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Cyrus C.Y. Chu Hung-Ken Chien*

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Abstract

We study the pricing problem of a durable-goods monopolist. With network effects, consumption externalities among heterogeneous groups of consumers generate a discontinuous demand function. Consequently, the lessor has to offer a low price in order to reach the mass market, whereas the seller has the option to build a customer base by setting a lower initial price and raise the price later in the mass market, which explains the practice of introductory pricing. Contrary to the existing literature, we show that profits from selling network goods may be higher than from leasing. Further, the seller in fact over-invests in R&D and makes the product more durable than necessary.

JEL codes: L1, L11.

Keywords: Penetration pricing, network externality.

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1. INTRODUCTION

In this paper, we shall consider the dynamic problem of a monopolist who produces durable goods with *network effects*. For instance, the goods in question may be a new computer software or an operating system. The network effect involved is in the sense of Rohlfs (1974), that the more total buyers there are in the market, the higher each individual's willingness to pay is.

Suppose, as it is intuitively appealing, that a new high-tech product has several periods of durability, and that the adoption of high-tech goods takes time. Normally the better-informed professional group adopts it first; and then the information spreads out gradually, leading to the adoption by less-informed, less professional groups, etc. Thus, in each period, the monopolist faces a new group of potential buyers, who just begin to be interested by this product. In general, as time goes, the total market demand increases.[1] This is different from the traditional durable-goods analysis where the total demand size is fixed, which in turn leads to the implication (as in Bulow (1982)) that current demand *crowds out* future demand.

Since the product in question has a network effect, it is natural to assume that the monopolist would exercise introductory pricing by setting a low price initially.[2] This strategy is in sharp contrast to Coasian dynamics that suggest nonincreasing prices for durable goods. Evidently, the key to reconcile the two seemingly contradictory propositions is to analyze the network effect explicitly in the dynamic pricing problem. Several papers approach this problem, but most of them cannot characterize the practice of introductory pricing by a monopolist with perfect information.[3] We shall provide a simple and tractable model that incorporates network effects for a durable-goods monopolist. By analyzing the dynamic pricing problem in this model, we are able to establish that introductory pricing is indeed optimal in the *selling* scenario. We will also demonstrate how network externalities would affect the monopolist's other strategies.

Our model differs from the previous work due to the selection of the market equilibrium. As pointed out in Katz and Shapiro (1986), consumer heterogeneity gives rise to the possibility of multiple equilibria. In other words, the potential buyers' expectations might sustain different network sizes for the same price. In the presence of multiple equilibria, which equilibrium would emerge determines the market demand, and hence the monopolist's pricing strategy. By assuming that consumers are able to coordinate their actions, the conventional approach is to select the equilibrium that maximizes consumers' total surplus. In this paper, we envision a situation where coordination is difficult and consumers are more conservative about their expectations of network growth. Accordingly, we select the stable equilibrium with the smallest network size. Our selection criterion results in a discontinuous demand function—a small decrease in price at the critical point generates a big jump in sales. Therefore, when the market expands, the monopolist should exercise penetration pricing by "offering a low price to

[1] Although Waldman (1993) considered a similar setup with different groups of consumers joining the market sequentially, consumers' willingness to pay in his model is assumed to be identical within each group. Therefore, pricing strategy is irrelevant in the monopolist's decision.

[2] For instance, when releasing MS-DOS, Microsoft briefly reduced the license fees by half. See Cabral et al. (1999) for other examples in computer software.

[3] For example, Katz and Shapiro (1985, 1986) studied a duopoly market and showed that introductory pricing emerges from the duopolists' competition to establish a network of users ahead of their opponents. For a monopolist model, Cabral et al. (1999) found introductory pricing in equilibrium. But their result derives from the presence of either imperfect information about consumers' valuations, or "large" consumers who have strategic interactions with each other.

invade another market" (Shapiro and Varian (1999), p. 288).[4] In effect, the demand discontinuity places a restriction on the monopolist's choice of quantities—she can either access the elite market (small quantity) with a high price, or reach the mass market (large quantity) with a low price. We shall call it the *penetration-pricing constraint* henceforth.

The constraint applies in both leasing and selling scenarios. However, unlike the monopolist lessor, the seller has the option to establish a large customer base, which changes the residual demand and enables the seller to raise the price and take full advantage of the network growth. Essentially, the seller's optimal strategy is to sacrifice her profits in the early stage in order to capitalize the gains later. Our model here explains the practice of introductory pricing. We show that sometimes a selling scenario may generate higher profits than leasing, in contrast to the previous results.

Furthermore, the fact that the market willingness to pay expands over time suggests a positive incentive to attract new buyers. This incentive countervails the inertia of devoting R&D expenses that has been argued by Bulow (1982). In general, the R&D activity has two effects on the monopolist's profits. It helps to curtail production costs but also reduces the revenue as the consumers expect a lower price. The net effect can be over-investment or under-investment in R&D. The presence of network externalities further mitigates the disincentive from the price-reduction effect. Indeed, we show that if R&D efforts are directed at improving product durability instead of reducing costs directly, then the monopolist seller always over-invests so that the product will be more durable than necessary, a conclusion contrary to the existing literature. It is important to reiterate that all of these implications hold only when the monopolist is constrained by the penetration-pricing restriction.

The rest of this paper is arranged as follows. In Section 2, we present the basic model that characterizes network effects. By employing this model, we provide a theoretical foundation for inverted-U shaped demands that are commonly seen in network-effect literature. In Section 3, we provide a numerical example of dynamic pricing to illustrate our point. In Section 4, we formulate and solve the general dynamic-pricing problems faced by a monopolist. When the monopolist has to take into account the restrictions placed by penetration pricing, we derive the optimal pricing strategies and the corresponding implications. In Section 5, we discuss the monopolist's incentive of devoting R&D expenses to reduce production costs. Although R&D is motivated to save costs, it also hurts the monopolist's profits when the early adopters expect a lower price due to the lower cost. In equilibrium, the monopolist seller may over-invest or under-invest in R&D, depending on which effect dominates. In Section 6, we consider the problem of endogenous obsolescence, in which the monopolist can choose how much durability to build in the product. We show that the monopolist seller always over-invests to make the product too durable. Section 7 discusses the results and concludes.

2. NETWORK EFFECT: A CHARACTERIZATION

2.1. *Consumer preferences*

Following Rohlfs (2001, p. 209), we assume that an individual's value for the good in question is composed of two parts: one is the generic value and the other is the

[4] We make a distinction between introductory and penetration pricing so that the former characterizes the change of prices between periods, while the latter identifies one's pricing strategy at a given time.

magnification of the network size. We further assume that heterogeneous groups of consumers comprise the economy. Consumer preferences for members of group g are represented by a willingness-to-pay function

$$p_g(x,n) = q_g(x) f_g(n), \quad g \in \{1, 2, ..., G\},$$

where G is the number of groups; n is the size of total consumption in the economy; and x is the index for an individual in group g.[5] Leaving aside the network effect, $q_g(x)$ is the generic valuation by consumer x in group g; and f_g is the network effect perceived by group-g members. By construction, individuals in each group are ordered decreasingly in terms of their generic willingness to pay, so that $q'_g(\cdot) \leq 0 \ \forall g$. We also assume that $f'_g(\cdot) > 0 \ \forall g$, which characterizes the underlying network effect.

To simplify our dynamic analysis, in what follows we shall assume that there are only two groups: $g = 1, 2$. The product is durable and never depreciates. There are two periods, and the discount rate is assumed to be zero. Following Bulow (1982), we assume that a perfect secondhand market exists.[6] It eliminates the possibility of price discrimination.

We shall consider the simplest form of generic demand: for the first and second groups, suppose $q_1(x) = A_1 - a_1 x$ and $q_2(x) = A_2 - a_2 x$.[7] To simplify the algebra, we assume that the network effects for both groups are the same: $f_1(n) = f_2(n) = n$. Assuming there is no income effect for any individual, we shall focus on the equilibrium analysis of the market in the presence of network effects.

2.2. *Inverted U-shaped Demands and the Critical Mass*

In the conventional analysis, a demand function associates any given price with a desired quantity of the commodity. Nonetheless, with network effects, a price might correspond to multiple quantities. To derive the demand correspondences in our model, note that if a consumer indexed x_g is willing to buy the object, anyone in the same group with a lower index $x < x_g$ will also demand it. Consequently, the total demand in the economy is determined by the marginal consumers from both groups:

$$p = (A_1 - a_1 x_1)(x_1 + x_2) = (A_2 - a_2 x_2)(x_1 + x_2). \tag{1}$$

In period 1, we assume that only the first group of people are informed. In that case, the inverse demand is simply

$$p_1(x) = (A_1 - a_1 x)x. \tag{2}$$

The maximum sustainable price is $\frac{A_1^2}{4a_1} (= \max p_1(x))$. For any price $p \in (0, \frac{A_1^2}{4a_1})$, p associates with three possible equilibria: $x = 0$ or the roots for the quadratic function $(A_1 - a_1 x)x = p$ (see Figure 1).[8] As is well known, the larger root $x_{\max}$ in the downward-sloping part of the parabola is a stable equilibrium (see the Appendix of Rohlfs (2001)),

[5] As in Rohlfs (2001, p. 205), each individual is assumed to make a simple yes-no decision about whether to buy a unit, therefore the index x of individuals also characterizes the demand size of the commodity. The assumption is not restrictive, because the same individual is allowed to make repetitive yes-no decisions.

[6] For instance, internet marketplaces such as Half.com or Amazon.com allow people to trade used computer software.

[7] An alternative interpretation for the generic demand functions is to assume that a member of group-g draws her generic value from $[0, A_g]$ uniformly, and the size of group-g population is A_g/a_g.

[8] Since the network effect is multiplicative, the willingness to pay by *any* consumer would be zero if she expects no one else to buy the product. Therefore, $x = 0$ is an equilibrium.

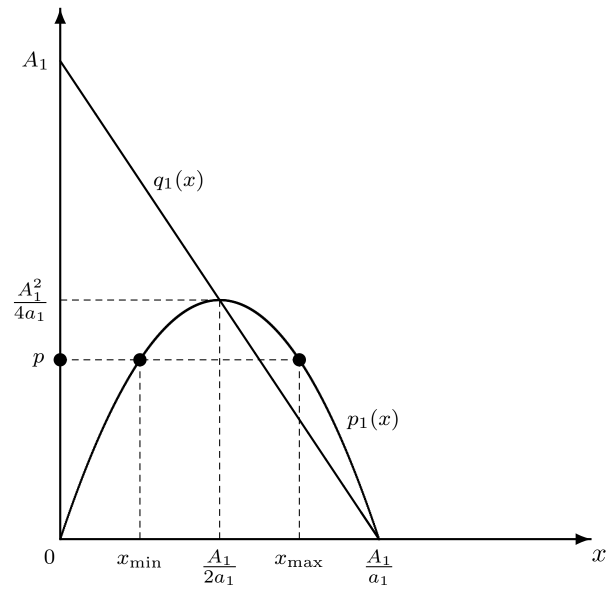

Figure 1: Single-hump Demand Curve in the First Period

and so is $x = 0$. Meanwhile, the smaller root $x_{\min}$ on the upward slope is an unstable equilibrium. The unstable equilibrium is usually called a critical mass, which the seller has to overcome in order to reach the stable and more profitable equilibrium.

In period 2, both groups are aware of the product. The aggregate demand correspondence is thus determined by equation (1). Alternatively, one can first derive the aggregate inverse *generic* demand as $\tilde{q}(x)$. The aggregate inverse demand is then given by $\tilde{p}(x) = \tilde{q}(x)x$. In Figure 2, $\tilde{q}(x)$ is the dashed line, which is obtained by summing $q_1(x)$ and $q_2(x)$ horizontally; while $\tilde{p}(x)$ is the double-hump shaped curve.

2.3. *The Penetration Pricing Constraint*

Specifically, $\tilde{p}(x)$ is equal to $A_1 x - a_1 x^2$ if $x \leq \hat{x}_1$, otherwise

$$\tilde{p}(x) = \tilde{A}x - \tilde{a}x^2 = \frac{a_2 A_1 + a_1 A_2}{a_1 + a_2} x - \frac{a_1 a_2}{a_1 + a_2} x^2, \tag{3}$$

where $\tilde{A}$ and $\tilde{a}$ denote the coefficients $\frac{a_2 A_1 + a_1 A_2}{a_1 + a_2}$ and $\frac{a_1 a_2}{a_1 + a_2}$, respectively. From Figure 2, one observes that $\tilde{q}(x) = q_1(x)$ for $x \leq \hat{x}_1$, where the critical value $\hat{x}_1 \equiv \frac{A_1 - A_2}{a_1}$ is determined by $q_1(\hat{x}_1) = q_2(0)$. The aggregate generic demand therefore has a flatter slope when $x > \hat{x}_1$. As such, the corresponding $\tilde{p}(x)$ has two humps if and only if $\frac{A_1}{2a_1} < \hat{x}_1 < \frac{\tilde{A}}{2\tilde{a}}$.[9] The double-hump shape of demand is similar to the one drawn in Rohlfs (2001, p. 219), except that we provide a theoretical justification for it here.[10]

[9]Specifically, a *hump* is a local maximum of the $\tilde{p}(\cdot)$ curve. In Figure 2, $\tilde{p}(x)$ reaches local maxima $\frac{A_1^2}{4a_1}$ and $\frac{\tilde{A}^2}{4\tilde{a}}$ at $x = \frac{A_1}{2a_1}$ and $\frac{\tilde{A}}{2\tilde{a}}$, respectively. However, if $\hat{x}_1 \leq \frac{A_1}{2a_1}$, $\tilde{p}(x)$ is increasing $\forall x \in [0, \hat{x}_1]$, and thus has only one local maximum at $x = \frac{\tilde{A}}{2\tilde{a}}$. The argument for the case with $\hat{x}_1 \geq \frac{\tilde{A}}{2\tilde{a}}$ is analogous. Note that $\frac{A_1}{2a_1} < \frac{\tilde{A}}{2\tilde{a}} = \frac{A_1}{2a_1} + \frac{A_2}{2a_2}$.

[10]Rohlfs (2001, p. 215) assumed $q(x) = cx^{\beta}$ and $f(n) = kn$. However, this is not going to generate the inverted-U figure he suggested. For under his specification, $p(x) = q(x)f(x) = ckx^{\beta+1}$, which

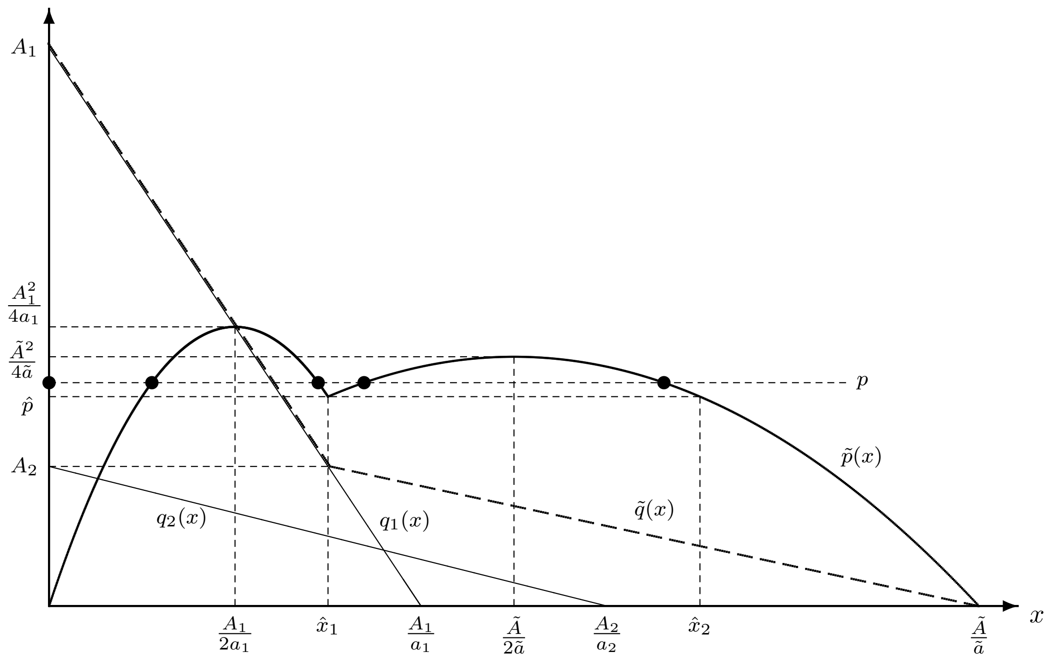

Figure 2: Double-hump Demand Curve in the Second Period

According to Rohlfs (2001) and Shapiro and Varian (1999), the monopolist can *penetrate* the section of $\tilde{p}(x)$ beyond $\hat{x}_1$ in the second period only if the price is set lower than or equal to $\hat{p}$. This is the *penetration pricing constraint* referred to in the literature.

Due to the double-hump shape of demand, for any price p between $\hat{p} \equiv \tilde{p}(\hat{x}_1)$ and $\frac{\tilde{A}^2}{4\tilde{a}}$, p associates with five equilibria (see the bullets in Figure 2), two (the third and the fifth from the left) of which are stable with positive quantities. In the presence of multiple stable equilibria, the criterion for equilibrium selection proves to be critical in our analysis of a monopolist's market strategy, as we will show in the next few sections.

3. DYNAMIC PRICING: A NUMERICAL EXAMPLE

Consider a monopolist who invents a new operating system and wants to release it to the market. This operating system lasts for two periods. As assumed, in the first period only group 1 (the professional engineers) access it, and in the second period, both groups of potential consumers access the product. We want to see how the analysis is different from the literature in various aspects. Let us look at the pricing strategy first. The following numerical example illustrates our point.

Let $x_{i\ell}$ be the quantity produced by the monopolist lessor up to the ith period, and x_{iS} the quantity produced by the monopolist seller up to the ith period.[11] Consider the following example.

(depending on the size of β) is either increasing or decreasing in x. Thus, the willingness to pay by the marginal consumer would be increasing or decreasing in the *whole range* of x; there would never be any inverted-U shaped curve.

[11] We adopt the notations so that $x_{2\ell}$ or x_{2S} represents the total number of units on the period-2 market. Period-2 production is therefore $x_{2j} - x_{1j}$ for $j = \ell$ or S.

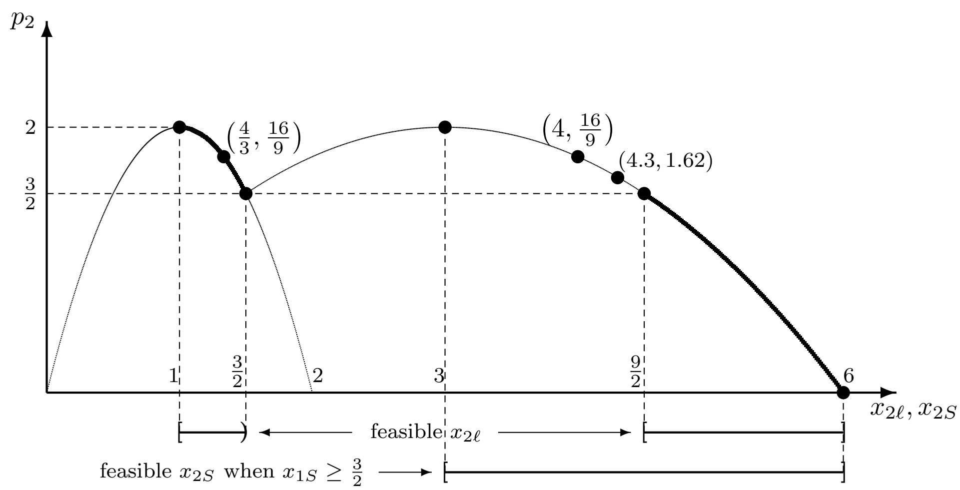

Figure 3: Penetration Pricing (Example)

Example: *Suppose $A_1 = 4$, $a_1 = 2$, $A_2 = 1$, and $a_2 = \frac{1}{4}$. One obtains $\tilde{A} = \frac{4}{3}$ and $\tilde{a} = \frac{2}{9}$. Assuming zero production costs, the monopolist lessor leases $x_{1\ell} = \frac{4}{3}$ in period 1, and $x_{2\ell} = \frac{9}{2}$ in period 2. Meanwhile, the monopolist seller sells $x_{1S} = \frac{3}{2}$ and $x_{2S} - x_{1S} \simeq 2.8$ in periods 1 and 2, respectively. It can be seen that selling is more profitable than renting for the monopolist.*

If the monopolist wants to lease the product, she chooses $x_{1\ell}$ and $x_{2\ell}$ to maximize

$$\pi_\ell = p_1(x_{1\ell}) \cdot x_{1\ell} + \tilde{p}(x_{2\ell}) \cdot x_{2\ell}.$$

It is easy to see that, given our numerical specifications in the example, the first-best prices are $p_1 = \frac{16}{9}$ $(x_{1\ell} = \frac{4}{3})$ and $p_2 = \frac{16}{9}$ $(x_{2\ell} = 4)$, as shown in Figure 3. Nonetheless, as Rohlfs (2001) pointed out, the double-hump shaped demand in period 2 limits the range of feasible $x_{2\ell}$'s. In particular, $(x_{2\ell}, p_2) = \left(4, \frac{16}{9}\right)$ is not an achievable strategy because the second-group demand cannot be "penetrated" by the price $p_2 = \frac{16}{9}$. By setting $p_2 = \frac{16}{9}$, the lessor can only lease $x_{2\ell} = \frac{4}{3}$ rather than 4. In fact, any period-2 leasing amount beyond $x_{2\ell} = \frac{3}{2}$ is achievable only if the price is set lower than $\frac{3}{2}$.

The boldfaced curve in Figure 3 illustrates the demand curve faced by the lessor. It indicates that the lessor can only select $x_{2\ell}$ from $[1, \frac{3}{2})$ and $[\frac{9}{2}, 6]$. When the monopolist's feasible actions in period 2 is constrained in this fashion, we say that the *penetration-pricing constraint* is binding.

Imposition of this constraint is where we deviate from the conventional analysis. From the game-theoretic viewpoint, the constraint follows from the way we select equilibrium in the presence of multiple equilibria, as we will explain below. It has long been recognized that a market price can associate with multiple network sizes in equilibrium.[12] Following Katz and Shapiro (1986), the conventional approach is to select the equilibrium that is Pareto-preferred by consumers.[13] Applying to our example, this selection criterion implies a different demand curve faced by the lessor so that she can

[12] See, for example, Oren and Smith (1981), Oren et al. (1982), and Katz and Shapiro (1986).

[13] Other authors such as Oren et al. (1982) simply assumed a single-peaked willingness-to-pay function, which eliminates the possibility of multiple equilibria.

choose to lease any quantity from $[3, 6]$ in period 2. In this paper, however, we follow the literature of network effects and assume that consumers are more conservative in terms of their expectations of network growth. Consequently, we focus on the stable equilibrium with the smallest network size for any given price. The resulting demand corresponding with the price range $(\frac{3}{2}, 2]$ is $[1, \frac{3}{2})$ instead of $[3, \frac{9}{2})$.[14] One way to justify our criterion for equilibrium selection is that it validates the dichotomy of pricing strategies when marketing a new product, as proposed by Dean (1976). The first segment of demand for $x_{2\ell} \in [1, \frac{3}{2})$ represents the strategy of *skimming pricing* with which the monopolist targets the elite group of consumers; while the second segment of demand for $x_{2\ell} \in [\frac{9}{2}, 6]$ corresponds with the strategy of *penetration pricing* that directs at the general public. In this paper, we focus on the situation under which penetration pricing dominates skimming pricing.

With this constraint, the best strategy under leasing is to set $p_1 = \frac{16}{9}$ and $p_2 = \frac{3}{2}$ so that $x_{1\ell} = \frac{4}{3}$ and $x_{2\ell} = \frac{9}{2}$. As such, the highest achievable two-period profits under leasing are[15]

$$\pi_\ell = \frac{16}{9} \cdot \frac{4}{3} + \frac{3}{2} \cdot \frac{9}{2} \simeq 9.12.$$

Now consider the alternative of a selling strategy. The monopolist's second-period strategy is still restricted by the need to penetrate the market. However, being able to build a customer base in the first period alleviates the constraint for the seller. In our example, if $x_{1S} < \frac{3}{2}$, the corresponding range for feasible x_{2S}'s is $[x_{1S}, \frac{3}{2}) \cup [\frac{9}{2}, 6]$, and the penetration-pricing constraint is similar to that in the leasing scenario. On the contrary, if $x_{1S} \geq \frac{3}{2}$, the seller can select any x_{2S} from $[3, 6]$, as shown in Figure 3. In other words, the seller is able to charge p_2 higher than $\frac{3}{2}$ while reaching the second-group consumers if and only if period-1 sales are greater than the critical amount $\frac{3}{2}$.

When the installed base of users is large enough so that $x_{1S} \geq \frac{3}{2}$, the residual demand faced by the seller is free of the problem of multiple equilibria. Consequently, any x_{2S} from $[3, 6]$ is feasible so that the seller can fully capitalize the gains from the expanded network. In the leasing scenario, however, those who rented in period 1 have to decide whether they want to rent again in period 2. There is never an installed base of users in this scenario, and hence the monopolist can reach the group-2 consumers only by charging a price lower than $\frac{3}{2}$.

The total profits under the selling regime are

$$\begin{aligned}\pi_S &= (p_1(x_{1S}) + \tilde{p}(x_{2S})) \cdot x_{1S} + \tilde{p}(x_{2S})(x_{2S} - x_{1S}) \\ &= p_1(x_{1S}) \cdot x_{1S} + \tilde{p}(x_{2S}) \cdot x_{2S}.\end{aligned}$$

Note that the seller charges period-1 buyers $p_1(x_{1S}) + \tilde{p}(x_{2S})$ so that the marginal buyer is indifferent between buying in period 1 and waiting for the price cut.[16] Besides the penetration-pricing constraint, the seller also encounters a time-inconsistency problem, i.e., she cannot commit to the first-best strategy maximizing π_S. Instead, the seller maximizes $\tilde{p}(x_{2S})(x_{2S} - x_{1S})$ in period 2 taking x_{1S} as given (see Bulow (1982)). Let $x_2^p(x_{1S})$ denote the solution to the last optimization problem subject to the penetration-pricing constraint. It is easy to see that the constraint is binding when $x_{1S} < \frac{3}{2}$,

[14] According to our criterion, the critical price at $p = \frac{3}{2}$ should have associated with $x_2 = \frac{3}{2}$. However, the monopolist is often better off by setting a price lower than but arbitrarily close to $\frac{3}{2}$, so that x_2 approximates $\frac{9}{2}$. For convenience of exposition, we simply assume that $x_2 = \frac{9}{2}$ for $p = \frac{3}{2}$.

[15] Recall that the discount rate is assumed to be zero for simplicity.

[16] The marginal buyer in period 1 always purchases the product in period 2 because his generic value is higher than any buyer's in period 2.

and non-binding otherwise. Therefore, $x_2^p(x_{1S}) = \frac{9}{2}$ for $x_{1S} < \frac{3}{2}$, while $x_2^p(x_{1S}) = \arg\max_{x_{2S}} \tilde{p}(x_{2S})(x_{2S} - x_{1S})$ for $x_{1S} \geq \frac{3}{2}$. The seller's problem in the example is then given by

$$\max_{x_{1S}, x_{2S}} (p_1(x_{1S}) + \tilde{p}(x_2^p(x_{1S}))) \cdot x_{1S} + \tilde{p}(x_{2S})(x_{2S} - x_{1S}),$$

$$s.t. \qquad x_{2S} \in [x_{1S}, \frac{3}{2}) \cup [\frac{9}{2}, 6] \quad \text{for} \quad x_{1S} < \frac{3}{2},$$

$$or \qquad x_{2S} \in [3, 6] \quad \text{for} \quad x_{1S} \geq \frac{3}{2}.$$

The optimal sales to solve the above problem are $x_{1S} = \frac{3}{2}$ and $x_{2S} \simeq 4.3$. It follows that the highest profits available to the seller are

$$\pi_\ell \simeq \frac{3}{2} \cdot \frac{3}{2} + 1.62 \cdot 4.3 = 9.23,$$

which are greater than the leasing profits. The comparison can also be obtained from the fact that the optimal leasing quantities $\left(\frac{4}{3}, \frac{9}{2}\right)$ is a feasible pair of (x_{1S}, x_{2S}) for the seller as $x_2^p\left(\frac{4}{3}\right) = \frac{9}{2}$. In general, selling is *always* more profitable than leasing as long as the penetration-pricing constraints are binding.[17]

4. THE GENERAL ANALYSIS

We shall present the general results for monopolist pricing in this section. To highlight the implications from imposing the penetration-pricing constraint, we start with the case where the monopolist is not bounded by the constraint. Specifically, the monopolist in period 2 is allowed to select any quantity on the inverse demand $\tilde{p}(x_2)$ to maximize her profits.

4.1. *The Case without Penetration-Pricing Constraints*

The monopolist lessor's aggregate profits are

$$\pi_\ell = (p_1(x_{1\ell}) - c) \cdot x_{1\ell} + \tilde{p}(x_{2\ell}) \cdot x_{2\ell} - c \cdot (x_{2\ell} - x_{1\ell}),$$

where c is the marginal cost of production. The last term is due to the fact that the monopolist does not incur the costs for what had been produced in the previous period. By rearranging the terms on the right hand side, one observes that the firm

[17] In the framework considered in Bulow (1982), one can argue that leasing is more profitable than selling because a lessor can duplicate any selling strategy. This argument does not work here due to the penetration-pricing constraint. In particular, a seller can benefit from building the network gradually: a large existing network (from previous sales) induces new consumers to pay more. However, any network in the leasing scenario is temporary. Without assurance of an existing network, new consumers are willing to rent the product only when the price is very low. Professor Hal Varian suggests that a lessor can sign a contingent contract with period-1 consumers to circumvent the problem. For example, a lessor offers the following commitment in period 1: the initial leasing price is $\frac{3}{2}$; if the technology succeeds and the network expands beyond $x_2 = 4.3$, the period-2 leasing price would be raised to 1.62; otherwise, the price remains the same. Under this commitment, in effect the lessor is able to duplicate the seller's optimal strategy. In our following analysis we rule out this unlikely scenario that the monopolist not only has to make commitments, but also price-discriminates in period 2.

maximizes $p_1(x_{1\ell})\cdot x_{1\ell}$ and $(\tilde{p}(x_{2\ell})-c)x_{2\ell}$, respectively, in the two periods. An alternative formulation of the lessor's problem is[18]

$$\max_{x_{1\ell},x_{2\ell}} \pi_\ell = (p_1(x_{1\ell}) + \tilde{p}(x_{2\ell}) - c) \cdot x_{1\ell} + (\tilde{p}(x_{2\ell}) - c)(x_{2\ell} - x_{1\ell}). \tag{4}$$

Let $MR_1(x_1)$ and $MR_2(x_2)$ denote the marginal revenue functions for period-1 and aggregate production, respectively. We have

$$MR_1(x_1) \equiv p_1(x_1) + p_1'(x_1) \cdot x_1, \qquad MR_2(x_2) \equiv \tilde{p}(x_2) + \tilde{p}'(x_2) \cdot x_2.$$

The first order conditions for (4) can then be written in terms of $MR_i's$.

$$MR_1(x_{1\ell}) = 0, \qquad MR_2(x_{2\ell}) = c. \tag{5}$$

By solving the first order conditions (5), one obtains the interior solution as follows.

$$x_{1\ell}^* = \frac{2}{3}\frac{A_1}{a_1}, \qquad x_{2\ell}^* = \frac{\tilde{A}}{3\tilde{a}} + \frac{1}{3}\sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \frac{3c}{\tilde{a}}}. \tag{6}$$

Assumption 1. $c < \frac{\tilde{A}^2}{4\tilde{a}}$.

Assumption 2. $p_1(x_{1\ell}^*) \cdot x_{1\ell}^* + (\tilde{p}(x_{2\ell}^*) - c) \cdot x_{2\ell}^* \geq \max_x (2p_1(x) - c) \cdot x$.

Assumption 1 ensures that $x_{2\ell}^*$ is well-defined.[19] It also implies $x_{2\ell}^* > \frac{\tilde{A}}{2\tilde{a}}$ and $\tilde{p}(x_{2\ell}^*) > c$. To derive $x_{2\ell}^*$ in (6), we implicitly assume that $x_{2\ell} \geq \frac{\tilde{A}}{2\tilde{a}}$ so that some of group-2 consumers will lease the product in period 2. Otherwise, the lessor can exclude group-2 consumers in period 2, and focus on leasing to group 1 in both periods. Assumption 2 makes sure that $(x_{1\ell}^*, x_{2\ell}^*)$ generates higher profits than the case of excluding group-2 consumers.[20] Intuitively, the assumption holds when the addition of group-2 consumers contributes significantly to the monopolist's profits. Roughly speaking, it is true when either their willingness to pay is high enough (A_2 is high), or the population is large enough (a_2 is low).[21] We discuss such specific conditions in Appendix.

Proposition 1. *Given Assumptions 1 and 2,* $(x_{1\ell}^*, x_{2\ell}^*)$ *in* (6) *maximizes the monopoly lessor's profits in absence of penetration-pricing constraints.*

We now consider the seller's problem. As illustrated in Section 3, the monopolist seller encounters a time-inconsistency problem so that she cannot commit to the first-best strategy adopted in the leasing scenario. Instead, the seller has to take what happened in period 1 as given, and maximizes $(\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S})$ in period 2. Let $x_2^*(x_{1S})$ denote the solution to the second-stage optimization problem.

$$x_2^*(x_{1S}) \equiv \arg\max_{x_{2S}} (\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S}).$$

[18] It is possible that $x_{2\ell} < x_{1\ell}$, in which case the last term of (4) becomes $\tilde{p}(x_{2\ell})(x_{2\ell} - x_{1\ell})$ so that the lessor does not incur production costs in period 2. In equilibrium, $x_{2\ell} > x_{1\ell}$, and thus we adopt the current formulation for π_ℓ for simplicity.

[19] An appendix that discusses the relationship, consistency, and intuition behind the assumptions in this section is available from the authors.

[20] In Appendix, we show that the maximal profits attained by the alternative strategy are $\max_x (2p_1(x) - c) \cdot x$. That is, in absence of a growing market, the lessor's optimal strategy is to lease the same amount of product in both periods.

[21] Recall that the generic value for a group-2 member is drawn from a uniform distribution in $[0, A_2]$, and the size of group-2 population is $\frac{A_2}{a_2}$. See footnote 7.

Realizing her own choice in period 1 restricts her action in period 2, the seller's problem becomes

$$\max_{x_{1S},x_{2S}} \pi_S = (p_1(x_{1S}) + \tilde{p}(x_2^*(x_{1S})) - c)x_{1S} + (\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S}). \tag{7}$$

Program (7) is different from (4) in the term $\tilde{p}(x_2^*(x_{1S}))$, which represents the consumers' rational expectation. The first order conditions for (7) are

$$MR_1(x_{1S}) = -\frac{\partial \tilde{p}(x_2^*(x_{1S}))}{\partial x_{1S}} x_{1S}, \tag{8}$$

$$MR_2(x_{2S}) = c + \tilde{p}'(x_{2S})x_{1S}. \tag{9}$$

Let (x_{1S}^*, x_{2S}^*) denote the solution to (7). The following assumption ensures that one obtains (x_{1S}^*, x_{2S}^*) by solving the first order conditions.

Assumption 3. The profits attained by the solution to (8) and (9) are higher than those attained by maximizing π_S while excluding group-2 consumers.

The mathematical expression for Assumption 3 is provided in (A11). The right hand side of (8) is positive and attests to the fact that increasing period-1 production reduces the period-2 price, which in turn reduces the seller's revenue in period 1. Given this side effect of extra production in period 1, one concludes that $x_{1S}^* < x_{1\ell}^*$. As for (9), the last term is due to the fact that the seller only loses $\tilde{p}'(x_{2S})(x_{2S} - x_{1S})$ rather than $\tilde{p}'(x_{2S}) \cdot x_{2S}$ from price reduction. Since $\tilde{p}'$ is negative at stable equilibrium, the right hand side is less than c. Comparing with (5), one obtains a higher aggregate production in the selling scenario: $x_{2S}^* > x_{2\ell}^*$. The explicit formulation for (x_{1S}^*, x_{2S}^*) is more complicated, but we can make the following comparison:

Proposition 2. *Given Assumptions 1 and 3, the solution to* (8) *and* (9) *maximizes the monopolist seller's profits in absence of penetration-pricing constraints.*

Proposition 3. *Given Assumptions 1 to 3, the seller produces less than the lessor does in period 1, but the total quantity up to period 2 is higher; that is,* $x_{1S}^* < x_{1\ell}^*$ *and* $x_{2S}^* > x_{2\ell}^*$. *Moreover, leasing is more profitable than selling.*

In sum, when we incorporate network effects and a growing market into our theoretical framework, the patterns of pricing strategies are consistent with those in Bulow (1982), so long as the penetration-pricing constraint is ignored.

4.2. *The Case with a Penetration-Pricing Constraint*

The monopolist's pricing strategy will be much different when she takes into account the penetration-pricing constraint. Recall that $\hat{x}_1$ is the critical value at which $p_1(\hat{x}_1) = \tilde{p}(\hat{x}_1) \equiv \hat{p}$ (see Figure 2); $\hat{x}_1 = \frac{A_1 - A_2}{a_1}$. A related critical value is $\hat{x}_2 = \frac{A_2}{\tilde{a}}$, which implies the same price: $\tilde{p}(\hat{x}_2) = \hat{p}$.

For the monopolist lessor, the range for feasible $x_{2\ell}'s$ is $[\frac{A_1}{2a_1}, \hat{x}_1) \cup [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$. In other words, the lessor cannot penetrate the period-2 market unless the price is lower than

$\hat{p}$.[22] Therefore, the lessor's problem is

$$\max_{x_{1\ell},x_{2\ell}} \pi_\ell = (p_1(x_{1\ell}) + \tilde{p}(x_{2\ell}) - c) \cdot x_{1\ell} + (\tilde{p}(x_{2\ell}) - c)(x_{2\ell} - x_{1\ell}), \tag{10}$$
$$s.t. \qquad x_{2\ell} \in [\frac{A_1}{2a_1}, \hat{x}_1) \cup [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$$

Let $(x^p_{1\ell}, x^p_{2\ell})$ denote the solution to (10).

Assumption 4. $x^*_{2\ell} < \hat{x}_2$.

Assumption 5. $p_1(x^*_{1\ell}) \cdot x^*_{1\ell} + (\tilde{p}(\hat{x}_2) - c) \cdot \hat{x}_2 \geq \max_x (2p_1(x) - c) \cdot x$.

We are more interested in the case where the penetration-pricing constraint is binding, which requires the unconstrained solution $x^*_{2\ell}$ be lower than $\hat{x}_2$. As in Section 4.1, we further assume that some of the group-2 members lease the product in equilibrium so that π_ℓ is maximized at $x_{2\ell} \in [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$ rather than $[\frac{A_1}{2a_1}, \hat{x}_1)$. Since Assumption 4 implies that π_ℓ is decreasing in $x_{2\ell}$ for $x_{2\ell} \in [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$, one concludes that π_ℓ is maximized at $x^p_{2\ell} = \hat{x}_2$.

Proposition 4. *Given Assumptions 1, 4, and 5, the monopolist lessor's optimal strategy under the penetration-pricing constraint is* $(x^p_{1\ell}, x^p_{2\ell}) = (x^*_{1\ell}, \hat{x}_2)$.

For the monopolist seller, the restrictions imposed by the penetration-pricing constraint depend on how much she sells in period 1. If x_{1S} is lower than the critical amount $\hat{x}_1$, the seller faces a similar constraint as in (10), so that the feasible $x'_{2S}s$ are limited to $[x_{1S}, \hat{x}_1) \cup [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$. In other words, when $x_{1S} < \hat{x}_1$, the seller has two options: she can either charge a high price ($p_2 > \hat{p}$) that only appeals to group-1 consumers, or a low price ($p_2 \leq \hat{p}$) that reaches group-2 consumers. These options correspond to the intervals $[x_{1S}, \hat{x}_1)$ and $[\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$, respectively. On the contrary, if x_{1S} is greater than $\hat{x}_1$, the seller has built a customer base large enough to penetrate the market in period 2, so that she can select any x_{2S} from $[\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}]$. The penetration-pricing constraint has no effect in this case. For the constraint to be binding in equilibrium, we assume

Assumption 6. $x^*_{1\ell} < \hat{x}_1$.

Assumption 7. $x^*_2(\hat{x}_1) < \hat{x}_2$.

Now we look at the seller's problem. Given the time-consistency constraint, for any x_{1S}, the seller maximizes $(\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S})$ subject to the penetration-pricing constraint corresponding to x_{1S}. Let $x^p_2(x_{1S})$ denote the solution to the seller's constrained problem in period 2. Provided Proposition 3 ($x^*_{1S} < x^*_{1\ell}$), Assumption 6 implies that $x^*_{1S} < \hat{x}_1$. As we know that $x^*_2(\cdot)$ is an increasing function, Assumption 7 implies $x^*_2(x_{1S}) < \hat{x}_2 \ \forall x_{1S} < \hat{x}_1$. As such, one concludes that the unconstrained solution $x^*_{2S}(= x^*_2(x^*_{1S}))$ in Section 4.1 is lower than $\hat{x}_2$. In short, Assumptions 6 and 7 imply a binding penetration-pricing constraint so that (x^*_{1S}, x^*_{2S}) is not achievable.

Given these assumptions, one obtains $x^p_2(x_{1S})$ as follows. For $x_{1S} < \hat{x}_1$, the interior solution $x^*_2(x_{1S})$ is not achievable, and thus $x^p_2(x_{1S}) = \hat{x}_2$ or $x'_2(x_{1S})$.[23] For $x_{1S} \geq \hat{x}_1$,

[22] For the parameters assumed in the example, one can verify that $\frac{A_1}{2a_1} = 1$, $\hat{x}_1 = \frac{3}{2}$, $\hat{x}_2 = \frac{9}{2}$, and $\frac{\tilde{A}}{\tilde{a}} = 6$.

[23] $x'_2(x_{1S}) \equiv \arg\max_{x_{2S}} (p_1(x_{2S}) - c)(x_{2S} - x_{1S})$ maximizes the seller's period-2 profits when she excludes group-2 consumers; see (A10) in Appendix.

the seller is able to penetrate the market in period 2, and $x_2^p(x_{1S}) = x_2^*(x_{1S})$. Given $x_2^p(x_{1S})$, the seller's optimization problem is

$$\max_{x_{1S},x_{2S}} \pi_S^p = (p_1(x_{1S}) + \tilde{p}(x_2^p(x_{1S})) - c)x_{1S} + (\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S}),$$

$$s.t. \qquad x_{2S} \in [x_{1S}, \hat{x}_1) \cup [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}] \quad \text{for} \quad x_{1S} < \hat{x}_1, \tag{11}$$

$$or \qquad x_{2S} \in [\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}] \quad \text{for} \quad x_{1S} \geq \hat{x}_1.$$

To solve (11), we maximize π_S^p separately over $x_{1S} < \hat{x}_1$ and $x_{1S} \geq \hat{x}_1$. In the former case, $x_2^p(x_{1S}) = \hat{x}_2$ or $x_2'(x_{1S})$. It follows from Assumption 5 that $(x_{1\ell}^*, \hat{x}_2)$ maximizes π_S^p. Note that this solution is identical to that in the leasing scenario in Proposition 4. In the latter case, $x_2^p(x_{1S}) = x_2^*(x_{1S})$. Given that the unconstrained solution $x_{1S}^* < \hat{x}_1$ and that $\partial^2\pi_S/\partial x_{1S}^2 < 0$, the objective function π_S^p is decreasing in x_{1S} for the relevant range and therefore is maximized at $x_{1S} = \hat{x}_1$. One compares $\pi_S^p(x_{1\ell}^*, \hat{x}_2)$ against $\pi_S^p(\hat{x}_1, x_2^*(\hat{x}_1))$ to determine the solution to (11). In summary, we have

Proposition 5. *Given Assumptions 1, 5, 6, and 7, the monopolist seller's optimal strategy under the penetration-pricing constraint, (x_{1S}^p, x_{2S}^p), is either $(x_{1\ell}^*, \hat{x}_2)$ or $(\hat{x}_1, x_2^*(\hat{x}_1))$.*

When comparing $(x_{1\ell}^*, \hat{x}_2)$ and $(\hat{x}_1, x_2^*(\hat{x}_1))$, the seller trades off her profits in two periods. In the former strategy, the seller is bounded in period 2 by the penetration-pricing constraint, but she is able to maximize her period-1 profits. In the latter strategy, the seller sacrifices her period-1 profits in order to penetrate the market and capture bigger gains in period 2. If the period-2 market is lucrative enough, as we assume in this paper, the latter strategy will be optimal.

As we show above, one derives very different implications when taking into account the penetration-pricing constraint. First of all, leasing is no longer more profitable than selling. The leasing profits are reduced because the lessor has to lower the period-2 price in order to reach group-2 consumers. In contrast, the seller's loss occurs mostly in period 1. Therefore, the constraint affects the lessor's and the seller's profits differently.

Corollary 6. *Following Proposition 5, one obtains $\pi_\ell^p \leq \pi_S^p$ in equilibrium. Therefore, selling is more profitable than leasing under the penetration-pricing constraint.*

The assertion follows from the fact that the leasing solution $(x_{1\ell}^*, \hat{x}_2)$ is feasible in the selling regime, as $x_2^p(x_{1\ell}^*) = \hat{x}_2$. The inequality in the corollary holds strictly when A_2 is large enough or a_2 is small enough. Recall that $\frac{A_2}{2}$ is the average willingness to pay by group-2 consumers, and a_2 represents the size of population: the lower a_2 is, the larger population of group-2. Therefore, Corollary 6 implies that when the monopolist expects a significant growth of the market, she will prefer selling rather than leasing the product.

Another implication of Bulow (1982) is that in equilibrium $x_{1S} < x_{1\ell}$ while $x_{2S} > x_{2\ell}$, which is upheld when network effects are present but the penetration-pricing constraint is ignored. However, when the monopolist is bounded by the constraint, the comparisons are reversed.

Corollary 7. *Proposition 5 implies that $x_{1S}^p \geq x_{1\ell}^p$ and $x_{2S}^p \leq x_{2\ell}^p$ in equilibrium.*

The economic intuition for Corollary 7 is as follows. When A_2 is large or a_2 is small, the market in period 2 is lucrative. In absence of penetration-pricing constraints, both the lessor and the seller would like to charge a relatively high price in period 2. However, the constraint eliminates the feasibility of this strategy. Essentially, the seller is left with two options: she can either lower p_1 and sacrifice profits at the beginning in the hope that she can capture the period-2 market later; or, she sets a very low p_2 at $\hat{p}$ but is able to charge whatever she wants in period 1. With a large population for group 2, the former option is more profitable. In contrast, the lessor does not have access to this option so that she can only set her period-2 price at $\hat{p}$. Therefore, the comparisons are as indicated.

It is important to note that Corollary 7 implies a relatively high x_{1S}^p, which results in a low p_1. Therefore, the equilibrium outcome in the presence of the penetration-pricing constraint suggests the practice of introductory pricing. One often observes that the firms in a market with network effects adopt an aggressive pricing strategy initially in the hope to capture the market in the later stages. This commonly seen practice is justified theoretically in our setting only when the monopolist is confined by the penetration-pricing constraint.

5. THE OPTIMAL R&D DECISION

Another difficult problem faced by a monopolist, as Bulow (1982, pp. 321-323) showed, is the inertia of devoting R&D expenses to reduce marginal production cost. Here we discuss the issue taking into account the network effect. Suppose the monopolist originally faces a constant marginal cost c in both periods. If she devotes additional y resources to R&D in the first period, her marginal cost will reduce by $g(y)$. Thus, for the leasing scenario, the two-period total profits become

$$\pi_\ell = -y + p_1(x_{1\ell}) \cdot x_{1\ell} + (\tilde{p}(x_{2\ell}) - (c - g(y))) \cdot x_{2\ell}. \tag{12}$$

The efficient R&D, denoted y^*, can be obtained by maximizing π_ℓ with respect to $x_{1\ell}$, $x_{2\ell}$, and y. Bulow showed that this efficient y^* cannot be obtained under a selling scenario, because a lower second-period marginal cost implies a larger second-period sale, which in turn suggests a lower second-period price. Foreseeing this, the first-period buyers' willingness to pay will be lower.[24] Given this side effect of cost-reducing R&D, the monopolist would have less incentive to devote R&D expenditure to reduce the cost under the selling regime.

The situation would be very different under the scenario with network effects. Let us first look at the ideal case under leasing without any constraints. The first order conditions are

$$MR_1(x_{1\ell}) = 0, \quad MR_2(x_{2\ell}) = c - g(y), \quad \text{and } \ 1 = g'(y)x_{2\ell}. \tag{13}$$

The last condition implies that an extra dollar of investment reduces the total cost by one dollar so that the monopolist breaks even at y^*.

The monopolist seller is bounded by the time-consistency constraint. Disregarding

[24] Recall that the first-period buyers pay the sum of $p_1(x_1)$ and $\tilde{p}(x_2)$.

the penetration-pricing constraint, the seller's problem is

$$\max_{x_{1S},x_{2S},y} \pi_S = -y + (p_1(x_{1S}) + \tilde{p}(x_2^*(x_{1S},y)) - (c - g(y))) \cdot x_{1S} \\ + (\tilde{p}(x_{2S}) - (c - g(y)))(x_{2S} - x_{1S}), \quad (14)$$

where $x_2^*(x_{1S}, y) \equiv \arg\max_{x_{2S}} (\tilde{p}(x_{2S}) - (c - g(y)))(x_{2S} - x_{1S})$ is obtained by maximizing the seller's period-2 profits, taking x_{1S} and y as given. The first order conditions to (14) are

$$\begin{aligned} MR_1(x_{1S}) &= -\frac{\partial \tilde{p}(x_2^*(x_{1S},y))}{\partial x_{1S}} x_{1S}, \\ MR_2(x_{2S}) &= c - g(y) + \tilde{p}'(x_{2S}) \cdot x_{1S}, \\ 1 &= g'(y)x_{2S} + \frac{\partial \tilde{p}(x_2^*(x_{1S},y))}{\partial y} x_{1S}. \end{aligned} \quad (15)$$

The right hand side of the last condition consists of two different impacts from R&D activities in the presence of the time-consistency constraint. The first is cost reduction, represented by $g'(y)x_{2S}$. Since the cost-saving function g is usually assumed concave, we know that $g'(\cdot)$ is decreasing in y. Therefore, a higher aggregate quantity x_2 associates with a higher R&D investment level. As the seller will produce more in total ($x_{2S}^* > x_{2\ell}^*$), the cost-reduction effect induces more R&D by the seller. The second effect is what has been argued by Bulow (1982), and characterized here by $\partial \tilde{p}(x_2^*(x_{1S}, y))/\partial y \cdot x_{1S}$: more intensive R&D implies lower willingness to pay in the first period ($\partial \tilde{p}/\partial y < 0$), and thus the seller has less incentive for R&D input. With the two forces working in the opposite directions, how much the seller will invest comparing to y^* depends on which effect dominates. Proposition 8 below shows that the cost saving induced by R&D is not enough to compensate the loss from price reduction, and thus $y_S^* < y^*$.[25]

When the penetration-pricing constraint is in force, the R&D investment y^* is no longer optimal under the leasing regime. Assuming the saving in production cost is only marginal, the penetration-pricing constraint will still be binding, and thus R&D activities do not affect the total quantity: $x_{2\ell}^p$ is still equal to $\hat{x}_2$ even if the monopolist has lowered the marginal cost by $g(y)$. The first order condition, $1 = g'(y)\hat{x}_2$, determines the optimal level of investment in this scenario. One observes that only the cost-reduction effect is present, and thus the lessor will over-invest in R&D because $x_{2\ell}^p > x_{2\ell}^*$.

In the selling scenario, suppose the presumptions in Proposition 5 hold so that $(x_{1S}^p, x_{2S}^p) = (\hat{x}_1, x_2^*(\hat{x}_1))$. The impacts from R&D investment are both enhanced by the penetration-pricing constraint, albeit in the opposite directions. That is, the seller saves even more from cost reduction as the total sales are higher ($x_{2S}^p > x_{2S}^*$). Meanwhile, the seller loses more in period 1 as the first-period sales are also higher ($x_{1S}^p > x_{1S}^*$). Proposition 8 indicates that the net effect implies less incentive for R&D so that the level of investment y_S^p will be lower than y_S^*.

Proposition 8. *Given assumptions in Propositions 4 and 5, and further assuming that $x_{1S}^p = \hat{x}_1$, the levels of R&D investment are ranked as follows.*

$$y_S^p < y_S^* < y^* < y_\ell^p.$$

[25]Note that Bulow (1982) did not mention the cost-reduction effect explicitly. Nonetheless, for linear demand considered in Bulow's model or quadratic inverse demand in ours, one can show that the incentive due to cost reduction is dominated by the disincentive from price reduction, assuming penetration-pricing constraint is not in effect.

Note that we derive y_S^p in Proposition 8 by assuming that $(x_{1S}^p, x_{2S}^p) = (\hat{x}_1, x_2^*(\hat{x}_1))$. Depending on the demand parameters (A_g's and a_g's), the seller's optimal strategy could be $(x_{1\ell}^*, \hat{x}_2)$, which is identical to the lessor's strategy (see Proposition 5). In this case, $y_S^p = y_\ell^p > y^*$, and the seller will *over*-invest in R&D activities.

In sum, when determining the R&D investment, the monopolist seller has to take into account the cost-reduction effect in addition to the disincentive caused by the rational expectations of consumers. The concern of cost reduction encourages R&D input if and only if the seller expects to produce more by the second period. The aggregate outcome could be under-investment or over-investment by the seller, depending on which effect dominates. In contrast, the lessor always over-invests.

6. PLANNED OBSOLESCENCE

In reality, two-period durable goods may become out of order after one period of use. The probability of this obsolescence may be reduced if the monopolist devotes more resources to improve the product quality. Let z be the amount devoted to quality improvement, c be the marginal cost of production, and $h(z)$ be the probability of sustaining a two-period product life, with $h'(z) > 0$ and $h''(z) < 0$. Suppose x_1 is the period-1 production. Leaving aside consumers' strategic concerns, an efficient investment on z is to minimize $z + c(1 - h(z))x_1$, which generates the first order condition $1 - ch'(z)x_1 = 0$. The optimal z^* so derived will correspond to the efficient rate of obsolescence $1 - h(z^*)$.

Taking into account the rational expectations of consumers, however, Bulow (1986) showed that there is a factor that prevents the achievement of efficiency. Increasing durability has the impact of increasing the number of commodity units in the second period, which in turn implies a lower second-period price. This not only reduces the second-period profits, but also reduces the willingness to pay by rational first-period potential buyers. Thus, increasing durability has an extra negative effect of reducing profits.

When the commodity in question has a network effect, let us look at the leasing regime first. The lessor's problem is

$$\max_{x_{1\ell}, x_{2\ell}, z} -z + (p_1(x_{1\ell}) + h(z)\tilde{p}(x_{2\ell}) - c) \cdot x_{1\ell} + (\tilde{p}(x_{2\ell}) - c)(x_{2\ell} - h(z)x_{1\ell}),$$
$$s.t. \qquad x_{2\ell} \le \hat{x}_1 \quad or \quad x_{2\ell} \ge \hat{x}_2. \tag{16}$$

The first order conditions to the *unconstrained* problem are

$$MR_1(x_{1\ell}) = c(1 - h(z)), \quad MR_2(x_{2\ell}) = c, \quad 1 = ch'(z)x_{1\ell}. \tag{17}$$

We showed in Proposition 4 that the leasing scenario with the penetration-pricing constraint suggests $x_{1\ell}^p = x_{1\ell}^*$ and $x_{2\ell}^p = \hat{x}_2$. Note that the second-period production is $x_{2\ell}^p - h(z)x_{1\ell}^p$, where the last term being multiplied by $h(z)$ is due to the obsolescence loss of the first-period sales. When R&D investment z increases, $h(z)$ increases, but the second-period production is just to retain the total leasing amount $\hat{x}_2$; the second-period price $\tilde{p}(\hat{x}_2) = \hat{p}$ remains the same. Thus, commodity obsolescence does not affect the pricing strategy in period 2 under the leasing regime. Furthermore, one observes that the first and last conditions in (17) also determine the *constrained* solutions of x_1 and z. Therefore, the monopolist lessor can still achieve the first-best level of investment under the penetration-pricing constraint, and hence $z_\ell^p = z^*$.

We now move to the selling scenario. The monopolist seller's problem is

$$
\begin{aligned}
\max_{x_{1S}, x_{2S}, z} & -z + (p_1(x_{1S}) + h(z)\tilde{p}(x_2^p(x_{1S}, z)) - c)x_{1S} + (\tilde{p}(x_{2S}) - c)(x_{2S} - h(z)x_{1S}), \\
s.t. \quad & x_{2S} \in [h(z)x_{1S}, \hat{x}_1) \cup [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}] \quad \text{for} \quad h(z)x_{1S} < \hat{x}_1, \qquad (18) \\
or \quad & x_{2S} \in [\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}] \quad \text{for} \quad h(z)x_{1S} \geq \hat{x}_1,
\end{aligned}
$$

where $x_2^p(x_{1S}, z)$ is the solution that maximizes $(\tilde{p}(x_{2S}) - c)(x_{2S} - h(z)x_{1S})$ subject to the same constraints. The first order conditions to the *unconstrained* problem are

$$
\begin{aligned}
MR_1(x_{1S}) &= c(1 - h(z)) - h(z)\frac{\partial \tilde{p}(x_2^*(x_{1S}, z))}{\partial x_{1S}} x_{1S}, \\
MR_2(x_{2S}) &= c + h(z)\tilde{p}'(x_{2S})x_{1S}, \qquad (19) \\
1 &= ch'(z)x_{1S} + h(z)\frac{\partial \tilde{p}(x_2^*(x_{1S}, z))}{\partial z} x_{1S},
\end{aligned}
$$

where $x_2^*(x_{1S}, z) \equiv \arg\max_{x_{2S}}(\tilde{p}(x_{2S}) - c)(x_{2S} - h(z)x_{1S})$. The last condition in (19) suggests that the effort to enhance product durability has two consequences. The direct effect is cost-saving, as characterized by $ch'(z)x_{1S}$; and the indirect effect is the loss caused by the consumers' rational expectations, which is the main theme of Bulow (1982, 1986).

Unlike the case in Section 5 for R&D investment, both the direct and indirect effects mentioned above work in the same direction to reduce the seller's incentive to improve durability. In the last section, the monopolist saves on every unit she produces due to a lower marginal cost. In contrast, a lower obsolescence rate leaves less defective units to replace in the beginning of period 2. In other words, the cost-saving effect here only involves period-1 production. In the unconstrained equilibrium, the seller produces less in period 1 and more in total comparing to the lessor ($x_{1S}^* < x_{1\ell}^*$ and $x_{2S}^* > x_{2\ell}^*$). Therefore, the cost-saving effect discourages the seller to improve durability. Combining with consumers' expectations of price reduction, one concludes that $z_S^* < z^*$.

When considering the constrained problem in (18), one can see that the indirect effect disappears, and the optimal level of z is given by $1 = ch'(z)x_{1S}$. Following the argument for Proposition 5, the seller's goal is to establish a customer base large enough to penetrate the period-2 market. In particular, the first-period sale has to be $\hat{x}_1/h(z)$ so that the quantity after the obsolescence loss is $\hat{x}_1$. It is easy to see that $x_2^p(\hat{x}_1/h(z), z) = x_2^*(\hat{x}_1)$, which implies $\partial x_2^p(x_{1S}, z)/\partial z = 0$, and hence the condition $1 = ch'(z)x_{1S}$.

It follows that $z_S^p > z^*$ if and only if $x_{1S}^p > x_{1\ell}^*$, which is true under the presumptions similar to those in Proposition 5 that ensure $(\hat{x}_1/h(z), x_2^*(\hat{x}_1))$ solves (18). In other words, when the period-2 market is lucrative enough, the seller will over-invest to make the product too durable.

Proposition 9. *Given assumptions in Propositions 4 and 5, and further assuming that $x_{1S}^p = \hat{x}_1/h(z)$, the efforts devoted to quality improvement are ranked as follows.*

$$z_S^* < z_\ell^p = z^* < z_S^p.$$

The commodity obsolescence has one more implication that was not discussed in the literature. Under the selling regime, if the second-period price is $\tilde{p}(x_{2S}^p)$, the first-period willingness to pay would be

$$p_1(x^p_{1S}) + h(z^p_S)\tilde{p}(x^p_{2S}).$$

A higher obsolescence rate not only increases the production cost of the *out-of-order* goods, it also reduces the proportion of price capitalization of *all* period-1 sales. Recall that one reason we obtain the result that selling may be better than leasing in Section 4 is that, by setting the first-period production equal to the critical amount needed for penetration pricing, the monopolist can capitalize a higher second-period price. However, with commodity obsolescence, this capitalization is shrunk by a proportion of $h(z)$. In fact, it can be seen that a higher rate of obsolescence makes the leasing regime more attractive than the selling one, because the latter's maximum profits decrease more as $h(z)$ reduces.

7. CONCLUSION

In this paper, we study the dynamic pricing problem of a monopolist who produces durable goods with network effects. Assuming that the market grows over time, we derive double-hump shaped demands that are commonly seen in network-effect literature. Due to the shape of the demand, the monopolist encounters a penetration-pricing constraint. Nevertheless, the constraint imposes different restrictions depending on whether the monopolist leases or sells the product. In the leasing scenario, the lessor can only reach the less-informed consumers by lowering the price. In contrast, the seller has the option to flood the market early on, so that she can exercise her market power in the expanded marketplace. Given the seller's strategic flexibility, the profits from selling the product are higher than from leasing, as long as the expanded market is lucrative enough.

The working of our model can also be understood from the concept of commitment. The advantage of the seller derives from her ability to establish an installed base of users. In contrast, the lessor cannot imitate the same strategy due to the lack of commitment from the renters. Specifically, the renters in period 1 will have to make leasing decisions once again in period 2. In other words, they cannot promise to stay in the network, and thus undercut the network effects they generate. As a result, the lessor would be forced to exercise penetration pricing unless he can conceive some commitment device that "locks-in" his customers by raising switching costs or through contractual agreements, etc.

From this perspective, we can better understand the different results between our model and that of Coase-Bulow in which the time-inconsistency problem puts the seller in a disadvantageous position. First, without network effects, the lack of commitment power occurs only on the supply side (specifically, the seller) in the Coase-Bulow model, whereas under the influence of network effects it arises on both the supply (the seller) and demand (the renters) sides in our model. The supply-side commitment problem affects the seller's profitability, while the demand-side problem induces the penetration-pricing constraint and reduces the lessor's profits. In our model the latter effect can dominate the former so that leasing is less profitable than selling. Second, as far as the renters are concerned, their lack of commitment power actually benefits themselves as the lessor has to choose a lower equilibrium price and a larger network in period 2. Thus, the renters have no incentive to lock themselves in for a long term arrangement. The same lack of commitment power, in contrast, will disadvantage the seller and compel her to commit herself to a lower future production.

Another important result of this paper is that we establish introductory pricing as the monopolist seller's optimal strategy. This practice can be identified in two parts. First, a lower initial price. To achieve the building of an installed base, the seller adopts an aggressive pricing strategy that makes a much larger sale in period 1 than imputed by the first-best strategy. Second, the upward trend of prices. As we have repeatedly shown, due to the penetration pricing constraints, only in the selling scenario will the monopolist be able to take advantage of the growth with prices raised. The practice of introductory pricing can be intuitively understood in that the monopolist seller follows an aggressive strategy if and only if the potential of the expanded market is worthwhile for her to sacrifice the profits in the early stage. Such trade-off, as we have shown, is made necessary by the penetration-pricing constraint.

Furthermore, the monopolist's incentive in R&D activities also displays a unique dynamic in our model. If the R&D expenses contribute to reduction of marginal production cost, the lessor will over-invest in R&D, while the seller may under-invest or over-invest. Note that the cost saving induced by R&D associates with the aggregate production. Therefore, if the optimal aggregate production is very large, it may stimulate too much R&D investment.

The implication becomes even more dramatic if the R&D efforts result in better product durability. The costs saved by lowering the obsolescence rate associate only with the first-period production. However, the penetration-pricing constraint restricts the monopolist's choice of equilibrium prices in the second period. Consequently, the second-period prices are independent of the obsolescence rate, and the distortion from the consumers' expectation of overproduction disappears. Therefore, the monopolist seller will make the product too durable because of her large first-period production.

In sum, by incorporating network effects into the model, this paper demonstrates the crucial role played by the penetration-pricing constraint in the monopolist's dynamic pricing problem. Indeed, without the constraint, a growing market with network effects alone does not warrant our conclusions. In doing so, we provide a simplistic and tractable approach to model the market with network effects. It equips us for further studies of network-effect economy.

Appendix

Proof of Proposition 1

As we only consider the stable equilibria, the *effective* demand faced by the monopolist is the downward-sloping part of $p_1(x)$ or $\tilde{p}(x)$. Due to the double-hump shape of $\tilde{p}(x)$, in absence of penetration-pricing constraints, the effective inverse demand in period 2 consists of

$$p = (\tilde{A}x_2 - \tilde{a}x_2^2) \cdot x_2 \quad \text{for} \quad x_2 \in [\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}], \tag{A1}$$

$$p = (A_1 x_2 - a_1 x_2^2) \cdot x_2 \quad \text{for} \quad x_2 \in [\frac{A_1}{2a_1}, \tilde{x}_1), \tag{A2}$$

where $\tilde{x}_1$ is such that $p_1(\tilde{x}_1) = \max \tilde{p}(x)$ with $\tilde{x}_1 \geq \frac{A_1}{2a_1}$.[26] By solving the first order

[26] $\tilde{x}_1$ is not well-defined if $\max p_1(x) < \max \tilde{p}(x)$, in which case (A2) vanishes and the effective demand consists of only (A1).

conditions as in (5),

$$MR_1(x_{1\ell}) = 2A_1 x_{1\ell} - 3a_1 x_{1\ell}^2 = 0, \tag{A3}$$

$$MR_2(x_{2\ell}) = 2\tilde{A} x_{2\ell} - 3\tilde{a} x_{2\ell}^2 = c, \tag{A4}$$

one obtains the solution in (6) as follows.

$$x_{1\ell}^* = \frac{2}{3}\frac{A_1}{a_1}, \qquad x_{2\ell}^* = \frac{\tilde{A}}{3\tilde{a}} + \frac{1}{3}\sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \frac{3c}{\tilde{a}}}. \tag{A5}$$

Note that the solution to (A4) only takes into account the segment of demand in (A1). An alternative strategy for the lessor is to lease to group-1 consumers exclusively in both periods so that the second-period demand is represented by (A2). To show that (A5) indeed maximizes π_ℓ, we have to demonstrate (i) (A5) maximizes π_ℓ over $x_{2\ell} \in [\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}]$, and (ii) (A5) generates higher profits than the alternative strategy that excludes group-2 consumers.

The first statement is easily verified geometrically: the objective function $(\tilde{p}(x_{2\ell}) - c) \cdot x_{2\ell}$ is a degree-three polynomial that has a root of zero and two positive roots. $x_{2\ell}^*$ in (A5) is the larger root to the quadratic equation (A4), and thus achieves the local maximum. It remains to show that the maximum is positive. It is equivalent to showing that $\tilde{p}(x_{2\ell}^*) - c > 0$. The inequality reduces to $c < \frac{\tilde{A}^2}{4\tilde{a}}$, which is assured by Assumption 1.

For the second part of the proof, we first derive the optimal strategy when the lessor excludes group-2 consumers. In what follows, we will show that when the demand is static, the lessor's production in period 2 is zero so that $x_{2\ell} = x_{1\ell}$. For any leasing quantity $x_{1\ell}$ in period 1, the subsequent optimal strategy in period 2 depends on $x_{1\ell}$. Thus, we will derive the optimum in two stages. Define $x_1' \equiv \frac{A_1}{3a_1} + \frac{1}{3}\sqrt{(\frac{A_1}{a_1})^2 - \frac{3c}{a_1}}$. We consider the following three cases. For $x_{1\ell} \le x_1'$, the optimal strategy corresponding with $x_{1\ell}$ is $x_{2\ell} = x_1'$. For $x_{1\ell} \ge \frac{2A_1}{3a_1}$, the optimal $x_{2\ell}$ is $\frac{2A_1}{3a_1}$. Finally, for $x_{1\ell} \in (x_1', \frac{2A_1}{3a_1})$, the optimal $x_{2\ell}$ is equal to $x_{1\ell}$. One observes that π_ℓ is maximized at $x_{1\ell} = x_{2\ell} = x_1'$ if $x_{1\ell}$ is confined to be weakly lower than x_1'. Similarly, π_ℓ is maximized at $x_{1\ell} = x_{2\ell} = \frac{2A_1}{3a_1}$ if $x_{1\ell} \ge \frac{2A_1}{3a_1}$. One concludes that $x_{1\ell} = x_{2\ell}$ is necessary to maximize π_ℓ when the demand does not change over time. In short, the lessor's problem in this scenario is to maximize $(2p_1(x) - c) \cdot x$. Note that the solution lies between x_1' and $\frac{2A_1}{3a_1}$. Comparing the profits and $\pi(x_{1\ell}^*, x_{2\ell}^*)$, one requires

$$p_1(x_{1\ell}^*) \cdot x_{1\ell}^* + (\tilde{p}(x_{2\ell}^*) - c) \cdot x_{2\ell}^* \ge \max_x (2p_1(x) - c) \cdot x \tag{A6}$$

for (A5) to be the solution.

Define $x_\ell' \equiv \arg\max_x (2p_1(x) - c) \cdot x$. A sufficient condition for (A6) to hold is to assume that $p_1(x_\ell') \le \frac{\tilde{A}^2}{4\tilde{a}}$, or equivalently, $x_\ell' \ge \tilde{x}_1$. The left hand side is the price for the alternative strategy, while the right hand side is the local maximum of the second hump of $\tilde{p}(x)$. Given this inequality, (A6) would hold because there exists a strategy that is more profitable than (x_ℓ', x_ℓ'). Specifically, by choosing $x_{2\ell}'$ such that $\tilde{p}(x_{2\ell}') = p_1(x_\ell')$, $(x_\ell', x_{2\ell}')$ is more profitable than (x_ℓ', x_ℓ'). Note that $x_{2\ell}'$ is well-defined if $p_1(x_\ell') \le \frac{\tilde{A}^2}{4\tilde{a}}$.

The condition (A6) involves the parameters such as $A_g's$, $a_g's$, and c. In the special case with zero marginal cost, it reduces to $\frac{\tilde{A}^3}{\tilde{a}^2} \ge \frac{A_1^3}{a_1^2}$. ‖

Proofs of Proposition 2 and Proposition 3

The proof is analogous to the previous proof. We first derive (x^*_{1S}, x^*_{2S}) by maximizing π_S over $x_{2S} \geq \frac{\tilde{A}}{2\tilde{a}}$. Then we characterize the conditions under which this solution dominates those that exclude group-2 consumers.

Given the time-consistency constraint, for any x_{1S}, the seller has to maximize her profits in period 2 as follows.

$$\begin{aligned} x^*_2(x_{1S}) &\equiv \arg\max_{x_{2S}} (\tilde{A}x_{2S} - \tilde{a}x^2_{2S} - c)(x_{2S} - x_{1S}) \\ &= \frac{1}{3}\left(\frac{\tilde{A}}{\tilde{a}} + x_{1S}\right) + \frac{1}{3}\sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \left(\frac{\tilde{A}}{\tilde{a}}\right)x_{1S} + x^2_{1S} - \frac{3c}{\tilde{a}}}. \end{aligned} \tag{A7}$$

Note that the above formulation for $x^*_2(x_{1S})$ is the larger root to the quadratic first order condition. For the above objective function, the coefficient of x^3_{2S} is negative $(-\tilde{a})$, and thus (A7) achieves the local maximum. The period-2 profits attained by $x^*_2(x_{1S})$ are positive if and only if the objective function as a polynomial of x_{2S} has three roots, or equivalently, $(\tilde{A}x_{2S} - \tilde{a}x^2_{2S} - c)$ has two roots. Therefore, we require that $\tilde{A}^2 - 4\tilde{a}c > 0$ (or, Assumption 1: $c < \frac{\tilde{A}^2}{4\tilde{a}}$), which implies $\tilde{p}(x^*_2(x_{1S})) > c \ \ \forall x_{1S}$, and $x^*_2(x_{1S}) > \frac{\tilde{A}}{2\tilde{a}}$.

Given $x^*_2(x_{1S})$, one derives (x^*_{1S}, x^*_{2S}) by solving the following first order conditions (see (8) and (9)):

$$MR_1(x_{1S}) = 2A_1x_{1S} - 3a_1x^2_{1S} = -\frac{\partial(\tilde{A}x^*_2(x_{1S}) - \tilde{a}x^*_2(x_{1S})^2)}{\partial x_{1S}}x_{1S}, \tag{A8}$$

$$MR_2(x_{2S}) = 2\tilde{A}x_{2S} - 3\tilde{a}x^2_{2S} = c + (\tilde{A} - 2\tilde{a}x_{2S})x_{1S}. \tag{A9}$$

Note that the right hand side of (A8) is positive, given that $x^*_2(x_{1S}) > \frac{\tilde{A}}{2\tilde{a}}$ and $\frac{dx^*_2(x_{1S})}{dx_{1S}} > 0$. Comparing (A8) with (A3), one concludes that $x^*_{1S} < x^*_{1\ell}$ since the marginal revenue MR_1 is decreasing for the relevant range. Similarly, the right hand side of (A9) is less than c, and hence $x^*_{2S} > x^*_{2\ell}$.

Similar to the leasing scenario, an alternative strategy for the seller is to sell to group-1 consumers exclusively. The following equation system is parallel to (A7)—(A9), and characterizes the alternative solution.

$$\begin{aligned} x'_2(x_{1S}) &= \frac{1}{3}\left(\frac{A_1}{a_1} + x_{1S}\right) + \frac{1}{3}\sqrt{\left(\frac{A_1}{a_1}\right)^2 - \left(\frac{A_1}{a_1}\right)x_{1S} + x^2_{1S} - \frac{3c}{a_1}}, \\ 2A_1x_{1S} - 3a_1x^2_{1S} &= -\frac{\partial(A_1x'_2(x_{1S}) - a_1x'_2(x_{1S})^2)}{\partial x_{1S}}x_{1S}, \\ 2A_1x_{2S} - 3a_1x^2_{2S} &= c + (A_1 - 2a_1x_{2S})x_{1S}. \end{aligned} \tag{A10}$$

Let (x'_{1S}, x'_{2S}) denote the solution to (A10). For (x^*_{1S}, x^*_{2S}) to maximize the seller's profits, it is sufficient to require[27]

$$p_1(x^*_{1S}) \cdot x^*_{1S} + (\tilde{p}(x^*_{2S}) - c) \cdot x^*_{2S} \geq p_1(x'_{1S}) \cdot x'_{1S} + (p_1(x'_{2S}) - c) \cdot x'_{2S}. \tag{A11}$$

The condition would hold if, for example, $\frac{\tilde{A}^2}{4\tilde{a}} \geq \frac{A^2_1}{4a_1}$.

The comparison of profits under different regimes is straightforward. Note that (x^*_{1S}, x^*_{2S}) is feasible to the lessor, and thus the lessor's profits must be higher than the seller's.

[27]It is possible that x'_{2S} is greater than $\tilde{x}_1$, and hence not feasible according to (A2).

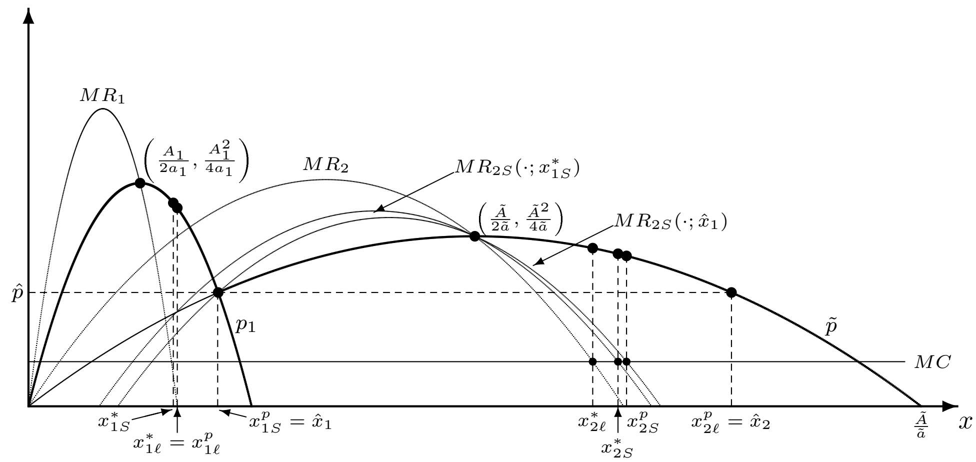

Figure 4: Optimal Dynamic Pricing

A graphical solution is provided in Figure 4. One observes that $MR_1(x)$ ($MR_2(x)$) intersects with $p_1(x)$ ($\tilde{p}(x)$) at $x = \frac{A_1}{2a_1}$ $\left(\frac{\tilde{A}}{2\tilde{a}}\right)$ and 0,[28] and that the marginal revenue function for the seller, $MR_{2S}(x; x_1) \equiv d(\tilde{p}(x)(x - x_1))/dx$, intersects with $\tilde{p}(x)$ at $x = \frac{\tilde{A}}{2\tilde{a}}$ and x_1.[29] To obtain the solution to (8) and (9), one first derives x_{1S}^* from (8), as the condition depends only on x_{1S}. Note that the marginal revenue MR_1 evaluated at x_{1S}^* is positive, as shown in Figure 4. Given x_{1S}^*, one can then plot the marginal revenue curve $MR_{2S}(x; x_{1S}^*)$ for the second period. The first order condition (9) can be rewritten as $MR_{2S}(x_{2S}; x_{1S}) = c$. Therefore, the intersection of the marginal revenue $MR_{2S}(\cdot; x_{1S}^*)$ and the marginal cost $MC = c$ determines the aggregate sales $x = x_{2S}^*$. $\|$

Proof of Proposition 4

Recall that $\pi_\ell = p_1(x_{1\ell}) \cdot x_{1\ell} + (\tilde{p}(x_{2\ell}) - c) \cdot x_{2\ell}$ when $x_{2\ell} \geq x_{1\ell}$. When the monopolist lessor has to take into account the penetration-pricing constraint, $x_{2\ell} \in [\frac{\tilde{A}}{2\tilde{a}}, \hat{x}_2)$ is no longer feasible. We are more interested in the case when the constraint is binding, and thus we assume that $x_{2\ell}^* < \hat{x}_2$ as in Assumption 4. It follows that π_ℓ is decreasing over $x_{2\ell} \in [\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$, and hence is maximized at $x_{2\ell} = \hat{x}_2$ for $x_{2\ell}$ in that interval. Meanwhile, the penetration-pricing constraint does not affect the lessor's strategy in period 1. Hence, $(x_{1\ell}^*, \hat{x}_2)$ is a plausible solution if $x_{2\ell}$ is limited to $[\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$.

The lessor's alternative strategy is to exclude group-2 consumers in period 2 and select $x_{2\ell}$ from $[\frac{A_1}{2a_1}, \hat{x}_1)$. As shown in the proof of Proposition 1, one obtains another candidate for the solution by maximizing $(2p_1(x) - c) \cdot x$. For $(x_{1\ell}^*, \hat{x}_2)$ to maximize π_ℓ, one requires

$$p_1(x_{1\ell}^*) \cdot x_{1\ell}^* + (\tilde{p}(\hat{x}_2) - c) \cdot \hat{x}_2 \geq \max_x (2p_1(x) - c) \cdot x, \tag{A12}$$

which is specified in Assumption 5. In sum, Assumptions 4 and 5 ensure that $(x_{1\ell}^p, x_{2\ell}^p) = (x_{1\ell}^*, \hat{x}_2)$ maximizes the lessor's profits under the penetration-pricing constraint. $\|$

Proof of Proposition 5

[28] $MR_1(x) = p_1(x)$ implies $p_1'(x) \cdot x = 0$, which in turn implies either $x = \frac{A_1}{2a_1}$ or $x = 0$.

[29] $MR_{2S}(x; x_1) = \tilde{p}(x)$ implies $\tilde{p}'(x)(x - x_1) = 0$, which in turn implies either $x = \frac{\tilde{A}}{2\tilde{a}}$ or $x = x_1$.

Recall that (x^*_{1S}, x^*_{2S}) maximizes π_S in absence of penetration-pricing constraints, and $x^*_2(x_{1S}) = \arg\max_{x_{2S}}(\tilde{p}(x_{2S})-c)(x_{2S}-x_{1S})$. We want to characterize the conditions under which $(\hat{x}_1, x^*_2(\hat{x}_1))$ maximizes x^p_S. Note that $x^*_2(x_{1S})$ is increasing in x_{1S} given Assumption 1. Thus, Assumption 7 ($x^*_2(\hat{x}_1) < \hat{x}_2$) implies $x^*_2(x_{1S}) < \hat{x}_2$, $\forall x_{1S} < \hat{x}_1$. Consequently, $x^*_2(x_{1S})$ is not feasible for any $x_{1S} < \hat{x}_1$. In particular, $x^*_{2S} = x^*_2(x^*_{1S})$ is not feasible as $x^*_{1S} < \hat{x}_1$, which is implied by Assumption 6 ($x^*_{1\ell} < \hat{x}_1$). In sum, Assumptions 6 and 7 implies a binding penetration-pricing constraint.

To solve the maximization problem in (11), we first derive $x^p_2(x_{1S})$ that maximizes the seller's period-2 profits subject to the penetration-pricing constraint. For $x_{1S} \geq \hat{x}_1$, the constraint is not binding, and hence $x^p_2(x_{1S}) = x^*_2(x_{1S})$. Meanwhile, for $x_{1S} < \hat{x}_1$, the constraint limits the feasible $x'_{2S}s$ to either $[x_{1S}, \hat{x}_1)$ or $[\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}]$. For the latter interval, the seller's period-2 profits are maximized at $\hat{x}_2$ since $x^*_2(x_{1S}) < \hat{x}_2$. For the former interval, it is maximized at $\min(x'_2(x_{1S}), \hat{x}_1)$, where $x'_2(x_{1S}) = \arg\max(p_1(x_{2S}) - c)(x_{2S} - x_{1S})$ maximizes period-2 profits when the seller excludes group-2 consumers (see the proof of Proposition 2). Apparently, $x_{2S} = \hat{x}_1$ is less profitable than $x_{2S} = \hat{x}_2$ as the prices are the same. One concludes that $x^p_2(x_{1S})$ is either $\hat{x}_2$ or $x'_2(x_{1S})$ for $x_{1S} < \hat{x}_1$.

Given $x^p_2(x_{1S})$, one obtains three candidates for (x^p_{1S}, x^p_{2S}) that maximizes π^p_S. For $x_{1S} \geq \hat{x}_1$, $x^p_2(x_{1S}) = x^*_2(x_{1S})$. One can show that $\partial^2\pi^p_S/\partial x^2_{1S}$ is negative. Therefore, π^p_S is decreasing over $x_{1S} \in [\hat{x}_1, \frac{A_1}{a_1}]$ as $\hat{x}_1 > x^*_{1S}$, and thus π^p_S is maximized at $(\hat{x}_1, x^*_2(\hat{x}_1))$ if x_{1S} is limited to $[\hat{x}_1, \frac{A_1}{a_1}]$.

For $x_{1S} < \hat{x}_1$, $x^p_2(x_{1S})$ is equal to either $\hat{x}_2$ or $x'_2(x_{1S})$. In the former case, π^p_S is maximized at $(x^*_{1\ell}, \hat{x}_2)$ because $x^p_2(x_{1S})$ is a constant and independent of x_{1S}. In the latter case, π^p_S is maximized at (x'_{1S}, x'_{2S}) that solves (A10). Assumption 5 implies that $(x^*_{1\ell}, \hat{x}_2)$ generates higher profits than (x'_ℓ, x'_ℓ) (see (A12)), which in turn dominates (x'_{1S}, x'_{2S}). In sum, (x^p_{1S}, x^p_{2S}) is either $(\hat{x}_1, x^*_2(\hat{x}_1))$ or $(x^*_{1\ell}, \hat{x}_2)$.

A graphical solution is illustrated in Figure 4. The monopolist lessor has to set the period-2 price at $\hat{p}$ due to the penetration-pricing constraint, and hence $x^p_{2\ell} = \hat{x}_2$. For the monopolist seller, x^p_{1S} is equal to $\hat{x}_1$ in order to penetrate the market.[30] Consequently, x^p_{2S} is determined by intersecting $MC = c$ with the parabola that passes through $(\hat{x}_1, \hat{p})$ and $(\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}^2}{4\tilde{a}})$. ‖

Proof of Proposition 8

We first show that $y^*_S < y^*$. Note that $x^*_2(x_{1S}, y)$ is given by

$$\frac{1}{3}\left(\frac{\tilde{A}}{\tilde{a}} + x_{1S}\right) + \frac{1}{3}\sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \left(\frac{\tilde{A}}{\tilde{a}}\right)x_{1S} + x^2_{1S} - \frac{3(c-g(y))}{\tilde{a}}},$$

and one obtains $\partial x^*_2(x_{1S}, y)/\partial y$ accordingly. The last condition of (15) can then be reduced to the following.

$$1 = g'(y)\left(x_{2S} + \frac{(\frac{\tilde{A}}{2\tilde{a}} - x_{2S})x_{1S}}{\sqrt{(\frac{\tilde{A}}{\tilde{a}})^2 - (\frac{\tilde{A}}{\tilde{a}})x_{1S} + x^2_{1S} - \frac{3(c-g(y))}{\tilde{a}}}}\right). \tag{A13}$$

Substituting x_{2S} with $x^*_2(x_{1S}, y)$ further simplifies the above condition. Comparing the right hand side of (A13) with

$$g'(y)x^*_\ell = g'(y)\left(\frac{\tilde{A}}{3\tilde{a}} + \frac{1}{3}\sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \frac{3(c-g(y))}{\tilde{a}}}\right),$$

[30] In Figure 4, we assume that $\pi^p_S(\hat{x}_1, x^*_2(\hat{x}_1)) > \pi^p_S(x^*_{1\ell}, \hat{x}_2)$.

one concludes that the former is less than the later, and hence $y_S^* < y^*$.

To show that $y_S^p < y_S^*$, note that the right hand side of (A13) after substitution is a function of x_{1S} and y. Furthermore, the partial derivative of this function with respect to x_{1S} is negative. It follows that a higher first-period production corresponds to a lower R&D investment. Since $x_{1S}^p = \hat{x}_1$ is independent of y and greater than x_{1S}^*, one obtains $y_S^p < y_S^*$. ‖

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