Parallel axiomatizations of majority and unanimity

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Abstract
The relative majority rule and the unanimity rule are characterized for the case in which there are only two alternatives. The main axioms are motivated by a principle of binary representativeness: the aggregation of the preferences of n voters is the result of splitting the n voters into two groups, aggregating the preferences of the voters of each group, selecting for each group a representative that adopts the preference of the group, and finally aggregating the preferences of the two representatives. The two characterizations are shown to differ from each other in just one axiom, expressing strategyproofness (unanimity) or group strategyproofness (majority).

Keywords: Social welfare function, relative majority rule, unanimity rule, representativeness, axiomatic characterization, two alternatives.

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1. Introduction

A collective decision can be considered democratic when the decision has enough support in the collective. When there are only two alternatives α and β involved, one may define α to have enough support if more individuals prefer α to β than β to α. This definition leads to the relative majority rule as a source for democratic decisions. Characterizations of this rule for the case of two alternatives have been provided by, for instance, May (1952, p. 682), Fishburn (1973, p. 58), Aşan and Sanver (2002, p. 411), Woeginger (2003, p. 91; 2005, p. 9), Miroiu (2004, p. 362) and Xu and Zhong (2009). Going from one extreme to the other, one may define α to have enough support only if it has maximum support. The adoption of this principle leads to the unanimity rule. In between, there are the different versions of the absolute majority rule, where enough support means having the support of some percentage $p > 50\%$ of the individuals; see Llamazares (2006) and Houy (2007) for characterizations of absolute majority rules.

This paper provides characterizations of both the relative majority rule and the unanimity rule. Conceptually, the characterizations hinge on the principle of representative democracy, which is understood in the sense that the preference of a collective I can be obtained by aggregating the preferences of subgroups of the collective. It is also worth noticing that the two characterizations (see Proposition 3.8) differ from each other in just one axiom: whereas the specific axiom for the relative majority rule is a group strategyproof condition, it is an individual strategyproof property for the unanimity rule.

2. Definitions and axioms

Members of the set $\mathbb{N}$ of natural numbers are names for individuals. A society is a finite non-empty subset of $\mathbb{N}$. The alternatives are α and $\beta \neq \alpha$. A preference over $\{\alpha, \beta\}$ is represented by a number from the set $\{-1, 0, 1\}$. If the number is 1, α is preferred to β; if -1, β is preferred to α; if 0, α is indifferent to β. A preference profile for society I is a function $x_I : I \to \{-1, 0, 1\}$ assigning a preference over $\{\alpha, \beta\}$ to each member of I. For $x_I \in X$, $J \subseteq I$ and $i \in I$, x_J is the restriction of x_I to J and x_i abbreviates $x_I(i)$. For $n \in \mathbb{N}$, X_n is the set of all preference profiles x_I such that I has n members. The set X is the set of all preference profiles x_I such that I is a society. For $I = \{i, j\}$, $a \in \{-1, 0, 1\}$ and $b \in \{-1, 0, 1\}$, (a^i, b^j) is the member x_I of X_2 such that $x_i = a$ and $x_j = b$.

Definition 2.1. A social welfare function is a mapping $f : X \to \{-1, 0, 1\}$.

A social welfare function takes as input the preferences over $\{\alpha, \beta\}$ of all the members of any given society I and outputs a collective preference over $\{\alpha, \beta\}$. Specifically, for $x_I \in X$: (i) $f(x_I) = 1$ means that, according to f, society I prefers α to β; (ii) $f(x_I) = -1$, that society I prefers β to α; and (iii) $f(x_I) = 0$, that society I is indifferent between α and β. For $x_I \in X$ and $a \in \{-1, 0, 1\}$, $n_a(x_I)$ is the number of members of the set $\{i \in I: x_i = a\}$.

Definition 2.2. The relative majority rule is the social welfare function $\mu : X \to \{-1, 0, 1\}$ such that, for all $x_I \in X$: (i) if $n_1(x_I) > n_{-1}(x_I)$, then $\mu(x_I) = 1$; (ii) if $n_1(x_I) < n_{-1}(x_I)$, then $\mu(x_I) = -1$; and (iii) if $n_1(x_I) = n_{-1}(x_I)$, then $\mu(x_I) = 0$.

Definition 2.3. The unanimity rule is the social welfare function $\upsilon : X \to \{-1, 0, 1\}$ such that, for all $x_I \in X$: (i) if $n_{-1}(x_I) = n_0(x_I) = 0$, then $\upsilon(x_I) = 1$; (ii) if $n_1(x_I) = n_0(x_I) = 0$, then $\upsilon(x_I) = -1$; and (iii) otherwise, $\upsilon(x_I) = 0$.

Definition 2.4. Given a social welfare function f and $x_I \in X \backslash X_1$, the preference $f(x_I)$ is:

(i) determinable by representatives of two subsocieties if, for some subsociety J of I, $j \in I$ and $i \in I \backslash J$, $f(x_I) = f(f(x_{I \backslash J})^i, f(x_J)^j)$; and
(ii) determined by representatives of two subsocieties if, for every subsociety J of I, there are $j \in I$ and $i \in I \backslash J$ such that $f(x_I) = f(f(x_{I \backslash J})^i, f(x_J)^j)$.

When $f(x_I)$ is determinable by two representatives, the preference $f(x_I)$ can be explained as arising from the aggregation of preferences of two individuals representing two subsocieties (or districts) J and $I \backslash J$. When $f(x_I)$ is determined by two representatives, the preference $f(x_I)$ is obtained no matter the subsocieties chosen.

SPD. *Strict preference is determinable by two representatives*. For all $x_I \in X \backslash X_1$, if $f(x_I) \neq 0$, then $f(x_I)$ is determinable by representatives of two subsocieties.

IND. *Indifference is determined by two representatives*. For all $x_I \in X \backslash X_1$, if $f(x_I) = 0$, then $f(x_I)$ is determined by representatives of two subsocieties.

The asymmetric treatment of strict preference and indifference in SPD and IND presumes that indifference is less desirable than strict preference (for instance, because strict preference makes easier for the collective to select an alternative). IND requires that, if indifference is the outcome, then every possibility of obtaining a strict preference by redistricting has been exhausted. IND makes indifference robust: when indifference is the outcome, no other result could be obtained if two representatives must determine the collective preference.

3. Results

Remark 3.1. μ satisfies SPD and IND.

As regards SPD, if $x_I \in X\backslash X_1$ and $\mu(x_I) = a \neq 0$, then choose $k \in I$ such that $x_k = a$, so $\mu(x_{I\backslash\{k\}}) \in \{0, a\}$. Then, with $J = \{k\}$, $\mu(\mu(x_{I\backslash J})^i, \mu(x_J)^k) = \mu(b^i, a^k) = a = \mu(x_I)$, because $b \in \{0, a\}$. With respect to IND, let $x_I \in X\backslash X_1$ and $\mu(x_I) = 0$. Choose $J \subset I$, $j \in I$ and $i \in I\backslash J$. If $\mu(x_J) = 0$, then $\mu(x_I) = 0$ implies $\mu(x_{I\backslash J}) = 0$. Hence, $\mu(\mu(x_{I\backslash J})^i, \mu(x_J)^j) = \mu(0^i, 0^j) = 0 = \mu(x_I)$. If $\mu(x_J) = a \in \{-1, 1\}$, then $n_a(x_J) > n_{-a}(x_J)$. In this case, $\mu(x_I) = 0$ implies $n_a(x_{I\backslash J}) < n_{-a}(x_{I\backslash J})$, for which reason $\mu(x_{I\backslash J}) = -a$. Thus, $\mu(\mu(x_{I\backslash J})^i, \mu(x_J)^j) = \mu(-a^i, a^j) = 0 = \mu(x_I)$.

Remark 3.2. υ satisfies SPD and IND.

Since $\upsilon(x_I) = a \neq 0$ implies that, for all $i \in I$, $x_i = a$, it is evident that υ satisfies SPD. Concerning IND, let $x_I \in X\backslash X_1$ and $\upsilon(x_I) = 0$. Choose $J \subset I$, $j \in I$ and $i \in I\backslash J$. To prove that IND holds, it is enough to show that $\{\upsilon(x_{I\backslash J}), \upsilon(x_J)\} = \{1, -1\}$ or that $0 \in \{\upsilon(x_{I\backslash J}), \upsilon(x_J)\}$. If, for some $i \in I$, $x_i = 0$, then either $i \in J$ or $i \in I\backslash J$. Consequently, $\upsilon(x_{I\backslash J}) = 0$ or $\upsilon(x_J) = 0$. If, for all $i \in I$, $x_i \neq 0$, then there must be $j \in I$ and $k \in I\backslash\{j\}$ such that $x_j = 1$ and $x_k = -1$. If $\{j, k\} \subseteq J$, $\upsilon(x_J) = 0$. If $\{j, k\} \subseteq I\backslash J$, $\upsilon(x_{I\backslash J}) = 0$. If $j \in J$ and $k \in I\backslash J$, $\upsilon(x_J) \in \{0, 1\}$ and $\upsilon(x_{I\backslash J}) \in \{0, -1\}$. And if $j \in I\backslash J$ and $k \in J$, $\upsilon(x_J) \in \{0, -1\}$ and $\upsilon(x_{I\backslash J}) \in \{0, 1\}$.

Lemma 3.3. With $k \geq 2$, let f be a social welfare function such that $f = \mu$ on $X_1 \cup \ldots \cup X_k$. Then $f = \mu$ if and only if f satisfies SPD and IND.

Proof. "$\Rightarrow$" Remark 3.1. "$\Leftarrow$" Taking the fact that $f = \mu$ on $X_1 \cup \ldots \cup X_k$ as the base case of an induction argument, choose $n > k$ and, arguing inductively, suppose that $f = \mu$ on $X_1 \cup \ldots \cup X_{n-1}$. To show that $f = \mu$ on X_n, choose $x_I \in X_n$. Case 1: $\mu(x_I) = 0$. Suppose $f(x_I) = a \neq 0$. By SPD, there are $J \subset I$, $j \in I$ and $i \in I\backslash J$ such that $f(f(x_{I\backslash J})^i, f(x_J)^j) = f(x_I) = a$. By the induction hypothesis, $f(f(x_{I\backslash J})^i, f(x_J)^j) = \mu(\mu(x_{I\backslash J})^i, \mu(x_J)^j)$. Therefore, $\mu(\mu(x_{I\backslash J})^i, \mu(x_J)^j) = a$. By definition of μ, $a \in \{\mu(x_{I\backslash J}), \mu(x_J)\}$. Without loss of generality, suppose that $\mu(x_J) = a$. This means that $n_a(x_J) > n_{-a}(x_J)$. Since $\mu(x_I) = 0$ implies $n_a(x_I) = n_{-a}(x_I)$, it follows that $n_a(x_{I\backslash J}) < n_{-a}(x_{I\backslash J})$, so $\mu(x_{I\backslash J}) = -a$. But then $a = \mu(\mu(x_{I\backslash J})^i, \mu(x_J)^j) = \mu(-a^i, a^j) = 0$: contradiction.

Case 2: $\mu(x_I) = a \neq 0$. As $\mu(x_I) = a$, there is $k \in I$ such that $x_k = a$. By definition of μ, $\mu(x_{I\backslash\{k\}}) \in \{0, a\}$. Suppose $f(x_I) = 0$. By IND, for every $J \subset I$, there are $j \in I$ and $i \in I\backslash J$ such that $f(f(x_{I\backslash J})^i, f(x_J)^j) = f(x_I) = 0$. In particular, letting $J = \{k\}$, $f(f(x_{I\backslash\{k\}})^i, f(x_k)^k) = 0$. By the induction hypothesis, $0 = f(f(x_{I\setminus\{k\}})^i, f(x_k)^k) = \mu(\mu(x_{I\setminus\{k\}})^i, \mu(x_k)^k) = \mu(\mu(x_{I\setminus\{k\}})^i, a^k)$. As $\mu(x_{I\setminus\{k\}}) \in \{0, a\}$, $\mu(\mu(x_{I\setminus\{k\}})^i, a^k) = a$: contradiction. In view of this, $f(x_I) \in \{a, -a\}$. Suppose $f(x_I) = -a$. By SPD, there are $J \subset I$, $j \in I$ and $i \in I\setminus J$ such that $f(f(x_{I\setminus J})^i, f(x_J)^j) = f(x_I) = -a$. By the induction hypothesis, $f(f(x_{I\setminus J})^i, f(x_J)^j) = \mu(\mu(x_{I\setminus J})^i, \mu(x_J)^j)$. Thus, $\mu(\mu(x_{I\setminus J})^i, \mu(x_J)^j) = -a$. By definition of μ, $-a \in \{\mu(x_{I\setminus J}), \mu(x_J)\}$. Without loss of generality, suppose that $\mu(x_J) = -a$. This implies $n_{-a}(x_J) > n_a(x_J)$. Since $\mu(x_I) = a$, $n_a(x_I) > n_{-a}(x_I)$. As a result, $n_{-a}(x_{I\setminus J}) < n_a(x_{I\setminus J})$, which means that $\mu(x_{I\setminus J}) = a$. In consequence, $-a = \mu(\mu(x_{I\setminus J})^i, \mu(x_J)^j) = \mu(a^i, -a^j) = 0$: contradiction.■

Lemma 3.4. With $k \geq 2$, let f be a social welfare function such that $f = \upsilon$ on $X_1 \cup \ldots \cup X_k$. Then $f = \upsilon$ if and only if f satisfies SPD and IND.

Proof. "⇒" Remark 3.2. "⇐" Taking the fact that $f = \upsilon$ on $X_1 \cup \ldots \cup X_k$ as the base case of an induction argument, choose $n > k$ and, arguing inductively, suppose that $f = \upsilon$ on $X_1 \cup \ldots \cup X_{n-1}$. To show that $f = \upsilon$ on X_n, choose $x_I \in X_n$. Case 1: $\upsilon(x_I) = 0$. Suppose $f(x_I) = a \neq 0$. By SPD, there are $J \subset I$, $j \in I$ and $i \in I\setminus J$ such that $f(f(x_{I\setminus J})^i, f(x_J)^j) = f(x_I) = a$. By the induction hypothesis, $f(f(x_{I\setminus J})^i, f(x_J)^j) = \upsilon(\upsilon(x_{I\setminus J})^i, \upsilon(x_J)^j)$. Therefore, $\upsilon(\upsilon(x_{I\setminus J})^i, \upsilon(x_J)^j) = a$. By definition of υ, $\upsilon(x_{I\setminus J}) = \upsilon(x_J) = a$. This implies $n_{-a}(x_{I\setminus J}) = n_0(x_{I\setminus J}) = n_{-a}(x_J) = 0$, so $n_{-a}(x_I) = n_0(x_I) = 0$. In view of this, $\upsilon(x_I) = a$: contradiction. Case 2: $\upsilon(x_I) = a \neq 0$. By SPD or IND, there are $J \subset I$, $j \in I$ and $i \in I\setminus J$ such that $f(f(x_{I\setminus J})^i, f(x_J)^j) = f(x_I)$. By the induction hypothesis, $f(f(x_{I\setminus J})^i, f(x_J)^j) = \upsilon(\upsilon(x_{I\setminus J})^i, \upsilon(x_J)^j)$. By definition of υ, $\upsilon(x_I) = a$ implies that, for all $k \in I$, $x_k = a$. As a result, $\upsilon(x_{I\setminus J}) = \upsilon(x_J) = a$ and $\upsilon(\upsilon(x_{I\setminus J})^i, \upsilon(x_J)^j) = a$. As a consequence, $f(x_I) = a = \upsilon(x_I)$.■

Lemmas 3.3 and 3.4 suggest that, given SPD and IND, the differences between unanimity and relative majority can be traced back to the way they aggregate preferences in societies with one or two individuals. In particular, any characterization of $\delta \in \{\mu, \upsilon\}$ on $X_1 \cup X_2$ combined with SPD and IND will characterize δ. The results presented next make use of this possibility.

PAR$_2$. *Pareto efficiency for societies with at most two members*. For all $x_I \in X_1 \cup X_2$ and $a \in \{-1, 0, 1\}$, if, for all $i \in I$, $x_i \in \{a, 0\}$ and, for some $i \in I$, $x_i = a$, then $f(x_I) = a$.

VET$_2$. *No veto power for societies with two members*. For all $x_I \in X_2$ and $a \in \{1, -1\}$, if $I = \{i, j\}$, then $f(a^i, -a^j) \neq a$.

The condition VET$_2$ of no veto power is equivalent to Xu and Zhong's (2009) simple equal treatment condition.

Given a social welfare function f, society I and $a \in \{-1, 0, 1\}$, define $\pi_a(I)$ to be the number of preference profiles x_I such that $f(x_I) = a$.

FET$_2$. *Full equal treatment of outcomes in societies with two members*. For every society I with two members, $\pi_{-1}(I) = \pi_0(I) = \pi_1(I)$.

Lemma 3.5. A social welfare function $f : X \to \{-1, 0, 1\}$ satisfies PAR$_2$ and either FET$_2$ or VET$_2$ if and only if $f = \mu$ on $X_1 \cup X_2$.

Proof. "$\Rightarrow$" It is not difficult to verify that μ satisfies PAR$_2$, FET$_2$ and VET$_2$. "$\Leftarrow$" By PAR$_2$, for all $a \in \{-1, 0, 1\}$, $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$: $f(a^i) = a = \mu(a^i)$; $f(a^i, a^j) = a = \mu(a^i, a^j)$; $f(a^i, 0) = a = \mu(a^i, 0^j)$; and $f(-a^i, 0^j) = -a = \mu(-a^i, 0^j)$. By either FET$_2$ or VET$_2$, $f(a^i, -a^j) = f(-a^i, a^j) = 0 = \mu(a^i, -a^j) = \mu(-a^i, a^j)$.■

Proposition 3.6. A social welfare function f satisfies SPD, IND, PAR$_2$ and either FET$_2$ or VET$_2$ if and only if $f = \mu$.

Proof. Lemmas 3.3 and 3.5.■

UNA$_2$. *Unanimity for societies with at most two members*. For all $x_I \in X_1 \cup X_2$ and $a \in \{-1, 0, 1\}$, if, for all $i \in I$, $x_i = a$, then $f(x_I) = a$.

EQA$_2$. *Equal treatment of the two alternatives in societies with two members*. For every society I with two members, $\pi_{-1}(I) = \pi_1(I)$.

DIC$_2$. *Non-dictatorship for societies with two members*. For every society I with two members, there is no $i \in I$ such that, for all $x_I \in X_2$, $x_i \neq 0$ implies $f(x_I) = x_i$.

SSP$_2$. *Strong strategy-proofness for societies with two members*. For all $x_I \in X_2$ and $i \in I$, there is no $a \in \{-1, 0, 1\}$ such that $|f(a^i, x_{I\backslash\{i\}}) - x_i| < |f(x_I) - x_i|$.

GSP$_2$. *Group strategy-proofness for societies with two members*. For all $x_I \in X_2$, there is no $y_I \in X_2$ such that, for all $i \in I$ with $x_i \neq 0$, $|f(y_I) - x_i| < |f(x_I) - x_i|$.

Lemma 3.7. A social welfare function $f : X \to \{-1, 0, 1\}$ satisfies UNA$_2$, EQA$_2$, DIC$_2$, and

(i) GSP$_2$ if and only if $f = \mu$ on $X_1 \cup X_2$.

(ii) SSP$_2$ if and only if $f = \upsilon$ on $X_1 \cup X_2$.

Proof. (i) "$\Rightarrow$" It is not difficult to verify that μ satisfies UNA_2, EQA_2, DIC_2, and GSP_2. "$\Leftarrow$" By UNA_2, for all $a \in \{-1, 0, 1\}$, $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$, $f(a^i) = a = \mu(a^i)$ and $f(a^i, a^j) = a = \mu(a^i, a^j)$. Choose $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$. Let $a \in \{-1, 1\}$. If $f(a^i, 0^j) \neq a$, then, as shown, $f(a^i, a^j) = a$, which contradicts GSP_2. Consequently, for all $a \in \{-1, 1\}$, $f(a^i, 0^j) = a = \mu(a^i, 0^j)$. If $f(1^i, -1^j) = 1$, then, by EQA_2, $f(-1^i, 1^j) = -1$, contradicting DIC_2. If $f(1^i, -1^j) = -1$, then, by EQA_2, $f(-1^i, 1^j) = 1$, contradicting DIC_2. As a result, $f(1^i, -1^j) = 0 = \mu(1^i, -1^j) = 1$. Given this, by EQA_2, $f(-1^i, 1^j) = 0 = \mu(-1^i, 1^j)$.

(ii) "$\Rightarrow$" It is not difficult to verify that υ satisfies UNA_2, EQA_2, DIC_2, and SSP_2. "$\Leftarrow$" By UNA_2, for all $a \in \{-1, 0, 1\}$, $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$, $f(a^i) = a = \upsilon(a^i)$ and $f(a^i, a^j) = a = \upsilon(a^i, a^j)$. Choose $i \in \mathbb{N}$ and $j \in \mathbb{N}\backslash\{i\}$. Case 1: $f(1^i, -1^j) = 1$. By SSP_2, $f(1^i, 0^j) = 1$. Case 1a: $f(-1^i, 1^j) = 0$. By SSP_2, $f(-1^i, 0^j) = 0 = f(0^i, 1^j)$, contradicting EQA_2. Case 1b: $f(-1^i, 1^j) = -1$. By SSP_2, $f(-1^i, 0^j) = -1$, so DIC_2 does not hold. Case 1c: $f(-1^i, 1^j) = 1$. By SSP_2, $f(0^i, 1^j) = 1$. In view of this, EQA_2 does not hold. Case 2: $f(1^i, -1^j) = -1$. By SSP_2, $f(0^i, -1^j) = -1$. Case 1a: $f(-1^i, 1^j) = 0$. By SSP_2, $f(-1^i, 0^j) = 0 = f(0^i, 1^j) = 1$, which contradicts EQA_2. Case 1b: $f(-1^i, 1^j) = -1$. By SSP_2, $f(-1^i, 0^j) = -1$, so EQA_2 does not hold. Case 1c: $f(-1^i, 1^j) = 1$. By SSP_2, $f(0^i, 1^j) = 1$. Consequently, DIC_2 does not hold. Case 3: $f(1^i, -1^j) = 0$. By SSP_2, $f(1^i, 0^j) = 0 = f(0^i, -1^j)$. By cases 1 and 2, $f(-1^i, 1^j) \neq 0$ leads to a contradiction. Accordingly, $f(-1^i, 1^j) = 0$. By SSP_2, $f(-1^i, 0^j) = 0 = f(0^i, 1^j)$. In sum, for all $a \in \{-1, 0, 1\}$ and $b \in \{-1, 0, 1\}$, $f(a^i, b^j) = \upsilon(a^i, b^j)$.■

Proposition 3.8. A social welfare function f satisfies SPD, IND, UNA_2, EQA_2, DIC_2, and

(i) GSP_2 if and only if $f = \mu$.
(ii) SSP_2 if and only if $f = \upsilon$.

Proof. Lemmas 3.3, 3.4 and 3.7.■

Proposition 3.8 establishes that, given SPD, IND, UNA_2, EQA_2 and DIC_2, the choice between majority and unanimity can be reduced to the choice between, respectively, the group strategyproofness requirement GSP_2 and the individual strategyproofness condition SSP_2 (which is qualified as strong because an indifferent individual is presumed to be interested in having the collective preference to be indifference). It is somewhat paradoxical that GSP_2, a sort of "cooperative" axiom, yields the majority rule, which is not particularly cooperative in that not much consensus is needed to reach a decision. On the other hand, the rather "non-cooperative" axiom SSP_2, yields the unanimity rule, which is the quintessential cooperative rule, as overall consensus is necessary to reach a decision.

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