: the double dividend does not materialise. However, both proponents and critics of this hypothesis neglect the possibility of earmarking.

2. Static partial equilibrium analysis

Suppose a firm's cost function is $C = C(q)$, where q is the quantity of its product, with the property that $C' \geq 0$ and $C'' \geq 0$. The damage caused by pollution from the production process is $D = D(q)$ with the property that $D(0) = 0$, $D' \geq 0$ and $D'' \geq 0$. A Pigouvian tax, in the form of product tax, is set to be $t^* = D'(q^*)$. It is derived on the assumption that the tax revenue is a transfer from the firm to the public, and does not change the net social benefit. However, if the revenue can be used for pollution control activities, the optimal tax rate may be different.

For simplicity, it is assumed that the pollution-eliminating function is $E = E(S)$, where S is the spending on the pollution control project, and E is the reduction in damage by the project in monetary terms,[2] and that $E' \geq 0$, $E'' \leq 0$, $E(0) = 0$, $E'(0) > 0$. It is clear that a pure pollution tax scheme is not optimal if $E'(0) > 1$. So the social planner has to choose an appropriate level of environmental investment in addition to the tax rate. Suppose the levy rate is τ, product price is p, the firm's response function to a certain levy rate is $q = q(\tau)$. The social planner's problem is:

$$\begin{aligned} \max_{\tau S} \quad & pq(\tau) - C(q(\tau)) - D(q(\tau)) + E(S) - S \\ \text{s.t.} \quad & S \leq \tau q(\tau) \end{aligned}$$

The Kuhn-Tucker conditions for this problem are:

$$pq' - C'q' - D'q' + \lambda(q + \tau q') = 0; \tag{1}$$

$$E' - 1 - \lambda = 0; \tag{2}$$

$$\lambda(\tau q - S) = 0,\ \lambda \geq 0 \text{ if } S = \tau q; \tag{3}$$

where λ is the multiplier to constraint. If the constraint is binding, a fully earmarked levy scheme is optimal, $S = \tau q(\tau)$ and $\lambda \geq 0$ according to equation (3),[3] which in turn implies $E' \geq 1$ from equation (2). Using $p - C'(q) = \tau$ and equation (1), the optimal tax rate can be written as:

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[2] Usually the pollution-eliminating activity is affected by the pollution level. As the model does not specify the pollution level, we may use D as a proxy; that is, the E function could be written as $E = E(S, D)$. If so, the optimal tax rate becomes $\tau^* = (1 - E_D)D'/(E_S + (E_S - 1)/\varepsilon)$, where E_D and E_S are respectively the partial derivative of E with respect to D and S. It is clear that the qualitative result from the simplified model does not change although the marginal cost of pollution is adjusted to include the effect on reducing pollution.

[3] Accurately speaking, $S = \tau q(\tau)$ and $\lambda = 0$ are not conditions of binding constraint because the interior solution happens to be at the corner. However, as all revenues are used for the environment, we include the discussion in the fully earmarked scheme.

$$\tau^* = \frac{dD}{dq} - \frac{d(E-S)}{dS}\frac{dS}{dq}.$$

The above expression reflects two effects of the changes in production: impacts on the pollution damages and on tax revenues. The first effect is captured by marginal damage cost, and the second effect is relevant because the environmental spending is financed by the pollution tax which is affected by quantity of product. Note that E' is the marginal direct benefit of environmental investment, and $E' - 1$ is the marginal net benefit; while $d(\tau q)/dq = dS/dq$ is the marginal tax revenue of production. Therefore, $(E' - 1)d(\tau q)/dq$ or $(E' - 1)dS/dq$ is the marginal net benefit of environmental spending caused by changes in production.

To clearly see the difference between this and a Pigouvian tax, the optimal tax rate can be rewritten as:

$$\tau^* = \frac{D'}{E' + (E' - 1)/\varepsilon},$$

where $\varepsilon = q'(\tau)\tau/q(\tau)$ is the output elasticity of pollution levy rate. It can be seen that the conventional Pigouvian tax rate is not likely to be justified. If $E' > 1$, the optimal tax/levy rate is not equal to the marginal damage cost, unless $\varepsilon = -1$. If $\varepsilon > -1$, i.e. the output is inelastic to the tax/levy rate, the social planner can get more revenue by raising the tax rate, therefore the optimal tax rate is above the marginal damage cost. By contrast, if the output is elastic, the optimal tax rate will be below the marginal damage cost. These discussions can be summarised as:[4]

> **Result 1.** If $E'(0) > 1$, a pure Pigouvian pollution tax is not optimal. In addition, if $E'(S) \geq 1$, when $S = \tau q$, a fully earmarked pollution levy scheme is optimal. In this case, if $E'(S) > 1$, the optimal tax/levy rate is larger than, equal to or less than the marginal damage cost if the output is inelastic, unit-elastic or elastic to tax/levy.

When the budget constraint is sluggish, $\lambda = 0$ according to equation (3) and, using $p - C'(q) = \tau$, conditions (1) and (2) become: $\tau^* = D'(q)$ and $E'(S) = 1$. Because the budget constraint is not binding now, the decision on environmental investment has no effect on the production side. Therefore the optimal tax rate has the usual form. In this case, only a part of the tax/levy revenue is used for environmental purposes. Therefore neither a pure tax scheme nor a fully earmarked levy scheme is optimal. However, if they have the same rate, and the reduced damage or improved benefit is larger than the spending, an earmarked levy system is better.

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[4] Proofs of results are fairly easy, using just the definition and first-order conditions; therefore, they are omitted to save space and are available from the author on request.

> **Result 2.** If $E(S) > S$ holds and the tax can be treated as a pure transfer, an earmarked pollution levy is always better than a pure pollution tax with the same rate. Moreover, the optima of a fully earmarked pollution levy scheme are at least as good as the optima of a pure pollution tax scheme.

The result seems trivial; however, it is important to policy-makers. In the real world, it is often difficult to find enough information to decide the optimal tax rate and optimal amount of environmental spending. The result here suggests that they could be determined by analysing individual projects. This was a popular practice in European countries during the 1960s and 1970s: 'the authorities have employed schedules of fees that generate revenues sufficient to cover the costs of public pollution-abatement programs', and this is criticised as 'a most unsatisfactory method' (Baumol *et al.* 1979, p. 375). However, according to the discussion here, it has firm theoretical grounds.

In some countries, e.g. China, only part of the pollution charges goes to the pollution control project; the remainder goes to local environmental protection authorities to subsidise environmental protection personnel and equipment. This practice is a hybrid of pollution tax and levy. The part that is not directly used in pollution abatement can be seen as a replacement of the general tax. It can be easily shown that this hybrid system is still better than a pure pollution tax system.

3. Static general equilibrium analysis

3.1 The model

There are three agents in the economy. A representative firm employs labour (L), and environmental capacity whose use may produce harmful pollutants that have negative effects on the household's utility. The environmental unit is carefully chosen such that the environmental factor used in the firm can be represented by the pollutant emissions (D). Thus the production function can be written as $Y = f(L, D)$,[5,6] which has the usual properties. Suppose a pollution tax or levy is imposed at the rate t, the wage rate is w, and the

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[5] As the model is static at this stage, capital is not considered here for simplicity and will be included later in the dynamic model. However, it can be shown that including capital and other input factors will not affect the analytical result.

[6] The usual approach to model a firm's pollution and abatement activity assumes that pollution is a useless byproduct of the production process, and under certain environmental policies, the firm is forced to abate the pollution to some extent at its cost. The current approach also captures these properties, although the firm's abatement activity is not explicitly modelled.

output price normalised, the first-order conditions for profit maximisation are:[7]

$$f_L(L, D) = w, \quad f_D(L, D) = t. \tag{4}$$

A representative household tries to maximise its utility subject to the budget constraint. The utility comes from the consumption (C), leisure ($\tilde{L}$), and amenity of the environment (E). Leisure is defined as the difference between total time endowment ($\overline{L}$) and supply of labour (L). Therefore the utility function can be written as:

$$U(C, \overline{L} - L, E). \tag{5}$$

Again, it is assumed that the utility function has the usual properties.

The household owns labour and the firm, therefore its income comes from the labour income and profit of the firm ($\pi(w, t)$). It also receives transfers from the government (G). Thus the household's utility maximisation problem is to choose C and L to maximise equation (5) subject to:

$$C \leq \pi(w, t) + wL + G. \tag{6}$$

And the first-order conditions are:

$$U_C = \lambda, \quad U_{\tilde{L}} = \lambda w, \tag{7}$$

where λ is the Lagrangian multiplier of constraint (6).

The government tries to maximise the social welfare which could be represented by the household's utility function (5) through various government instruments. One of them is the exertion of state ownership over environment; that is, it decides the supply of environment. The government also decides how to allocate the environmental tax revenue.

Similarly to leisure, the environmental quality is defined as the difference between environmental endowment ($\overline{E}$) and supply of environmental absorption capacity of pollution (D). Investment in environment can increase the environmental amount as well. Therefore:

$$E = \overline{E} - D + e(S), \tag{8}$$

where S is the spending on environmental projects, and $e(S)$ the pollution abatement or environment improvement function. It is assumed that $e(0) = 0$, $e' > 0$, $e'' < 0$.

In line with the above-mentioned pollution tax schemes, the government's budget constraint has three forms: (1) under the general tax-income scheme,

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[7] Subscript denotes the partial derivative of the function relative to that variable, e.g. $f_L = \partial f / \partial L$.

the budget constraint is $S \geq 0$ and $S + G \leq tD$.[8] (2) As the pure tax scheme suggests, all revenue is transferred to household. This is equivalent to impose one more constraint $S = 0$. (3) If all revenue from supplying environmental capacity is earmarked for environment, another constraint $G = 0$ is added to the general case.

With more constraints imposed, according to optimisation theory, neither a pure tax scheme nor a fully earmarked levy scheme can achieve better results than a general scheme because the feasible set for choice variables becomes smaller.

Result 3. The optima of a pure pollution scheme and a fully earmarked levy scheme are no better than the optimum of the general pollution tax/income scheme.

3.2 The optimal pollution tax/bounty scheme

Outlay of pollution tax/levy

In order to compare these three schemes, the government's problem can be set as to maximise equation (5) subject to equations (6), (8), $S + G \leq tD$, $S \geq 0$ and $G \geq 0$. Government transfer (G) is set as a choice variable because the government anticipates that G affects household's income, and thus affects its utility. The household's budget constraint (6) is added in line with this fact. The Lagrangian is:

$$\begin{aligned} \pounds = {} & U(C, \overline{L} - L, \overline{E} - D + e(S)) + \lambda_1(tD - S - G) \\ & + \lambda_2(\pi + wL + G - C) + \lambda_3 S + \lambda_4 G \end{aligned}$$

The corresponding first-order conditions are:

$$\begin{gathered} U_E = \lambda_1 t, \quad U_E e'(S) = \lambda_1 - \lambda_3, \\ \lambda_1 = \lambda_2 + \lambda_4, \quad \lambda_3 S = 0, \quad \lambda_4 G = 0 \end{gathered} \tag{9}$$

Note that both λ in equation (7) and λ_2 in equation (9) are the marginal utility of increasing the budget available to the household; they should therefore be equal. A general tax-income scheme sets no limitation on S; that is the constraint $G \geq 0$ could be dropped, which is equivalent to $\lambda_4 = 0$. If an interior solution exists under the general scheme, $\lambda_3 = 0$. Thus the above conditions become:

$$U_E = \lambda_1 t, \quad U_E e'(S) = \lambda_1, \quad \lambda_1 = \lambda_2 = \lambda \tag{10}$$

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[8] Note that this constraint could end up with $S > tD$, therefore the transfer G could be negative. This case will be discussed in detail later.

If a corner solution, $S = 0$, exists in the general scheme, the pure tax scheme is justified. Then $\lambda_3 \geq 0$ and $\lambda_4 = 0$ as $G = tD > 0$, and the first-order conditions thus become:

$$U_E = \lambda_1 t, \quad U_E e'(S) \leq \lambda_1, \quad \lambda_1 = \lambda_2 = \lambda. \tag{11}$$

Using these conditions, the following result is derived:

Result 4. A pure pollution tax scheme is optimal if both the following conditions are satisfied:

Condition (1) $U_E e'(0) \leq U_C = \dfrac{U_{\bar{L}}}{f_L(L, D)}$; and

Condition (2) $\dfrac{1}{e'(0)} \geq f_D(L, D) = t.$

The above conditions make sense and are intuitive. Because spending on the environment will reduce the spending on consumption, the first part of condition (1) says that it is not optimal to invest in environmental projects if the marginal utility of environmental quality derived from environmental spending ($U_E e'(S)$) is less than the marginal utility of consumption.

Condition (1) also compares the marginal utility of environmental investment and the marginal utility of leisure. As spending on the environment increases, the household may want to supply more labour to increase its whole income to support a certain amount of consumption, and thus enjoy less leisure. $f_L(L, D)$ is the marginal product of labour, $1/f_L(L, D)$ is therefore the amount of labour needed to produce an extra (infinitesimal) unit of product or income. Thus this condition says it does not pay for society to invest in the environment if the resulting marginal utility is less than the marginal utility of leisure forgone to recover the extra unit of income.

Condition (2) deals with the efficiency of environmental investment. $e'(S)$ is the marginal environmental output of spending on environmental projects, so $1/e'(S)$ is the marginal cost of environmental goods, while $f_D(L, D)$ is the marginal product or income of the environmental factor. It is not optimal to invest in the environment if the former is greater than the latter.

It is known that $e'(0)$ is a large number because $e'(S) > 0$ and $e''(S) < 0$, therefore the marginal utility of environment (U_E) should be sufficiently low to make condition (1) hold. According to the property of the utility function, this means that environmental quality should be very good or environmental resources be abundant. Unfortunately, the environmental problem has become a global issue and caused great concern, therefore the conditions are not likely to be satisfied in the real world. As a result, a pure pollution tax system may not be optimal.

Theoretically, the optimal spending on the environment could be larger than the tax revenue from pollution. Because government has no other taxation revenues, this leads to $G < 0$, that is, there is a net lump sum transfer from household to government. In a model as simple as the one presented here, it seems that transfers from household to government do not cause much trouble. However, in the real world, government should find a cost-free way to raise revenue beyond the pollution taxation revenue. If it fails to do so, it may be better to set a budget constraint for government, that is, $S \leq tD$. Clearly this will be a second-best option.[9]

If the fully earmarked scheme is considered, $S = tD$. From $S + G = tD$, this is equivalent to $G = 0$, which implies $\lambda_4 \geq 0$. Therefore the equation (9) becomes:

$$U_E = \lambda_1 t, \quad U_E e'(S) = \lambda_1, \quad \lambda_1 \geq \lambda_2 = \lambda. \tag{12}$$

Then the conditions justifying an earmarking scheme can be summarised as follows:

Result 5. All pollution tax/levy revenue should be used in environmental projects if all of the following conditions are satisfied:

Condition (1) $U_E e'(tD) \geq U_C = \dfrac{U_{\tilde{L}}}{f_L(L, D)}$;

Condition (2) $\dfrac{U_E}{U_C} \geq f_D(L, D) = t$; and

Condition (3) $\dfrac{U_E}{U_{\tilde{L}}} \geq \dfrac{f_D(L, D)}{f_L(L, D)} = \dfrac{t}{w}$.

Condition (1), which is opposed to condition (1) in Result 4, shows that it pays for the society to increase investment in the environment if the marginal utility of the environment resulting from environmental investment is larger than the marginal utility of consumption, or the marginal utility of leisure forgone to recover the spending on the environment. Conditions (2) and (3) show that the marginal rate of substitution of consumption or leisure for environmental amenity is larger than the related marginal rate of transformation.

These three conditions are likely to be satisfied when the marginal utility of environment is high, the pollution charges and the marginal utility of consumption are small. To apply these conditions, we can divide the world into several groups along three dimensions: abundance of environmental

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[9] It may be interesting to examine which one is better: spending on the environment less than the optimum due to the budget constraint or imposing some other taxes to finance the environmental expenditure. If the latter is proved to be better, the 'double-dividend hypothesis' is turned upside down.

Table 1 Application of conditions for earmarking

Environmental resources	Development degree	Pollution tax/levy revenue	Fully earmarking
abundant	low	low	ambiguous
		high*	not likely
	high	low	ambiguous
		high	not likely
scarce	low	low	likely
		high*	not likely
	high	low	likely
		high	ambiguous

Note: * Not likely to happen in the real world

resources; strength of pollution control; and degree of development (table 1). Possible examples of countries in certain categories are some African countries in row one and China in row five.

Optimal pollution tax/levy rate

From first-order conditions in (4), (7) and (10), the optimal tax rate under a general tax-income scheme can be derived as:

$$t^* = f_D = \frac{U_E}{U_C} = \frac{w^* U_E}{U_{\bar{L}}} = \frac{1}{e'}.$$

Most of the above expressions merely repeat the usual doctrines, and are thus not worth repeating here, except $t^* = 1/e'$; that is, the optimal tax rate should be equal to the marginal cost of public environment-improving or pollution-cleansing activities. It shows that the usual interpretation of Pigouvian tax is conceptually incomplete. Typically, a Pigouvian tax rate is set according to the marginal cost of pollution damage and, when implemented, leads to identical marginal cost of pollution abatement across firms. However, it does not consider the public environmental activities. Moreover, the relationship derived here has an important implication for policy-making, as discussed in partial equilibrium analysis. The view is prevalent that valuing environmental benefit or measuring pollution damage is highly subjective and very difficult. However, it is easier to count the cost of environmental projects. The policy-makers may be more confident in policy design if they can plan projects according to certain environmental targets and set the pollution tax rate based on the cost analysis of these projects.

If a pure pollution tax scheme is justified, most of the relationships are maintained except that $t^* \leq 1/e'(0)$. This has been indicated by condition (2) of Result 4. Because the marginal cost of public environment-improving or pollution-cleansing activities is very high, no tax revenue should be used in such activities; therefore the optimal tax rate is no higher than the marginal cost of environmental projects.

If a fully earmarked pollution levy scheme is justified, it is true that $t^* = f_D = 1/e'(tD)$, but $t^* \leq U_E/U_C = w^* U_E/U_{\tilde{L}}$. These have been explained in the discussion of Result 5, but it is still worth making a couple of points. Analogous to the case where $S = 0$, one would expect that $t \geq 1/e'(tD)$ because an upper limit is put on environmental spending. However, tD is not a fixed point like 0. The government chooses both D (therefore t) and S to maximise the objective function, and thus achieve equality between the optimal tax rate and the marginal cost of environmental project. This choice results in an imbalance between the optimal tax rate and the marginal rates of substitution (U_E/U_C and $w^* U_E/U_{\tilde{L}}$). Unlike the pure tax scheme where $S = 0$ is a fixed value, the government has the incentive to increase the tax revenue if the constraint on its environmental spending is binding.

3.3 Pollution tax versus emission permit

Because there is no transaction cost and uncertainty in the current model, the government can achieve the same results through either taxing the firm at an appropriate rate or directly issuing an appropriate number of emission permits (Weitzman 1974). However, there are still some problems with the permit scheme in the real world.

First, if there are many firms, the administrative cost of allocating permits efficiently to individual firms would be prohibitively high. Thus a tradeable permit system should be developed to ensure that the permits are allocated efficiently.

Second, the way in which permits are issued may make a difference. As the model implies, the household spends its income on consumption goods only. Therefore a grandfathered emission permit system works in the same way as a pure pollution tax scheme, while an auctioned permit system may follow the optimal general tax-income scheme.

3.4 Pre-existing taxes and 'double-dividend'

Now consider the existence of other distorting taxes. Before a pollution tax/levy scheme was introduced, the government taxed labour income at the rate ω to finance some public goods $\overline{G}$. With this $\overline{G}$, the utility function now becomes:

$$U(C, \overline{L} - L, \overline{E} - D + e(S), \overline{G}). \tag{13}$$

The government introduces a pollution tax/levy to tackle the environmental problem. To keep things simpler, and more realistic, it is assumed that tax revenues, pollution or income tax, will not be transferred to the household. The new first-order conditions are:

$$U_C = \lambda, \qquad U_{\bar{L}} = \lambda(w - \omega), \tag{14}$$

where λ is the Lagrangian multiplier of the new constraint $C \leq \pi + (w - \omega)L$.

The government chooses ω, $\overline{G}$, D and S to maximise equation (13), subject to:

$$\overline{G} + S \leq \omega L + tD, \quad C \leq (w - \omega)L + \pi(t, w), \quad 0 \leq S \leq tD.$$

The second constraint is added because the government anticipates that ω affects the household's budget and behaviour. The Lagrangian for this problem is:

$$\begin{aligned} \pounds = {} & U(C, \overline{L} - L, \overline{E} - D + e(S), \overline{G}) + \lambda_1(\omega L + tD - \overline{G} - S) \\ & + \lambda_2(\pi(w, t) + (w - \omega)L - C) + \lambda_3 S + \lambda_4(tD - S). \end{aligned}$$

The corresponding Kuhn-Tucker conditions are:

$$\begin{aligned} & U_{\overline{G}} = \lambda_1, \quad U_E = \lambda_1 t + \lambda_4 t, \quad \lambda_1 = \lambda_2 = \lambda, \\ & U_E e'(S) = \lambda_1 - \lambda_3 + \lambda_4, \quad \lambda_3 S = 0, \quad \lambda_4(tD - S) = 0. \end{aligned} \tag{15}$$

The relationship that $\lambda_1 = \lambda_2$ is derived from setting the partial derivative of the Lagrangian with respect to ω equal zero and represents the fact that a rise in the government's budget due to changing ω leads to an equal decline in the household's budget, while $\lambda_2 = \lambda$ comes from the fact that both λ_2 and λ are the multiplier of the household's budget constraint when maximising the same objective function.

If $S = 0$, that is, the whole pollution tax revenue is used to replace the distortional income tax, the following result is derived according to the above conditions:

Result 6. With pre-existing labour income tax, all pollution tax/levy revenue should be used to replace the income tax if both the following conditions are satisfied:

Condition (1) $U_E e'(0) \leq U_{\overline{G}} = U_C = \dfrac{U_{\bar{L}}}{f_L - \omega}$; and

Condition (2) $\dfrac{1}{e'(0)} \geq t = f_D$.

The first condition is similar to condition (1) in Result 4, except that the effect of labour income tax and the utility of public good are now considered.

Because the public good and environmental investment are jointly financed by tax revenue, a rise in environmental spending would reduce the funds available to the public good. Therefore, if the marginal utility from an infinitesimal change in environmental spending is less than the marginal utility of the public good, no tax revenue should be used in environmental projects; that is, all the revenue from the pollution tax should be used to replace the labour income tax. Because of the existence of income tax, the marginal income of labour received by the household should be adjusted by extracting the tax rate ω, and $U_E e'(0) \leq U_{\tilde{L}}/(f_L - \omega)$ states that if the marginal utility of an infinitesimal environmental spending is less than the marginal utility of leisure forgone to recover that extra unit of income, the environmental spending is not justified, thus all pollution tax revenue should be used to replace labour income tax.

On the other hand, if $S = tD$ is justified, a fully earmarked scheme is desirable, and it is likely to finance environmental spending by increasing income tax. The conditions are given in the following statement:

> **Result 7.** With pre-existing labour income tax, all pollution tax/levy revenue should be earmarked for environmental purposes if both the following conditions are satisfied:

$$\text{Condition (1)} \quad U_E e'(tD) \geq U_{\overline{G}} = U_C = \frac{U_{\tilde{L}}}{f_L - \omega}; \text{ and}$$

$$\text{Condition (2)} \quad \frac{U_E}{U_C} = \frac{U_E(w - \omega)}{U_{\overline{L}}} = \frac{U_E}{U_{\overline{G}}} \geq t = f_D.$$

These conditions are similar to those given in Result 5, except that the marginal labour income is adjusted due to the tax and the marginal utility of the public good is considered. The new terms in conditions of Result 7 indicate that a fully earmarked pollution levy scheme is justified if the marginal utility from environmental spending is larger than the marginal utility of public good and if the marginal rate of substitution of environmental good for public good is larger than the marginal product of environmental good.

Checking the situation that usually holds in practice against the conditions, it may be found that those in Result 6 are likely to be violated, while those in Result 7 are not. Therefore, the basis for the 'double-dividend hypothesis' is eroded. However, another version of the 'double-dividend hypothesis' may be proposed. Even without using the revenue to replace pre-existing taxes, the environmental tax might help to reduce the distortion of labour income tax through a substitution effect. After the environmental tax is imposed, the environmental factor is more expensive than before, so the firm may demand more labour, that is, the income tax base is enlarged. Given $\overline{G}$, this leads to a less distorting labour income tax rate.

4. Dynamic general equilibrium analysis

In the real world, the environment is a stock of certain elements and provides a flow of services, of which absorbing pollutants is one. More importantly, the environment itself has the capacity of natural assimilation which may affect earmarking in environmental projects. Therefore the above results should be examined in a dynamic setting.

4.1 The model

The framework is a modified Ramsey model with environment arguments in both production and utility functions. Like the static general equilibrium analysis, there are three agents in this model.

The firm employs three factors: labour, capital and environmental absorption capacity. The effective labour is increasing at an exogenous, constant rate n, and economy-wide variables are normalised by this effective labour level.[10] Capital is accrued by investment (i) made by the firm, and depreciated at a constant rate δ:

$$\dot{k}_t = i_t - (\delta + n)k_t \tag{16}$$

Assume the production function is constant returns to scale, so the output per effective labour unit is $f(k_t, d_t)$. If a pollution tax is imposed at rate τ, the firm's instantaneous profit is $\pi_t = f(k_t, d_t) - i_t - w_t - \tau_t d_t$. It chooses i and d to maximise its intertemporal profit $\int_t^\infty \pi_s e^{-(r_s - n)(s-t)} ds$, subject to the constraint given by equation (16).

The representative household owns the firm, supplies labour, and receives dividends and government transfers. It can also borrow and lend at the rate r to finance its spending on consumption. The dynamic budget constraint is therefore:

$$\dot{a}_t = (r - n)a_t + w_t + v_t - c_t, \tag{17}$$

where a is the household's assets and v is government transfer. The household's debt should also meet the No-Ponzi-Game condition $\lim_{t\to\infty} a_t e^{-(r-n)t} \geq 0$. The household chooses paths of consumption (c_t) to maximise its intertemporal utility:

$$\int_t^\infty e^{-(\theta - n)(s-t)} u(c_s, e_s) ds, \tag{18}$$

subject to equation (17).

The government collects pollution tax from the firm and allocates it on environmental spending and transfers. It is assumed that the government does not accrue any asset or debt, that is:

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[10] Following convention, lower case letters are used to denote these variables.

$$s_t + v_t = \tau_t d_t. \tag{19}$$

The environmental stock is eroded by pollution, and improved by environmental investment, while the environment itself has natural assimilation ability:[11]

$$\dot{e}_t = h(e_t) + g(s_t) - ne_t - d_t, \tag{20}$$

where e is environmental stock, d is pollutant discharge, $h(\cdot)$ is the natural assimilation function of the environment, and $g(\cdot)$ is the provision function by environmental investment. It is assumed these functions have the usual properties.

The government chooses paths of d_t, s_t and v_t to maximise the intertemporal social welfare represented by equation (18) subject to equations (17), (19) and (20). Constraint (17) is included because the government anticipates that its spending on the environment affects the household's budget, and thus utility.

4.2 Analytical results

The current Hamiltonian and first-order conditions for the above model are reported in table 2. Using these results, the conditions to justify pure tax and fully earmarked schemes are derived.

> **Result 8.** Along the optimal path, a pure tax scheme is adopted if $\lambda_1 \tau g'(0) = \lambda_2 g'(0) \leq u_c$ and $1/g'(0) \geq f_d(k, d) = \tau$ at every time; and a fully earmarked scheme is optimal if $\lambda_1 = \lambda_2 g'(s)|_{s=\tau d} \geq u_c$, and $\tau = 1/g'(s)|_{s=\tau d}$ at every time.

These conditions are virtually the same as those in the static model. Note that λ_1 is the marginal value of objective function (i.e. marginal utility) if the government budget is increased exogenously by one infinitesimal unit, λ_2 is the marginal utility of environmental stock, $g'(s)$ is the marginal environmental product of investment; therefore $\lambda_2 g'(s)$ and $\lambda_1 \tau g'(s)$ are the marginal utility of environmental spending, with the former arising through a direct increase in environmental stock and the latter through savings in government spending. It is clear that no tax revenue should be used in environmental projects if the marginal utility of such spending is less than the marginal utility of consumption. If the condition is reversed when all tax revenue is used in environmental projects, a fully earmarked scheme is justified.

It should be pointed out that these conditions do not imply that the

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[11] Pollution stock is not used as the state variable because, at the steady state, the total pollution stock will increase at the rate n, which is not a desirable result.

Table 2 First-order conditions for dynamic problem

Agent	Current Hamiltonian	First-order condition
firm	$f(k, d) - i - w - \tau d$ $+\lambda_0[i - (\delta + n)k]$	$f_d = \tau,\ f_k = r + \delta,\ \lambda_0 = 1,$ $\dot{k} = i - (\delta + n)k$
household	$u(c, e) + \lambda[(r - n)a + w$ $+v - c]$	$u_c = \lambda,\ \dot{\lambda} = (\theta - r)\lambda,$ $\dot{a} = (r - n)a + w + v - c$
government	$u(c, e) + \lambda_1(\tau d - s - v)$ $+\lambda[(r - n)a + w + v - c]$ $+\lambda_2[h(e) + g(s) - ne - d]$ $+\lambda_3 s + \lambda_4 v$	$\lambda_1\tau = \lambda_2,\ \lambda_1 = \lambda + \lambda_4,$ $\lambda_1 = \lambda_2 g' + \lambda_3,$ $\lambda_3 s = 0,\ \lambda_4 v = 0,$ $\dot{\lambda}_2 = (\theta - h')\lambda_2 - u_e,$ $\dot{e} = h(e) + g(s) - ne - d$

government should stick to one particular scheme along the optimal path. It is possible to shift from one to another.

The steady state is summarised in table 3. In deriving these results, the relations $w_t = f(k_t, d_t) - f_k k_t - \tau d_t$ and $a_t = k_t$ are used. The first comes from the assumption of constant returns to scale production. The second comes from the fact that, in equilibrium, there is neither lending nor borrowing (Blanchard and Fischer 1989, p. 50). These results are similar to those of static general equilibrium analysis.

Because a pure pollution tax can only reduce demand for the environment, rather than increase its supply, it may be unstable in the dynamic process. However, environmental investment can increase the environmental stock and thus may help to improve stability, and is likely to achieve sustainable growth. A simple example is presented to demonstrate this. Suppose the

Table 3 Steady state of general tax/income scheme

$s = 0$	$0 < s < \tau d$	$s = \tau d$
$1/g'(s) \geq f_d(k, d)$	$1/g'(s) = f_d(k, d)$	$1/g'(s) = f_d(k, d)$
$U_c f_d = U_e/(\theta - h')$	$U_c f_d = U_e/(\theta - h')$	$U_c f_d \leq U_e/(\theta - h')$
$U_c \geq U_e g'/(\theta - h')$	$U_c = U_e g'/(\theta - h')$	$U_c \leq U_e g'/(\theta - h')$
$f_k = \theta + \delta$	$f_k = \theta + \delta$	$f_k = \theta + \delta$
$f(k, d) = (\delta + n)k + c$	$f(k, d) = (\delta + n)k + c + s$	$f(k, d) = (\delta + n)k + c + s$
$h(e) = ne + d$	$g(s) + h(e) = ne + d$	$g(s) + h(e) = ne + d$

environmental regeneration function is $h(s) = \beta s$ with $0 < \beta < n$. According to equation (20), if there is no environmental investment, e will decrease forever, and a steady state cannot be found. However, when environmental investment is made, the steady state is possible.

5. Conclusion

This article demonstrates that the usual interpretation of the Pigouvian tax is misleading. An optimal pollution tax/levy program should include the possibility of spending the revenue on environment-improving or pollution-cleansing projects, that is, earmarking. A pure tax scheme cannot do better than a general tax-income or 'tax-bounty' scheme and, in fact, is not sufficient for an optimum. This article also investigates the conditions for a pure tax system and for earmarking all pollution tax/levy revenues into environmental projects. A pure tax system might be an optimum only if the marginal utility of environment is sufficiently small; that is, environmental resources are abundant. However, this situation is rare in practice. By contrast, a fully earmarked system is more likely to be a better choice when the marginal utility of environment is high, relative to that of consumption, and the pollution tax/levy revenue is small.

Conventionally, it is accepted that a Pigouvian pollution tax is set equal to the marginal damage cost of the negative externality. However, it is found that this is not necessarily optimal if the tax revenue can be used for environmental purposes. In the general equilibrium setting, under the general tax-income scheme, the optimal pollution tax rate is equal to the marginal cost of producing environmental goods and services. This may imply that less information than usually thought is required to design an efficient tax policy. Even in a partial setting, if all revenue is to be used in an environmental project, it is not necessary to fully internalise pollution damage to reach an optimum, and the optimal tax rate might be different from the marginal damage cost.

In the case with pre-existing distorting taxes, using all the pollution tax/levy revenue to replace the distorting taxes is usually not optimal. At least some of the revenue from the pollution tax/levy should be used in environmental activities. Considering the current situation, it is likely to be optimal to earmark the whole pollution tax/levy revenue into environmental projects.

These static results survive in the steady state of a dynamic setting. Moreover, because a pure pollution tax can only reduce the demand for environment, rather than increase its supply, a pure pollution tax may be unstable in the dynamic process. However, using the revenue from the tax for environmental investment may help to improve the stability, and be likely to achieve sustainable growth.

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